# Estimation of Steep fronted and full wave lightning Channel-Base-Current function parameters computed using a Genetic Algorithm

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**Abstract**

Typical negative first-return-stroke (FS) and subsequent-return-stroke (SS) lightning channel-base current (CBC) parameters are evaluated using a Genetic Algorithm (GA). Sum of two Heidler's function parameters are tuned for a typical value of peak current ($I_m \approx 12kA$), and maximum time derivative of current ($(di/dt)_{max} \approx 40kA/\mu s$), for different time to crest ($t_m$). These parameters are also tuned for median and severe negative FS and SS to accurately obtain full wave of CBC, including characteristics related to wave tail. It is thus shown that GA can be effectively applied for obtaining the lightning current parameters in accordance with data reported in the literature. Further, this approach is used to tune the parameters for steep fronted CBC waveforms. The CBC function parameters thus evolved using GA for severe cases will be useful in lightning related simulation study and research.

***Index Terms*** – Channel-base current; Genetic Algorithm; Heidler's function; Lightning; Negative first-return-stroke; Negative subsequent-return-stroke.

## 1 Introduction

The return stroke phase of lightning discharge is considered to be the major factor responsible for the indirect destructive effects, since most intense electromagnetic radiation occurs during this phase. Based on their characteristics (peak current ($I_m$) and maximum time derivative of current ($di/dt)_{max}$), lightning return strokes are grouped into first-return-stroke (FS) and subsequent-return-stroke (SS) [1, 2]. These parameters of lightning CBC are available in the literature. They are obtained by measurement, either (i) during natural lightning, or (ii) during artificially generated (triggered) lightning [3, 4]. The process of simulation and analysis of induced lightning electric and magnetic fields (LEMF) in general involves usage of reconstructed current wave-shapes, closely matching those of the field observed wave-shapes. Hence such a study needs an appropriate CBC function for the engineering return stroke models [5, 6]. Based on the field data [7, 8] the wave-front region of the return strokes currents can be characterized by three parameters ($I_m$, $(di/dt)_{max}$, $t_m$) for the purpose of modeling. A detailed discussion related to these parameters (and a few more) can be found in a recent publication [9]. Many analytical functions representing lightning CBC have been proposed in the literature [9, 10]. Lightning CBC is best approximated using the

mathematical function presented by Heidler et al. [11]. This function has been adopted by International Electro-technical Commission standard IEC 62305-1 [8] for CBC representation. Sum of two Heidler's functions has previously been used in representing the CBC wave shape of a SS [12, 13, 14, 15].

The Heidler's function has its own set of parameters which are to be identified to achieve the desired wave shape of the CBC. Identifying these parameters is not a straight forward procedure. Heidler et al. have given a graphical method to identify these parameters specific to a wave shape [11]. While adjusting the wave-front of the current waveform, Heidler's function allows nearly independent adjustment of $I_m$ and $(di/dt)_{max}$, but not time to crest $t_m$. There exists an interdependence between $I_m$, $t_m$, and $(di/dt)_{max}$ [19]. To overcome this issue, Javor and Rancic [16] have proposed a new channel base current (NCBC) function. NCBC allows independently adjust $I_m$, $t_m$, and $(di/dt)_{max}$. In the process NCBC exhibits a delay characteristic in the wave-front region (as compared to Heidler's function [12]). This delay is not only significant but also comparable with the $t_m$, particularly for higher values of $t_m$ and $(di/dt)_{max}$ [17]. It is demonstrated [18] that Genetic Algorithm (GA) is a useful tool in determining Heidler's function parameters (related to wave-front only) of the typical values provided in literature, [12, 13] with not attempting to adjust $t_m$.

In the present study, GA is used to tune the Heidler's function CBC parameters, with the objectives as (i) exploring the possibility of GA application for the wave-front region of a negative SS, with known, typical $I_m$ and $(di/dt)_{max}$ for different time-to-crest $t_m$ (an attempt to adjust all these three independently), (ii) using GA to adjust the full wave characteristics, by including the time to 50% of peak current , $t_{50}$ and the rate of current rise from 30-90% of peak current, $t_{30-90}$ and (iii) evaluating Heidler's Function parameters for a steep-fronted negative FS by adjusting $I_m$, $(di/dt)_{max}$, $t_m$ and $t_{50}$.

## 2   Computation of Heidler's function parameters

### 2.1   Genetic Algorithm

A Genetic Algorithm is a general-purpose optimization algorithm based on natural selection, the process accepted as the driving force of biological evolution. GA starts with no knowledge of the correct solution, and depends entirely on evolution operators such as selection, crossover and mutation to arrive at the best solution. In the algorithm, the selection of the natural environment is replaced by artificial selection based on a computed fitness for each design. The term fitness is used as an indicator of the chromosomes goodness; and it is, usually, the objective function of the optimization problem. In GA, strings of numerical values, representing members of the population, replace these chromosomes [20]. As the evolution proceeds through the generations, the fittest offsprings will survive.

GA being a parallel search method, can use more than one initial guess as the starting point. In the present study, the population size is chosen to be 50. The crossover probability is 0.8 and the crossover function chosen is of the type scattered, which generates a random binary vector and chooses the gene from parent A if the vector element is 0 and from parent B otherwise. The mutation probability is 0.001 and the mutation function is of the type adaptive-feasible, which generates an adaptive direction and step length depending on the constraints and the previous successful or unsuccessful operation. The convergence criterion is chosen as 100 generations or error below $10^{-6}$, whichever is attained earlier. The search is started with 50 randomly generated population values for each parameter given in (1). The accuracy of GA and convergence greatly depend on the search region for the parameters. This search region can be restricted by specifying appropriate bounds for the parameters. A flowchart explaining the process is given in Fig. 1.

## 2.2 Heidler's CBC Function

It has been reported in the literature that in case of indirect lightning strikes, steep current wave forms result in most intense LEMFs [13, 17]. The steep front is a characteristic of the return stroke phase of the subsequent return strokes. Thus, the modeling of return stroke is essential for lightning electromagnetic field (LEMF) calculation. Among the lightning return stroke models, most widely used one is the engineering model [5, 6]. In this model spatio-temporal distribution of the current along the lightning channel is related to CBC as a product of its delayed version and a decay factor [17]. Heidler et al. proposed a CBC function popularly known as Heidler's function. This function is a combination of power and an exponential function [12]. In this study, sum of two Heidler's functions is used to represent the CBC (1).

$$i(0,t) = \frac{I_{01}}{\eta_1} e^{-t/\tau_{21}} \frac{(t/\tau_{11})^{n_1}}{1 + (t/\tau_{11})^{n_1}} + \frac{I_{02}}{\eta_2} e^{-t/\tau_{22}} \frac{(t/\tau_{12})^{n_2}}{1 + (t/\tau_{12})^{n_2}}$$

where,

$$\eta_1 = exp(-\frac{\tau_{11}}{\tau_{21}} * \frac{n_1\tau_{21}}{\tau_{11}}^{1/n_1})$$

$$\eta_2 = exp(-\frac{\tau_{12}}{\tau_{22}} * \frac{n_2\tau_{22}}{\tau_{12}}^{1/n_2}) \tag{1}$$

Here, $I_{01}$, $I_{02}$ are constants which control the amplitude of the CBC; $\tau_{11}$, $\tau_{12}$ are front time constants; $\tau_{21}$, $\tau_{22}$ are the decay time constants; $n_1$, $n_2$ are dimensionless constants controlling the steepness, generally varying between 2 and 10; $\eta_1$ and $\eta_2$ are the amplitude correction factors.
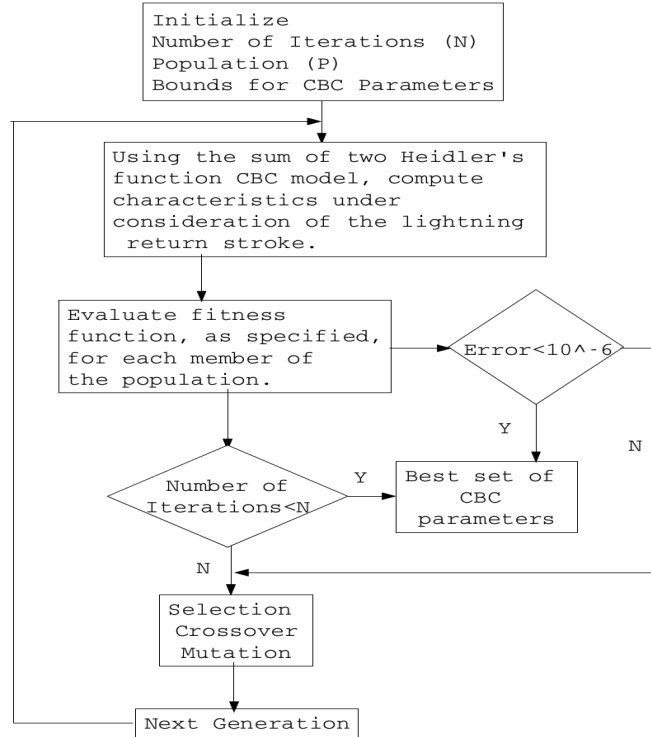


Figure 1: Flowchart to find Heidler's function parameters to achieve desired CBC characteristics.

## 2.3 Tuning wave-front of negative SS

The lightning current parameters for downward negative Cloud-to-Ground lightning are presented by Berger et al.[1]. This data is based on direct current measurements and reported field data . From this database, the typical parameters of negative SS is summarized in Table 1. To simulate this typical negative SS using sum of two Heidler's function the appropriate values of function parameters must be computed. Rachidi et al. [13], have identified these parameters $(I_{01}, I_{02}, \tau_{11}, \tau_{21}, \tau_{12}, \tau_{22}, n_1,$ and $n_2)$ which are summarized in Table 2. In the present study, using GA as the tool, a systematic and independent adjustment of $t_m$ around its typical value (0.8 $\mu$s) is attempted.

CBC Heidler's function parameters in (1) constitute the eight variables to be evaluated by GA; to achieve the desired $I_m$ and $(di/dt)_{max}$ corresponding to the SS for different $t_m$. Hence the objective function (fitness function) value is derived from the computed current peak, maximum time derivative of current, and time to crest of the SS lightning CBC as given in (2).

$$f = |\frac{I_{md} - I_{mc}}{I_{md}}| + |\frac{(di/dt)_{maxd} - (di/dt)_{maxc}}{(di/dt)_{maxd}}| + |\frac{t_{md} - t_{mc}}{t_{md}}| \qquad (2)$$

where,

$I_{md}$ is the desired peak current,

$I_{mc}$ is the computed peak current,

$(di/dt)_{maxd}$ is the desired maximum rate of rise of current,

$(di/dt)_{maxc}$ is the computed maximum rate of rise of current,

$t_{md}$ is the desired time to current peak,

$t_{mc}$ is the computed time to current peak,

Table 1: Important wave-front parameters associated with lightning discharge of negative SS [1].

| Current Parameter | Unit | Typical value of SS |
|---|---|---|
| Peak value of the current $(I_m)$ | kA | 12 |
| Maximum time derivative of the return stroke current $(di/dt)_{max}$ | kA/$\mu$s | 40 |
| Time to crest $(t_m)$ | $\mu$s | 0.8 |

As discussed in section 2.1, the objective for GA is set to minimize the deviation from the desired value of $I_m$, $(di/dt)_{max}$ and $t_m$. The bounds opted for tuning the wave-front of a typical negative SS are reported in Table 3. For the case of $t_m$=1.2 $\mu$s the bounds are as specified in brackets. In subsequent sections, the fitness function and bounds opted are appropriately modified, however, only the bounds opted pertaining to this section are reported.

Here all the three parameters controlling the wave-front are independently computed, as an extension of the earlier study [19]. The Heidler's function parameters are tuned for different values of $t_m$ (around its typical values of 0.8 $\mu$s) by retaining $I_m$=12 kA and $(di/dt)_{max}$= 40 kA/$\mu$s.

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Table 2: Heidler's CBC function parameters for "typical" negative SS [13].

| $I_{01}$ (kA) | $\tau_{11}$ ($\mu$s ) | $\tau_{21}$ ($\mu$s ) | $n_1$ | $I_{02}$ (kA) | $\tau_{12}$ ($\mu$s ) | $\tau_{22}$ ($\mu$s ) | $n_2$ |
|------|------|------|------|------|------|------|------|
| 10.7 | 0.25 | 2.5 | 2 | 6.5 | 2 | 230 | 2 |

Table 3: Bounds opted for Heidler's CBC function parameters in tuning the wave-front of negative SS.

| GA Variable | Unit | Lower Bound | Upper Bound |
|------|------|------|------|
| $I_{01}$ | kA | 10.0 | 13.0 |
| $I_{02}$ | kA | 7.00 (7.50) | 9.00 (9.50) |
| $\tau_{11}$ | $\mu$s | 0 (0.50) | 1.00 (1.50) |
| $\tau_{21}$ | $\mu$s | 0 (1.50) | 1.5 (3.00) |
| $\tau_{12}$ | $\mu$s | 0 (1.50) | 2 (3.50) |
| $\tau_{22}$ | $\mu$s | 210 | 300 |
| $n_1$ | – | 2.00 | 10.0 |
| $n_2$ | – | 2.00 | 2.50 |

These resultant CBC waves obtained using GA tuned parameters are depicted in Figs 2 and 3. This proves the capability of GA in independently tuning CBC function parameters to achieve all the three parameters describing the wave-front.
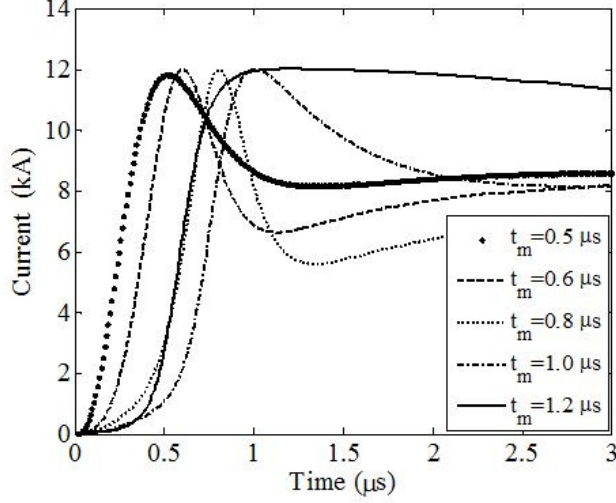
Figure 2: Plot of GA tuned Hediler's CBC function with $t_m$ as parameter (for a fixed "typical" $I_m$ of 12 kA and $(di/dt)_{max}$ of 40kA/$\mu$s) of SS.


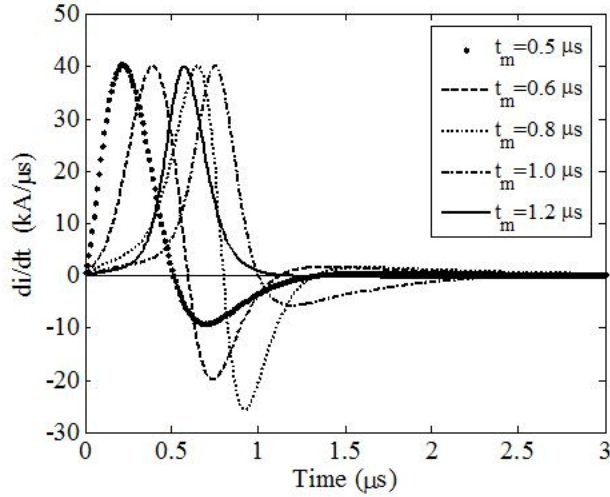
Figure 3: Plot of time rate of change of Heidler's CBC as a function of time, corresponding to the plots given in figure 2.

## 2.4 Tuning of CBC function parameters for negative FS and SS, including those controlling wave-tail

It is reported in literature [7, 9] that a lightning stroke can be almost completely characterized by 5 (or in some cases, 6) parameters, namely, $I_m$, $(di/dt)_{max}$ , $t_m$, $t_{50}$, $(di/dt)_{30-90}$ and $Q$. From extensive field measurements, the standards concerning lightning protection systems [8] report that these parameters can be closely approximated to follow a log-normal distribution, and provide the median (50%) and severe (5%) values . In this section, we discuss the application of GA to compute the CBC parameters, for negative FS and SS, so as to obtain the reported lightning characteristics.

In case of negative FS, GA is applied to tune 4 parameters, namely, $I_m$, $(di/dt)_{max}$, $t_m$ and $t_{50}$, for both the median and severe case. The fitness function used in this GA application is presented in (3).

$$f = (|\frac{I_{md} - I_{mc}}{I_{md}}|) + (|\frac{(di/dt)_{maxd} - (di/dt)_{maxc}}{(di/dt)_{maxd}}|) + (|\frac{t_{md} - t_{mc}}{t_{md}}|) + (|\frac{t_{50d} - t_{50c}}{t_{50d}}|) \quad (3)$$

where,

$t_{50d}$ is the desired time to 50% current peak,

$t_{50c}$ is the computed time to 50% current peak,

and, other parameters are same as those presented in (2)

For negative SS, an additional parameter, the current rate of rise from 30 to 90 percent $(di/dt)_{30-90}$ is also included. The fitness function for this case is presented in (4).

$$f = (|\frac{I_{md} - I_{mc}}{I_{md}}|) + (|\frac{(di/dt)_{maxd} - (di/dt)_{maxc}}{(di/dt)_{maxd}}|) + (|\frac{t_{md} - t_{mc}}{t_{md}}|) +$$
$$(|\frac{t_{50d} - t_{50c}}{t_{50d}}|) + (|\frac{(di/dt)_{30-90d} - (di/dt)_{30-90c}}{(di/dt)_{30-90d}}|) \quad (4)$$

$(di/dt)_{30-90d}$ is the desired rate of rise of current from 30 to 90 percent ,

$(di/dt)_{30-90c}$ is the computed rate of rise of current from 30 to 90 percent,

and other parameters are same as that presented in (3)

## 3 Results

### 3.1 Wave-front region of negative SS

Typical SS (12 kA peak, 40 kA/$\mu$s maximum time derivative and 0.8 $\mu$s time to crest) CBC, for which the Heidler's function parameters exist in the literature [13], is taken as the example to evaluate Heidler's function parameters using GA as a tool. This is to assess the effectiveness of GA as a tool in tuning Heidler's CBC, by comparing with the established results available in the literature (Rachidi et al. [13]).

In continuing with typical cases of SS parameters discussed above, the present study is focused on tuning of Heidler's function parameters by systematically changing the time-to-crest of the CBC $(t_m)$ of SS, by keeping peak $(I_m)$ and maximum time rate of change of CBC $(di/dt)_{max}$ constant. For 5 different $t_m$, computed parameters of the Heidler's function are as given in Table 4. Five simulation test runs are performedto minimize the objective function (2). The best case (out of the 5 test runs of GA with randomized initial population) GA computed parameters, $I_{01}$, $I_{02}$, $\tau_{11}$, $\tau_{21}$, $\tau_{12}$, $\tau_{22}$, $n_1$, and $n_2$ for negative SS are given in Table 4. The resultant $I_m$ and $(di/dt)_{max}$ waveform variation with $t_m$ as a parameter is depicted in Figs 2 and 3.

### 3.2 Concerning complete wave shape adjustment of negative FS and SS

For the adjustment of tail end of the lightning return strokes, it is necessary to tune the time to half the peak, $t_{50}$. This data has been reported in IEC standard 62305-1 [8]. By appropriately modifying the fitness function, it has been demonstrated that the Heidler's function parameters for the CBC can be computed accurately by GA. The GA tuned Heidler's function parameters for negative FS and SS, for both the median and severe cases are summarized in Table ??. Figs 4

Table 4: GA computed Heidler's CBC function parameters for "typical" $I_m$ (=12 kA), $(di/dt)_m$ (=40 kA/$\mu$s) and for different $t_m$.

| SS CBC parameters estimated using GA | | | | | | | | $I_m$ | $(di/dt)_m$ | $t_m$ | Error |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $I_{01}$ | $I_{02}$ | $\tau_{11}$ | $\tau_{21}$ | $\tau_{12}$ | $\tau_{12}$ | $n_1$ | $n_2$ | $I_m$ | $(di/dt)_m$ | $t_m$ | Error |
| (kA) | (kA) | ($\mu$s) | ($\mu$s) | ($\mu$s) | ($\mu$s) | | | (kA) | (kA/$\mu$s) | ($\mu$s) | (%) |
| 11.855 | 8.418 | 0.678 | 0.189 | 0.468 | 288.892 | 3 | 2 | 11.829 | 40.288 | 0.510 | 4.146 |
| 12.412 | 8.099 | 0.693 | 0.126 | 0.703 | 274.830 | 6 | 2 | 12.000 | 40.000 | 0.600 | 0.003 |
| 12.535 | 7.205 | 0.829 | 0.130 | 0.927 | 250.568 | 10 | 2 | 11.998 | 40.038 | 0.800 | 0.107 |
| 10.144 | 8.666 | 0.802 | 0.719 | 1.525 | 298.190 | 10 | 2 | 11.996 | 40.054 | 1.000 | 0.165 |
| 10.119 | 8.307 | 0.604 | 2.527 | 1.783 | 225.727 | 8 | 2 | 12.003 | 39.955 | 1.200 | 0.139 |

through 7 contain plots of these CBC and their respective time derivatives for four different cases considered (Severe FS, Median FS, Severe SS, Median SS). In this study, the charge transfer (Q) during the lightning stroke has not been included. This is due to the fact that $I_m$ and $t_{50}$ show a high degree of interdependence with parameter Q (for severe cases), making it difficult to tune using sum of two Heidler's function.

Table ?? summarizes the Heidler's function parameters obtained using GA for the median (50%) and severe (5%) FS and SS reported in the literature [7, 8, 9]. Table 6 compares the literature reported and GA-computed (present work) data for median and severe case, for both, negative first and subsequent return stroke obtained using GA tuned Heidler's parameters given in Table ??.

Table 5: GA computed Heidler's function parameters corresponding to lightning FS and SS, those listed in the standard [8].

| Type of stroke | $I_{01}$ | $I_{02}$ | $\tau_{11}$ | $\tau_{12}$ | $\tau_{21}$ | $\tau_{22}$ | $n_1$ | $n_2$ |
|---|---|---|---|---|---|---|---|---|
| | (kA) | (kA) | ($\mu$s) | ($\mu$s) | ($\mu$s) | ($\mu$s) | – | – |
| 50% FS | 9.230 | 10.803 | 5.452 | 5.879 | 96.898 | 98.807 | 24 | 42 |
| 5% FS | 40.324 | 49.719 | 20.016 | 19.877 | 245.753 | 261.835 | 60 | 56 |
| 50% SS | 5.618 | 6.204 | 0.241 | 0.357 | 40.011 | 47.056 | 4 | 3 |
| 5% SS | 18.605 | 10.156 | 1.801 | 1.185 | 177.645 | 183.602 | 62 | 2 |

Table 6: Comparison of lightning parameters from literature [8] with those computed using GA-tuned Heidler's CBC.

| Return Stroke Parameter | Measured value (Data from [8]) | | Modeled value (GA-tuned) | | % Error | |
|---|---|---|---|---|---|---|
| | 50% | 5% | 50% | 5% | 50% | 5% |
| Negative FS | | | | | | |
| Peak Current (kA) | 20 | 90 | 19.996 | 89.922 | 0.022 | 0.087 |
| Time to current peak (10-90 percent) ($\mu$s) | 5.5 | 18 | 5.512 | 17.848 | 0.218 | 0.844 |
| Maximum rate of current rise (kA/$\mu$s) | 24.3 | 65 | 24.300 | 64.022 | 0.001 | 0.344 |
| Time to decay to half peak value ($\mu$s) | 75 | 200 | 74.995 | 199.080 | 0.007 | 0.460 |
| Negative SS | | | | | | |
| Peak Current (kA) | 11.8 | 28.6 | 11.854 | 28.802 | 0.455 | 0.708 |
| Time to current (10-90 percent) peak ($\mu$s) | 1.1 | 4.5 | 1.080 | 4.496 | 0.089 | 1.818 |
| Maximum rate of current rise (kA/$\mu$s) | 39.9 | 161.5 | 38.957 | 161.518 | 0.011 | 2.364 |
| Time to decay to half peak value ($\mu$s) | 32 | 140 | 32.000 | 133.195 | 0 | 4.861 |
| 30 to 90 percent rate of current rise (kA/$\mu$s) | 20.1 | 98.5 | 20.100 9 | 98.380 | 0 | 0.122 |
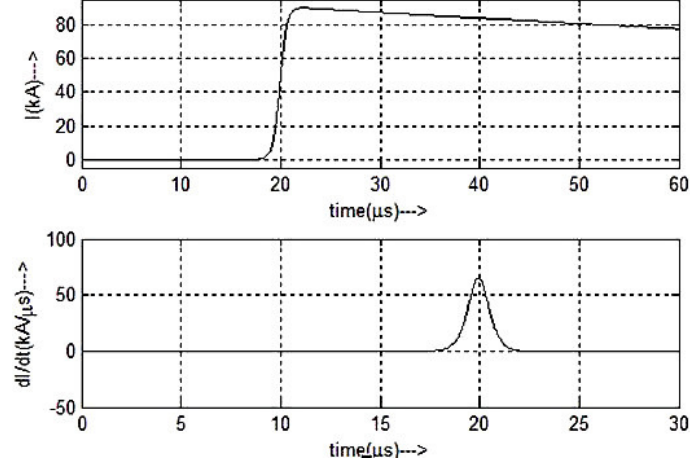
Figure 4: Severe negative FS current (peak of 90 kA) and its peak time derivative (65 kA/$\mu$s) specified in [8] obtained using GA tuned Heidler's function parameter.
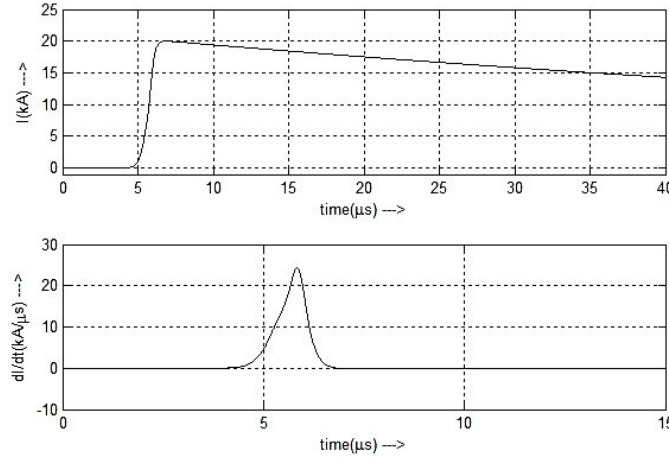


Figure 5: Median negative FS current (peak of 18 kA) and its peak time derivative (24.3 kA/$\mu$s) specified in [8] obtained using GA tuned Heidler's function parameter.

## 3.3  Tuning of lightning CBC having steep wave front

Recently, Gamerota et al [9] have recommended a set of salient lightning parameters for positive and negative cloud–to–ground lightning discharges. In case of negative median FS, they have used sum of six Heidler's functions to model the lightning CBC. Sum of six Heidler's function has become necessary since the value of $(di/dt)_{max}$ chosen is very high. In the present study, the same desired steepness of CBC (of 100 kA/$\mu$s) has been achieved using just two Heidler's functions through the application of GA. The GA tuned Heidler's function parameters corresponding to CBC data (32 kA, 100 kA/$\mu$s) of reference [9] are given in Table 7. The computed CBC parameters lie within $\pm 5\%$ of the desired values. A comparison of the lightning CBC, and its time derivative, evolved by GA and those existing literature [9] is presented in Fig. 8.
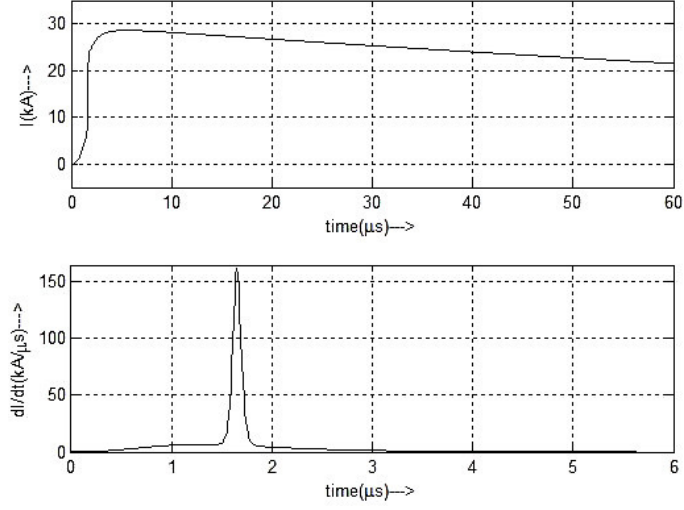
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Figure 6: Severe negative SS current (peak of 28.6 kA) and its peak time derivative (161.5 kA/$\mu$s) specified in [8] obtained using GA tuned Heilder's function parameter.
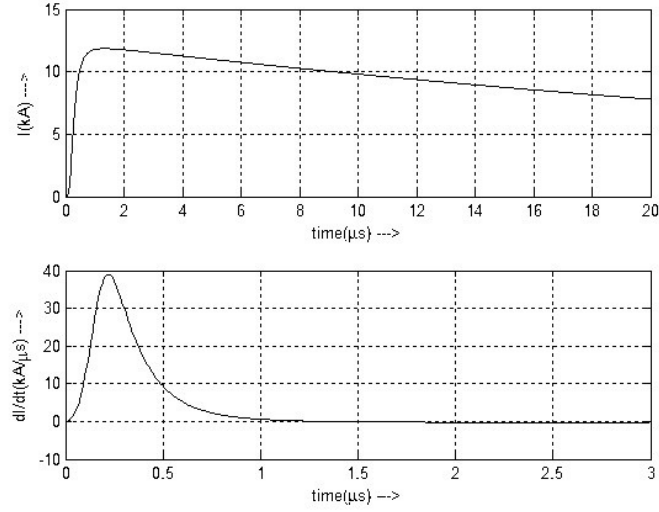


Figure 7: Median negative SS current (peak of 11.8 kA) and its peak time derivative (39.9 kA/$\mu$s) specified in [8] obtained using GA tuned Heidler's function parameter.

Table 7: GA computed Heidler's CBC function parameters corresponding to the steep fronted CBC [9].

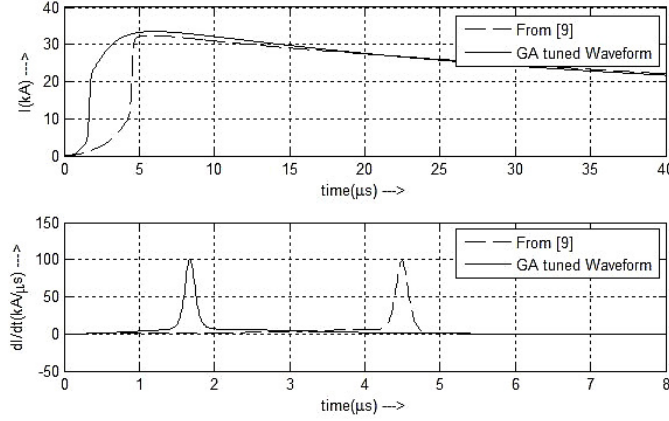| $I_{01}$ (kA) | $I_{01}$ (kA) | $\tau_{11}$ ($\mu$s) | $\tau_{12}$ ($\mu$s) | $\tau_{21}$ ($\mu$s) | $\tau_{22}$ ($\mu$s) | $n_1$ – | $n_2$ – |
|---|---|---|---|---|---|---|---|
| 16.363 | 20.210 | 1.680 | 2.510 | 23.924 | 247.146 | 38 | 3 |



Figure 8: Median negative FS current (steep wave-front as given in reference [9]) and its time derivative compared with GA tuned CBC.

## Conclusions

GA is successfully applied in computing the lightning CBC Heidler's function parameters ($I_{01}, I_{02}, \tau_{11}, \tau_{21}, \tau_{12}, \tau_{22}, n_1$ and $n_2$) and to independently adjust all the five parameters of the lighting CBC wave, probably for the first time. It is shown that it is possible to achieve the required CBC wave shape by using GA as the tool with only two Heidler's functions (With consequent reduction in number of parameters). It has also been demonstrated that the wave decay region can also be accurately modeled, in case of both negative FS and negative SS. The worst case errors in the $I_m$, $(di/dt)_{max}$, $t_m$, $t_{50}$ and $t_{30-90}$ thus obtained using GA are less than $\pm 4.8\%$. The Heidler's function parameters for median and severe case waves described in IEC lightning protection standard of FS and SS are evaluated using GA and are reported; which would find their application in lightning research.

## Acknowledgment

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