Crop Insurance – Revenue protection, Product design and Pricing

A Dissertation as a Course requirement for

Master of Science (Mathematics)

with specialization in Actuarial Science

Venkata Sai Praneeth P

(Regd.no. 18009)

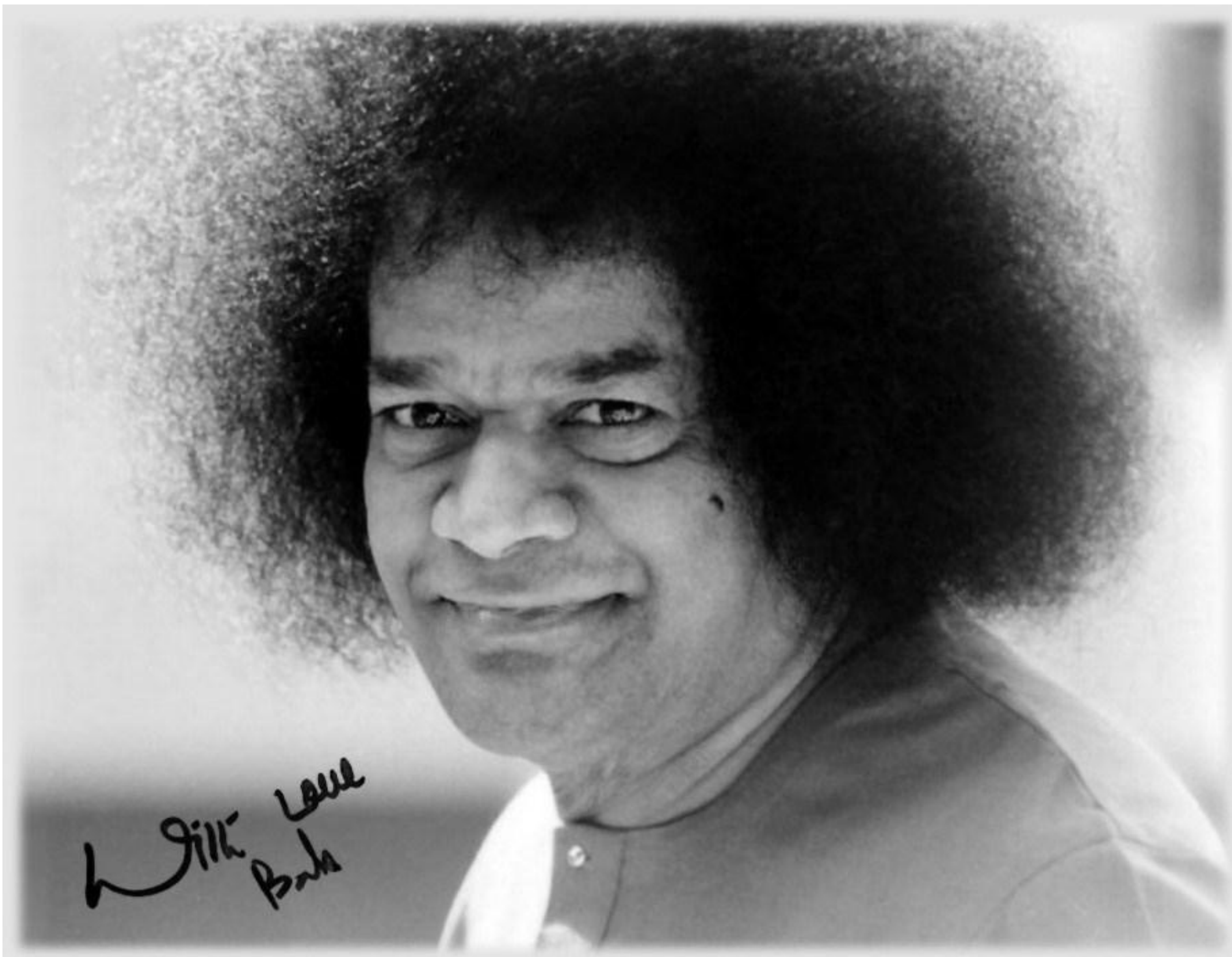
SRI SATHYA SAI INSTITUTE OF HIGHER LEARNING

(Deemed to be University)

Department of Mathematics and Computer science

Prasanthi Nilayam Campus

September 2019



DEDICATED TO THY LOTUS FEET

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
Sri Sathya Sai Institute of Higher Learning
(Deemed to be University)
Prashanthi Nilayam Campus, A.P.-515134

CERTIFICATE

This is to certify that this Dissertation titled Crop Insurance Revenue Protection, Product Design and Pricing submitted by Venkata Sai Praneeth P, 18009, Department of Mathematics and Computer Sciences, Prasanthi Nilayam is a bonafide record of the original work done under my supervision as a Course requirement for the Degree of Masters in Mathematics with Specialization in Actuarial Sciences.

Sri Sathya Sai Mudigonda
Project / Dissertation Supervisor

Dr. Pallav Kumar Baruah
Project / Dissertation Co-Supervisor

Counter Signed by

Place: Prasanthi Nilayam
Date: 12th April, 2020

Dr. Raghunatha Sharma
Head of the Department

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
Sri Sathya Sai Institute of Higher Learning
(Deemed to be University)
Prashanthi Nilayam Campus, A.P.-515134

DECLARATION

The Dissertation titled Crop Insurance Revenue Protection, Product Design and Pricing was carried out by me under the supervision of Sri Satya Sai Mudigonda, Department of Mathematics and Computer Sciences, Prasanthi Nilayam as a Course requirement for the Degree of Masters in Mathematics (Specialization in Actuarial Sciences) and has not formed the basis for the award of any degree, diploma or any other such title by this or any other University.

Place: Puttaparthi

Date: April 12, 2020

Venkata Sai Praneeth P
Regd. No. 18009
II M.Sc. (Mathematics)
Prashanthi Nilayam Campus

Acknowledgement

First and foremost my sincere heartfelt gratitude to my God and Master, Bhagawan Sri Sathya Sai Baba.

I thank my loving parents for their constant encouragement and support during the course of my dissertation.

My heartfelt gratitude to my dissertation supervisor, Sri Sathya Sai Mudigonda, Senior Tech Actuarial Consultant, Visiting faculty, Department of Mathematics and Computer Science (DMACS). I am extremely thankful to him for sharing his expertise, valuable guidance and also for the continuous encouragement that he extended to me throughout the semester.

I thank Dr. R Raghunatha Sarma, Head, DMACS for providing me with all the necessary facilities for my dissertation.

I am grateful to Phani Krishna and TVS Aditya, visiting faculties, for taking out time from their busy schedule and providing valuable insights and constant support and guidance which were useful for my dissertation.

I would like to thank Sri Pranav Sai and Sri Rohan Yashraj Gupta, Doctoral Research Scholars, DMACS, for all their support and help.

I would like to take this opportunity to express my sincere gratitude to all the faculty members of DMACS for their constant motivation and support.

I would like to thank DMACS and Sri Sathya Sai Institute of Higher Learning (SSSIHL) for providing me with all the resources necessary to carry out this work.

I thank the administration of Sri Sathya Sai Senior Boys Hostel for making my stay so memorable and eventful.

Thanks a million to my classmates, Narayana Rao, Bishal Gupta, Aman Sharma, Manideep Vasa, Bhanu Prakash, Sagar Shivakoti and Rishi Rao for making the classroom lively and a fun place to learn.

Last but not the least, I also place on record, my sense of gratitude to one and all, who directly or indirectly, have lent their hand in this venture.

Abstract

In India, the current Pradhan Mantri Fasal Bhīma Yojana (PMFBY) is a broad risk insurance to protect yield losses due to the risks which are inevitable. PMFBY covers the yield risk faced by the farmers, but does not cover the price risk taken on by the government. The loss due to price risk occurs when government accepts to procure the harvest at Minimum Support Price (MSP) and market price is lower than MSP. This study, proposes a revenue protection insurance product which would help in effectively transferring some portion of the price risk to insurance and reinsurance companies, to reduce the losses faced by the government due to price risk (i.e. the market price being lower than minimum support price).

Table of Contents

Chapter 1 Introduction

1.1 Motivation

The individuals who are insured by the insurance corporations are covered from the losses resulting from uncertain situations. This does not imply that, by buying an insurance, the risks would be totally eliminated. This risk now is taken up by the insurance companies. Premium is paid by the insured individuals in return for this. Insurance companies strive to diversify away the risks by collecting the risks which are diverse. The end result of this is that, the financial burden of the policyholders is now shared by a large variety of policyholders who are insured for similar activities but are not in risk at that moment.

Collection of many unbiased risk units is a prerequisite to cover a risk. When there is a strong positive correlation between many people at the same time, the insurance companies would undergo huge losses. For this very reason, systematic risks are not insured by the insurance companies. The revenue protection plan incorporates an enormously systemic risk - price volatility and hence is an exception. Revenue Protection insurance protects the farmers against loss of revenue caused due to many factors.

The insurance companies can fail if the reimbursements are brought on by low costs. This would force the companies to default all or most of its policyholders. In such situations the company may be proclaimed insolvent.

To know, how the insurance companies hedge their risk, is very important. The crop insurance companies of the United States hold a large amount of risk even if they have a standard reinsurance agreement. These companies use different types of derivatives to cover their risks.

In India, the current Pradhan Mantri Fasal Bhīma Yojana (PMFBY) is a broad risk insurance to protect yield losses due to the risks which are inevitable. This is similar to Yield protection insurance provided in US. PMFBY covers the yield risk faced by the farmers, but does not cover the price risk taken on by the government. The loss due to price risk occurs when government accepts to procure the harvest at Minimum Support Price (MSP) and market price is lower than MSP.

To reduce the losses faced by the government due to price risk (i.e. the market price being lower than minimum support price), we propose a revenue protection insurance product which would help in effectively transferring some portion of the price risk to insurance and reinsurance companies.

The GOI guarantees a Minimum Support Price (MSP) to the farmers to sell their crop. The MSP differs by the type of the crop / commodity and also the season year of cultivation. The sales price in the market may be lower or higher than the MSP and hence, the GOI is exposed to risk by virtue of offering the price guarantee.

One way to hedge this risk is to invest in futures market. The GOI also provides a market for futures (NCDEX) so that the farmers can participate and transfer the risk of fall in prices to capital markets. Ideally, market price and futures price should converge. In reality, MSP, market price and futures prices differ since the factors that influence them are different. This leads to uncertainty in expected income.

This can be calculated by modeling:

- Futures Price movement
- Correlation between futures price and claims cost

Future price movement in the market determines the size and timing of investment required to hedge the risk. The movement of the futures contract depends on a lot of factors, most of which would directly or indirectly impact the expected price of the crop. The price of the future would move towards the market price of the crop as the harvest time nears. This may be higher or lower compared to the MSP. Higher the expected prices in the market, higher is the price of the futures and vice versa.

The objective of this project is to:

- Design and price an insurance solution to meet farmers' needs
- Suggest a suitable methodology for insurers to invest in commodity futures and hedge the risk

1.2 Challenges and Opportunities

One of the greatest challenges for the development of crop revenue insurance is the Systemic risk. The farmers tend to sell the crop at lower prices than expected if there is a downfall in the crop prices at the time of harvest. The main problem is that, this risk cannot be diversified. This in turn causes the difficulty in pricing the crop revenue insurance products. Another major estimation which needs to be made is of the extent of coverage and also the conditions of claims payments that is the product terms and conditions play a major role in setting up the price of the price protection insurance product. The evaluation of loss could become quite complex given there is a systematic risk in the market. Maintaining trust and confidence among the parties for reasonable settlements is challenging.

As with Multi-Peril Crop Insurance (MPCI), Revenue Insurance (RI) suits the most for large production units and main crop types (e.g. rice, corn, wheat, cotton, soybean) where reliable and longer time series of commodity prices exist, e.g. from future markets. Pricing and underwriting of RI products is a bit challenging because we need to simulate farm-yields and model the volatility in the prices and at the same time we need to perform joint yield-price simulations and make sure that the correlations are constant.

Rating errors may reflect incorrect assumptions on yield and price distributions, data inconsistencies and the length of the combined yield price time series as well as more fundamental issues with the rating methodology as such. The severe rating errors for revenue insurance products imply that there are insurance products in the market that are not efficient. Difficulties in pricing price volatility due to lack of reliable data and liquid markets and the relatively high premium rates often prevent the wider development of revenue insurance.

In countries with government schemes that provide minimum commodity prices, RI will be an additional and relatively costly risk management measure and makes economic sense only if it can complement or replace existing government-supported price schemes. Depending on the size of an RI scheme and the number of crop types covered, the systemic exposure from yield volatility and even more so from price fluctuations creates large liabilities, which will require government-backed catastrophe protection.

1.3 Problem Statement

The aim of this work is to come up with a suitable methodology for the insurers to invest in the commodity futures and hedge the risk and also to design and price an insurance solution to meet farmers' needs. We have studied many research papers and articles from which we have come up with the concepts behind basic pricing methodologies and designing an insurance product. We have also come across many methodologies and actuarial techniques for the insurers to invest in the futures and the options to hedge their risks.

1.4 Thesis Outline

This work is organized as follows:

Chapter 2 explains the literature that were surveyed for the study

Chapter 3 discusses various terms and conditions required to design a revenue insurance product and various ways in which the risk faced by the insurance companies can be diversified

Chapter 4 gives a brief introduction to what is an actuarial pricing model and explains few of the models

Chapter 5 explains what a hedging model is and discusses a hedging model in detail

Chapter 6 gives a brief description of the data which we have consolidated

Chapter 7 explains the methodology we have used to fit various pdfs to different attributes and also presents few results for the same

Chapter 8 explains the liability projection methodology

Chapter 2 Literature Review

2.1 Crop Revenue Insurance

2.1.1 Revenue protection

Crop revenue insurance is obtained by slightly deviating from the already existing crop insurance for yield. In the case of crop revenue insurance, instead of reimbursement for yields, we have reimbursement for the sudden changes in the prices of the commodities. The insurer insures the policyholders against the decline in the predicted income below a revenue which is guaranteed.

2.1.2 Revenue protection an aid for farmers

There are farmers who enter into the futures contracts or forward contracts to sell a portion of their output much before the harvest. They may also end up in a loss if the actual output is less than their expected output. In that case, crop revenue insurance plan significantly benefits the farmers by assisting them not only in paying for the manufacturing decline but also in offsetting the increase in the market prices.

2.1.3 Revenue insurance indemnities

The calculation of the expected revenue is very important for the farmers to set the strike price for their futures or forward contracts. One important point to be noted is that this calculation has to be made even before the planting process. The Expected revenue is calculated as follows:

$$\textit{Expected revenue} = \textit{Average Historical production} \times \textit{Expected Harvest Price Index}$$

Where, the harvest price index comprises of futures market prices and expected cash.

$$\textit{Harvest price index} = \textit{AVERAGE (harvest period futures contract prices calculated earlier than planting)}$$

Base Price = Expected harvest price. The Revenue Guarantee depends on Expected Revenue and a pre-selected coverage level.

Final Revenue Guarantee = MAX (Base Price, Estimated Harvest Price)

The Estimated Harvest Price = AVERAGE (end-of-season harvest futures contract prices)

Using actual production and the harvest price, we calculate the Actual revenue. A revenue insurance reimbursement is paid if:

The Actual Revenue < Final Revenue Guarantee.

2.1.4 Pricing a Revenue protection Insurance

The amount of revenue which is pre-decided is ensured in the revenue protection insurance. On an average, the change in rate and the change in the yield of agricultural crops tend to move in the opposite directions and hence there is a negative correlation between the price and the yield. It is very important to factor this into the equation for rating a revenue policy.

The gist of rating a crop revenue policy is as follows:

- We need to calibrate the annual producer probability density functions to the corresponding provincial yield time series and
- At the same time we need to incorporate the likelihood of a correlated price change.

Price risk is usually considered to follow LOGN (μ, σ) distribution. The mean (μ) and stdev (σ) estimated using the option market prices volatility. We can use the historical data to estimate the correlations between the price and yield movements. If on a correlation basis, we can combine the price and the yield density functions, a Monte Carlo simulation technique can be implemented to calculate the revenue income loss costs over a range of coverage levels.

2.2 Systematic risk, reinsurance and failure of crop insurance markets

Unfortunately, the private crop insurance markets are floundering due to the systemic weather effects on the farms. The problem lies in the high correlation induced by these weather effects among the crop yields. The bigger problem arises if the insurance companies do not have an affordable reinsurance plan. Without the reinsurance companies, the insurance companies are unable to pool the risks across the farms.

The researchers, M. J. Miranda and J. W. Glauber, used an empirical model of crop insurance market of the United States and found that the portfolios of the crop insurers are 20 – 50 times riskier than what they should be if yields are independent stochastically, across different farms. They also found that, if the insurers take up reinsurance contracts, it would cover most of their systemic crop or revenue loss risk, reducing their risk exposures drastically.[1]

2.3 Revenue protection insurance for producer hedging

Due to the development of new types of crop insurance products, there are many tools available now to the producers to choose and manage their risk. There is a very little information about the interactions between these products and futures and options.

The researchers, K. H. Coble, R. G. Heifner, and M. Zuniga, in their analysis have examined the optimal futures and put ratios in the presence of four alternative insurance coverages. They investigated the comparative statics of the relationship between hedging and insurance using an analytical model. They conducted an additional numerical analysis that incorporated futures price, basis, and yield variability. They found that the yield insurance has a positive effect on different levels of hedging. The results showed that the demand for revenue insurance was lesser than the yield insurance. [2]

2.4 Pricing a Crop Revenue Insurance Product

As suggested by Turvey and Amanor-Boadu, Black-Scholes option pricing model can be used for the pricing of premiums for Gross Revenue Insurance Products (GRIP). They used this model for put options on Canadian producers' gross revenue when price and yield are stochastic. The model was not very appropriate because of the many underlying assumptions of the Black-Scholes model for option pricing. The empirical studies show that farm gross revenues are non-tradable assets which contradicts the very fundamental assumption that all the assets are tradable in a perfect market. The log normality assumption is also contradicted as the empirical studies show that the actual gross revenue is not consistent with log normal distributions.[3]

2.5 Use of Options Market to Hedge the Price Risk of Crop Revenue Insurance Product

The researchers, S. Tiwari, K. H. Coble, A. Harri, and B. J. Barnett, did a research on whether the crop insurers can use put options to hedge their risks. They used a behavioral model to analyze a crop producer's hedging optimization behavior with crop insurance to analyze the optimal hedge ratio of a company selling revenue insurance. They found that the insurers will not go for hedging their risks if they want to maximize their utility even though they are risk averse. They came to a conclusion that, for the insurers, hedging the price - risk using put options may not be the best option. [4]

The researchers, K. H. Coble, M. Zuniga, and R. Heifner, examined the interaction of alternative insurance designs with futures hedging and the purchase of options. They found that the forward pricings are influenced by alternative insurance designs. By using alternative behavioral assumptions, they found that the optimal hedge is influenced by the assumptions. They also discovered that, yield insurance generally compliments forward pricing while pure revenue insurance has a negative. [5]

The researchers David Buschena and Lee Ziegler, in their research, evaluated the predictive reliability of using price distribution inferred from the options markets to rate the revenue insurance products. They found out that the price distributions inferred from the options trades offer greater reliability than distributions based on historical futures trade for the periods early in the crop growing seasons.

The study analyses the difference between the usefulness of the information given by the options market early in the growing season and other sources of information. This was done by evaluating the distributions implied by the options over various commodities. It was found that the most reliable price distributions were estimated during the first half of the growing season. The most reliable distributions were the Burr and the log normal distributions. [6]

The researchers, Paul L. Fackler and Robert P. King, in their research, developed a method to evaluate the option based price probability assessment based on the calibration concepts. In their study, they conducted empirical tests using goodness of fit criteria on four agricultural commodities. In their study they found out that the assessment in the corn and live cattle markets are reliable but such assessments overstate the volatility of soybean prices and understate the location of hog prices. [7]

2.6 Designing Crop Revenue Insurance Product

The researchers Joseph B. Cole and Richard Gibson, in their research highlighted the important factors in successful contract writing and presented a robust analytical procedure for assessing combined crop yield and price risks. Correlated multivariate analysis is required for the analysis of the indemnity equation because the revenue insurance adds price changes to the indemnity equation. They used Monte-Carlo modeling for the analytical procedure for measuring the revenue risk, wherein the state space comprises of correlated yield and price risks. Finally, they identified five main criteria for a successful index and these five factors are as follows:

1. Standardization
2. Verifiable pricing
3. Frequent price dissemination
4. Competitive price determination
5. Value representation

Properly constructed and rated, revenue-based crop insurance can provide superior risk management performance to producers. [8]

An optimal crop revenue insurance is dependent only on the gross revenue of the farmer whereas the indemnity schedule depends on the individual yield and farmers' price. If the function which determines the coverage level depends on the estimators of the individual prices which are imperfect, the design of the contract may not be as efficient as expected. The optimal indemnity schedule largely depends on the extent of basis risk of an individual. The results show that if there is a dependence of indemnity schedule on the individual yield and prices, then the optimal insurance contract is contingent only on the individual gross revenue. [9]

Chapter 3 Product Design Revenue Protection

There are a large number of points to be considered while designing a crop revenue insurance product. First of all the terms and conditions of the product must be decided. The conditions for insurability of a risk at a reasonable cost must be identified. Some of these conditions are as follows:

- The existence of a large number of similar risks that allows risk diversification through the law of large numbers.
- Individual risks to be preferably independent or with a low correlation with other risks.
- Loss amounts to be measurable and to be reasonably estimated.
- Loss distributions to be sufficiently stationary or predictable over time.
- The ability to calculate the frequency and the severity of possible losses through judgement or the underlying probabilities of the risks.
- Moral hazard and adverse selection to be controllable.

The systematic risk in the market can be diversified by following the conditions given below:

- Balanced or diversified insurance portfolios can be achieved by the insurers if the losses are uncorrelated. By diversifying away the risks, the insurers can charge affordable premiums rates.
- The asymmetric information like adverse selection and moral hazard must be controlled and managed. This is usually done by risk classification and risk adequate insurance.
- The premium which the policy holder pays in return for the insurance must be equal to the cost of mitigating their risk. The government helps the farmers to pay the premium in case they are unable to pay. This makes the insurance affordable and achieve a sustainable insurance penetration, which lowers adverse selection and market failure.
- To avoid market failure the insurance companies must always be in a position where in their assets are more than their liabilities.
- The insurer must be able to compute risk-adequate premium rates through loss distributions, including loss frequency and severity which is based on either historical or modelled data. Agricultural insurers have developed specific pricing and modelling approaches; however, the wealth and access of data from developed markets often

limit application in less developed markets, where new approaches need to be developed according to data availability and the market context.

- The insurer must be able to price risks among different risk classes to avoid concentrations of high-risk insurance buyers that drive up the costs of insurance and can ultimately lead to market failure. Limited data and the difficulty in classifying risks have been a challenge in agricultural insurance, particularly in low-income countries.

Agricultural insurance is often driven by demand and supply and may depend on a large number of factors such as:

- Risk awareness
- Affordability
- Alternative risk management options
- Economic conditions
- Market conditions

Important factors that determine demand are:

- The value perceived by a potential policyholder in the way insurance increases income and stabilizes earning variability
- The affordability of an available cover

In the economic theory, a producer's decision to buy crop insurance is a measure of insurance demand and affordability. This depends largely on the producer's

- Utility function of income in that insurance will increase the income and reduce the variance of future incomes.
- The current income, while the cost a producer attaches to risk typically declines with increasing profits.
- Subjective perception of the frequency distribution of future income.
- The cost and coverage of the insurance contract.

In insurance terms, a potential policyholder is the one, who buys an insurance protection only if they see some benefit in it. The insurers assume by default that an agricultural insurer is a potential policyholder and hence are to some degree risk averse. This means that a risk investment will be done only if there is a strictly positive return. So the insurance premiums must be commercially affordable and attractive in order to attract a potential policyholder who is willing to pay for the insurance. A potential policyholder will be willing to buy an insurance policy only if the price is below certain amount which satisfies the policyholder's utility.

In revenue insurance, the expected revenue is established before the planting is done by using farm based yields. The expected revenue is calculated by multiplying the expected historical yields with the expected harvest price. The expected harvest price is the average of the futures prices. The same approach is used by the area based revenue insurance the only difference being the area based yields are used instead of historical yields.

When the individual yields are insured, the coverage levels which the producers are allowed to select are usually between 65-70% of the average historical farm yields. But, when it comes to the situation where in all the fields are to be covered, there will be a diversification effect and hence the risk can be handled in a much more efficient way. Hence in this case the coverage levels will increase to 80-85% typically. The premium rates are determined from the guaranteed revenue which is the product of guaranteed yield and the expected harvest prices and the area insured. All of these calculations are based of the expected revenue and the coverage levels selected. The final revenue guaranteed is calculated during the harvest time as a function of the actual harvest price and the guaranteed yield. The actual harvest price is established by averaging over a time period which is agreed before at the time of the contract and multiplied by the area insured. The actual revenue is also is calculated simultaneously as a product of actual yield and actual harvest price and the area insured. The insured will receive an indemnity in case the actual revenue is lesser than the final guaranteed revenue.

The harvest prices vary a lot from the local prices as these harvest prices come from the futures market or national commodity exchanges for example NCDEX in India. For example the prices paid by the grain elevators may be totally different from the prices in the futures markets. Hence these difference may cause some risk as the prices cannot be predicted perfectly. Basis is an insurance term used for these differences in prices. The basis risk is the variability of this basis over time. The imperfect correlation of actual prices with price indices leads to basis risk.

Chapter 4 Introduction to Actuarial Pricing models for Revenue Protection

Pricing establishes the technical premium the insurer should charge to cover costs and leave a certain profit margin, which is typically undertaken by an actuary. Costing is the process that establishes commercial terms based on the technical premium and is a different process to pricing. While pricing needs statistical expertise to extrapolate loss potentials from past claims data and benchmarks, costing requires an understanding of the competitive environment and the underwriting cycle.

Actuarial pricing estimates premiums of individual risks and portfolios based on probabilities of loss occurrences from statistical distributions and historical claims information. Additionally, actuarial pricing addresses solvency ratios, risk retention levels and risk transfer through portfolio modelling. With advanced computation, multivariate pricing allows the use of multiple variables that explain a certain risk to be included in pricings through generalized linear models.

Machine learning and big data algorithms are increasingly used to identify factors that determine risk and correlations among risk parameters, which gradually leads to more tailored insurance terms and conditions for policyholders. Often several pricing approaches are combined to reflect future loss expectations in function of historical experience.

It is necessary to have an accurate data of the crop yields to price the revenue insurance product. For estimating the expected revenue and guaranteed revenue, the forward or historical commodity prices are required. Pricing and underwriting of revenue insurance products is challenging because we need to simulate farm-yields and model the volatility in the prices and at the same time, perform joint yield-price simulations and make sure that the correlations are constant.

To price any revenue insurance product, the four commonly used steps are as follows:

1. The yield probability density function and the probability density function for the prices need to be established.
2. Once the probability density functions are obtained, the yield-price correlation needs to be estimated.

3. Once the correlation is obtained, different price and yield events need to be jointly simulated such that the yield-price correlation is constant.
4. Loss must be estimated taking into consideration various demographics and coverage levels.

4.1 Factors involved in pricing a crop revenue insurance product

There are many factors which needs to be taken into consideration for pricing an insurance product for crop revenue insurance:

- Cost of production
- Changes in input prices
- Input-output price difference
- Trends in market prices over a period of time (3-6 months)
- Demand and supply of commodities
- Inter-crop price rates
- Effect on the industrial cost structure
- Effect on cost of living of the population
- Effect on general price level in the market
- International prices of the commodities
- Parity between prices paid and prices received by the farmers.
- Effect on issue prices for subsidy
- Cost of cultivation per hectare in various regions of the country.
- Cost of production per quintal in various regions
- Prices of various inputs
- Market prices of products
- Prices of commodities sold by the farmers
- Supply-related information like area, yield, imports, exports and domestic availability with procurement corporations
- Demand related information like total and per capita consumption
- Prices in the international market

- Prices of farm derivatives like sugar, jute goods, oils and cotton yarn
- Cost of processing of agricultural products
- Cost of marketing - storage, transportation, processing, marketing services etc.
- Macroeconomic factors like general level of prices, consumer price indices etc.

These figures are arrived at by taking into account real factors of production and include all actual expenses in cash incurred by the farmer like rent paid for the leased land, the value of family labor, the interest value of other capital assets (excluding land), depreciation on farm machinery and other miscellaneous expenses.

4.2 Sources of commodity prices

- Futures or the commodity markets are one of the reliable sources from where the prices for various commodities can be obtained. But there are few crops that are not listed in the futures market. For these crops the historical prices are used to establish the expected prices over the insurance period.
- One problem with these historical prices is that, if the market is semi-strong or for that matter strong, the estimates of the future expected prices would be imperfect.
- A subscription is required for obtaining the available information on the prices of options, spot markets and futures from the financial market terminals and platforms.
- There are few open source platforms such as Agmark website which give access to the commodity data.

4.3 Calculation of price volatility

4.3.1 Present Value method

The present value method assumes an arbitrary discount rate to obtain the present value of the expected indemnity which is the price of insurance contracts. Under the present value method, the value of the price of a revenue insurance contract is written for a guarantee level (G_t) at the time of planting. This depends on the crop revenue index Y_t and the payment of any insurance indemnities.

We have,

Premium rate = (1 + γ)*E[S]

Where, γ is the loading factor and S is the claim amount and E[S] is the expected claim amount.

Here the claim amount is:

S = max {(G_t – Y_t), 0}

Which implies:

E[S] = $\int_{0}^{Gt} (Gt - Yt) h(Yt) dYt$

Let $P_{t-1}(G_t)$ be the premium at time t-1. Hence, the premium is given by,

$P_{t-1}(G_t) = (1 + \gamma_t)*\beta_t*E[S] = (1 + \gamma_t)*\beta_t*\int_{0}^{Gt} (Gt - Yt) h(Yt) dYt$

4.3.2 Black Scholes method

Under the Black Scholes Option Pricing Method, a policyholder holding a revenue insurance contract has right but not obligation to exercise the insurance by claiming a fixed amount on a loss event. The probability of loss can only be determined by the observed distribution of the yield of crop and commodity prices. There is a lot of similarity between the European put options and the revenue insurance products. European put options are financial derivatives and are valued by the Black Scholes option pricing method with an assumption that the prices follow lognormal distribution and the market is perfect.

The assumptions underlying the Black Scholes option pricing method are as follows:

1. The prices of the underlying commodity follows a geometric Brownian motion.
2. There are no risk free arbitrage opportunities.
3. The risk free rate of interest is constant.
4. The risk free rate of interest is same for all maturities and the same for borrowing or lending.
5. Unlimited short selling is allowed.
6. There are no taxes or transaction costs.
7. The underlying assets can be traded continuously and in infinitesimally small number of units.

There are many problems regarding these assumptions. The commodity prices can jump contradicting the first assumption that the commodity prices follow geometric Brownian motion. The risk free rate of interest rate does vary and in an unpredictable way. Unlimited short selling may not be allowed. Commodity prices can only be dealt in integer multiples of one unit and not continuously. There also be some transaction cost in commodity trading. The assumption of no taxes is totally invalid. Distribution of commodity prices tend to have fatter tails than suggested by the lognormal models. Hence contradicting the lognormal assumption.

4.3.3 Geometric Brownian motion method

The Geometric Brownian motion has two components:

1. The drift term which represents the expected return on a stock over a period of time. The drift component is certain.
2. Volatility term which is included in a stochastic process. This is sometimes referred to as the implied volatility. The volatility component is uncertain.

The geometric Brownian motion method is used to model commodity prices and to price the revenue insurance product. The expected return on capital in the crop season is used as drift and the implied volatility is derived from the capital markets.

4.3.4 Calculation of Yield-Price correlation

The yield-price correlation tends to be negative in the regions where the global market consists mainly of large commodity production regions. This implies that as the demand for commodity increases the prices go high and the demand increases when the production is low. If there is a high production, the supply increases and in turn the demand decreases which leads to lower prices.

In the markets where less importance is given to the production at local and global level, there will not be any relation between the yield and the prices. As suggested by the United States revenue insurance products, there is a strong correlation between the yield and price for certain crops at local as well as global levels. These correlations are used in joint yield price simulations which was taken from the idea of copula function.

4.3.5 Pricing Methodology

Due to the nature of product features, the liabilities would need to be hedged by appropriate investments in derivative markets. Due to unavailability of matured derivative market for the given commodities in India, there exists uncertainty in the return and the extent of hedging. Hence insurance company needs to consider the uncertainty related to both claims cost and expected income to arrive at the premium charged.

Chapter 5 Hedging Models

People take insurance to safeguard themselves from some kind of risk. When it comes to hedging, it is a risk management strategy to offset losses in investment strategies. When people decide to hedge, they are insuring themselves against a negative event to their finances. Hedging techniques generally involve the use of financial instruments known as derivatives, the two most common of which are options and futures. The risk taken and the profits obtained are directly proportional whereas the hedging and profits are inversely proportional which means, more the risk you take more the profits you obtain but at the same time you may end up getting a huge loss. If we hedge our risk more, we end up taking less risk which in turn gives us minimal profits.

We have done a lot of research on various hedging models to come up with a methodology for the insurers to hedge their risk by investing in commodity markets. One of those models is presented here.

5.1 The Behavioral Model

Defining the terms:

y – Yield obtained

y_g – yield guarantee level (percentage of the expected yield)

W – end-of-season wealth

W_H – end-of-season wealth for above the yield guarantee

W_L – end-of-season wealth for below the yield guarantee

W_0 – initial wealth

A – Crop acres

p – Crop prices

p_0 – known planting time expectation of harvest price

p_1 – stochastic harvest time price

C – Non-stochastic production cost

$R(y_g)$ – crop insurance premium (increasing function of yield guarantee)

h – Quantity of crop forward priced with a future hedge

The end of season wealth will be conditional on whether insurance payouts are made.

If $y < y_g$, $W_L = W_0 + A[p_1 y - C - R(y_g) + h(p_0 - p_1) + p_g(y_g - y)]$

Otherwise, $W_L = W_0 + A[p_1 y - C - R(y_g) + h(p_0 - p_1)]$

A growing crop can be hedged by selling futures contracts equal to a portion of the expected crop before harvest and purchasing an equal number of futures contracts later when the actual crop is sold.

Let random harvest-time price and yield be normal variates. Assuming bivariate normality, the joint cumulative distribution is F(y, p). The objective function of a producer choosing the optimal hedge level may then be written as follows:

$$\operatorname*{Max}_{h} L = \int_{\underline{p}}^{\bar{p}} \int_{\underline{y}}^{y_g} U(W_L) f(y, p_1)\, dy dp_1 + \int_{\underline{p}}^{\bar{p}} \int_{y_g}^{\bar{y}} U(W_H) f(y, p_1)\, dy dp_1 .$$

In this model, the producer's utility is evaluated as the sum of expected utility in the W_L states where yield is certain, and the W_H states where yield is random. The producer's choice variable (h) represents the quantity of production to hedge given that the producer is insured. By maximizing this objective function, we are maximizing the utility of the person who is hedging as a function of h. There are a large number of optimization techniques available such as to obtain the optimal hedging strategy. This model is developed for the yield insurance but can certainly be extend to the revenue insurance by changing certain variables.

Chapter 6 Data

For the purpose of this project, we have collected the following data:

6.1 Data 1 (Commodity data)

This data set is downloaded from the agmarknet.gov.in website. This data set again has two subsets. One is the daily data and the other is the monthly data of commodities Barley, Bengal gram, Chana, Maize, Mustard, Soybean and Wheat.

The daily data consists of Arrivals of the commodities in tones, Minimum price per quintal, Maximum price per quintal, Modal price per quintal and the reported date, for each of the commodities mentioned above. This data is taken from 01 Jan 2008 to 30 Jun 2019 for each State, District, Market, Variety of the commodity and the Group the commodity belongs to.

The monthly data consists of monthly arrivals of each commodity which are mentioned above, in each state in India from January 2014 to December 2019.

6.2 Data 2 (Futures Price Movement data)

The below data has been collected from NCDEX website for each of the commodities Barley, Chana, Wheat, Maize, Mustard and Soybean:

- Futures price movement by commodity since 2008
    - Commodity
    - Date
    - Open, min, max and close prices
    - Date of contract settlement
    - Quantity traded
- Quantity sold in the market by commodity in each state over the last one-year April 2018 to March 2019
- Settlement prices by future contract since 2008

Chapter 7 Fitting PDFs

The minimum price of a commodity on a particular day is the lowest price at which the commodity was sold in one particular market on a particular day. The maximum price of a commodity on a particular day is the highest price at which the commodity was sold in one particular market on a particular day. The modal price is the price of a commodity on a particular day is the price at which the commodity was sold maximum number of times in a particular market on a particular day.

For each of these prices, three distributions; lognormal, gamma and Weibull distributions were fit. The following table consists of the estimated parameter values for each distribution and their standard errors for each of the six commodities namely; Maize, Chana, Barley, Bengal gram, Wheat and soya bean. There are four tables given below. The first one is for the minimum prices, the second one is for the Maximum prices, the third table is for the modal prices and the last one is for the arrivals. The arrivals on a particular day in particular market is the quantity of the commodity which has arrived in that market on that particular day.

7.1 Minimum Price of the commodity

Crop	Distribution	Parameters	Estimate	Standard Error
Maize	lognormal	meanlog	7.0227484	0.000346625
		sdlog	0.2766132	0.000245086
	gamma	shape	13.7832648	2.15E-02
		rate	0.01184424	1.87E-05
	Weibull	shape	3.145339	0.002171857
		scale	1275.18609	0.534963483
Chana	lognormal	meanlog	8.5675103	0.001984872
		sdlog	0.2686258	0.001403429
	gamma	shape	14.066674	4.99E-02
		rate	0.00258089	7.55E-06
	Weibull	shape	3.343133	0.01621177
		scale	6018.50076	14.14370797
Barley	lognormal	meanlog	7.062408	0.000571801
		sdlog	0.241842	0.000404293
	gamma	shape	17.1828745	5.15E-02
		rate	0.01429499	4.32E-05
	Weibull	shape	2.888293	0.003249277
		scale	1305.9419	1.122677471
Bengal gram	lognormal	meanlog	5.8529791	0.000498983
		sdlog	0.3865464	0.000352823
	gamma	shape	6.83595904	1.19E-02
		rate	0.01821109	3.27E-05
	Weibull	shape	2.501864	0.002162314
		scale	421.983261	0.230511621
Wheat	lognormal	meanlog	7.14962	0.000207281
		sdlog	0.196233	0.000146553
	gamma	shape	25.5393316	3.50E-02
		rate	0.01966023	2.71E-05
	Weibull	shape	1.744851	0.001036911
		scale	1536.169	1.134295116
Soybeans	lognormal	meanlog	7.8570946	0.000377486
		sdlog	0.2740328	0.000266907
	gamma	shape	14.6871581	1.85E-02
		rate	0.00549198	6.79E-06
	Weibull	shape	4.22312	0.004110394
		scale	2928.56953	1.007142651

Table1: Parameter estimates and their standard errors for minimum prices of the commodities

7.2 Maximum price of the commodity

Crop	Distribution	Parameters	Estimete	Standard Error
Maize	lognormal	meanlog	7.1077591	0.000328755
		sdlog	0.2623529	0.00023245
	gamma	shape	14.66596161	2.26E-02
		rate	0.01160068	1.81E-05
	weibull	shape	2.778148	0.001730777
		scale	1381.573314	0.655911598
Chana	lognormal	meanlog	8.6357706	0.001730964
		sdlog	0.2342628	0.001223876
	gamma	shape	17.77615034	6.95E-02
		rate	0.00306939	1.07E-05
	weibull	shape	3.509526	0.01664502
		scale	6366.521089	14.2797612
Barley	lognormal	meanlog	7.108448	0.000565292
		sdlog	0.239089	0.00039969
	gamma	shape	17.28681913	5.14E-02
		rate	0.01373736	4.12E-05
	weibull	shape	2.794148	0.003133013
		scale	1368.568164	1.217446471
Bengal gram	lognormal	meanlog	5.9511917	0.000483409
		sdlog	0.3744821	0.000341811
	gamma	shape	7.17167817	1.24E-02
		rate	0.01737885	3.11E-05
	weibull	shape	2.42751	0.001950548
		scale	462.48038	0.259912157
Wheat	lognormal	meanlog	7.26384	0.000236233
		sdlog	0.223642	0.000167027
	gamma	shape	19.18826144	0.024960708
		rate	0.01309186	1.71532E-05
	weibull	shape	1.955547	0.00081904
		scale	1749.09478	1.122892639
Soyabean	lognormal	meanlog	7.9781327	0.000346196
		sdlog	0.2513184	0.00024478
	gamma	shape	16.89559728	2.07E-02
		rate	0.005622459	6.77E-06
	weibull	shape	2.400396	0.001205562
		scale	3573.1401	2.471754817

Table2: Parameter estimates and their standard errors for maximum prices of the commodities

7.3 Modal price of the commodity

Crop	Distribution	Parameters	Estimete	Standard Error
Maize	lognormal	meanlog	7.0732685	0.000328054
		sdlog	0.2617929	0.000231954
	gamma	shape	14.98687204	2.34E-02
		rate	0.01228027	1.94E-05
	weibull	shape	3.135964	0.002118954
		scale	1334.841661	0.561922662
Chana	lognormal	meanlog	8.6068105	0.001782734
		sdlog	0.2412692	0.001260486
	gamma	shape	16.84812533	6.55E-02
		rate	0.002990045	1.02E-05
	weibull	shape	3.463451	0.01651646
		scale	6202.467218	14.08038331
Barley	lognormal	meanlog	7.0889589	0.000560437
		sdlog	0.2370355	0.000396257
	gamma	shape	17.6965268	5.29E-02
		rate	0.0143482	4.33E-05
	weibull	shape	2.886489	0.003241554
		scale	1339.567818	1.152711623
Bengal gram	lognormal	meanlog	5.9133623	0.000483524
		sdlog	0.3745709	0.000341892
	gamma	shape	7.21564909	1.25E-02
		rate	0.01816846	3.26E-05
	weibull	shape	2.546067	0.002183936
		scale	445.994084	0.239451705
Wheat	lognormal	meanlog	7.2107497	0.000213834
		sdlog	0.2024372	0.000151187
	gamma	shape	23.67785945	3.19E-02
		rate	0.01712029	2.32E-05
	weibull	shape	1.996247	0.000735452
		scale	1637.475928	1.019511206
Soyabean	lognormal	meanlog	7.9428089	0.000340262
		sdlog	0.2470105	0.000240584
	gamma	shape	17.72472839	2.26E-02
		rate	0.006119086	7.69E-06
	weibull	shape	4.122363	0.003053891
		scale	3137.874428	1.09448377

Table3: Parameter estimates and their standard errors for modal prices of the commodities

7.4 Arrivals of the commodity

Crop	Distribution	Parameters	Estimete	Standard Error
Maize	lognormal	meanlog	2.233056	0.002828785
		sdlog	2.257424	0.002000252
	gamma	shape	0.315673294	4.35E-04
		rate	0.003875716	9.29E-06
	weibull	shape	0.4708835	0.000430431
		scale	28.2352311	0.079571316
Chana	lognormal	meanlog	1.538908	0.012653702
		sdlog	1.712509	0.008947505
	gamma	shape	0.4388543	0.003737466
		rate	0.02148898	0.000300749
	weibull	shape	0.5743176	0.00306184
		scale	11.2144341	0.15321513
Barley	lognormal	meanlog	1.765995	0.004604036
		sdlog	1.947268	0.003255541
	gamma	shape	0.316912082	8.35E-04
		rate	0.006275697	3.01E-05
	weibull	shape	0.4865014	0.00079538
		scale	15.8031979	0.081567414
Bengal gram	lognormal	meanlog	-0.7174724	0.002603568
		sdlog	2.0169036	0.001840999
	gamma	shape	0.3463866	0.00050519
		rate	0.1015079	0.000267337
	weibull	shape	0.5017347	0.000464471
		scale	1.3405626	0.003658169
Wheat	lognormal	meanlog	0.8661693	0.002249062
		sdlog	2.1291883	0.001590325
	gamma	shape	0.28275674	0.000331175
		rate	0.01013608	2.31047E-05
	weibull	shape	0.4543841	0.000334245
		scale	6.9805225	0.017228556
Soyabean	lognormal	meanlog	3.017082	0.002955742
		sdlog	2.145698	0.002090023
	gamma	shape	0.352393736	5.18E-04
		rate	0.002562297	5.44E-06
	weibull	shape	0.501684	0.000508078
		scale	58.361035	0.169707957

Table4: Parameter estimates and their standard errors for arrivals of the commodities

7.5 Discussion

Looking at the parameter estimates and their standard errors, it can be seen that lognormal model is the best fit among the three models tested for all the six commodities. The gamma model fits better than the Weibull model. The results are as expected since our assumptions in all the models are that the prices follow the lognormal model. Same is the case for the maximum prices and the modal prices. For the arrivals data, the gamma model fits better than any other models for all the six commodities. For some commodities, weibull distribution fits better than the lognormal distribution.

Chapter 8 Liability Projection

Liability projection is a very important step in the premium pricing. The liability cashflows are on the best estimate basis. For each year the net cashflows are generated. This net cashflow is the difference between the cash outflow and the cash inflow. For each year and for every policy, the net liability is generated. For every policy, the total net liability is summed for the respective years.

Large part of the liability consists of the expected claims cost. This depends on two factors:

- Expected shortfall of market price to MSP for each group
- Expected quantity that is sold at MSP for each crop

Expected claims cost = $\sum Quantity\ of\ crop\ sold\ at\ MSP\ \times (MSP\ - Market\ Price)$

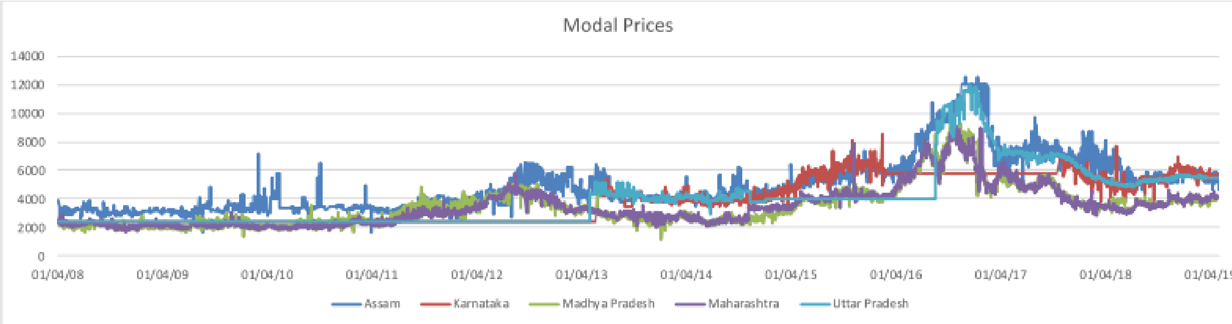
The summation is over all the crops and the MSP – Market price > 0 for each crop.

Liabilities under the product is the total shortfall of the price when compared to the MSP. Size of expected liabilities can be established as the expected amount of shortfall of the actual market price when compared to MSP. This would depend on the quantity of the commodity sold at MSP and also the prevailing market prices. We need to hence, estimate both the price movement over the coming year and the quantity traded during the next year. The gap of this price when compared to the MSP will help establish the liability during the year. We have taken the below steps in this regard:

- We have used the data from agmarknet.gov.in to obtain the market data for commodities. The dataset downloaded is in the below format.

Sl no.	District Name	Market Name	Commodity	Variety	Min Price (Rs./Quintal)	Max Price (Rs./Quintal)	Modal Price (Rs./Quintal)	Price Date
1	Sirohi	Abu Road	Bengal Gram(Gram)(Whole)	Other	3300	3350	3325	08-Jun-18
2	Sirohi	Abu Road	Bengal Gram(Gram)(Whole)	Other	3475	3525	3500	29-Jan-18
3	Sirohi	Abu Road	Bengal Gram(Gram)(Whole)	Other	3675	3725	3700	18-Dec-17
4	Sirohi	Abu Road	Bengal Gram(Gram)(Whole)	Other	4375	4425	4400	13-Nov-17
5	Sirohi	Abu Road	Bengal Gram(Gram)(Whole)	Other	4375	4425	4400	10-Nov-17
6	Ajmer	Ajmer (Grain)	Bengal Gram(Gram)(Whole)	Other	5030	6350	6320	23-Apr-16
7	Ajmer	Ajmer (Grain)	Bengal Gram(Gram)(Whole)	Other	4320	4650	4530	27-Oct-15
8	Ajmer	Ajmer (Grain)	Bengal Gram(Gram)(Whole)	Other	4130	4380	4350	16-Sep-15
9	Ajmer	Ajmer (Grain)	Bengal Gram(Gram)(Whole)	Other	3810	4150	3990	08-Jul-15

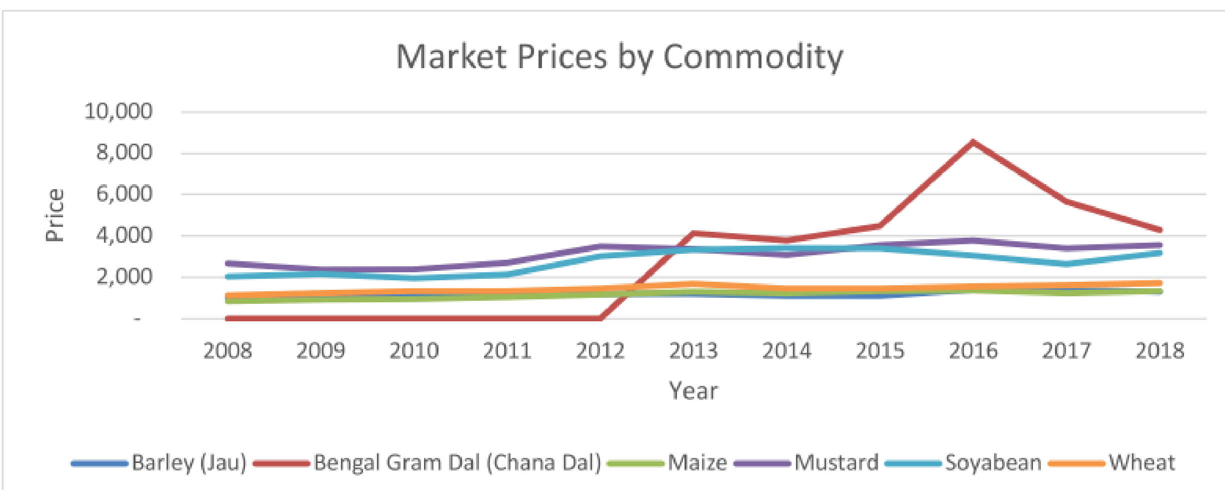
- We have the list of prices for each commodity in each market within districts across states.
- We have calculated the price at commodity level by taking an average of prices of all the markets within the state.



- We have computed the volume of production in each state over the last few years.
- Using the volume data as the weight, we have calculated the overall modal price for each state on a daily basis.
- Monthly seasonality in price movement has been observed. Price movements have been separately modelled for each month within the calendar year.
- We have seen volatile trends in the market prices year on year. There has been a drop in the prices for most commodities in 2018.

- We have computed the volume of production in each state over the last few years.
- Using the volume data as the weight, we have calculated the overall modal price for each state on a daily basis.
- Monthly seasonality in price movement has been observed. Price movements have been separately modelled for each month within the calendar year.
- We have seen volatile trends in the market prices year on year. There has been a drop in the prices for most commodities in 2018.

- Overall commodity wise average prices are as below:



Correlation% table is as below. We see significant correlation between Maize, Mustard, Soybean and Wheat. The high correlation may be attributed to the weather conditions and other macro-economic factors.

Commodity	Barley (Jau)	Bengal Gram Dal (Chana Dal)	Maize	Mustard	Soyabean	Wheat
Barley (Jau)	100%	75%	55%	55%	17%	66%
Bengal Gram Dal (Chana Dal)		100%	53%	71%	-51%	-1%
Maize			100%	92%	89%	90%
Mustard				100%	83%	79%
Soyabean					100%	72%
Wheat						100%

Next steps

Liabilities Modeling

- Consider the below qualitative factors into consideration when projecting price:
    - Projected demand and supply of crop and hence price of the product
    - Trends in improvement of infrastructure and its impact on cost of production
    - International trends in supply and demand of that commodity
- Drastic impact can be seen when there is a huge increase or decrease in the price since the previous year.
    - Shortage in rainfall or weather events can lead to shortfall of supply and hence increase in the price. This should however, lead to a fall in the liability under this product as the MSP will most likely be lower than the price.
    - Bumper crop or intervention of the government at the wrong time can lead to lower market prices than MSP. This will lead to increase in liability
- Anomalies that can lead to extreme fluctuation in prices
- Apply Bayesian credibility theory framework to the past data experience to ensure appropriate mix of experience and exposure assumptions
- Current model is a deterministic time series model. Use extreme value distributions and generalized Pareto to model the stochastic market price process. This would account for tail end extreme events with suitable probability.
- Back testing results using past data and establish mean square errors for randomly split segments within the portfolio.
- Establish 90-95% confidence interval of price movements for each month for the next 12-month period.

Futures modeling

- Current projected market prices and MSP have been able to predict movement in futures to the tune of 80%-85%.
- The balance 15% can be attributed to other factors specific to demand and supply of futures in the market.

- Further accuracy can be achieved by modelling futures separated by tenure of the future and at daily level instead of month level. This will add additional complexity into the model.
- Sense test the result through parametric approaches using distributional assumptions for movement in futures price.
- Additional qualitative allowance would need to be given to:
    - Fluctuations in the market due to less participation
    - Inability to liquidate assets during extreme movements of price

These additional factors would mean additional assets would need to be invested over and above the expected claims cost.

Policy design, Conditions and wordings

Following aspects are being explored as part of product design:

- Defining liability
- Possibility of integrating Revenue protection with Fasal bhima yojana
- Comparing revenue protection products in other markets like USA
- Customizing and designing appropriate product for Indian market in discussions with NCDEX and others
- Defining policy conditions and wordings
- Designing reinsurance structure – profit sharing and loss sharing
- Reviewability of Premium
- Risk sharing
- Reinsurance support
- Claims settlement process

References

[1] M. J. Miranda and J. W. Glauber, “Systemic Risk, Reinsurance, and the Failure of Crop Insurance Markets,” *Am. J. Agric. Econ.*, vol. 79, no. 1, pp. 206–215, 1997.

[2] K. H. Coble, R. G. Heifner, and M. Zuniga, “Implications of Crop Yield and Revenue Insurance for Producer Hedging,” *J. Agric. Resour. Econ.*, vol. 25, no. 2, pp. 432–452, 2000.

[3] J. R. Stokes, W. I. Nayda, and B. C. English, “The Pricing of Revenue Assurance,” *Am. J. Agric. Econ.*, vol. 79, no. 2, p. 439, 1997.

[4] S. Tiwari, K. H. Coble, A. Harri, and B. J. Barnett, “Hedging The Price Risk of Crop Revenue Insurance Through the Options Market,” *J. Chem. Inf. Model.*, vol. 53, no. 9, pp. 1689–1699, 2013.

[5] K. H. Coble, M. Zuniga, and R. Heifner, “Evaluation of the interaction of risk management tools for cotton and soybeans,” *Agric. Syst.*, vol. 75, no. 2–3, pp. 323–340, 2003.

[6] D. Buschena and L. Ziegler, “Reliability of Options Markets for Crop Revenue Insurance Rating,” *J. Agric. Resour. Econ.*, vol. 24, no. 2, pp. 398–423, 1999.

[7] P. L. Fackler and R. P. King, “Calibration of Option-Based Probability Assessments in Agricultural Commodity Markets,” *Am. J. Agric. Econ.*, vol. 72, no. 1, p. 73, 1990.

[8] J. B. Cole and R. Gibson, “Analysis and feasibility of crop revenue insurance in China,” in *Agriculture and Agricultural Science Procedia*, 2010, vol. 1, pp. 136–145.

[9] B. D. Wright and O. Mahul, “Designing Optimal Crop Revenue Insurance,” *Am. J. Agric. Econ.*, vol. 85, no. 3, pp. 580–89, 2003.

[10] G. B. K., “Challenges in the design of crop revenue insurance,” *Agric. Financ. Rev.*, vol. 75, no. 1, pp. 19–30, Jan. 2015.