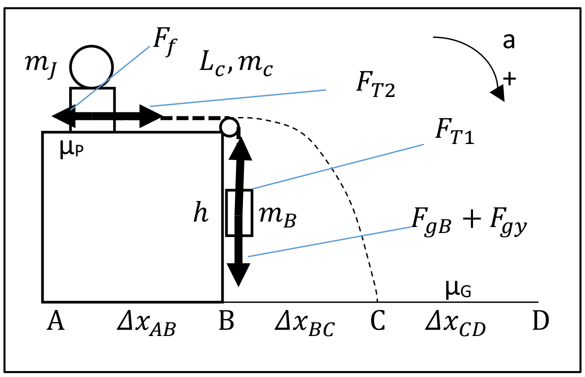
Praneet Mekala I Section 10/30/17

Uber Problem – "Jerky" Jerry's Jabberwocky Jumper

Description:

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (See diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pullet. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.

Diagram



Givens

$m_J = 60$ kg	$h = 14$ m
$m_B = 149$ kg	$\mu_P = 0.23$
$m_C = 62$ kg	$\Delta x_{BD} = 88$ m
$L_C = 5$ m	$\mu_G = ??$

Strategy: To set up this problem, I split it up into 3 main sections. The first section was the part of the trip where the rocks were accelerating Jerry and the jumper (AB). Given all the masses, it was possible to find the forces acting on the system and therefore the acceleration as a function of position. Then using that acceleration function, the velocity was found as a function of position. Then, the velocity right before Jerry was launched off was found. The second part of the trip was when Jerry was in projectile motion (BC). Using the kinematics formulas, the final velocity of Jerry was found. The final part of the trip was the point at which Jerry is sliding on the ground (CD). The force of friction was found by using the acceleration of Jerry. From the force of friction, the coefficient of friction was calculated.

Section 1: Pulley

Let the part of the chain that is hanging off the platform have a distance y meters. From the diagram, a net force equation can be made:

$$\Sigma F = F_{gB} + F_{gy} - F_{T1} + F_{T2} - F_f = m_T a$$

$$\Sigma F = m_B g + m_y g - F_{T1} + F_{T2} - \mu_P F_{NJ} = m_T a$$

The weight of the part of the rope that is hanging off the platform is the fraction of mass of the total chain is proportional to the length of chain hanging off, $m_y = \frac{y}{L_c} m_C$

Also, since both tensions are on the same rope, $F_{T1} = F_{T2}$

Since Jerry does not accelerate in the vertical direction, his normal force is equal to his weight ($F_{NJ} = F_{GJ}$)

$$\Sigma F = m_B g + \frac{y}{L_C} m_C g - \mu_P F_{gJ} = m_T a$$

$$\Sigma F = m_B g + \frac{y}{L_C} m_C g - \mu_P m_J g = m_T a$$

$$(149)(9.8) + \frac{y}{5}(62)(9.8) - (0.23)(60)(9.8) = (60 + 149 + 62)a$$

$\underline{a[y] = 0.448413y + 4.88915}$

Now finding the velocity function:

$$v\,dv = a\,dy$$

$$\int_{v_0}^{v} v\,dv = \int_{y_0}^{y} a[y]\,dy$$

$$\int_{v_0}^{v} v\,dv = \int_{y_0}^{y} 0.448413y + 4.88915\,dy$$

$$\left[\frac{v^2}{2}\right]_0^v = \left[\frac{0.44813y^2}{2} + 4.88915y\right]_0^y$$

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$$\left(\frac{v^2}{2}-\frac{0^2}{2}\right)=\left(\frac{0.44813y^2}{2}+4.88915y\right)-\left(\frac{(0.44813)(0^2)}{2}+(4.88915)(0)\right)$$

$$\frac{v^2}{2}=\frac{0.44813y^2}{2}+4.88915y$$

$$v^2=0.44813y^2+9.7783y$$

$$v[y]=\sqrt{0.44813y^2+9.7783y}$$

The final velocity occurs when the entire chain is hanging off the platform ($y=5$)

$\underline{v[5]=7.7521\text{ m/s}}$

Section 2: Projectile Motion

Starting in the y direction:

$$y_f=\frac{1}{2}a_yt^2+v_{iy}t+y_0$$

Since the final position is the ground and the original position is the top of the tower, $y_f=0$ and $y_0=h=14$

$$0=\frac{1}{2}(-9.8)t^2+(0)(t)+14$$

$$0=-4.9t^2+14$$

$$4.9t^2=14$$

$\underline{t=1.6903\text{ s}}$

Finding the y-component of the velocity just before Jerry hits the ground:

$$v_{fy}=v_{0y}+a_yt$$

$$v_{fy}=0+(-9.8)(1.6903)$$

$\underline{v_{fy}=16.565\text{ m/s}}$

Since the x-component of Jerry's velocity is always constant, $\underline{v_{fx}=7.7521\text{ m/s}}$

Finding the magnitude of the sum of the two vectors v_{fy} and v_{fx}:

$$|v_f|=\sqrt{|v_{fx}|^2+|v_{fy}|^2}$$

$$|v_f|=\sqrt{7.7521^2+16.565^2}$$

$\underline{|v_f|=18.2892\text{ m/s}}$

Also, since Jerry's horizontal velocity was a constant 7.7521 m/s and he travelled for 1.6903s, his horizontal displacement:

$$\Delta x_{BC}=v_xt$$

$$\Delta x_{BC}=7.7521*1.6903$$

$\underline{\Delta x_{BC}=13.1034\text{ m}}$

Section 3: Ground

Since 75% of Jerry's speed is converted to the horizontal direction, the initial velocity of the ground trip is $|v_{iG}|=0.75|v_f|=0.75*18.2892=13.717\text{ m/s}$

$\underline{|v_{iG}|=13.717\ m/s}$

Also, $\Delta x_{BD}=\Delta x_{BC}+\Delta x_{CD}$

$$88=13.1034+\Delta x_{CD}$$

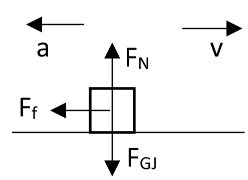
$\underline{\Delta x_{CD}=74.8966\text{m}}$

Using the kinematic equations to find the acceleration during the trip:

$$v_{fG}^2=v_{iG}^2+2a_G\Delta x_{CD}$$

$$0=13.717^2+(2)(a_G)(74.8966)$$

$$a_G=-1.256\text{ m/s}^2$$



$$\Sigma F=-F_f=m_Ja_G$$

$$-\mu_GF_N=m_Ja_G$$

$$-\mu_GF_{GJ}=m_Ja_G$$

$$-\mu_G\cancel{m_J}g=\cancel{m_J}a_G$$

$$-\mu_G(9.8)=-1.256$$

$$\boxed{\mu_G=0.1282}$$