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Effects of operator learning on production output: a Markov chain approach

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We develop a Markov chain approach to forecast the production output of a human-machine system, while encompassing the effects of operator learning. This approach captures two possible effects of learning: increased production rate and reduced downtime due to human error. In the proposed Markov chain, three scenarios are possible for the machine at each time interval: survival, failure, and repair. To calculate the state transition probabilities, we use a proportional hazards model to calculate the hazard rate, in terms of operator-related factors and machine working age. Given the operator learning curves and their effect on reducing human error over time, the proposed approach is considered to be a non-homogeneous Markov chain. Its result is the expected machine uptime. This quantity, along with production forecasting at various operator skill levels, provides us with the expected production output.

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1. Introduction

In a human-machine system, as operators learn, there may be two benefits: (1) improved production output rate, and (2) reduced human error rate. Both of these factors result in higher performance of the system. The study of the effects of human learning on the performance of the human-machine system has received much attention; studies such as Yelle (1979) and Dutton and Thomas (1984) state that the time to produce a single unit continuously decreases with the processing of additional units. Operator learning certainly plays a role in manufacturing environments and learning effects have been proven to exist by many empirical studies (eg Venezia, 1985; Webb, 1994). Considering the learning of the individual workers and using it as a variable in performance optimization can have many benefits for an organization. Onkham *et al* (2012) discuss the benefits an organization may realize by providing training to the employees, resulting in increased skill and knowledge as well as reducing human error. This negative relationship between knowledge and failure rate is also discussed by Kiassat *et al* (2013) and Fritzsche (2012).

We aim to forecast the production output of an operator over a planning horizon, considering the effects of learning, among other factors. The operator gains in expertise based on the amount of time he spends working on the machine, while the machine is operational. The learning curve is proportional to machine uptime. As the operator gains in expertise, he is less likely to make mistakes; therefore, the probability of success at various stages along the planning horizon is not constant. This probability can be calculated at each stage based on the appropriate value of operator-related factors, including those affected by learning. The analysis is performed in intervals and the value at each interval depends on the values of Machine-related (MR) and Human-related (HR) factors at the previous interval. This is the reason we choose a Markov chain approach. A Markov chain is a stochastic process that possesses the Markov property: when we know the present state of the process, the future development is independent of anything that has occurred in the past (Rausand and Hoyland, 2004). In our case, we have to use a non-homogenous Markov chain as operator expertise, learning, and working conditions are a function of time.

Our Markov Chain model quantitatively captures the positive effects of learning in terms of improved skill, leading to increased output, as well as reduced human error, leading to decreased downtime. In general, machine downtime can be caused by MR or HR factors. There are many reliability and failure risk analysis models that deal with the machinery. However, there are few that incorporate the role of the human operators on uptime and overall performance. To obtain the probability of machine failure due to both MR and HR factors, we use the proportional hazards model (PHM), a commonly used tool to model equipment time of failure (Jardine *et al*, 1989; Vlok *et al*, 2002; Kiassat and Safaei, 2009).

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There can be numerous applications for calculating the expected value of an operator's production output, considering the effects of learning. One possible application discussed in this paper is operator assignment (OA). We can calculate the expected production output for each operator on each machine and use the results as input factors into the objective function of a mathematical programming model for optimal OA. Our aim is to maximize performance, thus maximizing system revenue.

This paper is organized as follows. In Section 2, we perform a literature review and identify the gap we are filling with this paper. Section 3 covers the development of the Markov chain approach, and is then followed by Section 4, where a case study of a manufacturing company is used to develop our model and use the results to optimally assign the operators. This case study serves to demonstrate our model and validate its results.

2. Literature review

When one aims to analyse the effects of HR factors on failure risk analysis of human-machine systems, there are numerous studies such as those using human reliability analysis techniques (Cacciabue, 2005; Chang and Wang, 2010) or failure modes effects and analysis (Pillay and Wang, 2003; Seyed-Hosseini *et al*, 2006). However, when this analysis is to be used for equipment uptime or system performance analysis rather than just risk management, the literature is sparse. Horberry *et al* (2010) discuss human factors and their effects on operations and maintenance in a mining context but do not attempt failure prediction. Similarly, Kolarik *et al* (2004) develop a model to monitor and predict an operator's performance using a fuzzy logic-based assessment. But the purpose of their work is solely to provide a human reliability assessment, without providing any methods for risk reduction. Blanks (2007) discusses the need for improving reliability prediction, paying special attention to human error causes and prevention, but does not mention any predictive techniques for human reliability. Carr and Christer (2003) and Dhillon and Liu (2006) focus on the maintenance workforce performing repair work at times when machines are not being used for production purposes. Reer (1994) discusses human reliability in emergency situations, not regular production. There are works in the literature, such as those by Peng and Dong (2011) and Iakovou *et al* (1999), that use a Markov chain approach for uptime prediction. However, none use HR factors in their failure risk analysis and the calculation of transition probabilities. Therefore, the focus of all aforementioned works differs from ours in that we aim to predict the uptime of production equipment, based on the analysis of the risk of failure stemming from the human operator.

In addition to the scarcity of the previous works analysing the performance of human-machine systems from a human perspective, there are even fewer that do so while considering human learning. Biskup (2008) performs a state-of-the-art review on the effects of learning on production scheduling. Li and Cheng (1994) and Teyarachakul *et al* (2011) also study production scheduling and consider the effects of both learning and forgetting. But neither study has a focus on failure risk analysis; nor do they focus on effects of learning on decreased human error rate and improved system performance.

Malhotra *et al* (1993) discuss the role of learning on system performance. But the discussions are based on optimizing the cross-training of employees to strike a balance between flexibility and throughput loss due to forgetting and training. Similar to the aforementioned studies in the area of scheduling, this study does not have a detailed focus on failure risk analysis or increased production output as a result of operator learning. Similarly, there are works by Nembhard and Norman (2002), Leopairote (2003), and Vidic (2008) that discuss the effects of work-sharing and job-rotation on operator learning and forgetting. But these models aim to optimize operator learning and forgetting as a result of work-sharing and job-rotation. Unlike our scope, the aforementioned works do not attempt to predict equipment uptime using a failure risk analysis based on HR factors. Nor do they use learning curves for skill levels of individual operators. Adamides *et al* (2004) consider human learning, among other factors, to lead to increased productivity of maintenance activities. However, they do not distinguish among the various individuals and consider one rate of productivity improvement for all individuals based on a certain duration working on the particular product.

3. Markov chain approach

We aim to calculate the expected production output of an operator on a machine, over a planning horizon. To achieve this, we need to know the possible states of the system at the end of the planning horizon, along with the state probabilities and their corresponding production output levels.

To turn our problem into a feasible one to solve, we take the simplifying yet realistic step to discretize the planning horizon into individual time intervals; the intervals are then analysed through a Markov chain. For each of the intervals, the probability of failure/survival is calculated. We start from an initial condition with a certain machine working age and an initial level of operator expertise. At the first interval, there can only be one of two outcomes: failure or survival. There is a transition probability associated with each outcome and this probability is calculated using a PHM.

The PHM is a common tool for failure risk analysis. This is especially true when the PHM is parametrized using the Weibull baseline (Jardine *et al*, 1989). Weibull PHM is also the accelerated life model (Crowder *et al*, 1991). The general form of the Weibull PHM is defined as follows:

$$h(t) = h(t, z) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[\sum \gamma_j z_j(t)\right] \tag{1}$$

The PHM relates the time of an event, such as failure, to a number of explanatory variables known as covariates, $z_i(t)$, that may depend on time. Several factors, including the equipment age or specific system characteristics, may influence the equipment's hazard rate, which is the rate of transition out of a non-failed state to a failed state. The hazard rate, $h(t)$, is proportional to the (instantaneous) conditional probability of failure at time t; it can be affected by factors specific to the machine, the environment, or the humans within the human-machine system. Covariates can easily be incorporated into the PHM and their effects on the hazard rate evaluated. This is the most important advantage of the PHM over other statistical approaches in modelling the time of failure (Si *et al*, 2011). The convenience of the model accommodating the inclusion of HR covariates, in addition to MR factors, makes the PHM quite a suitable model for our analysis.

If the machine survives one interval, it reaches the next one and once again faces the two outcomes of survival or failure. However, if the machine fails over an interval, there is only one path to follow. It must remain in repair for a certain period of time, D. Therefore, in the general framework of our approach, multiple states are possible at each time interval. Each state can lead to at least one path, for repair, and at most two paths, for failure/survival. There is a transition probability associated with the paths leading from each state.

A simplifying assumption we make about D is that it is fixed, regardless of the type of failure. A realistic example is to set D equal to the Mean Time To Repair (MTTR). There are two other assumptions we make about our framework. Our first assumption is similar to the work by Gasmi *et al* (2003), where we assume the machine is brought back to zero-age following the repair. The second assumption is that operator learning occurs only when the machine is operational. The accumulated knowledge occurs cumulatively over the operational periods, but is not sensitive to any one specific time interval.

We design a certain Markov process to incorporate our assumptions. To fulfill the design conditions, we represent the state space of the Markov chain by a three-dimensional vector: (i, a, d). In essence, we construct a Markov process for the convenience of incorporating our assumptions. The planning horizon is discretized into N intervals. The quantity n, where $0 \leqslant n \leqslant N$, is a discrete indexing parameter and can be thought of as the machine's global age, calculated from the start of the planning horizon. The first variable, i, represents the cumulative number of time units that the machine has been operational at any given moment, n. On the basis of our assumption that operators learn only when the machine is operational, i determines the appropriate PHM covariate level, used to calculate the hazard rate. A higher value of i indicates a higher skill score of the operator, resulting in a lower hazard rate, all else being equal. In the case of operator expertise, the values are determined using learning curve equations. The second variable, a, represents the machine working age since the previous repair. At each failure, a is reset to zero and remains at zero during repair time. The last variable, d, $d \geqslant 0$, represents the remaining repair time. A positive d indicates the repair procedure is still in-progress. When $d = 0$, it means a repair has just been completed or that the machine is operational. The variable d is simply to track whether or not the machine is in repair, and if so, how long is remaining in the repair period. The other two variables, a and i, are designed to represent their respective information in a cumulative manner. Variable a is a counter for machine age, cumulatively since the last failure. Variable i is the cumulative number of operational periods and is measured cumulatively since calendar age zero.

For an interval $[n-1, n)$, the values of the three variables are considered at the instant in time immediately before n. According to this convention, the range for values for d is $0 \leqslant d \leqslant D-1$. If we experience a failure at $n-1$, by the time we reach the end of the interval at n, we have already had 1 time unit of repair completed. This is due to our assumption that the failure occurs at the beginning of the interval $[n-1, n)$. Therefore, $D-1$ time units remain.

The three state variables range as follows: $0 \leqslant i \leqslant N$, $0 \leqslant a \leqslant N$, $0 \leqslant d \leqslant D-1$. Given the range, as well as the fact that in this paper, we are analysing a specific planning horizon, both parameter space and the state space are discrete and finite. The possible transitions for the individual variable are: $(i \to i)$, $(i \to i+1)$, $(a \to 0)$, $(a \to a+1)$, $(d \to d-1)$, $(d = 0 \to D-1)$, but not all combinations of these individual transitions are possible. The state probability at n is based on: (1) all the states at $n-1$ that can lead to the state at n; and (2) the path from all these previous states to n. Figure 1 displays a state space diagram for an example where $D = 3$ for a planning horizon of $N = 7$. As evident, each state can be sufficiently described by relying strictly on the previous state(s) leading to it, the inherent property central to a Markov chain.

Let $p(i, a, d \mid n)$ be the probability to have i operational intervals at stage n, on a machine that has a current working age of a, with d time units remaining in the repair interval. At the initial moment, for simplicity, we assume an operational machine, with zero working age, and operators with an initial set of skills but no work experience. Therefore, the initial

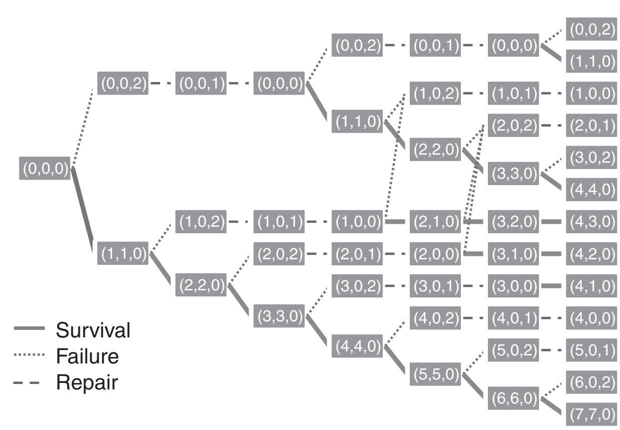

Figure 1 Typical state space for $N = 7$.

condition at $n = 0$ is expressed as follows:

$$p(i, a, d \mid 0) = \begin{cases} 1, & i = a = d = 0 \\ 0, & \text{otherwise} \end{cases}.$$

We can progress recursively to calculate the state probability at each future state. The recursive function expresses the state and the transition probability. Failure transition probability, q, is calculated using the PHM discussed in the previous section. Factors pertinent to the PHM are the number of operational time units, i, (affecting operator skill), as well as working age, a. Global age, n, does not play a direct role in calculating the hazard rate. As a result, $q(i', b|n)$, representing the probability of failure while transitioning from a state at n, with conditions i' and b, to a state at $n+1$, is expressed as $q(i', b)$. The recursive formula representing the feasible space of the proposed Markov chain is as follows:

$$p(i, a, d \mid n)$$
$$= \begin{cases} p(i, 0, d+1 | \, n-1), & a = 0 \wedge (0 \leqslant d < D-1) \\ \sum_{b=0}^{i} q(i, b) \times p(i, b, 0 \mid n-1), & a = 0 \wedge d = D-1 \quad a \leqslant i \leqslant n \leqslant N \\ (1 - q(i-1, a-1)) \times p(i-1, a-1, 0 \mid n-1), & a > 0 \wedge d = 0 \end{cases}$$

Where, $q(i', b)$ is the probability of failure and calculated using Equation (1) as:

$$q(i', b) = 1 - \exp\left[- \int_{b}^{b+1} h(i', t) dt \right]$$
$$= 1 - e^{\left\{ -e^{\sum \gamma_j z_j(i')} \left[\left(\frac{b+1}{\eta} \right)^{\beta} - \left(\frac{b}{\eta} \right)^{\beta} \right] \right\}},$$

The first term of recursive function represents the *repair scenario*. The working age, a, is reset to zero and remains at zero until the repair is completed and the machine is operational again. During the repair period which lasts D time units, d is reduced by 1 time unit as the machine progresses over subsequent stages.

The second term of the recursive function represents the *failure scenario*. This is where the machine starts a new repair interval. As such, at stage n, working age, a, is set to zero and remaining repair time is set to $D-1$. This means the machine is operational just before stage $n-1$ and can be in any state with $b > 0$, with probability $p(i, b, 0|n-1)$ and fails in $[n-1, n)$, with probability $q(i, b)$.

The last term of the recursive function represents the *survival scenario*. The fact that the machine is operational at n, with $a > 0$, and $d = 0$ means that in the previous stage, $n-1$, the machine was operational or it had just finished a repair interval. Therefore, the state probability at $n-1$ is $p(i-1, a-1, 0 \,|n-1)$ and the machine did not fail over $[n-1, n)$, with probability $1 - q(i-1, a-1)$.

We continue to use the recursive equations over the entire planning horizon until we calculate the probabilities of all possible states at stage N. We have obtained some properties (presented in the Appendix) of the recursive formula that can reduce the calculations required over the entire horizon.

At each instant, the output level, y, given i operational time units, can be modelled by a method such as regression. y is expressed as a function of operator characteristics, depicting the sensitivity of the machine to HR factors. Consider the case where there are k operator characteristics, w: $(w_1, w_2, \ldots, w_k)$. Each w_k is a function of i, since the level of expertise depends on how many operational periods the operator has experienced. $y(i)$ represents the production output for a single time unit and we forecast this output as a function of operator characteristics $w_k(i)$: $y(i) = a_0 + a_1 w_1(i) + \cdots + a_k w_k(i)$. $y(i)$ is affected by the characteristics of the operator currently working on the machine. A low i can be the consequence of frequent failures caused by the operator in the past, resulting in many non-operational intervals. Thus through the inclusion of skill components (which are functions of i) in the PHM, an increase in i results in a lower failure rate. Second, an operator may have struggled and not learned well (a flat or low-sloped learning curve). This results in a currently low skill score. Both of these scenarios result in a low $w_k(i)$, in turn resulting in a low $y(i)$.

At the final stage, $n = N$, we are interested in the expected output value. We introduce $L(i)$ to represent the total production output over i operational time units by $L(i) = \Sigma_{j=0}^{i-1} y(j)$, $L(0) = 0$. If there is no learning involved, $y(i)$ can be represented by a constant value, y_c. In this case, the total expected output over the horizon, $E[L]$, would be the product of y_c and the expected number of total operational time units over the planning horizon, $E[I]$, that is $E[L] = y_c \times E[I]$. But since operator learning is considered, we expect to have $y(i) \leqslant y(i+1)$, for every i. To calculate $y(i)$, we use the learning curves to forecast the expertise levels for the appropriate number of operational time units, i, that has helped the operator gain knowledge.

To calculate the expected output at N, we need to find $P(i \,|\, N)$, the probability of having i operational periods at N. The probabilities of the possible states for each i are summed, $P(i \mid N) = \Sigma_{a,d} p(i, a, d \mid N)$. We can calculate the expected number of operational time units over planning horizon: $E[I] = \Sigma_{i=1}^{N} i \times P(i \mid N)$. This probability distribution, $P(i|N)$, along with the output level, is used to calculate the expected total output value for a particular operator on a certain machine, over the planning horizon: $E[L] = \Sigma_{i=1}^{N} L(i) \times P(i \mid N)$.

In addition to calculating $E[L]$, a decision maker may be interested to calculate some other characteristics such as the variance of each operator's production over the horizon: $\text{Var}[L] = E[L^2] - (E[L])^2$. All else being equal, an operator with a lower production variance is preferred due to the resulting stability. Lower variance results in decreased uncertainty and this can positively serve the decision maker in several ways, such as raw material purchasing, inventory management, and shipping schedules. For further planning purposes, a decision maker may also be interested in calculating the prediction interval, with a certain probability. In this case, this is achieved as follows:

$$P(r \leqslant L \leqslant s \mid N) = \sum_{i:r \leqslant L \leqslant s} p(i \mid N) = \varepsilon = 0.95.$$

As demonstrated, we focus on a stochastic environment where the machine may continue to operate or fail at any time. The PHM is the method to calculate the probability of failure at each interval within the planning horizon. As we go through the various intervals, with the passage of time, the operators gain experience and may improve their skill set. We use learning curves to capture this dynamic nature of skill. The PHM includes HR factors as covariates and reflects their effects on the risk of failure. After considering all the intervals up to the end of the planning horizon, the decision maker can forecast the expected number of operational periods and the associated production output.

4. Empirical study

We consider an industrial setting, an automotive manufacturing company in Ontario, Canada, to illustrate our proposed approach. We use historical production data and operator skill assessments to develop a PHM, regression equations, and learning curves, and use them to develop a Markov chain model to predict the production output of each operator. There can be many applications for our model, such as production output maximization, maintenance cost minimization. We consider one such application, OA optimization, with the aim of production output maximization.

Within the company considered, a new transmission gear manufacturing department has recently started its operations. For confidentiality reasons, this department is referred to as Alpha. This human-machine system consists of three machines, all of the Kappa type, hereafter referred to as AA, BB, and CC. They differ in external cutting tools but have the same operational procedures, thus getting affected by differing MR factors, such as working age, but the same HR factors, such as operator experience level. On each shift, one operator is assigned to each machine. The three machines operate on three shifts per day; therefore, there are a total of nine operators to consider. In order for each operator to get assigned a skill score, everyone is assessed quarterly by two system experts. These skill scores are used to build the learning curves for the operators. We consider skill to be the ability of the operator to operate and troubleshoot the machine and similar to the skill categorization by Blau and Kahn (1996), and Huang *et al* (2009), categorize skill into sub-components: analytical knowledge, social interaction, and experience level, represented by z_1, z_2, and z_3, respectively.

System experts are consulted to identify the most important factors affecting machine failures. In addition to the three skill components, other factors are shift work and days of the week. Two indicator variables, (X_1, X_2), are used to represent the shifts. Day shift is the baseline, $(0, 0)$; afternoon shift and night shifts are represented by $(1, 0)$, and $(0, 1)$, respectively. The next two indicator variables, (V_1, V_2), represent the three segments of the week: 'Monday', 'Friday', and 'the rest', represented by $(1, 0)$, $(0, 1)$, and $(0, 0)$ respectively. Lastly, the three machines are treated as control variables and are represented by two indicator variables (Y_1, Y_2).

The three machines have a combined total of 3049 records, which include operational periods, production output per shift, failure causes and durations, as well as attendance records. There are 119 failures and 11 suspensions, providing a total of 130 events. The assessments on the operators are performed quarterly for the 9-month period. These assessments provide skill scores for each operator, which are then used as input values into the PHM. The PH modelling is done using the software EXAKT, designed specifically for proportional hazards modelling in industrial settings (Jardine *et al*, 1997; Jardine and Banjevic, 2005). The operational and failure data are used in EXAKT, using maximum likelihood estimators, to determine the significance of the covariates in the development of a PHM. The detailed procedure of developing a PHM, including the likelihood function, is discussed in Vlok *et al*, 2002 and Lin *et al*, 2005.

In the face of our large dataset, including the consistently tracked HR variables, as well a sufficiently large number of failure events, the usage of covariate-based models and Markovian-based models are appropriate (Si *et al*, 2011).

The following PHM is developed using primarily the backward selection method, complemented with the Akaike's Information Criterion (AIC) to help avoid bias in the model fitting process (Burnham and Anderson, 2004):

$$h(t) = \frac{1.086}{0.063}\left(\frac{t}{0.063}\right)^{0.086} e^{(-0.128z_1 - 0.138z_2 + 2.485z_3 + 1.427z_4 - 1.561z_5 - 1.813z_6 + 6.402z_7 + 0.002z_8 + 1.774z_9 - 0.101z_{10})} \quad (2)$$

Table 1 adds further detail to Equation (2).

All covariates included in the PHM have coefficients that are found to be significantly different than zero. The standard errors are also displayed in Table 1. The exception is the estimate for the Shape parameter, β. It is not found to be significantly different than 1. However, since this case study is not the central contribution of the paper and only serves to demonstrate our theoretical discussions, we choose to use the obtained estimate to reflect the machine working age, despite its low effect. The obtained PHM adequately represents the data set; when we perform a K-S test on the model fit, the p-value is 0.2 and as a result, the model fit is not rejected. This model has the lowest

AIC compared to any other models obtained using the same set of main effects and two-way interactions.

To forecast the production output at each i, we may use regression equations. When the machine is operational, each equation forecasts the hourly output in terms of the components of skill (Experience, E; Social, S; Analytical, A). In addition to having skill components as main effects, their pair-wise interactions are also considered. The general form of the regression equations is defined in Equation (3):

$$y_\vartheta(i) = \beta_0^{(\vartheta)} + \sum_j \beta_j^{(\vartheta)} w_j(i) + \sum_{j,k} \beta_{j,k}^{(\vartheta)} w_j(i) w_k(i), \quad (3)$$

where: $y_\vartheta(i)$: Output per unit time for machine ϑ, given i operational time units; $\vartheta \in \{1, \ldots, v\}$, v: number of machines; $\beta_j^{(\vartheta)}$: coefficients of the main effects, as applicable to machine ϑ; $\beta_{j,k}^{(\vartheta)}$: coefficients of the interaction terms, as applicable to machine ϑ; j and k: Indices iterating through the various operator characteristics considered. $j,k \in \{1, 2, \ldots, l\}$, where l is the total number of operator characteristics considered; $w_j(i)$: Represents the value of operator characteristic j, after i operational periods.

Information from Alpha's data set, consisting of each shift's production count and its corresponding HR values, is used in the SPSS software to build the regression equations. The coefficient estimates are displayed in Table 2, with their standard error in brackets. An approach similar to the one used to develop the PHM is used where we combine the backward selection method with AIC. To validate the obtained equations, we looked at R^2 values for assessing model fit and checked for linearity of residuals, homoscedasticity, independence of errors, testing for influential cases, and no perfect multicollinearity. We also split the data (70/30) to cross-validate the models and obtain a high Pearson's correlation coefficient of 0.874.

On the basis of quarterly skill assessments, learning curves for each operator's skill component are developed by fitting curves to the historical skill scores, assigned through expert evaluations. The learning curves for each operator are not machine-specific. Table 3 displays an example of such learning curves for one operator, expressed as a function of m, cumulative number of hours the operator has worked on Kappa machines:

4.1. Markov chain model calculation

The planning horizon considered is the fourth quarter and the unit of time is hours. Therefore, $N = 480$ for each operator, based on 24-h days, 5-day weeks, and 12-week quarters. The model is developed based on an MTTR of 3 h: $D = 3$. Applying the recursive function developed in Section 3, and using the PHM, regression equations, and learning curves developed for Alpha, we calculate the expected production output of each operator on each machine (Table 4).

Table 1 Summary of estimated parameters

Covariate	*Parameter*	*Estimate*	*Standard error*	*p-value*
—	Scale, η	0.063	0.14	
—	Shape, β	1.086	0.08	0.27
$z_1 = \text{Social}$	γ_1	−0.128	0.04	<0.01
$z_2 = \text{Analytical}$	γ_2	−0.138	0.05	<0.01
$z_3 = X_1$	γ_3	2.485	0.47	<0.01
$z_4 = X_2$	γ_4	1.427	0.48	<0.01
$z_5 = Y_1$	γ_5	−1.561	0.34	<0.01
$z_6 = Y_2$	γ_6	−1.813	0.33	<0.01
$z_7 = V_1$	γ_7	6.402	1.56	<0.01
$z_8 = z_1 z_2$	γ_8	0.002	0.0007	0.014
$z_9 = z_4 z_7$	γ_9	1.770	0.53	<0.01
$z_{10} = z_1 z_7$	γ_{10}	−0.101	0.0257	<0.01

Hypothesis: Shape = 1 tested, Gamma (cov) = 0 tested, based on 5% significance level.

Table 3 Learning curves of an operator's skill components

Analytical skill	*Social interaction*	*Experience level*
$L = 14.238m^{0.238}$	$L = 54.031m^{0.009}$	$L = 50.007 + 0.298m$

Table 4 Expected production output for each operator on each machine

Operator #	*Expected production output*		
	AA machine	*BB machine*	*CC machine*
1	20 926	21 821	19 030
2	17 223	19 636	16 919
3	14 888	17 616	15 109
4	20 768	21 752	18 973
5	20 125	20 711	17 747
6	17 929	20 478	17 623
7	20 211	20 890	18 146
8	18 621	20 256	17 622
9	20 266	20 782	17 957

Table 2 Regression equation coefficients and standard errors, significant at $p < 0.01$

	β0	*βE*	*βS*	*βA*	*βE,S*	*βS,A*
AA	18.664 (0.517)	0	0.051 (0.01)	0.214 (0.006)	0	0
BB	29.935 (0.322)	0	0	0.086 (0.007)	0.001 (0.000)	0
CC	23.721 (0.376)	0.090 (0.005)	0	0	0	0.001 (0.000)

As an application of our model, we perform an OA optimization. We take on a mathematical programming approach and define the objective function as the total revenue over the period. Revenue is maximized over the planning horizon by optimally assigning operators to each machine. The binary decision variable $x_{\vartheta,s}=1$ when operator s is assigned to machine ϑ (making product ϑ) and is zero otherwise. This model is a simple OA problem with each operator being assigned to one machine, and each machine having the correct number of operators, 3, to run three shifts per day.

To date, Alpha has not used any decision criteria for its OA. Positions have traditionally been filled on a naïve First-Come-First-Assigned (FCFA) basis. On the basis of an FCFA scenario, we use the expected operator output of our Markov chain approach, expressed in Table 4, to calculate Alpha's quarterly revenue. As an example, under the FCFA scenario, operators 1, 2, and 3 are assigned to the AA machine. Therefore, in calculating the total revenue for the FCFA scenario, the values used from Table 4 for operators 1, 2, and 3 are 20 926, 17 223, and 14 888. The values calculated for these three operators for the BB and CC machines are ignored.

In addition to comparing our optimal Integer Programming (IP) approach to the FCFA model, we can compare it to a 'simple skill-ranking' model where the DM assigns the operators to the machines based on their current skill level, without accounting for learning curves. Under this simple skill-ranking assignment policy, if operator A's *current* skill score is higher than operator B, operator A gets assigned to machine 1 whose product has a higher sale price than machine 2. The learning curves of the two operators are not considered. Consequently, this may be a sub-optimal decision if operator B has a steep positively sloped learning curve compared to operator A's flat learning curve, resulting in more production by B over the entire length of the quarter.

We can use the results of our Markov chain approach in an IP model for an optimal OA that maximizes Alpha's revenue. Using the expected production output from our Markov chain model, and an IP model, we calculate the quarterly revenue. We solve the IP model using the branch-and-bound method, embedded in LINGO, an optimization software application. The result of our model is compared to the other scenarios (Table 5).

We re-run our Markov chain model, this time ignoring the effect of learning. Skill scores are treated as a flat value across the quarter. The OAs are different than when learning is considered. System revenue is calculated to be US$16 532 620; this is $366 630, or 2.2% less revenue over the quarter. This is a relatively large difference in revenue and points to better model results when we are able to include more information in the model. The additional information included in the model in this case is the learning effects. It should be noted that the standard errors in various coefficient estimates will filter through for some inherent error in the revenue figures. Knowing more about the errors is important in comparing the revenues obtained under various approaches. As a reviewer suggested, one can find out this error by running simulations that use parameters randomly chosen from the coefficient's sampling distribution. We have not performed these simulations since the case study is not central to this paper but rather serves to demonstrate our theoretical discussions. But this analysis can be undertaken in more detailed studies.

Table 5 Comparing Alpha's revenue under various OA approaches

Approach	*Quarterly revenue*	*Revenue difference with optimal OA*	
	($)	*($)*	*(%)*
FCFA	16 333 880	565 370	3.5
Simple skill-ranking	16 197 900	701 350	4.3
Optimal assignment	16 899 250	—	—
Worst case	16 090 980	808 270	5.0

We can use our model to determine the effect if the decision-maker is to provide additional training to an operator prior to allowing him to work independently on a machine. We perform this sensitivity analysis by repeating the expected uptime calculation for the lowest skilled operator, starting the planning horizon with an analytical skill score that is 10% higher. This results in additional quarterly revenue of up to $54158, or as low $8400, depending on which machine the operator is assigned to. This information can be used as a cost-benefit analysis tool on providing machine-specific training to the operators.

4.2. Model validation

For the purposes of validating our model, we performed a data-split. We developed our model based on 9 months of data, obtaining the PHM, the learning curve equations, and the regression equations. For the data split exercise, we considered the first 7 months to obtain new PHM and regression equations and used them to re-run the Markov model. We were then able to compare the model results with actual production results for the last 2 months of the data set, not used for model development.

Looking at the actual data records, we determined the number of hours worked by each operator and used the equivalent number of hours obtained from the Markov model. On the basis of these operator hours, the production output for both cases is presented in Table 6.

During the regular course of production, the operators were only assigned to one machine and did not work on the others. As an example, an operator working on the CC machine worked there for the entire 9-month period. Therefore, when we compare the Markov chain results with the actual ones, we are left with many blank cells in the Table. But the nine comparison results we have obtained are quite promising. For example, in comparing Operator 1's forecasted *versus* actual production output, the difference is only 3.2%. When we compare the two sets of model *versus* actual, we obtain a Pearson R^2 of 0.917, significant at the

Table 6 Comparing model results with actual production volumes

Operator	*AA*		*BB*		*CC*	
	Model	*Actual*	*Model*	*Actual*	*Model*	*Actual*
1	11 479	11 848				
2	9288	10 094				
3	8623	9056				
4			11 013	11 851		
5			11 025	12 440		
6			10 302	9899		
7					8603	8498
8					9036	8911
9					9037	8925

0.01 level (two-tailed). This serves to validate the appropriateness of the models' results.

5. Concluding remarks and future work

In this paper, we have developed a Markov chain approach to forecast the production output of a human-machine system, considering HR factors and operator learning. A planning horizon is considered and discretized; each time interval can have multiple states, survival, failure, and repair, for which a state space is defined and transition probabilities are calculated. With the passage of time, an operator gains experience on the machine, likely leading to an improvement of expertise. This in turn leads to the possible reduction of downtime due to human error. We use the learning curves for each operator to forecast the appropriate level of skill at a particular instant in time and use this as the covariate value in the PHM to calculate the hazard rate.

Using a recursive formula and a Markov chain approach, we proceed one interval at-a-time and at the end of the planning horizon, all the possible states, along with their probabilities, are used to calculate the expected uptime over the entire horizon. This quantity, along with the production output for each state, is used to calculate the expected production output over the entire planning horizon.

Our work can have many applications such as the expected output of each operator or the variance in his production output. To demonstrate our model and one of its many possible applications, we have discussed the case study of a manufacturing organization. The application discussed is that of OA optimization. Nine months of production data are analysed using the framework of our Markov chain model. Our model's results is compared to that of the assignment approach currently practiced in the company as well as an assignment solely based on the current level of skill, disregarding operator learning. In both cases, applying our model would result in higher total system revenue.

A possible future work stemming from the work in this paper is to consider manufacturing environments where there are different machine types. This can lead to an additional complexity of operators having different learning curves on different machines. Another future work is to extend the assumption of a fixed duration for all repairs and have random repair durations. There can be a failure distribution to draw from. In doing so, we introduce greater complexity in the model but make the model more realistic. Alternatively, we can stay with the fixed duration for a repair, but consider several repair scenarios. We can select the top five most common failure modes, have an MTTR for each, add one more dimension to the Markov chain state for the type of failure, and implement the same type of analysis described in this chapter. Considering several failure modes will yield a more realistic analysis scenario, resulting in a more accurate estimate of production output.

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Appendix

Model properties

Property 1: At each stage n, where $n=i+mD$, probability of any state with $i=a>0$ and $d=0$ can be calculated as follows:

$$p[i,i,0 \mid n=i+mD] = q_0^m \times p[i,i,0 \mid n=i],$$
$$m=1,2,\ \ldots$$

In stage $n=i+mD$, $a=i$ indicates the machine must be operational in the last i stages, and have been non-operational at all other previous ones. For this scenario, the machine fails immediately after starting at $n=0$, with an initial failure probability q_0, goes through the repair period, and fails immediately again. The failures are repeated m times in total before the machine becomes operational for i stages (Figure A1). Failure cannot occur during the last i intervals, otherwise, a cannot equal i. The probability that the machine is operational in the last i stages is $p[i,i,0 \mid n=i]$.

Property 2: At any stage n, the probability of any state with $a=0$ and $d=j$ can be derived as follows:

$$p[i,0,j \mid n] = p[i,0,D-1 \mid n-D+j+1],$$
$$j=1,2,\ \ldots\ ,D-1.$$

Since $j>0$, the machine is in a repair period at stage n. This indicates that in the previous stage, $n-1$, $d=j+1$, since one less time interval had elapsed on repairing the machine. This relationship between '$n-k$' and '$j+k$' will go on over all previous stages until $d=D-1$. In this case, $d=D-1=j+k$, leading to $k=D-1-j$.

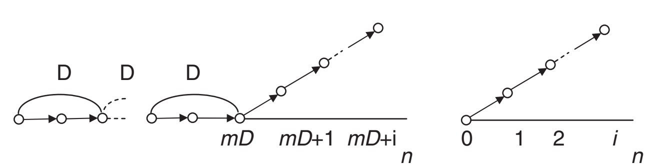

Figure A1 Relationship between $p[i,i,0 \mid n=i+mD]$and $p[i,i,0 \mid n=i]$. (a) stage $n=i+mD$, with m initial failures and repair periods (b) stage $n=i$, with i operational followed by i operational intervals intervals and no failures.

Once $d = D - 1$, we are no longer following from just one state from the previous stage. In the previous stage, the machine may be operational and fail just before n, or it may fail immediately after the completion of a repair period. Therefore, the recursive formula has to be used for $p[i, 0, D-1 \mid n - D + j + 1]$.

Property 3: At any stage n, the probability of any state with $0 < a < i$, $d = 0$ can be calculated as follows:

$$p[i, a, 0 \mid n] = \sum_{b=0}^{i-a} p[i-a, b, 0 \mid n-a-D] \times q(i-a, b) \times p[a, a, 0 \mid a], \qquad i \leqslant n - D.$$

The machine has been running for the last a stages; however, since $a < i$, there must be at least one previous failure. Given the total number of intervals n, the number of operational intervals must be $i - a$ at the point before the occurrence of the last failure (Figure A2).

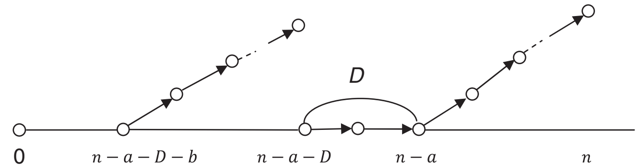

Figure A2 Depiction of a possible history of state (i, a, d) where $i > a, d = 0$.

The value of d must also be zero whether the failure occurred after an operational period or the completion of a repair. The value of a, however, is bounded between zero and $i - a$ since $a \leqslant i$, $\forall a$. The probability of failure in this scenario is affected by the expertise gained by the operator, having worked $i - a$ intervals.

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