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MARKOV CHAIN APPROACH TO FAILURE COST ESTIMATION IN BATCH MANUFACTURING

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Key Words

Cost modeling; Activity-based costing; Costs of poor quality; Markov chain.

Introduction

Failure costs are incurred when a product does not conform to its design specifications. A failure cost is internal if nonconformities are detected before the product is out to the marketplace; otherwise, it is external. Correcting or compensating for poor quality after a product has been delivered to the customer can be very expensive. According to Gupta and Campbell (1), companies may have to spend between 50% and 90% of their quality budget in order to correct defects in a finished product.

Many failure costs (2,3) are driven by variables which vary with the defective level of a product. Some of these cost drivers measure the frequency of defect correction activities (e.g., the number of product returns for factory repair or rework). Reshipping, rehandling, and updating of a parts database are examples of other activities which drive failure costs. Unlike certain unit-level cost drivers such as the direct labor of product inspection and field testing, activity-based cost drivers are not proportional to the number of product units produced. Therefore, these types of failure costs cannot be accurately estimated by the conventional unit-based cost systems (4).

In this article, we consider the activities that normally involve both manufacturing companies and their customers in the defect-correction process. These activities pertain to the repair or rework of defective products, customer inspection and field tests, shipping and handling for returned products, reinspection, work-in-process delays, and database updating following each defect-correction activity. Actually, when a defect is first detected, its cause is uncertain, so that the subsequent diagnostic and fault-correction (or quality assurance) process is stochastic. A defective unit of product may then go through several activities, each of which occurs with a certain probability, before it is finally classified in one of the following absorption states: defect free, scrapped, or downgraded (or regraded for alternative operation). However, the number of state transitions to absorption is not known beforehand. This type of discrete-time stochastic process can be appropriately modeled as a Markov chain (5). The current article is a significant extension of the original cost model in Ref. 6 to include a more insightful interpretation of the Markov chain results in light of the activity-based costing concept. We maintain the same system of notations used in the previous article.

The objective of this article is to show how a Markov chain model may be used to estimate the aforementioned activity-based failure costs. Following this introduction, background information of related research is presented. Next, we derive the Markov chain model and use it to formulate the various failure costs. A numerical example is then given to illustrate the application of the methodology.

Background

Estimates of certain cost changes that are a function of manufacturing activities are usually not available in conventional cost accounting systems. One has to employ appropriate quantitative analysis for cost estimation. For example, queuing theory can be used to model internal failure costs that are driven by manufacturing delays of defective products (7). Other failure costs such as lost sales and loss of customer goodwill may be estimated by Taguchi's quality loss functions (8).

Nandakumar et al. (7) argue that defective products require longer throughput times and that poor quality increases a company's inability to meet its delivery schedules. They call such costs of poor quality "demand side costs," which are defined as the sum of production delay and tardiness costs. Thus, rework, reinspection, and repair operations on a defective product would result in a longer throughput time as well as a larger inventory holding cost. They assume availability of the delay and tardy cost estimates, in dollars per order per unit time, and a lead-time probability distribution for each product. Apparently, the demand side costs are internal failure costs because they involve internal manufacturing operations. The authors do not categorize external failure costs according to the associated activities; they simply assume a single external failure cost per product unit. They calculate the aggregate external failure cost by multiplying the unit failure cost by the probability that a defective product is sent to the customer. A drawback of this unit-cost approach is that the estimate is not accurate because the various activity-dependent external failure cost drivers are not considered.

Problem Description

We will present a Markov chain model and formulate failure costs for a single-product system in which the finished assembly requires final inspection and functional testing before customer delivery. The factory may use either a sampling or a 100% inspection plan. When a unit of this product fails to pass the final test, a number of repairing or reworking operations to improve the quality is necessary. After the test, units of product that meet the factory quality standards are then shipped to the customer. If the repair/rework operation

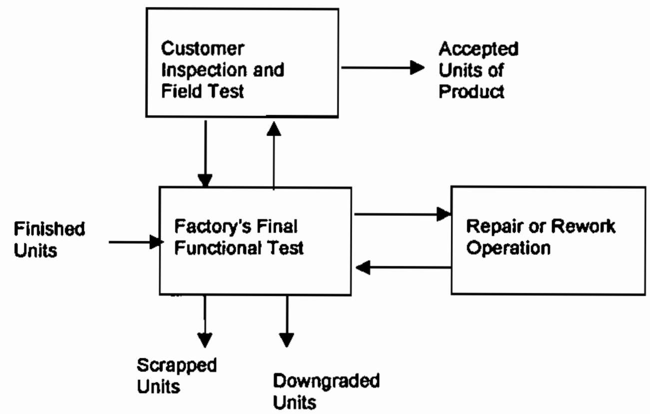

Figure 1. The Factory–Customer quality assurance system.

is not successful, it will be decided eventually whether the units which fail the functional testing should be scrapped or regraded for other applications (downgraded). The units of product received by the customer are inspected and put through a field-testing operation. The customer will return all units that do not pass its quality requirements. Figure 1 summarizes this problem.

The Markov Chain Model

Assume that a certain type of finished product has a fixed defective rate at the time of a planned delivery. It is generally known that even when a company adopts the 100% final inspection plan, it does not guarantee that all nonconforming units would be detected (9,10). Thus, some of the hidden defects may be discovered later by the customer's field-test operation. It is also conceivable that the same defective unit may be returned to the company a number of times. The two-way flow between the two sites in Figure 1 indicates this possibility. Similarly, the two-way flow between the final functional testing and repair/rework blocks denotes that repeated repair/rework operations may be required for some defective product units.

At any given time, a unit of finished product that enters the quality assurance system may be in one of the following six states:

State 0: undergoing a factory's functional test (a transient state)

State 1: undergoing repair/rework operation (a transient state)

State 2: undergoing customer's field testing (a transient state)

State 3: classified as scrap (an absorption state)

State 4: downgraded (an absorption state)
State 5: defect-free condition (an absorption state)

The defect-correction activities of the quality assurance system cause the product unit to move from one state to another. The state transition stops when the process enters any of the three absorption (trapping) states 3, 4, or 5. Associated with a state change is the state transition probability. The one-step transition probabilities for the process in Figure 2 are defined as follows:

p_{01} = repair/rework probability
p_{02} = factory-standard-satisfaction probability (this is the factory quality level)
p_{03} = scrapping probability
p_{04} = downgrading probability
p_{20} = customer returning probability
p_{22} = field-testing probability
p_{25} = customer acceptance (defect-free) probability

Let $P = \{p_{ij}\}$ be the matrix of transition probabilities, where p_{ij} is the probability that a product unit is in state j at time $t + 1$, given that it is in state i at time t. The usual assumption is that the transition probabilities are stationary. The transition probability matrix that characterizes the state transition in Figure 1 is given by

$$P = \begin{bmatrix} 0 & p_{01} & p_{02} & p_{03} & p_{04} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ p_{20} & 0 & p_{22} & 0 & 0 & p_{25} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

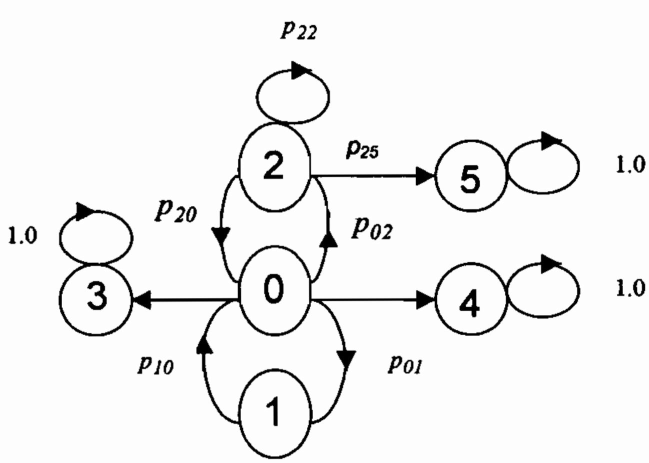

Figure 2. State transition diagram.

The Markov chain approach (5) is useful for determining the probability a product unit enters any absorption state from a given transient state. We can also compute an average number of visits to a given transient state from another transient state. Using these quantities, we can calculate the frequency of occurrence for each defect-correction activity, or activity rate, that is a failure cost driver. The activity-based cost drivers to be formulated by the Markov chain approach are described as follows:

u_{ik} = probability that a product unit is trapped in a state k (3, 4, or 5), given that it initially starts in a transient state i (0, 1, or 2).
v_i = expected number of transitions from a transient state i to a trapping state.
w_{ij} = expected number of visits to a transient state j prior to absorption, starting from a transient state i. This quantity also is the mean duration a unit spent in state j prior to absorption given that it starts in a transient state i.

Interpreting the above quantities in the framework of the six system states, we can define a number of the cost drivers, which are relevant to the problem, as follows:

u_{03} = fraction of the total number of product units of a given batch that are finally scrapped.
u_{04} = fraction of the total number of product units that are finally downgraded.
u_{05} = fraction of the total number of the product units that are defect-free; thus, $(1 - u_{05})$ represents the product defective rate.
u_{25} = fraction of the total number of units that are defect-free.
v_0 = average number of record changes per product unit after it leaves the factory final testing station; this quantity measures the frequency of record updating by the factory.
w_{22} = average number of times a product unit revisits the customer's field-testing operation.
w_{20} = average number of factory inspections per product unit initially received by the customer. Therefore, the average number of customer's returns (rejects) per product unit is $(1 - p_{22})w_{22}$. It can be shown that the expected number of field-testing operations performed on a product unit is given by $w_{22} - p_{02}w_{20}$.
w_{11} = average number of repair/rework operations per product unit.
w_{21} = average number of reworking operations per returned product unit.

By means of the law of total probability and the first-step analysis (5), $u_{ij} = p_{ij} + \Sigma_{k \neq j} p_{ik} u_{kj}$ and $v_i = 1 + \Sigma_{j \neq i} p_{ij} v_j$, we can obtain a system of equations from which we can solve for the activity-based cost drivers. The solution of the following systems of linear equations is given in the Appendix:

$$v_0 = 1 + p_{01}v_1 + p_{02}v_2, \qquad v_1 = 1 + v_0, \qquad v_2 = 1 + p_{20}v_0 + p_{22}v_2; \tag{2}$$

$$\begin{aligned} u_{03} &= p_{03} + p_{01}p_{10}u_{03} + p_{02}u_{23} = p_{03} + p_{01}u_{03} + p_{02}u_{23}, \\ u_{23} &= p_{20}u_{03} + p_{22}u_{23}, \\ u_{04} &= p_{04} + p_{01}p_{10}u_{04} + p_{02}u_{24} = p_{04} + p_{01}u_{04} + p_{02}u_{24}, \\ u_{24} &= p_{20}u_{04} + p_{22}u_{24}, \\ u_{05} &= p_{01}p_{10}u_{05} + p_{02}u_{25} = p_{01}u_{05} + p_{02}u_{25}, \\ u_{25} &= p_{25} + p_{22}u_{25} + p_{20}u_{05}; \end{aligned} \tag{3}$$

$$\begin{aligned} w_{00} &= 1 + p_{01}w_{10} + p_{02}w_{20}, \\ w_{01} &= p_{01}w_{11} + p_{02}w_{21}, \\ w_{02} &= p_{01}w_{12} + p_{02}w_{22}, \\ w_{22} &= 1 + p_{20}w_{02} + p_{22}w_{22}, \\ w_{10} &= w_{00}, \qquad w_{11} = 1 + w_{01}, \qquad w_{12} = w_{02}, \\ w_{21} &= p_{20}w_{01} + p_{22}w_{21}, \\ w_{20} &= p_{20}w_{00} + p_{22}w_{20}. \end{aligned} \tag{4}$$

Estimation of the Failure Costs

The cost drivers given by the Markov chain are useful for the formulation of failure costs that are a function of the defect-correction activity rate. For instance, suppose that a direct cost of D_1 dollars per operation is incurred at the repair/rework station whenever a defective unit is detected. How would we determine the expected total repair cost per batch?

Let r_{ij} be the probability that a unit of the product visits state j for the first time given that it is initially in state i. Note that when state j is a trapping state, $r_{ij} = u_{ij}$. Further, let X be the expected number of units that are reworked for the first time for a batch of size M. Thus, $X = r_{01}M$. We can solve for r_{01} by the first-step analysis, which yields

$$r_{01} = \frac{(1 - p_{22})p_{01}}{1 - p_{22} - p_{02}p_{20}}.$$

Because, on average, w_{11} reworking operations are required per unit of the product in the batch, the average cost is $w_{11}D_1$ dollars per product unit and the total reworking cost is $w_{11}r_{01}D_1M$ per batch.

In addition to D_1, other direct cost quantities (available from cost accounting systems) are as follows:

D_2 = customer's field testing cost, \$ per operation, per product unit
R = compensation, penalty, and warranty cost, \$ per unit rejected
S = shipping and handling cost, \$ per return of a rejected unit
U = database updating cost, \$ per update, per product unit
V = product scrapping cost, \$ per unit scrapped
W = product downgrading cost, \$ per downgraded unit

The first-step analysis is surprisingly simple and yet very powerful for determining the probability u_{ik}, r_{ij}, and the rate w_{ij} for processes that do not possess steady-state probabilities. Note that w_{ij} quantities are averages over all units in the batch. Suppose that G is the number of defect-free units required by the customer for an upcoming order. The batch size M is the expected number of units to be produced and to enter the final testing station, N is the expected number of scrapped units, and P is the expected number of units downgraded. Thus, $M - G$ equals the expected number of defective units and $M = P + N + G$. By observing that $u_{05} + u_{03} + u_{04} = 1$, the quantities P and N can be expressed in terms of G as follows:

$$\begin{aligned} G &= u_{05}M, \\ N &= u_{03}M = \frac{u_{03}}{u_{05}}G, \\ P &= u_{04}M = \frac{u_{04}}{u_{05}}G. \end{aligned} \tag{5}$$

Let Y denote the expected number of units that are returned by the customer per batch. This quantity is given by $Y = r_{02}r_{20}M$. Again, by applying the first-step analysis, $r_{02} = p_{02} + p_{01}r_{12}$, $r_{12} = r_{02}$, and $r_{20} = p_{20} + p_{22}r_{20}$. It follows that $r_{02} = p_{02}/(1 - p_{01})$ and $r_{20} = p_{20}/(1 - p_{22})$.

The Markov chain results (see the Appendix) can now be used to formulate the following failure costs, in \$ per batch:

Repair/rework activity:

$$F_1 = w_{11}r_{01}D_1M = \frac{(1 - p_{22})p_{01}D_1M}{L} \tag{6}$$

Scrapping operation:

$$F_2 = NV = u_{03}VM = \frac{p_{03}(1 - p_{22})VM}{L} \tag{7}$$

Downgrading operation:

$$F_3 = PW = u_{04}WM = \frac{p_{04}(1 - p_{22})WM}{L} \tag{8}$$

Field test:

$$F_4 = (w_{22} - p_{02}w_{20})r_{02}D_2M = \frac{(1 - p_{01} - p_{02}p_{20})p_{02}D_2M}{(1 - p_{01})L} \tag{9}$$

Compensation, penalty, and warranty:

$$F_5 = (1 - p_{22})w_{22}YR = \frac{p_{02}p_{20}}{L}MR \tag{10}$$

Assume that the factory also has to pay for the shipping and handling of returned units.

Shipping and handling:

$$F_6 = (1 - p_{22})w_{22}YS = \frac{p_{02}p_{20}}{L}SM \tag{11}$$

Database updating:

$$F_7 = v_0UM = \frac{(1 - p_{22})(1 + p_{01}) + p_{02}}{L}UM \tag{12}$$

The total of these seven items does not include other internal failure costs that are a function of manufacturing delays caused by poor product quality. In this study, we assume that the total inventory holding cost accounts for the holding delays of returned product units at the factory inspection station and the reworking station, and the inventory holdings of scrapped and downgraded units. To formulate this total inventory cost, we need additional quantities:

n = number of production batches per planning period
H = the unit holding cost per period
t = expected time a defective unit spent in the quality assurance system

The inventory holding cost per batch is given by

$$F_8 = Ht(M - G) + (1 - p_{22})w_{22}YHt + w_{01}r_{01}MHt = Ht\left(M - G + \frac{p_{02}p_{20}M}{L} + \frac{p_{01}^2(1 - p_{22})^2M}{(1 - p_{22} - p_{02}p_{20})L}\right). \tag{13}$$

The total failure cost, \$ per period, is given by

$$F = n\sum_{i=1}^{8} F_i. \tag{14}$$

A Numerical Example

To illustrate the application of the failure cost model, the cost data shown in Table 1 will be used. For a specified factory quality level, the repair/rework probability and the downgrading probability are assumed equal to five and four times the scrapping probability, respectively. It is also assumed that the average time spent in the quality system per defective unit is equal to 2 weeks (0.039 years) and the factory operates 52 weeks per year. Using Table 1 data, the cost drivers are computed by means of the formulas in the Appendix and the results are input to the failure cost equations (6)–(14). Tables 2 and 3 show the results of the computation for the factory quality level of 0.75. It may be noted that the defective rate, $1 - u_{05}$, is a function of both the factory quality level and the customer quality requirements.

Table 1. Quality Cost Data

Annual demand, G	5000 units
Unit production cost, C	\$1000.00
Annual interest rate, i	10%
Unit holding cost/year, $H = iC$	\$100
Total batches per year, n	20
Repair/rework cost per operation, D_1	\$25 per unit
Field test/inspection cost per operation, D_2	\$20 per unit
Scrap cost, V	\$15 per unit
Downgrading cost, W	\$50 per unit
Penalty and warranty claim cost, R	\$150 per unit rejected
Shipping and handling cost, S	\$10 per unit per handling
Database updating cost, U	\$0.5 per unit per update
Repair/rework probability, p_{01}	0.125
Scrapping probability, p_{03}	0.025
Downgrading probability, p_{04}	0.10
Factory quality acceptance level (probability of passing factory final test), p_{02}	0.75
Customer rejecting probability, p_{20}	0.100
Customer field test/inspection probability, p_{22}	0.100
Customer-satisfaction probability, p_{25}	0.800
Expected time a defective unit spends in the quality system, t (years)	0.039 (= 2 weeks)

Table 2. Computed Quantities in Eqs. (2)–(4); Factory Quality Level = 75%

$u_{03} = 0.032$	$1 - u_{05} = 0.158$	$v_1 = 3.474$	$w_{20} = 0.140$	$Y = 565.48$
$u_{04} = 0.126$	$1/u_{05} - 1 = 0.19$	$v_2 = 1.386$	$w_{22} = 1.228$	$N = 187.5$
$u_{05} = 0.842$	$1/u_{25} - 1 = 0.007$	$w_{01} = 0.158$	$w_{11} = 1.158$	$P = 750$
$u_{25} = 0.982$	$v_0 = 2.474$	$w_{02} = 1.053$	$X = 809.66$	$M = 5937.5$

Table 3. Failure Costs ($ per Batch) and Percentages of Total; Factory Quality Level = 75%

REPAIR/ REWORK (1)	SCRAP (2)	DOWN-GRADE (3)	CUSTOMER FIELD TEST (4)	PENALTY CLAIM (5)	SHIPPING/ HANDLING (6)	DATABASE UPDATING (7)	INVENTORY HOLDING (8)
23,437.5	2,812.5	37,500	114,285.7	93,750	6,250	3,164.1	6,507.8
8.15%	0.98%	13.03%	39.72%	32.59%	2.17%	1.10%	2.26%

Table 4. Percentages of the Total Failure Cost Versus Factory Quality Level

	FACTORY QUALITY LEVEL			
	0.80	0.85	0.90	0.95
Defective rate $1 - u_{05}$	0.123	0.09	0.06	0.03
Repair/rework cost	17,578.1 (6.51%)	12,408.1 (4.87%)	7,812.5 (3.24%)	3,700.7 (1.6%)
Scrapping cost	2,109.4 (0.78%)	1,489 (0.58%)	937.5 (0.39%)	444.1 (0.19%)
Downgrading cost	28,125 (10.4%)	19,852.9 (7.8%)	12,500 (5.2%)	5,921.1 (2.6%)
Field testing cost	113,888.9 (42.2%)	113,513.5 (44.6%)	113,157.9 (47%)	112,820.5 (49.3%)
Claim/warranty cost	93,750 (34.7%)	93,750 (36.8%)	93,750 (38.9%)	93,750 (41%)
Shipping cost	6,250 (2.3%)	6,250 (2.45%)	6,250 (2.6%)	6,250 (2.7%)
Database cost	3,093.8 (1.1%)	3,023.4 (1.2%)	2,953.1 (1.2%)	2,882.8 (1.3%)
Holding cost	5,410.4 (2%)	4,475.4 (1.8%)	3,676.2 (1.5%)	2,992.1 (1.3%)
Total, $/batch	270,205.5	254,762.3	241,037.2	228,761.2

In order to assess the impact of other factory quality levels, the above calculation is extended to factory quality levels in the range 0.75–0.95. It is assumed that the customer uses the same quality requirements (i.e., $p_{20} = p_{22} = 0.10$ and $p_{25} = 0.80$) regardless of the adopted quality level at the factory. The results are presented in Table 4.

Discussion

The approach of this study is applicable to any manufacturing systems that make consumer durable goods, which must meet customer's functional requirements (e.g., automotive airbags, sound systems, computers, etc.). Despite a high factory quality standard, the chance that such a product may be returned for factory repair/rework is still never equal to zero. In this manufacturing environment, the product defective rate must accurately reflect the interaction between the factory quality standard and the customer's acceptance policy. The above numerical example demonstrates that not only is the Markov chain approach able to capture such interaction, but it can also provide realistic estimates of the activity-based failure costs.

Past history of the frequencies of occurrence of the quality assurance activities should be used for the estimation of the transition probabilities (percentages of occurrence) at a specified factory quality level. However, subjective estimates of the probabilities might be preferable to the estimates based on past data when certain activities cannot be explained well by the historic estimates.

The expected time spent in the quality assurance system is assumed known in our present study. In practice, however, t can be determined by queuing analysis (e.g., Ref. 7), or simply by a work measurement approach of industrial engineering.

Appendix

Let $L = (1 - p_{22})(1 - p_{01}) - p_{02}p_{20}$. Solving Eqs. (2)–(4) yields the following activity-based cost drivers:

$$u_{03} = \frac{(1 - p_{22})p_{03}}{L}, \tag{A1}$$

$$u_{23} = \frac{p_{20}p_{03}}{L}, \tag{A2}$$

$$u_{04} = \frac{(1 - p_{22})p_{04}}{L}, \tag{A3}$$

$$u_{24} = \frac{p_{20}p_{04}}{L}, \tag{A4}$$

$$u_{05} = \frac{p_{02}p_{25}}{L}, \tag{A5}$$

$$u_{25} = \frac{p_{25}}{L}(1 - p_{01}), \tag{A6}$$

$$v_0 = \frac{(1 - p_{22})(1 + p_{01} + p_{02})}{L}, \qquad v_1 = 1 + v_0, \tag{A7}$$

$$v_2 = \frac{1 - p_{20}v_0}{1 - p_{22}},$$

$$w_{01} = \frac{p_{01}(1 - p_{22})}{L}, \tag{A8}$$

$$w_{00} = \frac{1 - p_{22}}{L} = w_{10}, \tag{A9}$$

$$w_{20} = \frac{p_{20}}{L}, \tag{A10}$$

$$w_{21} = \frac{p_{01}p_{20}}{L}, \tag{A11}$$

$$w_{11} = 1 + w_{01} = 1 + \frac{(1 - p_{22})p_{01}}{L}, \tag{A12}$$

$$w_{02} = \frac{p_{02}}{L}, \qquad w_{22} = \frac{1 - p_{01}}{L}. \tag{A13}$$

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