Numerical Methods

Solution of Non-Linear Equations

➥ Fixed Point Iteration Method

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May 2021

Institute of Engineering,
Tribhuvan University, Nepal

Fixed Point Iteration Method

Mechanism:

$$f(x) = 0$$

$$\Downarrow$$

$$x = g(x)$$

$$\Downarrow$$

Iteration formula

$$\Downarrow$$

$$x_{i+1} = g(x_i)$$

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Example:

$$\cos(x) + 3x - 2 = 0$$

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$$x = \frac{2 - \cos(x)}{3}$$

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Iteration formula

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$\because |x_6 - x_5| < 0.000005$

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$\therefore$ root $= 0.35400$

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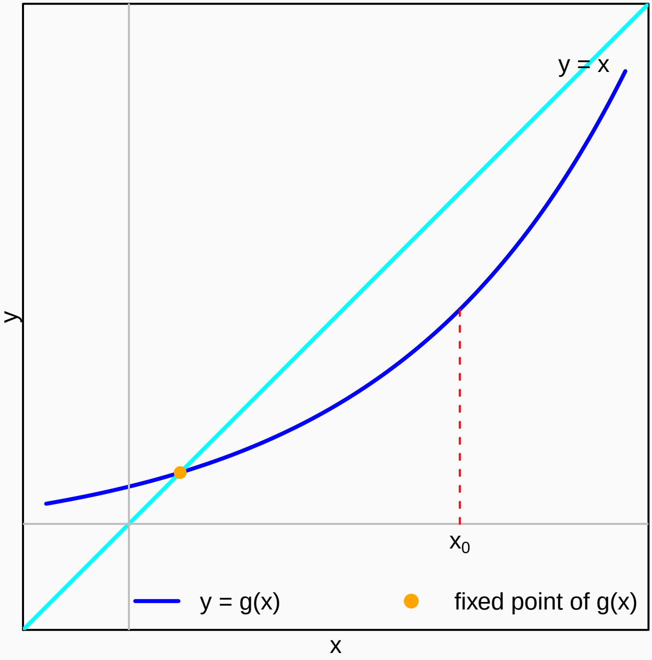
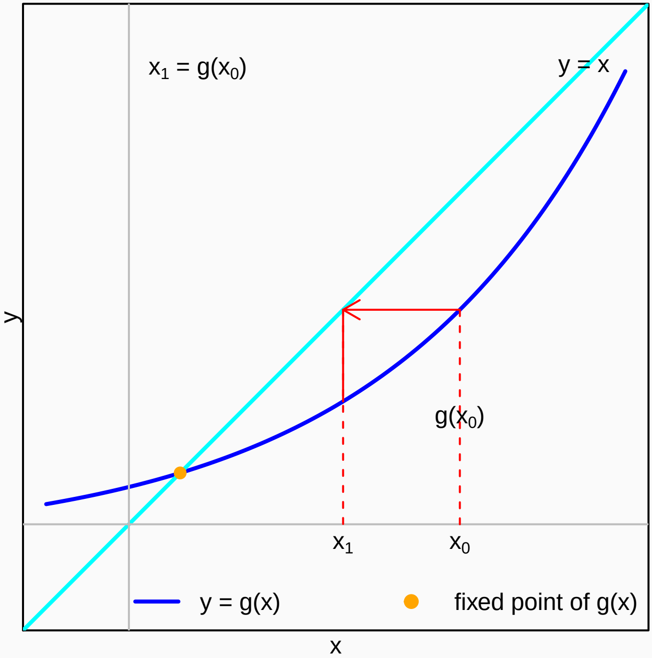
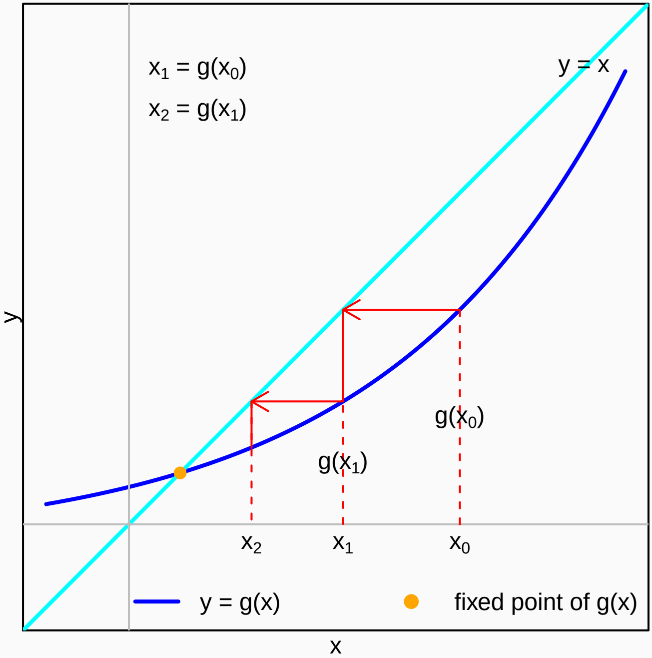

y = x
y
x_0
y = g(x)
fixed point of g(x)
x



$x_1 = g(x_0)$
y = x
y
$g(x_0)$
x_1
x_0
y = g(x)
fixed point of g(x)
x


$x_1 = g(x_0)$
$x_2 = g(x_1)$
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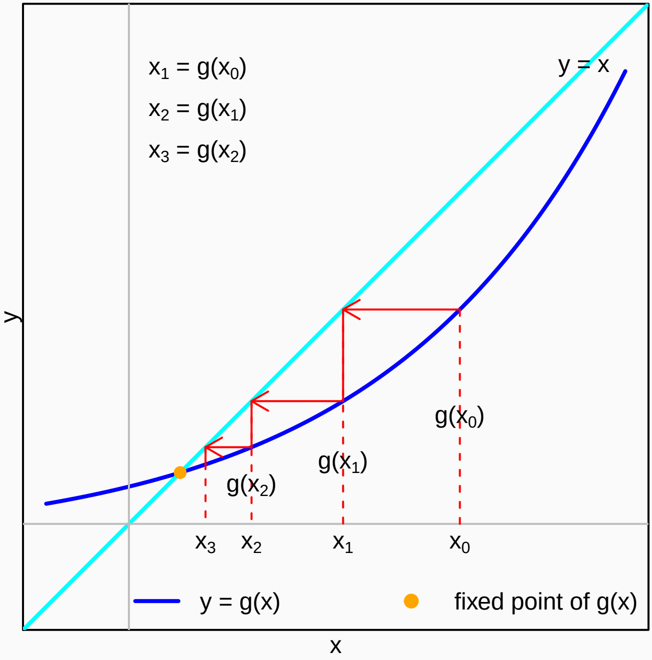
y
$g(x_0)$
$g(x_1)$
x_2
x_1
x_0
y = g(x)
fixed point of g(x)
x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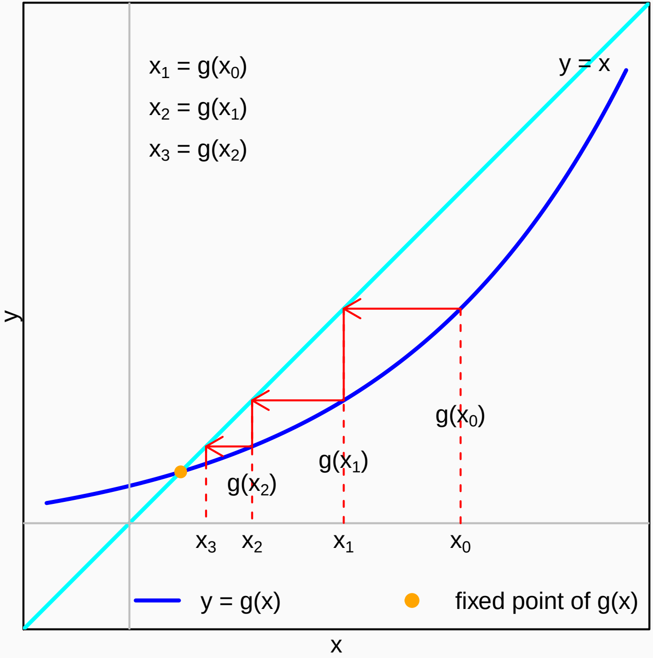
$x_2 = g(x_1)$
$x_3 = g(x_2)$
y = x
y
$g(x_0)$
$g(x_1)$
$g(x_2)$
x_3
x_2
x_1
x_0
y = g(x)
fixed point of g(x)
x



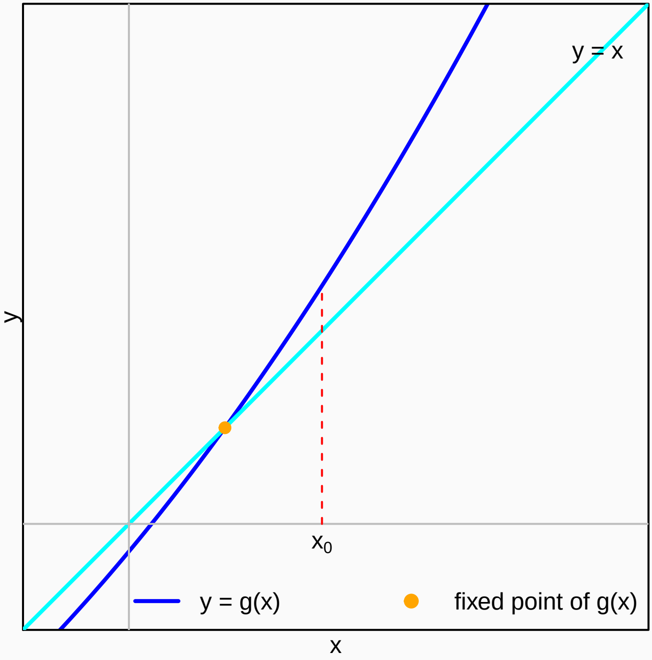
Case I

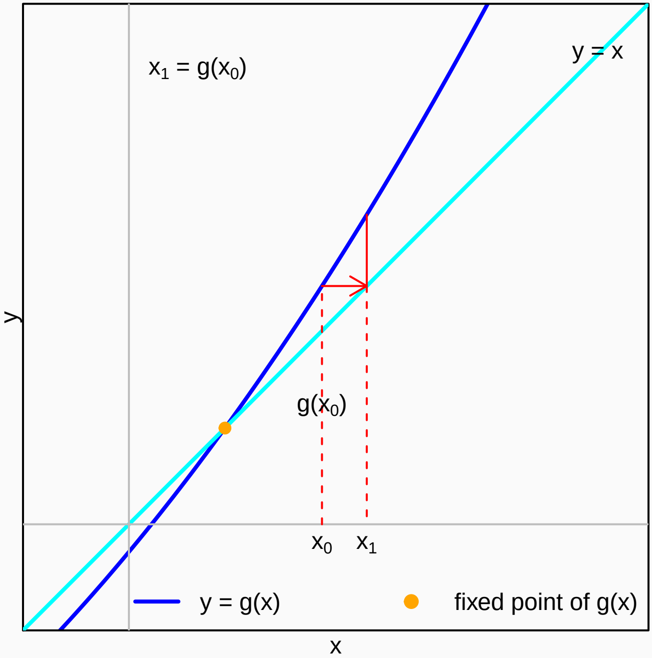
Near the root:

$0 < g'(x) < 1$

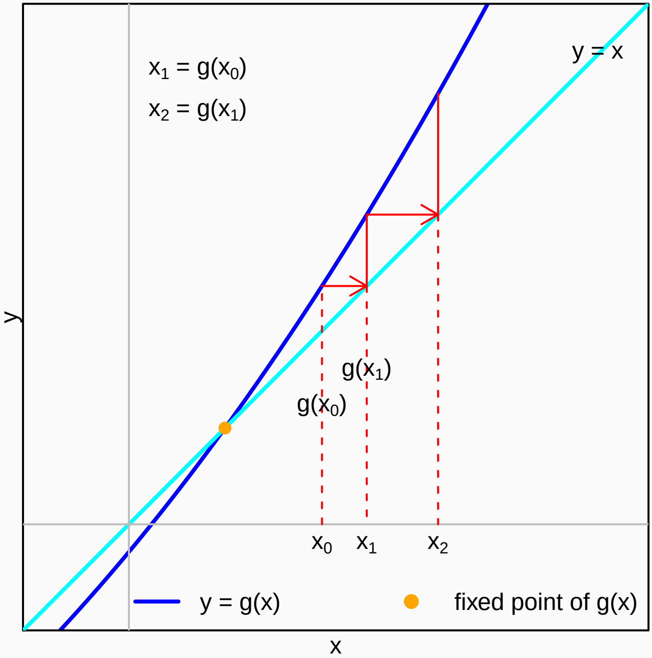
Resulting in:

Monotonic Convergence

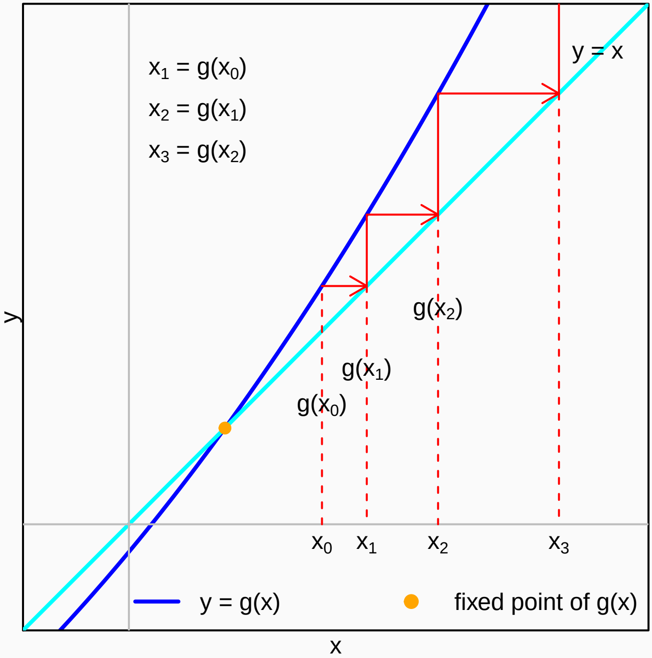

y = x
y
x_0
y = g(x)
fixed point of g(x)
x



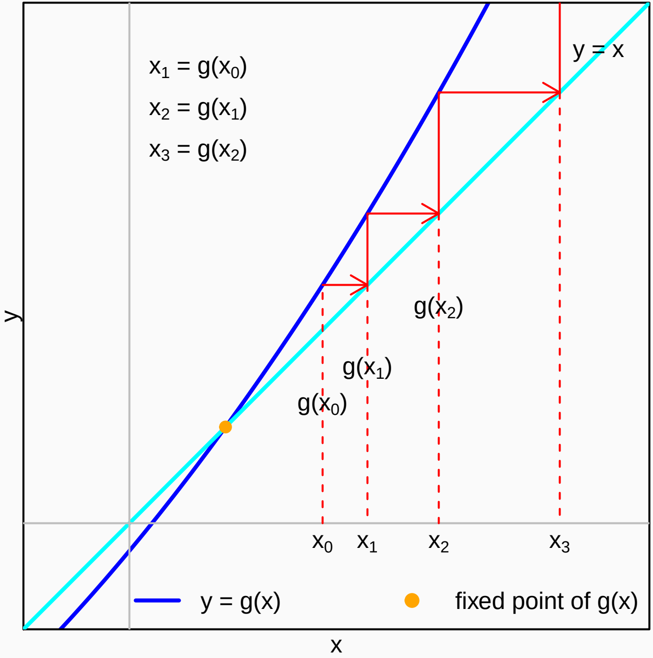
x₁ = g(x₀)
y = x
g(x₀)
x₀
x₁
y
x
y = g(x)
fixed point of g(x)



x1 = g(x0)
x2 = g(x1)
y = x
y
g(x1)
g(x0)
x0
x1
x2
y = g(x)
fixed point of g(x)
x



$x_1 = g(x_0)$
$x_2 = g(x_1)$
$x_3 = g(x_2)$
y = x
y
$g(x_2)$
$g(x_1)$
$g(x_0)$
x_0
x_1
x_2
x_3
y = g(x)
fixed point of g(x)
x



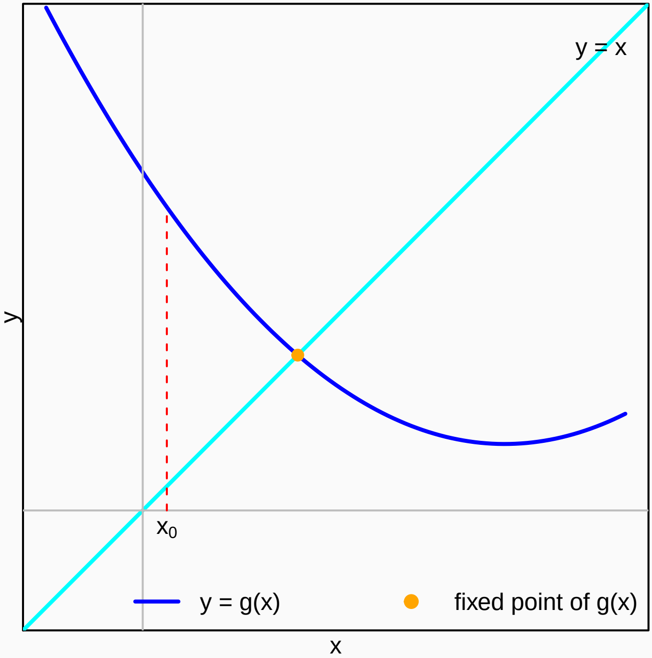
Case II

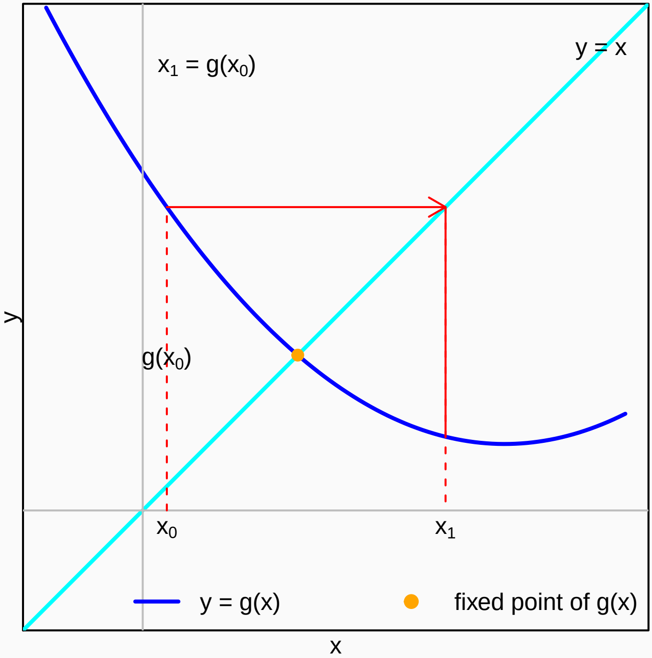
Near the root:

$g'(x) > 1$

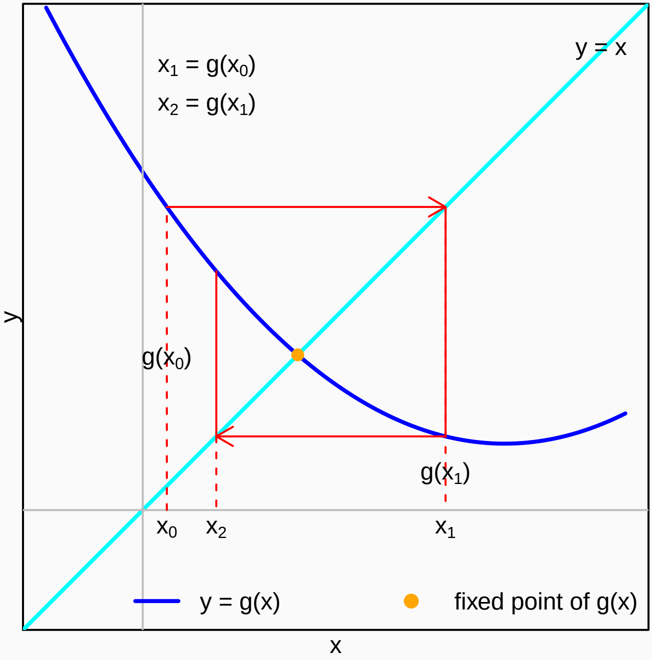
Resulting in:

Monotonic Divergence

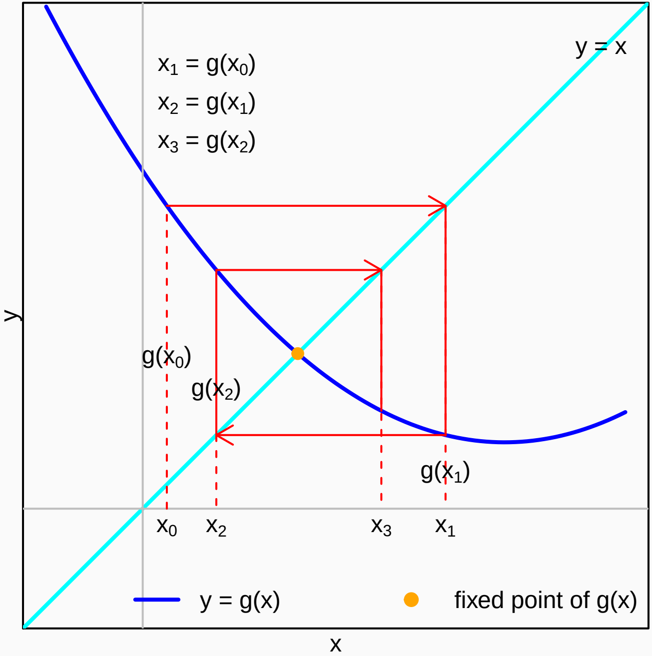

y = x
y
x_0
x
y = g(x)
fixed point of g(x)



$x_1 = g(x_0)$
y = x
g(x_0)
x_0
x_1
y = g(x)
fixed point of g(x)
y
x



x1 = g(x0)
x2 = g(x1)
y = x
y
g(x0)
g(x1)
x0
x2
x1
y = g(x)
fixed point of g(x)
x



$x_1 = g(x_0)$
$x_2 = g(x_1)$
$x_3 = g(x_2)$
y = x
y
$g(x_0)$
$g(x_2)$
$g(x_1)$
x_0
x_2
x_3
x_1
y = g(x)
fixed point of g(x)
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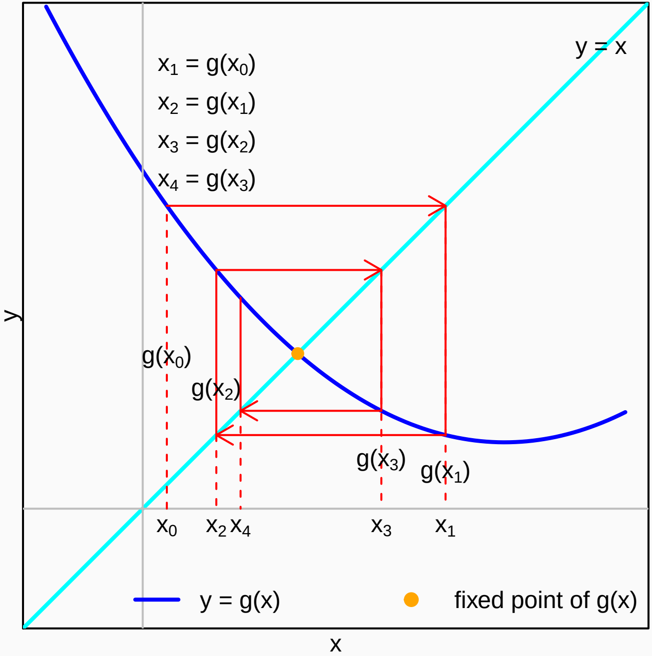
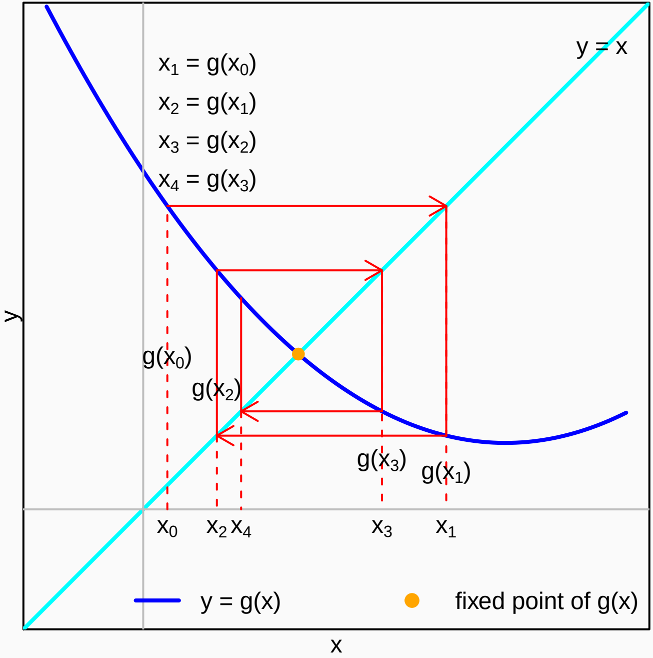


$x_1 = g(x_0)$
$x_2 = g(x_1)$
$x_3 = g(x_2)$
$x_4 = g(x_3)$
y = x
y
$g(x_0)$
$g(x_2)$
$g(x_3)$
$g(x_1)$
x_0
x_2
x_4
x_3
x_1
y = g(x)
fixed point of g(x)
x



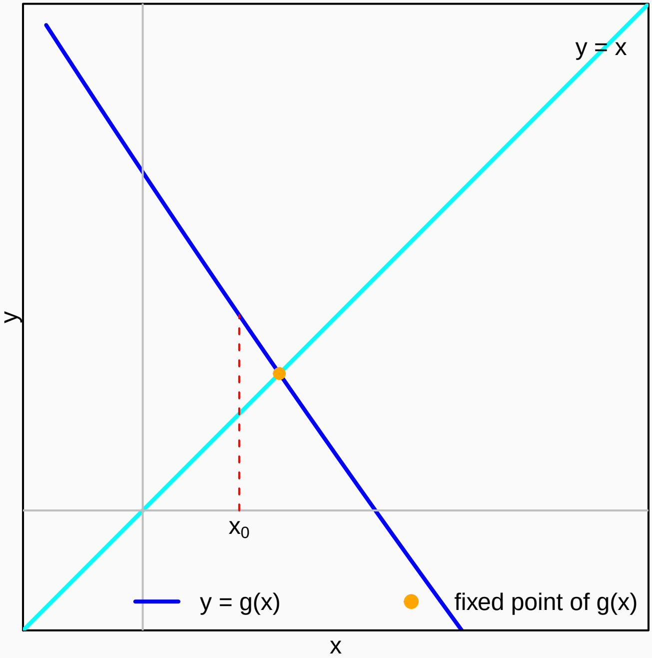
Case III

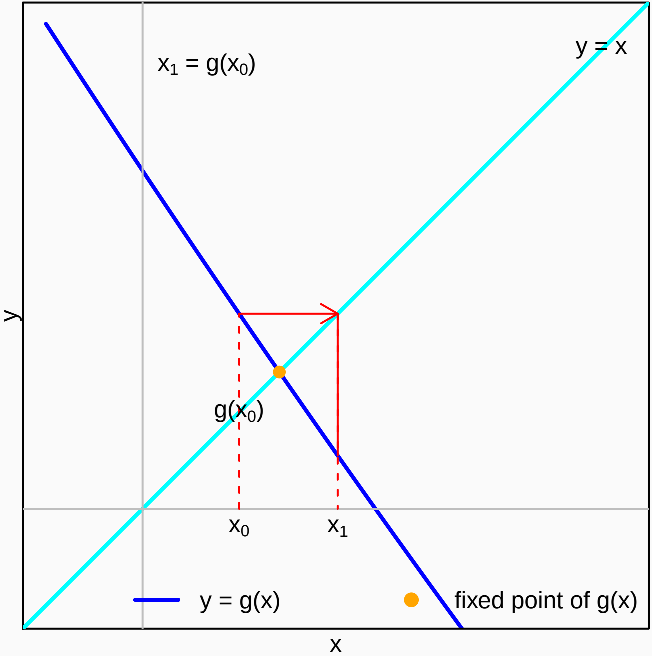
Near the root:

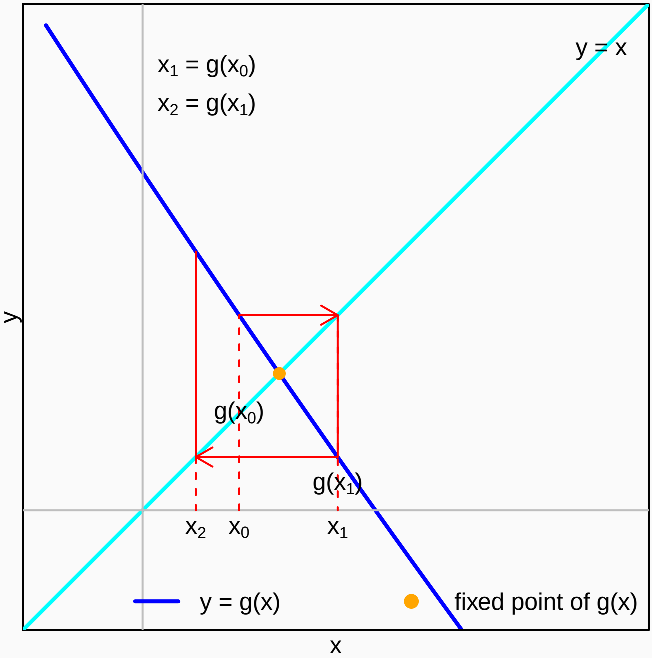
$-1 < g'(x) < 0$

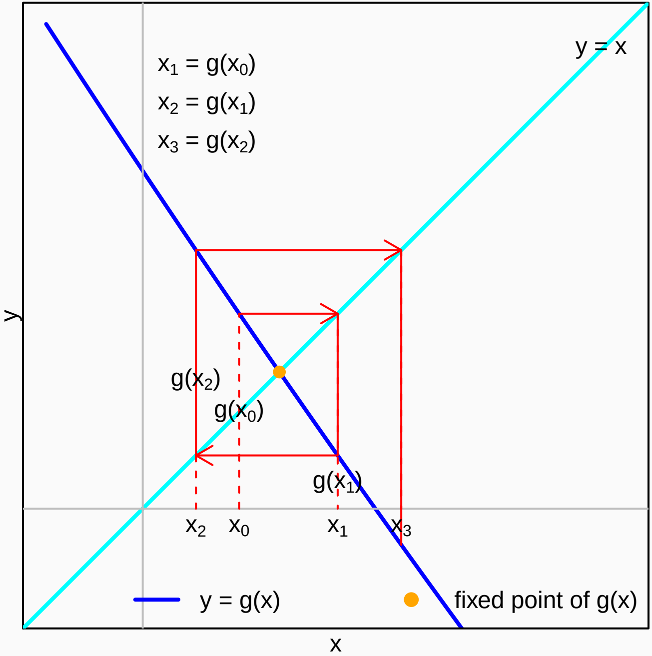
Resulting in:

Oscillating Convergence

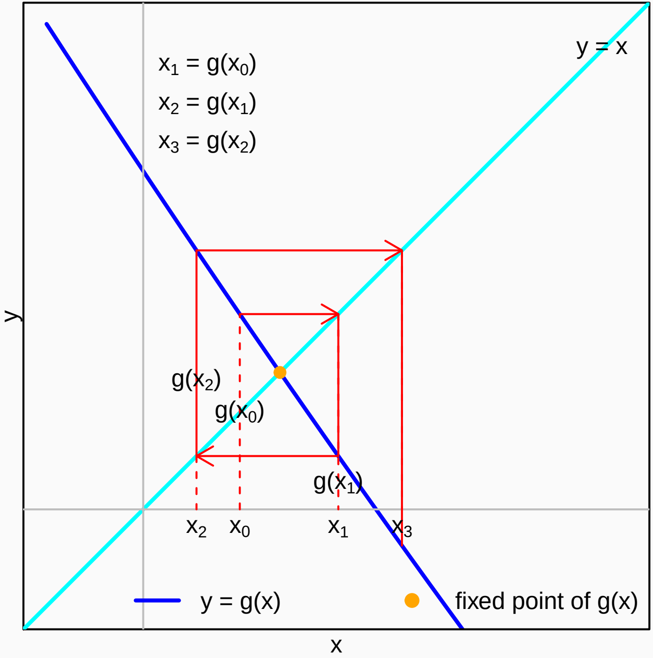

y = x
y
x₀
y = g(x)
fixed point of g(x)
x


$x_1 = g(x_0)$
y = x
y
$g(x_0)$
x_0
x_1
y = g(x)
fixed point of g(x)
x


$x_1 = g(x_0)$
$x_2 = g(x_1)$
y = x
y
$g(x_0)$
$g(x_1)$
x_2
x_0
x_1
y = g(x)
fixed point of g(x)
x



y = x
$x_1 = g(x_0)$
$x_2 = g(x_1)$
$x_3 = g(x_2)$
y
$g(x_2)$
$g(x_0)$
$g(x_1)$
x_2
x_0
x_1
x_3
y = g(x)
fixed point of g(x)
x



Case IV

Near the root:

$g'(x) < -1$

Resulting in:

Oscillating Divergence

Convergence Properties

i)	$0 < g'(x) < 1$	$\Rightarrow$	Monotonic convergence
ii)	$g'(x) > 1$	$\Rightarrow$	Monotonic Divergence
iii)	$-1 < g'(x) < 0$	$\Rightarrow$	Oscillating/Spiral Convergence
iv)	$g'(x) < -1$	$\Rightarrow$	Oscillating/Spiral Divergence

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i)	$0 < g'(x) < 1$	$\Rightarrow$	Monotonic convergence
ii)	$g'(x) > 1$	$\Rightarrow$	Monotonic Divergence
iii)	$-1 < g'(x) < 0$	$\Rightarrow$	Oscillating/Spiral Convergence
iv)	$g'(x) < -1$	$\Rightarrow$	Oscillating/Spiral Divergence

From i) and ii):

$g'(x) = +$ve $\quad \Rightarrow \quad$ Monotonic

Convergence Properties

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Convergence Properties

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From ii) and iv):

$|g'(x)| > 1 \quad \Rightarrow \quad$ Divergent

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From i) and iii):

$|g'(x)| < 1 \quad\Rightarrow\quad$ **Convergent**

Example 2

Find a real root of $x^3 + x^2 - 1 = 0$ correct to 6 decimals using the fixed point method.

Solution:

$f(x) = x^3 + x^2 - 1$

$f(0) = -ve$ and $f(1) = +ve$

$x = g(x)$	$g'(x)$	$g'(0)$	$g'(1)$	$g'(0.5)$	Useful?
$x = (1 - x^2)^{\frac{1}{3}}$	$-\dfrac{2x}{3(1-x^2)^{2/3}}$	0	$-\infty$	-0.4038	✗
$x = (1 - x^3)^{\frac{1}{2}}$	$-\dfrac{3x^2}{3(1-x^3)^{1/2}}$	0	$-\infty$	-0.4009	✗
$x = x^3 + x^2 + x - 1$	$3x^2 + 2x + 1$	1	6	2.75	✗
$x = (x+1)^{-\frac{1}{2}}$	$-\dfrac{1}{2(x+1)^{3/2}}$	-0.5	-0.1768	-0.2722	✓

[Please complete this problem]

Finding Square Root of a positive real number using basic math operations

Example: $\sqrt{5} = ?$

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Standard form for $\sqrt{N}$:

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What about cube root and higher roots?

Assignments

1. Complete the problem given in Example 2.
2. Find a real root of $e^x \tan x = 1$ correct to five decimal places using the Fixed Point Method.
3. Find the cube root of 10 using fixed point iteration method. Note that the iteration formula should consist of no other mathematical operations than the basic math operations (addition, substraction, multiplication, division).
4. Formulate a general iteration formula using the fixed point technique to find m^{th} root of N and use it to compute $\sqrt[5]{100}$ correct to 5 decimal places.