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3CS-5

PARAMETER ESTIMATION

Q1: Let $(X_1, X_2 \ldots X_n)$ be random sample of size 'n' taken from a normal population with parameters, Mean $= \theta_1$, Variance $= \theta_2$. Find Max. likelihood estimates of these 2 parameters.

$$MLE(\theta_1) = \text{joint pdf}(x_1, x_2 \ldots x_n \mid \theta_1)$$

$$MLE(\theta_2) = \text{joint pdf}(x_1, x_2 \ldots x_n \mid \theta_2)$$

$$MLE(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

LOG ON BOTH SIDES

$$\ln(\theta_1, \theta_2) = \sum_{i=1}^{n}\left[-\ln(\sqrt{2\pi\theta_2}) - \frac{(x_i-\theta_1)^2}{2\theta_2}\ln(e)\right]$$

$$= -n\ln(\sqrt{2\pi\theta_2}) - \frac{\Sigma x_i^2}{2\theta_2} - \frac{n\theta_1^2}{2\theta_2} + \frac{\theta_1}{\theta_2}\Sigma x_i$$

Diff. w.r.t θ_1

$$-\frac{2n\theta_1}{2\theta_2} + \frac{1}{\theta_2}\Sigma x_i = 0$$

$$\left[\theta_1 = \frac{\Sigma x_i}{n}\right]$$

$\theta_1 = \overline{x}$ ~~#~~ Sample Mean of y

Diff. w.r.t θ_2

$$= \frac{-n}{\sqrt{2\pi\theta_2}} \times \sqrt{2\pi} \times \frac{1}{2\sqrt{\theta_2}} + \frac{\Sigma x_i^2}{2\theta_2^2} + \frac{n\theta_1^2}{2\theta_2^2} - \frac{\theta_1 \Sigma x_i}{\theta_2^2} = 0$$

$$\frac{-n\theta_2 + \Sigma x_i^2 + n\bar{x}^2 - 2\bar{x}\Sigma x_i}{2\theta_2^2} = 0$$

$$\theta_2 = E(x_i^2) + E^2(x) - 2E^2(x)$$

$$= E(x_i^2) - E^2(x)$$

$$\begin{bmatrix} \theta_2 = \text{Variance}(x) \\ \theta_1 = \text{Mean}(x) \end{bmatrix}$$

Q2. Let $x_1, x_2, \ldots x_n$ be random sample from $B(m, \theta)$ dist. where $\theta \in (0,1)$ is unknown and m is a known +ve integer. Compute θ using M.L.E.

pdf Binomial $= {}^mC_x \, \theta^x (1-\theta)^{m-x}$

$$L(\theta \mid x_1 \ldots x_n) = \text{Joint pdf}(x_1 \ldots x_n \mid \theta)$$

$$= \prod_{i=1}^{n} {}^mC_{x_i} \, \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking $\ln$ on both sides

$$= \sum_{i=1}^{n} \left[\ln({}^mC_{x_i}) + x_i \ln\theta + (m-x_i)\ln(1-\theta) \right]$$

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Diff. wrt to θ

$$= \frac{\sum x_i}{\theta} + \frac{nm - \sum x_i}{1-\theta}(-1) = 0$$

$$\frac{\sum x_i}{nm} = \theta$$

$$\left[\theta = \frac{\bar{x}}{m} \right]$$

$\bar{x} \Rightarrow$ mean of sample