4. EQUATION SOLVING

e) Solving systems using Cramer's rule

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1 INTRODUCTION

Cramer's rule is an explicit formula for solving systems of linear equations.

Consider the system of equations $a_1x-b_1y+c_1z=d_1$, $a_2x+b_2y-b_3z=d_2$ and $a_3x+b_3y+c_3z=d_3$

$A = \text{Coefficient matrix} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$

$B = \text{Constant matrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$

$X = \text{Variable matrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

Cramer's rule states that $x_i = \frac{|A_i|}{|A|}$, where $i = 1, 2, 3$, and A_i is the resultant matrix when you replace the ith column of A with the column vector B.

2 PYTHON CODE

Solving equation systems using Cramer's rule. Hence, consider the following in the context of simultaneous equations.

[1]:
```
import numpy as np
import copy as cp
def cramersRule(A, B):
# A is the matrix of coefficients.
# B is the column matrix of constant sums.
    X = []
    solutions = [] # Intended list of solutions.
    nVars = len(A[0]) # Length of a row => Number of variables.
    for i in range(0, nVars + 1):
        X.append(cp.deepcopy(A))
        X[i][:, i] = B
        solutions.append(np.linalg.det(X[i])/np.linalg.det(A))
    return solutions
```

3 EXAMPLE

$4x + y = 6200$

$3x + 3y = 5600$

[2]:
```
A = np.matrix([[4, 1], [3, 3]]) # Matrix of coefficients.
B = np.matrix([[6200], [5600]]) # Matrix of constant sums.
cramersRule(A, B)
```

[2]:
```
[1444.4444444444437, 422.22222222222246]
```