5. SETS OF VECTORS FROM VECTOR SPACE

c) Linear independence (method 2)

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1 AIM

One result states that a set of vectors is linearly dependent if and only if a vector in the set can be expressed as a linear combination of the other vectors. Using this result, we can identify whether the set of vectors in linearly independent or not.

Support functions...

[1]:
```
from sympy import Matrix, zeros, pprint
def inputPositiveInteger(prompt):
    while True:
        try:
            i = input(prompt)
            if i == "x": return 0
            i = int(i)
            if i <= 0: i = 1/0
            return i
        except:
            print("Invalid integer, please re-enter.")
def floatInput(prompt):
    while True:
        try:
            i = float(input(prompt))
            return i
        except:
            print("Invalid number, please re-enter.")

def matrixInput(nRow, nCol):
    print("\nEnter row by row, each element in the row separated by comma...")
    A, i = zeros(nRow, nCol), 0
    while i < nRow:
        row = input("R{0}: ".format(i + 1)).split(",")
        if "x" in row: break # To stop inputting anymore
        if len(row) != nCol:
            print("ERROR: You must only enter", nCol, "per row")
            continue
        for j in range(0, nCol):
            try:
```

```
                A[i, j] = float(row[j])
            except:
                print("ERROR: Non-numeric inputs.")
                j = -1
                break
        if j != -1: i = i + 1
    return A
```

Main…

[8]:
```
M = []
print("\nINPUT VECTORS (1 ROW, 3 COLUMNS)")
for i in range(0, 4):
    M.append(matrixInput(1, 3))
print("\nYOUR VECTORS")
for i in range(0, 4):
    print(M[i])
N = Matrix(M)
print("\nMATRIX OF YOUR VECTORS")
pprint(N)
r = N.rank()
print("Rank: ", r)
if(r < 4): print("At least one vector is a linear combination of the others.")
else: print("None of these vectors are linear combinations each other.")
```

```
INPUT VECTORS (1 ROW, 3 COLUMNS)

Enter row by row, each element in the row separated by comma…
R1: 1,2,3

Enter row by row, each element in the row separated by comma…
R1: 2,5,3

Enter row by row, each element in the row separated by comma…
R1: 2,7,4

Enter row by row, each element in the row separated by comma…
R1: 3,8,9

YOUR VECTORS
Matrix([[1.00000000000000, 2.00000000000000, 3.00000000000000]])
Matrix([[2.00000000000000, 5.00000000000000, 3.00000000000000]])
Matrix([[2.00000000000000, 7.00000000000000, 4.00000000000000]])
Matrix([[3.00000000000000, 8.00000000000000, 9.00000000000000]])

MATRIX OF YOUR VECTORS
 1.0  2.0  3.0
```

```
 2.0  5.0  3.0

 2.0  7.0  4.0

 3.0  8.0  9.0
Rank:  3
At least one vector is a linear combination of the others.
```

2 CONCLUSION

To get the rank of a matrix, you need to reduce to row echelon form. To reduce to row echelon form, you need to do row operations, and row operations involve linear combination of vectors. Now, if one of the rows becomes 0 in the process, that means the vector that was in that row was equal to some linear combination of the other vectors (which is why it was cancelled out). So, since there are 4 rows, if the rank is less than 4, we know that one or more rows have been cancelled out, hence at least one vector is a linear combination of the others. Since we have that if a vector in a set of vectors can be expressed as a linear combination of the other vectors, then the set of vectors is linearly dependent, we have that the above set of vectors is linearly dependent.