5. SETS OF VECTORS FROM VECTOR SPACE

a) Linear span

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1 AIM

Span of a set of vectors S is the set of all vectors that can be expressed as a linear combination of the elements of S. Our aim in this record is to use linear equation system solving methods to find out whether a vector belongs to the linear span of a set of vectors, or equivalently, if a vector can be expressed as a linear combination of a set of vectors.

2 Does a given vector belong to the span of a set of vectors?

To see if a vector belongs in the span of a set of vectors, we must see if any linear combination of the set of vectors equals the given vector.

[25]:
```
import numpy as np
from sympy import *
print("\nGiven vector u:")
u = np.matrix([4, 7, 1])
print(u)

print("\nMatrix of vectors {v1, v2, v3} (vectors are the columns):")
v1 = np.matrix([[5], [-2], [7]])
v2 = np.matrix([[6], [-8], [3]])
v3 = np.matrix([[4], [7], [-1]])
V = np.hstack([v1, v2, v3])
print(V)

print("\nCoefficients of v1, v2 and v3 so that their linear combination is u:")
print(np.linalg.solve(V, u.T))
```

```
Given vector u:
[[4 7 1]]

Matrix of vectors {v1, v2, v3} (vectors are the columns):
[[ 5  6  4]
 [-2 -8  7]
 [ 7  3 -1]]
```

```
Coefficients of v1, v2 and v3 so that their linear combination is u:
[[ 0.35491607]
 [-0.20623501]
 [ 0.86570743]]
```

The presence solutions means that there is some linear combination of the set of vectors V = $\{v_1, v_2, v_3\}$ that results in the given vector i.e. $(4, 7, 1)$. Hence, the given vector is in the linear span of V.

3 Is a given vector a linear combination of a set of vectors?

[26]:
```
print("\nGiven vector y")
y = np.matrix([6, 4, 3])
print(y)

x1 = np.matrix([[1], [2], [1]])
x2 = np.matrix([[3], [1], [2]])
x3 = np.matrix([[3], [2], [1]])

print("\nMatrix of vectors {x1, x2, x3}:")
X = np.hstack([x1, x2, x3])
print(X)

print("\nCoefficients of x1, x2 and x3 so that their linear combination is y:")
print(np.linalg.solve(X, y.T))
```

```
Given vector y
[[6 4 3]]

Matrix of vectors {x1, x2, x3}:
[[1 3 3]
 [2 1 2]
 [1 2 1]]

Coefficients of x1, x2 and x3 so that their linear combination is y:
[[0.5       ]
 [0.66666667]
 [1.16666667]]
```

3.0.1 Expressing the above fact

Hence, we have that $(6, 4, 3) = \frac{1}{2}(1, 2, 1) + \frac{2}{3}(3, 1, 2) + \frac{7}{6}(3, 2, 1)$

(NOTE: 0.666666... = 2/3 and 1.1666666... = 7/6)

4 CONCLUSION

A linear combination of a set of vectors can be expressed as a system of equations, where the nth equation represents the sum of the scalar multiples of the elements in the nth position in each vector. So, for a linear combination $c_1a + c_2b = c_1(a_1, a_2) + c_2(b_1, b_2)$, equation 1's left hand side would be $c_1a_1 + c_2b_1$, and equation 2's left hand side would be $c_1a_2 + c_2b_2$. The right hand side is the nth element of the given vector, for which you should check if it lies in the linear span or not.