4. EQUATION SOLVING

a) System of linear equations

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1 INTRODUCTION

A linear equation for n variables is an equation wherein each variable appears only as a simple linear function of itself i.e. it is simply multiplied by a constant coefficient (which can also be 0 and 1). Apart from these variables, constants will also be present (at least 0 will be present, maybe after some remodelling).

1.1 Matrix method

Consider the system of equations $a_1x - b_1y + c_1z = d_1$, $a_2x + b_2y - b_3z = d_2$ and $a_3x + b_3y + c_3z = d_3$

$$A = \text{Coefficient matrix} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

$$B = \text{Constant matrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

$$X = \text{Variable matrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

We have $AX = B$, which means $X = A^1B$. Note that $X[0] = x, X[1] = y, X[2] = z$ Hence, we need to find the inverse of A, and perform matrix multiplication between it and B.

[1]:
```
import numpy as np
```

Solve the system of equations $2x - 3y + 5z = 11$, $5x + 2y - 7z = -12$ and $-4x + 3y + z = 5$

[20]:
```
# Defining the coefficent matrix
A = np.matrix([[2, -3, 5], [5, 2, -7], [-4, 3, 1]])
B = np.matrix([[11], [-12], [5]])
# Find the inverse of A
A_inverse = np.linalg.inv(A)
# Applying the formula
X = np.dot(A_inverse, B)
print("x = {0}, y = {1}, z = {2}".format(X[0][0], X[1][0], X[2][0]))
```

```
x = [[1.]], y = [[2.]], z = [[3.]]
```

1.2 Direct method (using inbuilt function)

[2]:
```python
import numpy as np
```

Solve the system of equations $2x - 3y + 5z = 11$, $5x + 2y - 7z = -12$ and $-4x + 3y + z = 5$

[3]:
```python
# Defining the coefficent matrix
A = np.matrix([[2, -3, 5], [5, 2, -7], [-4, 3, 1]])
B = np.matrix([[11], [-12], [5]])
# Applying the function
X = np.linalg.solve(A, B)
print("x = {0}, y = {1}, z = {2}".format(X[0][0], X[1][0], X[2][0]))
```

```
x = [[1.]], y = [[2.]], z = [[3.]]
```

1.3 NOTES

Note that A is a 3 x 3 matrix, while B and X are 3 x 1 matrices i.e. 3 rows, 1 column