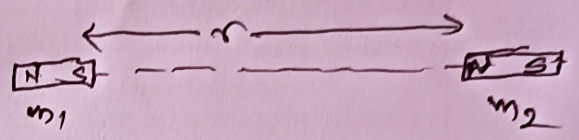
Magnetic fields :-

Coulomb law in magnetism:

Coulomb law in magnetism states that "the forces of attraction or repulsion between any two magnetic pole pieces with pole strength m_1 and m_2 is,



1. directly proportional to product of their pole strength

   i.e. $F \propto m_1 m_2$ 1

2. inversely proportional to square of distance between them

   i.e. $F \propto 1/r^2$ 2

   combining 1 and 2 equations

   $F \propto m_1 m_2/r^2$

Or, $F = \frac{\mu_0 m_1 m_2}{4\pi r^2}$ where $\frac{\mu_0}{4\pi} = 10^{-7} N A^{-2}$ or $wbA^{-1}m^{-1}$ is proportionality constant and $\mu_0 = 4\pi X 10^{-7}$ $wbA^{-1}m^{-1}$ is permeability value of free space.

In vector form;

$$\vec{F} = \frac{\mu_0 m_1 m_2}{4\pi r^2}\hat{r}$$

or, $\vec{F} = \frac{\mu_0 m_1 m_2}{4\pi r^3}\vec{r}$ { being $\hat{r} = \frac{\vec{r}}{r}$ is unit vector along distance between two magnetic poles centers}

~~Electric~~ magnetic Permeability (μ):-

It is the property of medium, that determines the tendency of magnetic lines of forces spread out in medium and hence determined for coulomb force between two poles of magnet in given medium. More is the value of permeability of medium, it refers to stronger force between two poles of magnet in that medium.

Magnetic field:-

A magnet consists of spread of magnetic lines of force. Its direction is from north to south pole outside the magnet in space or medium while its direction is south to north pole inside magnet.

The space or region around the magnet upto which magnetic lines of forces are spread up and any magnetic pole feel coulomb force of magnetism if any present is called magnetic field.

Hence, a magnet with larger magnitude of pole strength (m) have strong magnetic field strength.

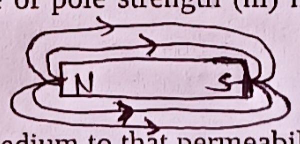


fig:- direction of magnetic lines in bar magnet

Relative permeability (μ_r):-

The ratio of magnetic permeability of medium to that permeability of air (space) is known as relativity permeability of medium.

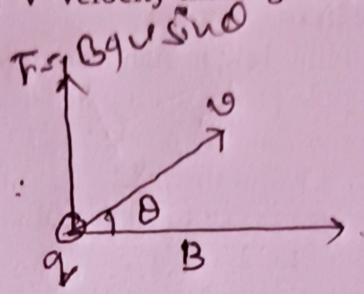
$$\mu_r = \frac{\mu}{\mu_0} = \frac{F_{medium}}{F_{air}}$$

It is dimensionless quantitty and signify the strength of medium to allow magnetic lines of force in medium in comparision to free space or air.

Force on a moving charge in a magnetic field (Lorentz force):-

When ever a charge moving in magnetic field, it experience a force and this force is called Lorentz force.

For , 'q' be moving charge in magnetic field with 'v' velocity making an angle 'θ' with magnetic field direction then,



Force experience by moving charge is

1. directly proportional to magnetic field strength

   i.e. $F \alpha B$ 1
2. directly proportional to amount of moving charge

   i.e. $F \alpha q$ 2
3. directly proportional to velocity of moving charge

   i.e. $F \alpha v$ 3
4. directly proportional to sine of angle between magnetic field strength and velocity of charge

   i.e. $F \alpha \sin\theta$.......................... 4

Combining all relations; we get,

$F = Bqv\sin\theta$ where, proportionality constant is of unit magnitude value.

In vector form;

$$F = q(\vec{v} \times \vec{B})$$

The direction of this force is perpendicular to the plane containing to velocity of charge and magnetic field and is given by left hand rule.

According to Left hand rule if we stretched thumb, fore-finger and middle-finger mutually perpendicular to each other such that fore-finger pointing to direction of magnetic field, middle-finger towards velocity of charge then thumb shows direction of Lorentz force on charge.

Case I:- When charge is moving parallel or opposite (anti-parallel) to the direction of magnetic field i.e. $\theta=0^0$ or $\theta=180^0$ then,

$F = Bqv\sin 0$ or $Bqv\sin 180$

$= 0$

Hence, a moving electron don't experience any forces. This is condition for minimum force on moving charge.

Case II:- When charge is moving perpendicular to the direction of magnetic field i.e. $\theta=90^0$ then,

$F = Bqv\sin 90$

$= Bqv$

Hence, force experience by moving charge is maximum.

Case III:- When charge is at rest i.e. v= 0; then,

$F = Bqv\sin 0 = 0$

Thus, a rest charge don't experience a force in magnetic field.

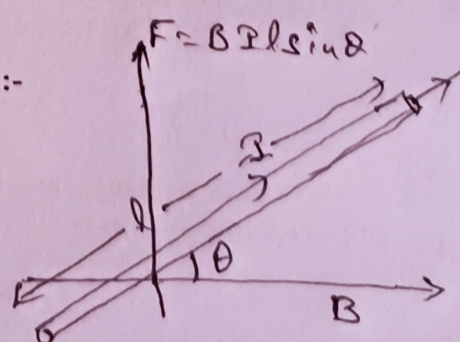
Case IV:- When a charge body is neutral i.e. q=0 then,

$F = Bqv\sin 0 = 0$

Thus, a charge less particles like neutron moving in magnetic field don't experience a force.

Force on a current carrying conductor in a magnetic field:-

Let, us consider a straight conductor of length 'L', cross-sectional area 'a' and having 'n' number of electron per unit volume is carrying 'I' current, placed in a magnetic field of strength 'B' by making an angle 'θ' as shown in fig;



Then,

total charge in a conductor (q)= total number of electron X charge of each electron

or, q= nAL x e

or, q = neAL 1

Further, for v_d be the drift velocity of electron, then , the force experienced by current carrying conductor in magnetic field is equal to total Lorentz force experience by moving chargew inside conductor. So,

$$F = Bqv_d \sin\theta$$
$$\text{or, } F = B\, neAL\, v_d \sin\theta$$
$$\text{or, } F = BIL \sin\theta \;\ldots\ldots\ldots\ldots\ldots\ldots 2$$

where, $I = neAv_d$ is current flowing in conductor.

In vector form;

$$F = I(\vec{L} \times \vec{B}) \;\ldots\ldots\ldots\ldots\ldots\ldots 3$$

The direction of this force is perpendicular to the plane containing to length element and magnetic field and is given by Fleming's left hand rule.
According to Fleming's Left hand rule if we stretched thumb, fore-finger and middle-finger mutually perpendicular to each other such that fore-finger pointing to direction of magnetic field, middle-finger towards direction of current then thumb shows direction of force on conductor.

Case I:- When current carrying conductor is perpendicular to the direction of magnetic field i.e. $\theta = 90^0$ then,

$$F = BIL \sin 90$$
$$= BIL$$

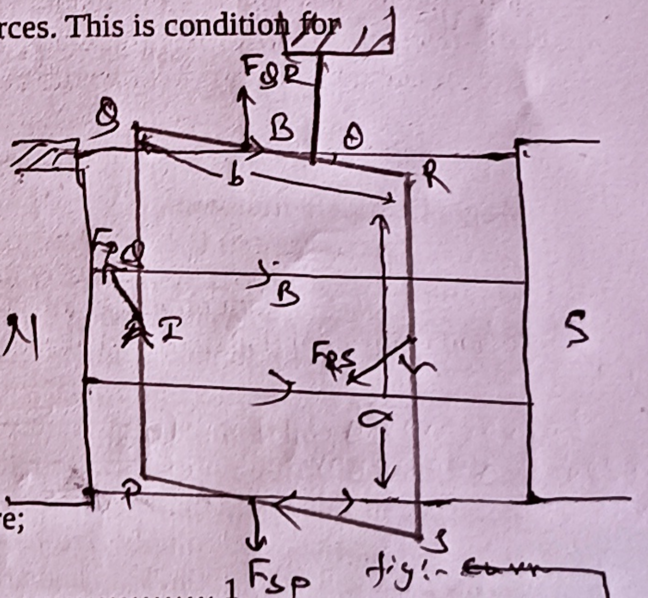
Hence, force experience by current carrying conductor is maximum.

Case II:- When current carrying conductor is parallel or opposite (anti-parallel) to the direction of magnetic field i.e. $\theta = 0^0$ or $\theta = 180^0$ then,

$$F = BIL \sin 0 \quad \text{or} \quad BIL \sin 180$$
$$= 0$$

Hence, a conductor don't experience any forces. This is condition for minimum force on moving charge.

Torque on a current carrying rectangular coil (loops):-



Let, us consider a rectangular coil PQRS of length 'L' , breadth 'b', containing 'N' number of turns and carrying 'I' current in a clockwise direction is placed in a uniform magnetic field of strength 'B' through suspension point between magnetic pole pieces 'N' and 's' as shown in figure.

For, 'θ' be the angle made by plane of coil with magnetic field. Then,
Force on current carrying conductor arm 'QR' and 'SP' are;

$F_{QR} = BIb \sin\theta$ in upward direction.
$F_{SP} = BIb \sin\theta$ in downward direction. 1

As these two forces are equal in magnitude and oppositely directed at same line of action of force, So they cancel each other.
Further, length of coli and magnetic field direction is always perpendicular to each other at each position of coil So, there is maximum force on conductor arms 'PQ' and 'RS' and are given by;

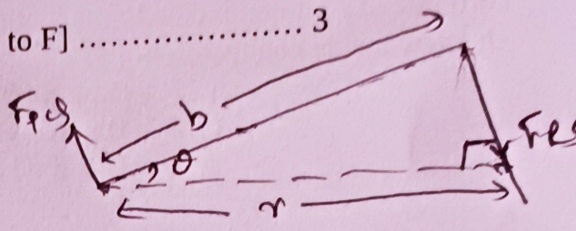
$F_{PQ} = BIL$ in inward direction to plane of paper.
$F_{RS} = BIL$ in outward direction to plane of paper.. 2

These two forces are equal in magnitude and oppositely acted in coil at different line of action of force. This formed couple in coil and hence result to torque. Thus, coil rotates about the axis of suspension when it is placed inside magnetic field.

fig:- Rectangular current carrying loop ~~in~~ ~~a~~ suspended in magnetic field.

Now, Torque on coil is;

$$\tau = \vec{r} \times \vec{F}$$
$$= r F \sin\theta$$
$$= r F \qquad \text{[being r is perpendicular to F]} \quad \ldots\ldots\ldots\ldots 3$$



Further, r is the perpendicular distances between two forces So, from fig;

$$r = b\cos\theta \quad \ldots\ldots\ldots\ldots 4$$

So, using equations 2, 4 in 3, we get;

$$\tau = BILb\cos\theta$$
$$= BIA\cos\theta \qquad \text{where } A = L \times b \text{ is an area of coil.}$$

Thus, for a coil having 'N' number of turns, the total torque on coil is

$$\tau = BINA\cos\theta \quad \ldots\ldots\ldots\ldots 5$$

For 'α' is the angle made by normal of plane of coil with magnetic field 'B', then

$$\theta = 90^0 - \alpha$$

Hence, torque in terms of angle between normal of coil and magnetic field is;

from eqn 5;

$$\tau = BINA\cos(90^0 - \alpha)$$
$$\text{or,} \quad \tau = BINA\sin\alpha \quad \ldots\ldots\ldots\ldots 6$$

Equations 5 and 6 are required expression for the torque on current carrying rectangular loop in magnetic field.

Case I:- When plane of coil is pendicular to the direction of magnetic field i.e. $\theta = 90^0$ then,

$$\tau = BINA\cos 90$$
$$= 0$$

This is the condition where coil experience minimum force.

Case II:- When plain of coil is parallel to the direction of magnetic field i.e. $\theta = 0^0$ then,

$$\tau = BINA\cos 0 \quad \text{or} \quad BINA\cos 180$$
$$= BINA$$

Thus, a coil experience maximum torque at this condition.

Magnetic dipole moments:-

According to Biot and Savert's law, a current carrying circular loop is associated with magnetic field at its center. For, a current flowing in anticlockwise direction, its center acts as north pole while in case of clockwise direction, center acts as South pole of magnet.

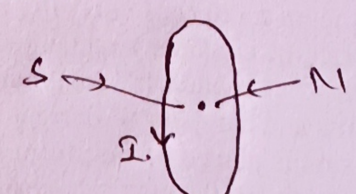


fig:- circular current carrying loop acting as dipole magnet.

If a coil with current is observed from two side then a clockwise current view from one side becomes to be anticlockwise at same time from other side. Thus, a current carrying loop possess a magnetic dipole.

Further, such dipole also have moment and called magnetic dipole moment and is define as the product of current in loop and area of loop. It is denoted by 'μ'

$$\mu = IA \quad \ldots\ldots\ldots\ldots 1$$

When such current carrying loop is kept in external field, then it experiences torque which is given by;

$$\tau = BIA\sin\alpha \quad \ldots\ldots\ldots\ldots 2$$

where, 'α' is an angle between magnetic field with normal to plane of coil.

So, from eqn 1. and 2.

$$\tau = B\mu\sin\alpha$$

$$\tau = \vec{\mu} \times \vec{B} \quad \ldots\ldots\ldots\ldots 3$$

This shows that the direction of torque on circular current loop is perpendicular to the plane containing its dipole moment (μ) and applied magnetic field direction (B).

The magnitude of torque on dipole is maximum at $\alpha=90^0$ i.e. $\tau = \mu B$

Energy stored in dipole moment:-

When magnetic dipole is placed in an external magnetic field then it experiences a torque due to which it align along field direction. Hence, the work done in dipole to change its orientation against the torque is stored in dipole in the form of dipole potential energy (P.E).

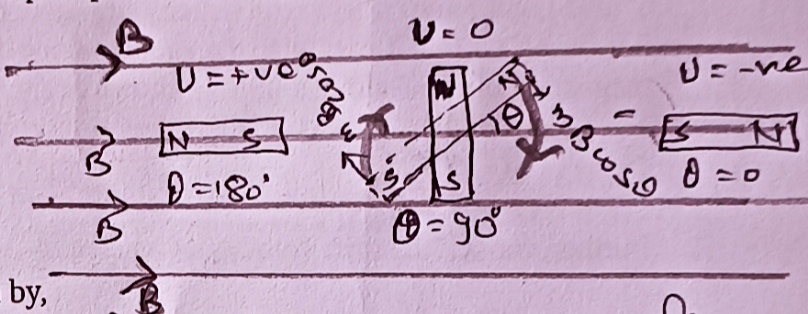


fig:- Torque on magnetic dipole in magnetic field.

Let, us consider a magnetic dipole with dipole moment 'μ' is placed in an external magnetic field 'B' as shown in figure. For, 'dθ' be the small turn or angular displacement made on dipole to rotate it against the torque existing on it.

Then, this small amount of work done is given by,

$$dW = \tau \, d\theta$$

$$\text{or, } dW = \mu B \sin\theta \, d\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Then, the work done in rotating dipole from 90^0 to 'θ' angle is,

$$W=\int_{90^0}^{\theta} dW=\int_{90^0}^{\theta} \mu B \sin\theta . d\theta$$

$$\text{or, } W=\mu B\int_{90^0}^{\theta} \sin\theta . d\theta$$

$$\text{or, } W=-\mu B[\cos\theta]_{90}^{\theta}$$

$$\text{or, } W=-\mu B(\cos\theta-\cos 90)$$

$$\text{or, } W=-\mu B\cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

This work done against torque to rotated magnetic dipole is stored in the form of dipole P.E.

Case I:- When dipole moment of magnetic dipole" and magnetic field are opposite (anti-parallel) i.e. $\theta=180^0$ then,

$W= -\mu B \cos 180$

$= \mu B$

Hence, at this condition dipole moment is maximum.

Case II:- When dipole moment of magnetic dipole" and magnetic field are parallel i.e. $\theta=0^0$ then,

$W= -\mu B \cos 0$

$= -\mu B$

Hence, at this condition dipole moment is minimum.

Hall Effect:-

When a current carrying conductor is placed in a magnetic field perpendicular to direction of current in cionductor then a voltage or potential difference is developed along the direction perpendicular to both magnetic and current direction. This effect on a current carrying conductor is known as hall effect and thus corresponding developed potential is called hall potential.

The electric field developed due to hall potential is called hall field.

Let, us consider a rectangular slab conductor with dimension 'l x b x t', cross-sectional area 'A=l x b' and containing 'n' electron density (number of electrons per unit volume).

For, I_x be current flowing in conductor along x-axis which is placed in a perpendicular uniform magnetic field 'B_z' along z-axis as shown in fig;

Then, current density in conductor is,

fig:-

$$J_x=\frac{I_x}{A} \quad \text{..........................1}$$

Further, for 'v_d' be drift velocity of an electron inside conductor, from relation ; $I=neAv_d$

$$v_d=\frac{J_x}{ne} \quad \text{.......................... 2}$$

As electron is moving inside a conductor is at external magnetic field it experiences maximum Lorentz force is given by,

$$F=B_z e v_d \quad \text{...................... 3}$$

The direction of this Lorentz force is directing towards downward direction in a conductor. Thus, there is a collection of -ve charge at bottom of slab and this produces an electric field in y-axis called hall field. Further, due to this, an upper side of conductor induces positive charges and hence there is potential difference arises inside conductor called hall potential. The strength of hall field increases with increase of charge and that opposes the further forward moment of electron at bottom of conductor.

Coulomb repulsive force due to 'E_H' hall field is

$$F= e E_H \quad \text{........................ 2}$$

Finally, the condition of equilibrium is achieved called hall condition at which;

Lorentz force on electron = Repulsive Coulomb force due to hall field

$$B_z e v_d = e E_H \qquad \text{(from 1 \& 2)}$$

$$\text{or, } E_H = B_z v_d$$

$$\text{or, } E_H=\frac{B_z J_x}{ne} \quad \text{..................... 3}$$

$$\text{or, } E_H=\frac{B_z I_x}{neA} \quad \text{..................... 4}$$

eq^{ns} 3 and 4 are required expression for hall field in terms of current density and current inside a conductor.

Further, for 'V_H' be hall potential developed inside conductor across 'b' width then,

$$E_H=\frac{V_H}{b} \quad \text{............................ 5}$$

then , using eq 5 in eqns 3 and 4 , we get;

$$\frac{V_H}{b}=\frac{B_z J_x}{ne}$$

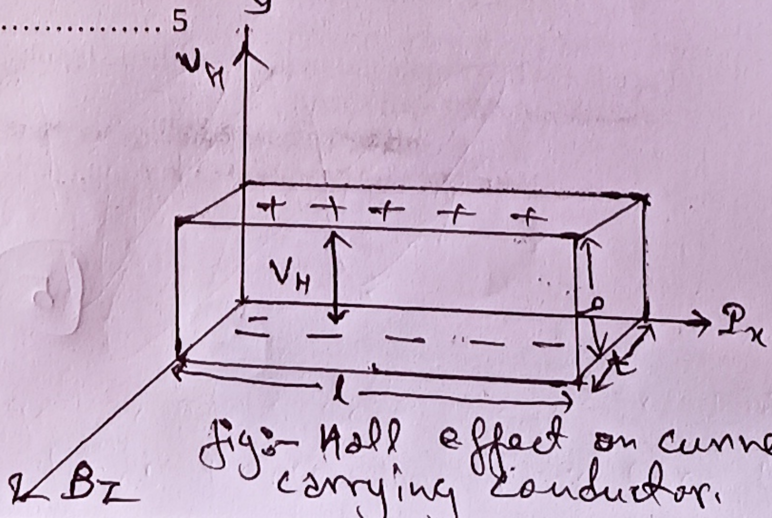


fig:- Hall effect on current carrying conductor.

$$\text{or, } V_H = \frac{B_z J_x b}{ne} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 6$$

$$\& \quad \frac{V_H}{b} = \frac{B_z I_x}{neA}$$

$$\text{or, } V_H = \frac{B_z I_x \cancel{b}}{ne\cancel{b} X t}$$

$$\text{or, } V_H = \frac{B_z I_x}{net} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 7$$

Hence, eqns 6 and 7 are the required expression for hall voltage developed in a given conductor in terms of current density and current in conductor.

Further, as $R_H = \frac{1}{ne}$ is a constant term for a given particular metal called coefficient of Hall effect.

Then, from eqn 7 and 3;

The hall voltage and hall field is given in terms of hall coefficient as,

$$V_H = \frac{B_z I_x R_H}{t} \quad \& \quad E_H = B_z J_x R_H \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 8$$

So,

$$R_H = \frac{E_H}{B_z J_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 9$$

Hall mobility (μ) :-

when an electric field is applied in a conductor, then electrons move in the direction opposite to applied electric field with an average velocity known as drift velocity.

Thus, Hall mobility of electron is define as the drift velocity gain by electron per unit applied electric field across conductor. It is denoted by ‘μ’ and given by,

$$\mu = \frac{v_d}{E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 10$$

$$\text{or, } \mu = \frac{J_x}{neE} \qquad \left[\text{being } v_d = \frac{J_x}{ne} \text{ ; from eqn 2 } \right]$$

$$\text{or, } \mu = \frac{\sigma \cancel{E}}{ne\cancel{E}} \qquad [\text{ being } J_x = \sigma E \text{ ; from ohm's law}]$$

$$\text{or, } \mu = \sigma R_H \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 11$$

$$\text{or, } \mu = \frac{R_H}{\rho} \qquad [\text{ being conductivity } (\sigma) = 1/\text{ resistivity } (\rho)\ ] \ldots\ldots\ldots\ldots\ldots\ 12$$

Thus, eqns 11 and 12 are required expression for hall coefficient in terms of conductivity and resistivity of conductor.

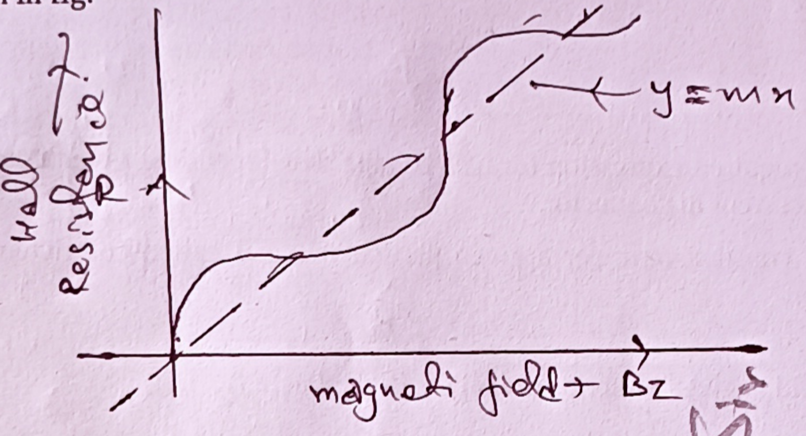
Hall resistance (R) :-

The ration of hall voltage developed to current flowing in conductor is called hall resistance and is expressed as,

$$R = \frac{V_H}{I_x}$$

or, $R = \frac{B_z \cancel{I_x}}{net \cancel{I_x}}$

or, $R = \frac{1}{net} B_z$ which is of y=mx , equation of straight line form with R Vs B_z with slope value 1/net. Hence, graph of 'B_z' versus 'R' is obtained with a line passing through origin as shown in fig.



But, experimentally it is found to be non-linear graph as shown in fig because of quantum hall effect.

Application or significance of Hall effect:-

1. It is used to determine nature of material regarding their conduction properties as conductor, semiconductor or insulator.
2. It is used to find sign or nature of charge carriers in conductor or to say identify the p-type and N-type semiconductor.
3. It is useful for calculating carriers concentration 'n', hall coefficient 'R_H' and also to find mobility value of electron in particular conductor.

Electromagnetic field :-

An electromagnetic field is the wave that is produced by an oscillating charge (electron) in space. This wave doesnot required medium for its propagation and run with speed of light $c=3x10^8$ m/s. This wave has wide spectrum distribution from 10^{-13} m wavelength to 0.1 m wavelength (10^{21} Hz to about 10^4 Hz) i.e from gamma rays to long radio signal waves, including visible light range 400nm to 700 nm wavelength ($7.5x10^{14}$ Hz to $3.8x\ 10^{14}$ Hz).

Further, The cross product of vectors $\vec{E} X \vec{B}$ gives the diection and magnitude of momentum vector for propagation of wave and clearly, occurs in perpendicular direction to both electric band magnetic field as shown in figure.

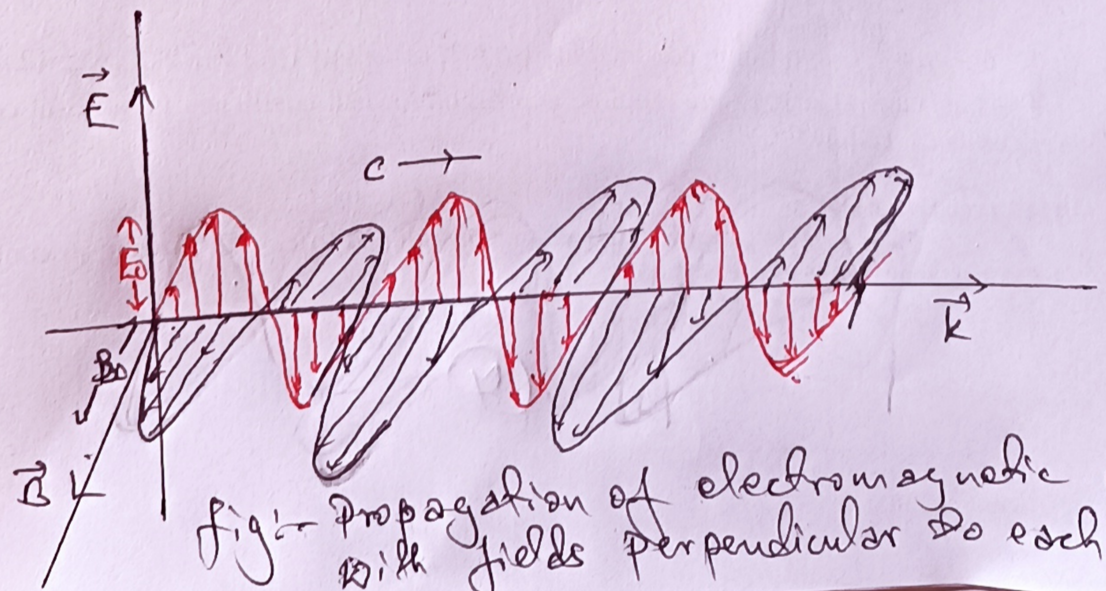


fig:- Propagation of electromagnetic wave with fields perpendicular to each other.

The vibration of charge creates Sinusoidally time varying both an electric and magnetic field components and that are mutually perpendicular. The electric and magnetic field are inter conversible during propagation of wave and are mathematically expressed along 1-dimension as;

$$\vec{E} = E_0 \sin(\omega t - kx)$$

$$\vec{B} = B_0 \sin(\omega t - kx)$$ where E_0 and B_0 are amplitudes of electric and magnetic fields, 'ω' and 'k' are the angular frequency and angular wave number (propagation wave vector) of wave.

By

THANK YOU