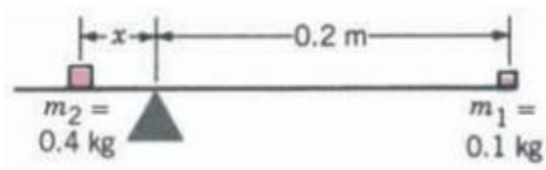
*1.1 A balance scale consisting of a weightless rod has a mass of 0.1 kg on the right side 0.2 m from the pivot point. See Fig. 1.2. (a) How far from the pivot point on the left must 0.4 kg be placed so that a balance is achieved? (b) If the O.4-kg mass is suddenly removed, what is the instantaneous rotational acceleration of the rod? (c) What is the instantaneous tangential acceleration of the O.1-kg mass when the O.4-kg mass is removed? [**TU micro syllabus W 8.1**]*

Solution:

Here,



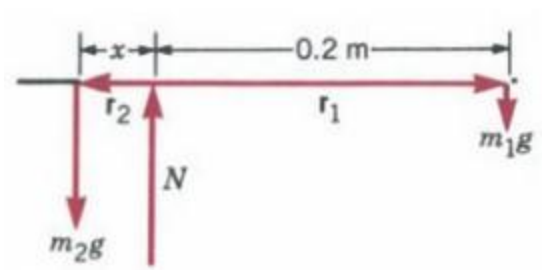


Fig 1.2

(a) For the balance point, a = 0 and therefore, $\Sigma\, \tau = 0 \Rightarrow \Sigma\vec{r} \times \vec{F} = 0$

i.e. Clockwise moment due to m_1g = Anticlockwise moment due to m_2g

Or, $F_1r_1\sin90^0 = F_2r_2\sin90^0$

Or, $m_1g\times 0.2 = m_2gx \Rightarrow x = 0.05$ m

(b) From $\tau = I\alpha$, $\alpha = \frac{\tau}{I} = \frac{F_1r_1\sin90^0}{m_1r_1^2} = \frac{m_1g\,r_1}{m_1r_1^2} = \frac{g}{r_1} = \frac{9.8}{0.2} = 49$ rad/s^2

(c) $a = a_T = r\alpha = 0.2\times 49 = 9.8$ m/s^2

1.2 *A bicycle wheel of mass 2 kg and radius 0.32 m is spinning freely on its axle at 2 rev/s. When you place your hand against the tire the wheel decelerates uniformly and comes to a stop in 8 s. What was the torque of your hand against the wheel? [**TU micro syllabus P 8.1**]*

Solution:

Here, initial angular velocity, $\omega_o = 2\pi f_o$

Final angular velocity, $\omega = 2\pi f = 0$

From $\omega = \omega_o - \alpha t$, $\alpha = \frac{\omega - \omega_o}{t} = \frac{0-2\times\pi\times2}{8} = \frac{-\pi}{2}$

Now, Torque, $\tau = I\alpha = mr^2\left(\frac{-\pi}{2}\right)$

$= 2\times0.32^2\times\left(\frac{-\pi}{2}\right)$

$= -0.32$ N

1.3 *Two masses, m_1 = 1 kg and m_2 = 5 kg, are connected by a rigid rod of negligible weight (see Fig. 1.3). The system is pivoted about point O. The gravitational forces act in the negative z direction. (a) Express the position vectors and the forces on the masses in terms of unit vectors and calculate the torque on the system. (b) What is the angular acceleration of the system at the instant shown in Fig. 1.3?* [***TU micro syllabus P 8.2***]

Solution:

Here, (a) $\overrightarrow{r_1} = -2\hat{j}$, 2 is in metre

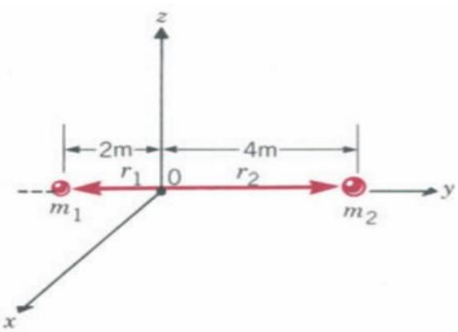
$\overrightarrow{r_2} = 4\,\hat{j}$, 4 is in metre.

(b) Torque, $\tau = \overrightarrow{r_1}\times\overrightarrow{F_1} + \overrightarrow{r_2}\times\overrightarrow{F_2}$

$= -2\hat{j}\times -10\hat{k}\ + 4\hat{j}\times -50\hat{k}$ $\because$ F = mg

$= 20\hat{i} - 200\hat{i}$

$\therefore\ \tau = -180\hat{i}$ Nm



Now from $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{\tau}{m_1 r_1^2 + m_2 r_2^2} = \frac{-180\hat{i}}{1\times2^2+5\times4^2} = -2.14i$ rads^{-2}

Fig: 1.3

1.4 A grindstone with I = 240 kg-m^2 rotates with a speed of 1 rev/s. A knife blade is pressed against it, and the wheel coasts to a stop with constant deceleration in 12 sec. What torque did the knife exert on the wheel?[***TU micro syllabus P 8.4]***

Solution:

Here, initial angular speed, $\omega_o = 2\pi f_o = 2\pi\times1 = 2\pi$ rad/s.

Final angular velocity, $\omega = 2\pi f = 0$

From $\omega = \omega_o - \alpha t$, $\alpha = \frac{\omega - \omega_o}{t} = \frac{0-2\pi}{12} = \frac{-\pi}{6}$

Now, Torque, $\tau = I\alpha = 240\times\left(\frac{-\pi}{6}\right)$

$= -125.6$ Nm

1.5 A uniform wooden board of mass 20 kg rests on two supports as shown in Fig. A 30-kg steel block is placed to the right of support A. How far to the right of A can the steel block be placed without tipping the board? [***TU micro syllabus P 8.7]*** *(Answer: 2 m)*

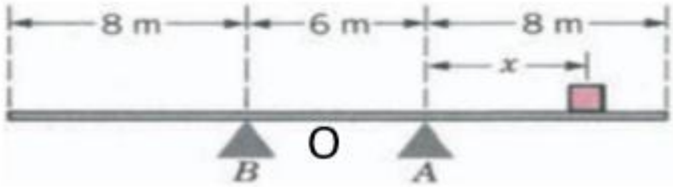


Fig: 1.4

(***Hint:*** *Centre of gravity of the board, say O lies at 3 m left of A. For not to tip at point A, 20 kg$\times$ $g \times OA$ = 30 kg$\times$ $g \times x$. And solve for x.)*

1.6 A spring (k = 200 N/m) is compressed 10 cm between two blocks of massm_1 = 1.5 kg and m_2 = 4.5 kg (see Fig. 1.17). The spring is not connected to the blocks, and the table is frictionless. What are the velocities of the blocks after they are released and lose contact with the spring? Assume that the spring falls straight down to the table. ***[TU Micro syllabus P10.10.18]***

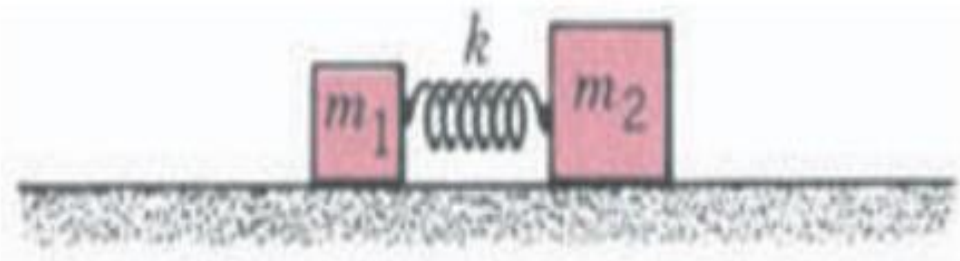


Fig 1.17

(***Hint:*** P.E on compression = K.E gained after release of the spring.

$\frac{1}{2}kx^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$. Apply conservation of energy; $m_1v_1 + m_2v_2 = 0 \Rightarrow v_1 = -3v_2$. Solve these two relations and find v_1 = -1 m/s and v_2 = 3.33 m/s).

*1.7 A large wheel of radius 0.4 m and moment of inertia 1.2 kgm^2, pivoted at the center, is free to rotate without friction. A rope is wound around it and a 2-kg weight is attached to the rope (see Fig. 1.6). When the weight has descended 1.5 m from its starting position (a) what is its downward velocity? (b) what is the rotational velocity of the wheel?****[TU Micro Syllabus W 8.2]***

Solution:

Here,

(a) Applying the principle of conservation of energy,

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

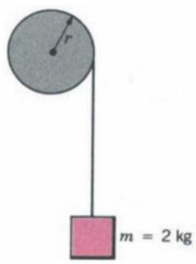


Fig 1.6

The velocity of the fall of the mass is equal to the tangential velocity on the rim of the wheel i.e. $v_T = v$

$\therefore$ mgh $=\frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} \Rightarrow v = [\frac{mgh}{\frac{1}{2}m+\frac{I}{2r^2}}]^{1/2}$

$$= [\frac{2\times9.8\times1.5}{\frac{1}{2}\times2+\frac{1.2}{2\times0.4^2}}]^{1/2}$$

$$= 2.5 \text{ m/s}$$

(b) The rotational velocity of the wheel, $\omega = \frac{v_T}{r}$

$$=\frac{v}{r}$$

$$= \frac{2.5}{0.4} = 6.2 \text{ rad/s}^2$$

1.8 Suppose the body of an ice skater has a moment of inertia $I = 4$ kg-m^2 and her arms have a mass of 5 kg each with the center of mass at 0.4 m from her body. She starts to turn at 0.5 rev/sec on the point of her skate with her arms outstretched. She then pulls her arms inward so that their center of mass is at the axis of her body, r = O. What will be her speed of rotation? ***[TU Micro syllabus P 8.4]***

Solution:

Let I_b and I_a respectively denote the moment of inertia of the body and the arm.
ω_o be the speed of rotation with arms outstretched and ω_f with arms folded.
If m be the mass of each arm, the total mass of the arms = 2m

From $(I\omega)_0 = (I\omega)_f$, $\Rightarrow (I_b + I_a)\omega_0 = I_b\omega_f \Rightarrow \omega_f = \frac{(I_b + Ia)\omega o}{I_b}$

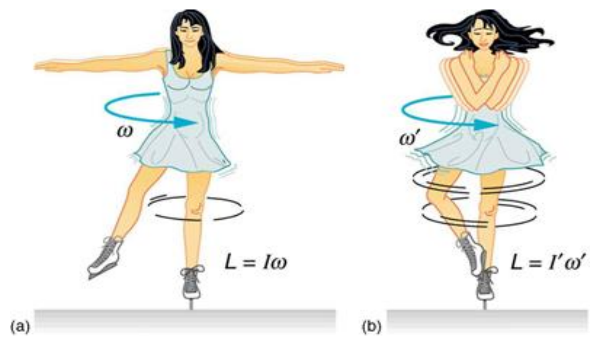


Fig: 1.8;

Or, $\omega_f = \left(\frac{I_{b+}\ 2mr^2}{I_B}\right)\omega_o$

$= \left(\frac{4+2\times5\times0.4^2}{4}\right) \times 0.5$

$= 0.7$ rev/s.

Test yourself

*1.9 A children's merry-go-round of radius 4 m and mass 100 kg has an 80-kg man standing at the rim. The merry-go-round coasts on a frictionless bearing at 0.2 rev/s. The man walks inward 2 m toward the center. What is the new rotational speed of the merry-go-round? What is the source of this energy? (The moment of inertia of a solid disk is $I = \frac{1}{2}mr^2$). (Answer: 0.37 rev/s.)[**TU micro syllabus P 8.18**]*

*(**Hint**: Apply $I_o\omega_o = I_f\omega_f$. $I_o = (M + m)\, r^2$, M and m are the masses of the merry – go-round and the man respectively, r = 4 m, $I_f = Mr^2 + mr_1^2$, r_1 = (4-2) m. Find ω_f*

*1.10 A given spring stretches 0.1 m when a force of 20 N pulls on it. A 2-kg block attached to it on a frictionless surface is pulled to the right 0.2 m and released. (a) What is the frequency of oscillation of the block? (b) What is its velocity at the midpoint? (c) What is its acceleration at either end? (d) What are the velocity and acceleration when x = 0.12 m, on the block's first passing this point? [**TU micro syllabus W 10.2, TU Model]***

Solution:

Here, $F = kx \Rightarrow k = \frac{F}{x} = \frac{20}{0.1} = 200$ N/m

a) We have, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2}} = 10$ rad/s

From $\omega = 2\pi\nu$, $\nu = \frac{\omega}{2\pi} = \frac{10}{2\pi} = 1.6$ Hz

b) Velocity is maximum at the mid-point and the maximum velocity, $v_{max} = \pm A\omega = \pm 0.2\times 10 = \pm 2$ m/s.

c) Since the velocity is maximum at the extreme position, $a = \pm A\omega^2 = \pm (0.2)(10)^2 = \pm 20$ m/s^2

d) For x = 0.12 m, we apply $x = A \cos \omega t$

Or, $\omega t = \cos^{-1}\frac{x}{A} = \cos^{-1}\frac{0.12}{0.2} = 53^0$

Now from $v = -A\omega \sin \omega t = -(0.2)(10) \sin 53° = -1.6$ m/s (moving to the left)

$a = -A\omega^2 \cos \omega t$

$= -(0.2)(10)^2 \cos 53° = -12$ m/s^2 (accelerating to the left).

[OR, you may apply $v = \sqrt{A^2 - x^2}$ and $a = -\omega^2 x$]

1.11 An oscillating block of mass 250 g takes 0.15 sec to move between the endpoints of the motion, which are 40 cm apart. (a) What is the frequency of the motion? (b) What is the amplitude of the motion? (c) What is the force constant of the spring? ***[TU micro syllabus P 10.5, TU 2074]***

1.12 The block of Example 1.11 is released from a position of $x_1 = A = 0.2$ m as before. (a) What is its velocity at $x_2 = 0.1$ m? (b) What is its acceleration at this position? ***[TU micro syllabus W 10.3]***

Solution:

Here, (a)

$$\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2$$

At the extreme point $x_1 = A$, $v_1 = 0$

$$\therefore v_2 = \sqrt{\frac{k(x_1^2 - x_2^2)}{m}} = \sqrt{\frac{200(0.2^2 - 0.1^2)}{2}} = 1.73 \text{ m/s}$$

(b) For acceleration, $F = ma \Rightarrow -kx = ma$

$$Or,\ a = \frac{-kx}{m} = \frac{-200\times 0.1}{2} = -10 \text{ m/s}^2$$

1.13 A block is oscillating with an amplitude of 20 cm. The spring constant is 150 N/m. (a) what is the energy of the system? (b) When the displacement is 5 cm, what is the kinetic energy of the block and the potential energy of the spring? ***[TU micro syllabus P10.13]***

Solution:

Here, (a) **Energy of the System**

The total mechanical energy E of a spring-mass system in simple harmonic motion is given by the potential energy at the maximum displacement (amplitude A).

$E_P = \frac{1}{2}kA^2$

Amplitude A=20 cm=0.20 m

Spring constant k=150 N/m

Now, substituting the values into the formula:

$E = \frac{1}{2} \times 150 \text{ N/m} \times (0.20 \text{ m})^2$

$\therefore E = 3$ J

So, the energy of the system is 3 J

(b) Kinetic Energy and Potential Energy at 5 cm Displacement

When the displacement x is 5 cm=0.05 m, we need to calculate the kinetic energy K.E and the potential energy E_P of the spring.

E_P at a displacement x is given by:

$E_P = \frac{1}{2}kx^2$

Substituting the values:

$E_P = \frac{1}{2} \times 150 \text{ N/m} \times (0.05 \text{ m})^2$

$= 0.19$ J

The total energy of the system E_{total} is the sum of the kinetic energy K.E and the potential energy E_P. Therefore, the kinetic energy K.E can be found by subtracting the potential energy E_P from the total energy E_{total}:

i.e. $K.E = E_{total} - E_P$

$= 3 \text{ J} - 0.19 \text{ J}$

$= 2.81$ J

Alternative Method: You may apply $K.E = \frac{1}{2} k (A^2 - x^2)$ because $v = \omega\sqrt{A^2 - x^2}$ and $k = m\omega^2$