By. S.G.

Electric and magnetic fields :-

Electric charge:

Electric charge is the intrinsic property or characteristics of fundamental particles like electron, proton, muons e.t.c. It is denoted by 'q' and S.I unit ios column.

$1C = 3X10^9$ Stat column (e.s.u) C.G.S unit

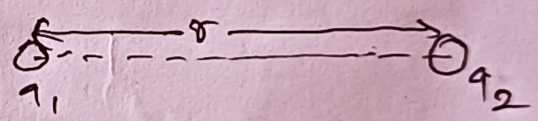
The fundamental amount of charge present in body is charge of electron about $1.6X10^{-19}C$.

Properties of electric charges:

1. like charge repel and unlike charge attract to each other.
2. Charge in a body is quantised i.e. charge of body can be expressed as integral multiple of fundamental charge of an electron i.e. $1.6X10^{-19}C$.
3. Charge is scalar quantity that can be added algebrically.
4. Charge in a system is conserved i.e. charge cannot be create nor be destroyed.
5. charge is independent to mass and speed of body.

Forces between electric charges:

The forces between two electric charges is given by coulomb and also called coulomb law of electrostatics. It states that, ' The force of attraction or repulsion between two charges q_1 and q_2 separated at r distance in space is,



1. directly proportional to product of their charges

   i.e. $F \propto q_1 q_2$ 1

2. inversely proportional to square of distance between them

   i.e. $F \propto 1/r^2$ 2

combining 1 and 2 equations

$$F \propto q_1 q_2 / r^2$$

Or, $F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$ where $\frac{1}{4\pi\epsilon_0} = 9 X 10^9 N m^2 C^{-2}$ is proportionality constant and $\varepsilon_0 = 8.85 X 10^{-12} C^2 N^{-2} m^{-2}$ is absolute permitivity value of free space.

In vector form;

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

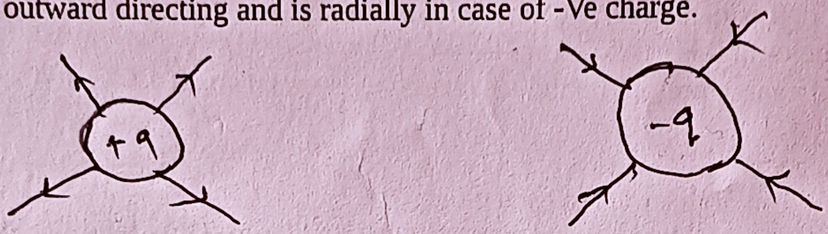
or, $\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^3}\vec{r}$ { being $\hat{r} = \frac{\vec{r}}{r}$ is unit vector along distance between two charge centers}

Electric Permitivity of medium(ε):-

It is the property of medium, that determines the tendency of electric lines of forces spread out in medium and hence determined for coulomb force between two charges in given medium. More is the value of permittivity of medium, it refers to weaker force between two charges in that medium. For example, water has 80 times greater value of permittivity than that of air So, electrostatic force in water is 80 times less between two charges at same distance than in air.

Electric field:-

A charge body consists of spread of electric lines of force. In case of +ve charge it is radially outward directing and is radially in case of -Ve charge.



The space or region around the charge body upto which electric lines of forces are spread up and any charges can feel electrostatic force if any present is called electric field.

Hence, a charge body with larger magnitude have more electric field and hence strong field strength.

Relative permittivity medium (ε_r):-

The ratio of electric permittivity of medium to that permittivity of air (space) is known as relativity permittivity of medium.

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$$\varepsilon_r = \frac{\epsilon}{\epsilon_0} = \frac{F_{air}}{F_{medium}}$$

It is dimensionless quantity and signify the strength of medium to resist electric lines of force in it in comparision to free space or air.

Electric field intenity (E):-

Electric field intensity at any point in electric field due to any source charge is define as the force experience by unit +Ve test charge placed at that point. i.e.

$$E = F = \frac{q}{4\pi\epsilon_0 r^2} \quad \text{.........1}$$

Or, further it can also be define as the force experienced by any charge body at given point per unit its charge.

For, q_0 be charge at 'r' distance from source charge '+q' then

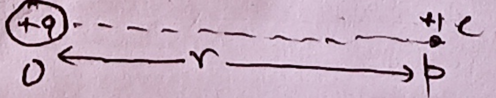
$$E = \frac{F}{q_0} = \frac{\frac{q q_0}{4\pi\epsilon_0 r^2}}{q_0}$$

or, $$E = \frac{q}{4\pi\epsilon_0 r^2} \quad \text{..................... 2}$$

Thus, electric field intensity at a point depend only on the magnitude of source charge.Electric field intensity is a vector quantity with unit NC^{-1} and its direction is always towards the direction of electric field of source charge.

Expression for electric field intensity due to point charge:-

Let, us consider a +ve point source charge 'q' placed in a space at 'O' point. For 'p' be the point in electric field region at 'r' distance from source charge where it is required to find the electric field intensity 'E'.



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Let, us take a +ve 'q_0' charge at point 'p' then the force experience by 'q_0' charge due to source charge 'q' is

$$F=\frac{qq_0}{4\pi\epsilon_0 r^2}$$

Then, from defination of electric field intensity at point 'p' is;

$$\text{or,}\quad E=\frac{F}{q_0}=\frac{\frac{qq_0}{4\pi\epsilon_0 r^2}}{q_0}$$

$$\text{or,}\quad E=\frac{q}{4\pi\epsilon_0 r^2}$$

In vector form;

$$\vec{E}=\frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\text{or}\quad \vec{E}=\frac{q_1q_2}{4\pi\epsilon_0 r^3}\vec{r}$$ { being $\hat{r}=\frac{\vec{r}}{r}$ is unit vector along radial distance}

For, a multi-charge system, q_1, q_2 , q_3 , ..., q_n in free space. Then, the net resultant electric field at a point in space due to multi charges can be obtained by vector addition of electric fields due to each charge at given point i.e.

$$\vec{E}=\vec{E}_1+\vec{E}_2+\vec{E}_3+\dots\dots\dots\dots+\vec{E}_n$$

Electric potential (V):-

Electric potential at any point in electric field region of any source charge is define as the total amount of work done in moving unit +ve test charge from infinity to given point against electrostatic force.

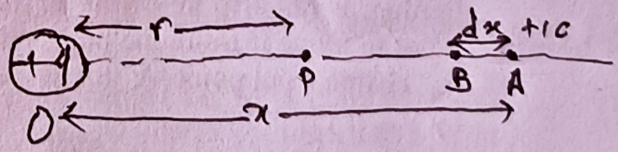
Or,

Further, it can also be define as the amount of work done over given charge per unit its charge in order to move it from infinity to that point.i.e. For 'W' be total work done in moving 'q_0' charge from infinity to a given point near to any source charge then electric potential is given by,

$$V=\frac{W}{q_0}$$ Its unit is J/C or Volt(V)

It is a scalar quantity.

Expression for electric potential due to point charge:-



Let, us consider a +ve point source charge 'q' placed in a space at 'O' point. For 'p' be the point in electric field region at 'r'distance from source charge where it is required to find the electric potential 'V'.

Let, us consider a unit +ve test charge at point 'A' which is at 'x' distance from source charge within its electric field region as shown in figure.

Then,

Columb force experience by given unit charge at 'A' point is

$$F=\frac{q.1}{4\pi\epsilon_0 x^2}=\frac{q}{4\pi\epsilon_0 x^2} \quad \dots\dots\dots\dots 1$$

For 'B' be a nearest point to 'A' at very small distance 'dx', then small amount of work done against force to move unit test charge through 'dx' distance is;

$$dW=-F.\,dx \quad \dots\dots\dots\dots\dots\dots\dots\dots 2$$

where, -ve sign is taken as displacement of charge is done against force and oppositely directed.

From eq^n 1. and 2.

$$dW = \frac{-q}{4\pi\epsilon_0 x^2} dx \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

Now, the total work done in moving unit charge from infinity to point 'p' is

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} \frac{-q}{4\pi\epsilon_0 x^2} dx$$

$$\text{or,}\quad W = \frac{-q}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{1}{x^2} dx$$

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$$\text{or,}\quad W = \frac{-q}{4\pi\epsilon_0} \left| \frac{-1}{x} \right|_{\infty}^{r}$$

$$\text{or,}\quad W = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$W = \frac{q}{4\pi\epsilon_0 r} = V$$

This is required expression for a electric potential due to point charge in space. Clearly, electric potential at a point in space depends on magnitude of source charge.

For 'n' number of charges (multi-charge system) q_1, q_2 , q_3 , ………………………, q_n in free space. Then, the net electric potential at a point in space due to multi charges can be obtained by algebric sum of electric potentials due to each charge at given point i.e.

$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots\ldots + V_n$$

Electric potential difference:-

Electric potential difference between any two points in electric field region about a source charge is define as the total amount of work done in moving unit +ve test charge from one point to another point against electrostatic force.

Or,

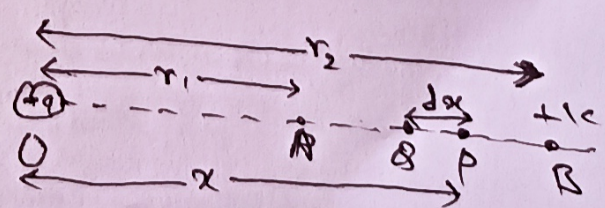
Further, it can also be define as the amount of work done over given charge per unit its charge in order to move it from one point to that another point. i.e. For 'W_{AB}' be total work done in moving 'q_0' charge from point 'A' to 'B' point then potential difference between point 'A' to 'B' is denoted by V_{AB} is given by,

$$V_{AB} = \frac{W_{AB}}{q_0} \quad \text{Its unit is J/C or Volt(V)}$$

It is a scalar quantity.

Expression for electric potential due to point charge:-

Let, us consider a +ve point source charge 'q' placed in a space at 'O' point. Further , 'A' and 'B' are the two points in electric field region of given source charge at distances 'r_1' and 'r_2' respectively. In order to find the electric potential difference between these two points. i.e.'V_{AB}'.



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let us consider a unit +ve test charge at point 'p' which is at 'x' distance from source charge and between two points 'A' and 'B' as shown in figure.

Then,
Columb force experience by given unit charge at 'p' point is

$$F=\frac{q.1}{4\pi\epsilon_0 x^2}=\frac{q}{4\pi\epsilon_0 x^2} \quad \text{............. 1}$$

For 'Q' be a nearest point to 'p' at very small distance 'dx', then small amount of work done against force to move unit test charge through 'dx' distance is;

$$dW = -F.\, dx \quad \text{.............................. 2}$$

where, -ve sign is taken as displacement of charge is done against force and oppositely directed.

From eq^n 1. and 2.

$$dW=\frac{-q}{4\pi\epsilon_0 x^2}dx \quad \text{................... 3}$$

Now, the total work done in moving unit charge from point 'B' to point 'A' is

$$W=\int_{r_2}^{r_1} dW=\int_{r_2}^{r_1}\frac{-q}{4\pi\epsilon_0 x^2}dx$$

$$\text{or,}\quad W=\frac{-q}{4\pi\epsilon_0}\int_{r_2}^{r_1}\frac{1}{x^2}dx$$

$$\text{or,}\quad W=\frac{-q}{4\pi\epsilon_0}\left|\frac{-1}{x}\right|_{r_2}^{r_1}$$

$$W=\frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]=V_{BA}$$

This is required expression for a electric potential between any two points due to point charge in space. Clearly, electric potential at a point in space depends on magnitude of source charge.

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Electric potential energy (U):-

Electric potential energy at any point in electric field region of any source charge is define as the total amount of work done in moving 'q' amount of charge from infinity to given point against electrostatic force.

For 'V' be electric potential at a point in space due to 'q_0' source charge (i.e. work done over unit test charge carrying from infinity to given point) then electric potential energy for test charge 'q' at given point is given by,

$U=qV$ Its unit is Joule (J) or electronVolt (eV).

essentially, 1 eV= 1.6 X 10^{-19} J

For a system of two charges 'q_1' and 'q_2' separated at r_{12} distance in space. The potential energy is given by;

$$U_{12}=q_1V_2$$

$$\text{or,}\quad U=\frac{q_1q_2}{4\pi\epsilon_0 r_{12}^2}$$

Simillarly, for system of three charges; electric potential energy is,

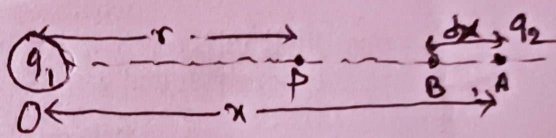
$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2} + \frac{q_2 q_3}{4\pi\epsilon_0 r_{23}^2} + \frac{q_1 q_3}{4\pi\epsilon_0 r_{13}^2}$$

For 'n' charge system with number of charges; q_1, q_2 , q_3 , ..., q_n ; total electric potential energy is,

$$U = \sum_{i=j=1;\, i\neq j}^{n} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}^2}$$

Expression for electric potential energy between two charge syatem:-

Let, us consider a +ve point charge 'q_1' placed in a space at 'O' point. For 'p' be the point in electric field region at 'r' distance from point 'O' where it is required to find the electric potential energy with another charge 'q_2'.



For this , Let, us consider 'q_2' charge at point 'A' which is at 'x' distance from charge 'q_1' within its electric field region as shown in figure.

Then,

Columb force experience by given charge 'q_2' at 'A' point is

$$F = \frac{q_1 \cdot q_2}{4\pi\epsilon_0 x^2} \quad \text{.....................} \; 1$$

For 'B' be a nearest point to 'A' at very small distance 'dx', then small amount of work done against force to move unit test charge through 'dx' distance is;

$$dW = -F.\, dx \quad \text{.........................} \; 2$$

where, -ve sign is taken as displacement of charge is done against force and oppositely directed.

From eq^n 1. and 2.

$$dW = \frac{-q_1 . q_2}{4\pi\epsilon_0 x^2} dx \quad \text{..............................} \; 3$$

Now, the total work done in moving unit charge from infinity to point 'p' is

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} \frac{-q_1 q_2}{4\pi\epsilon_0 x^2} dx$$

$$\text{or, } W = \frac{-q_1 q_2}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{1}{x^2} dx$$

$$\text{or, } W = \frac{-q_1 q_2}{4\pi\epsilon_0} \left| \frac{-1}{x} \right|_{\infty}^{r}$$

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$$\text{or, } W = \frac{q_1 q_2}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$W = \frac{q_1 q_2}{4\pi\epsilon_0 r} = U$$

This is required expression for a electric potential energy due to point charge in space. Clearly, electric potential energy at a point in space depends on magnitude of both charges in space.