Purpose & application of Instrumentation:-

1) In trade / commercial purpose
To ensure the commercial unit is producing standard commodities.

2) For monitoring purpose:
To monitor some critical requirements.
Eg- Temperature of green house
heartbeat of serious patient

3) Automatic control system
- to maintain room temperature.

Classification of ~~Instrumentation or~~ Measuring Instrument

Instruments (Device used for measuring unknown quantity)

- Absolute Instrument
- Secondary Instrument

Absolute Instrument → Determines the magnitude of the quantity to be measured in terms of instrument parameter. (constant) & deflection

→ Time consuming
→ Measuring quantity varies.
eg- Tangent Galvanometer.

Secondary Instrument:-

This instrument determines the value of the quantity to be measured directly. Examples are:- voltmeter, ammeter & wattmeter. Practically secondary instruments are suitable for measurement. (Generally these instruments are calibrated by comparing with another standard secondary instrument).

Secondary Instruments

- Indicating
- Recording
- Integrating
- Electromechanically Indicating

Indicating Instrument:-

These instruments make use of the dial & pointer for showing or indicating magnitude of the unknown quantity.

eg- Voltmeter, ammeter, wattmeters.

Recording:-

These instruments records the magnitude of the quantity to be measured continuously over a specified period of time.

For eg- we may have a recording voltmeter in a substation which keeps record of the variations of the supply voltage during the day.

Integrating:-

These instruments totalize events over a specified period of time. It measures the total quantity of electricity delivered over period of time.

eg- Ampere hour & watt hour (energy) meters.

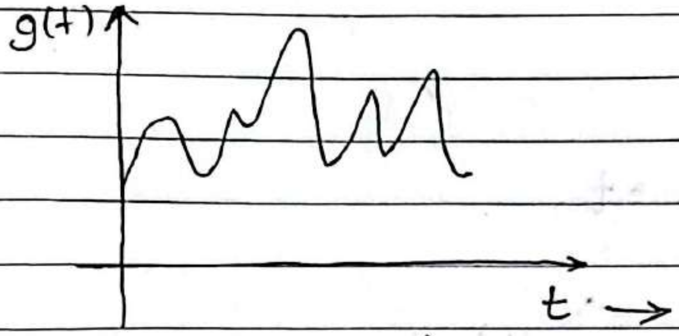
Types of Signals in Instrumentation:-

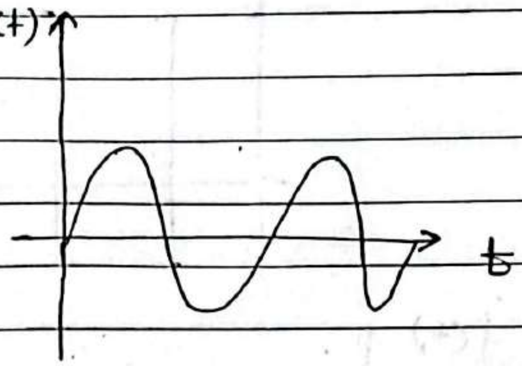
→ Any time varying physical quantity that convey information is called signal.

Classification of Signals:-

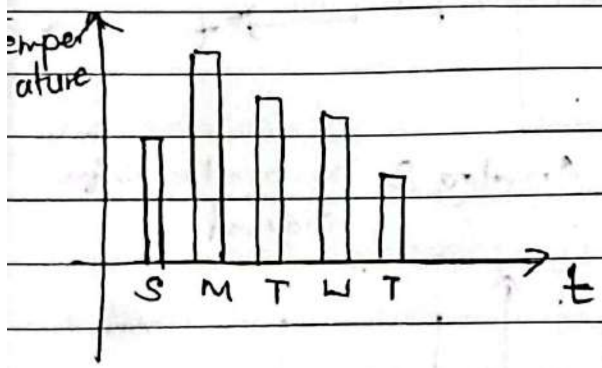
a) Continuous, discrete and digital signal & analog signal

→ If the signal is specified for every value in time, then it is known as the continuous time signal.

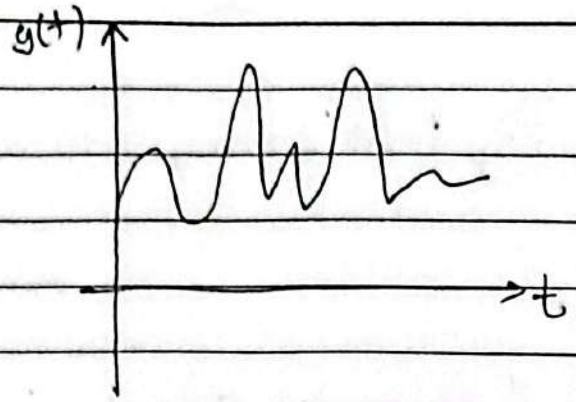


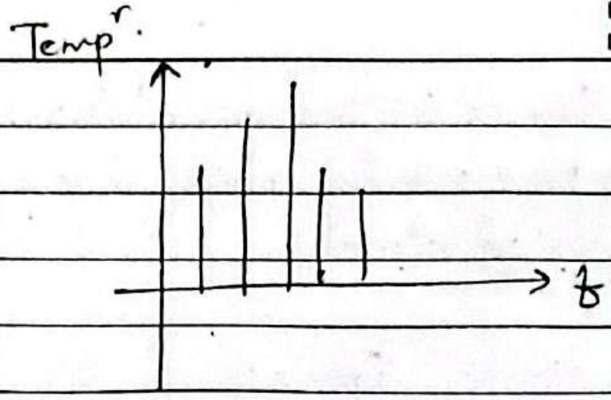


→ If the signal is specified only for discrete time instances then it is called discrete time signal.

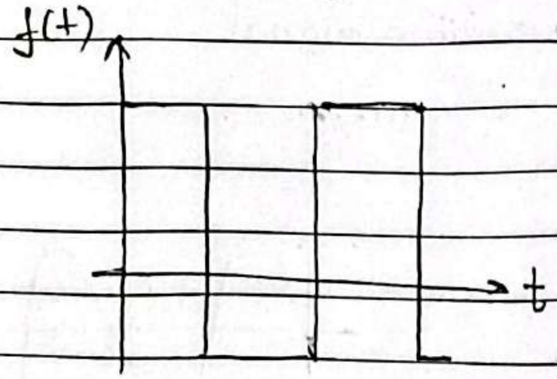


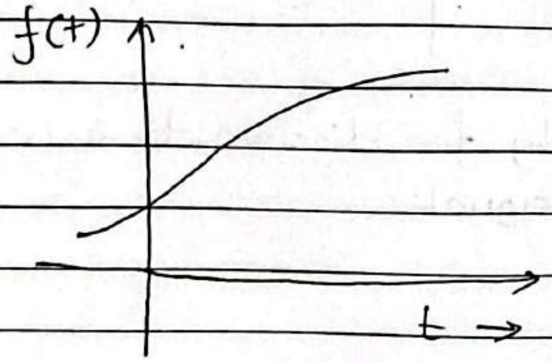
→ A signal whose amplitude (amplitude continuously varying w.r.t) can take any value in the continuous range is the analog signal.



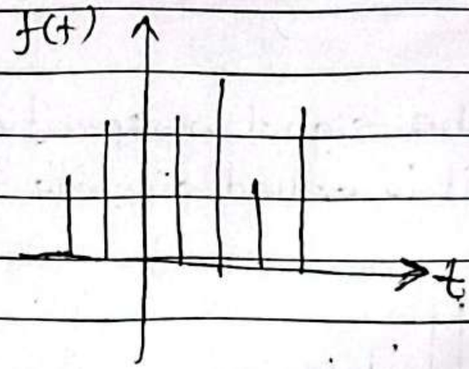


→ A signal whose amplitude can take only finite number of values is the digital signal.

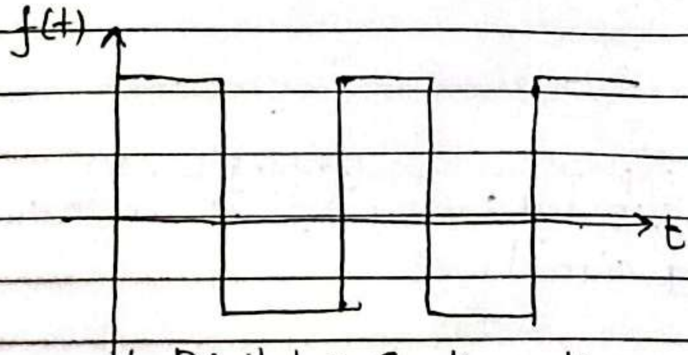




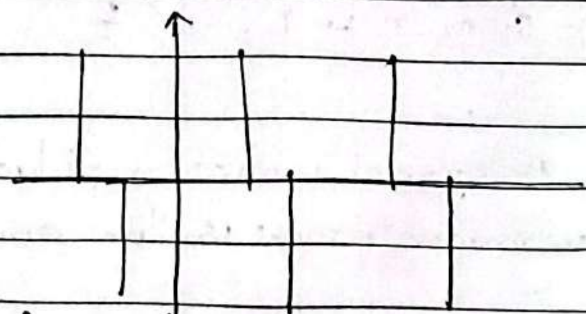
Analog & Continous time signal



Analog & Discrete time signal



Digital & Continous time signal



Digital & discrete time signal

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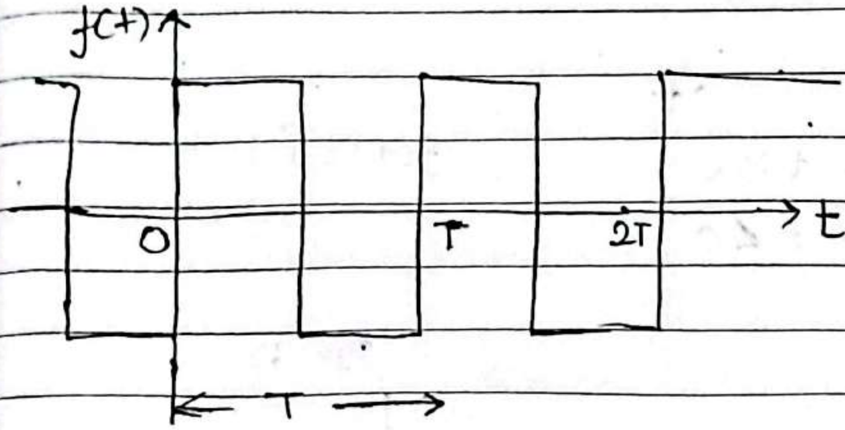
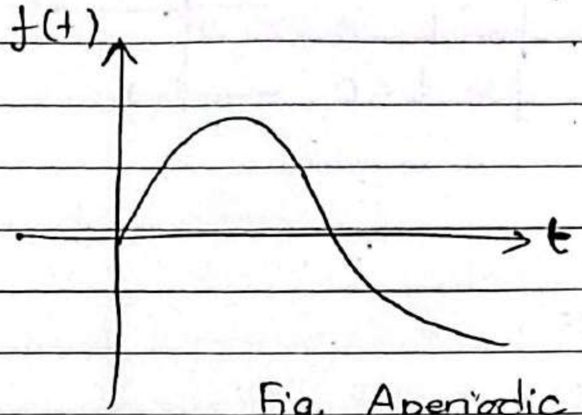
Periodic & Aperiodic Signals

A signal which repeats itself after finite time T is called periodic signal.



$$f(t) = f(t+T)$$

Fig. Periodic Signal



$$f(t) \neq f(t+T)$$

Fig. Aperiodic signal.

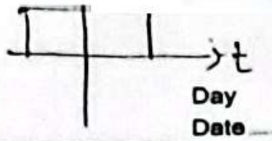
Deterministic & Random Signal:-

→ A signal whose physical description is known completely either in mathematical form or graphical form is known as the deterministic signal.

Random→ A signal which is known only in terms of the probabilistic description like mean, mean square value & distributions is known as the random signal.
eg- Noise Signal

Even & odd signals :- Even:- $x(t) = x(-t)$

odd :- $x(t) = -x(-t)$



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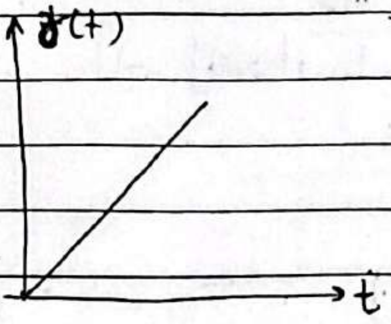
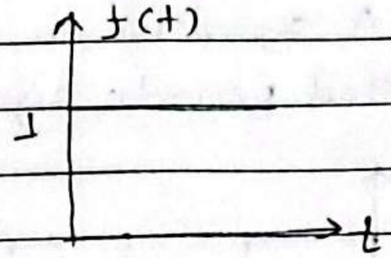
4) Ramp Signal :-



$$f(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

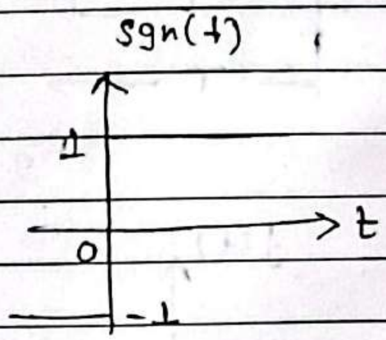
* Unit Step :-



$$f(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

5) Signum function

$$\text{sgn}(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t = 0 \\ -1 & \text{for } t < 0 \end{cases}$$



6) Exponential Signal

$$x(t) = e^{at}$$

i) $a = 0$, $x(t) = 1$

ii) $a < 0$, $x(t) = e^{-at}$

iii) $a > 0$, $x(t) = e^{at}$

Other signals includes rectangular signal, triangular signal, sinusoidal signal.

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Standard of measurements :-

Standards are fundamental reference for which all other measuring devices are compared.
On the basis of their function & application, the standards can be classified as:

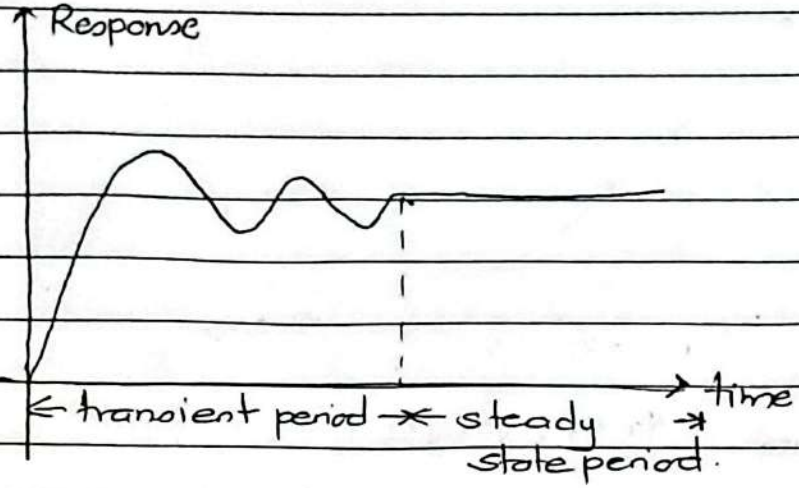
1) International Standard:-
These are maintained by International Bureau of weight & measurements (technological & scientific methods)

2) Primary Standard:-
They are maintained by national standards laboratories. Main function is calibration of secondary standard.

3) Secondary Standards:-
Basic reference standard used in industrial measurement laboratories & check against the primary standard on the periodic basis. They calibrate working standards.

4) Working Standards:
Used frequently in workshop & labs.

Static & dynamic characteristics of measurement system



When an input is applied to a system, the response doesn't take it's final value immediately, there will be some delay in the time known as transient time/period. The behaviour shown by the system during this period is known as dyamic characteristics. The time when the response takes it's steady value or maximum value is known as steady state period & the behaviour shown by the system during this period is known as static characteristics.

The static characteristics of the measurement system (set of criteria defined for the instruments which are used to measure quantities which are slowly varying with time or mostly constant) are :-

1) Accuracy
2) Precision or repeatability
3) Resolution
4) Sensitivity
5) Linearity
6) Hysteresis
7) Bias
8) Range or span
9) Tolerance
10) Stability
11) Threshold
11) Drift

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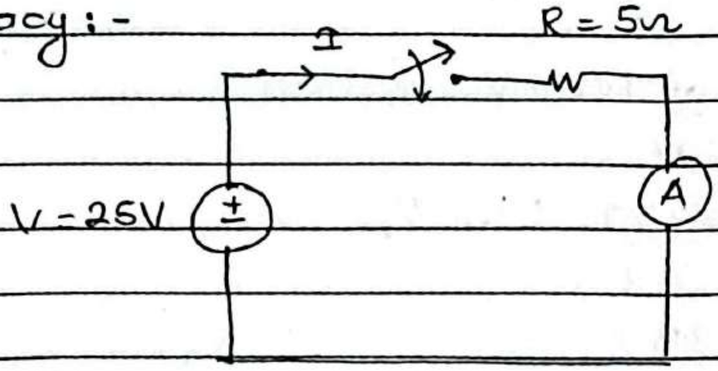
1) Accuracy :-



$$T.V \text{ of } I = \frac{V}{R} = \frac{25}{5} = 5A$$

$A_1 \rightarrow 4.7A$, $A_2 \rightarrow 3A$

The accuracy of a measurement specifies the difference between the measured value & true value of the quantity. Accuracy is the closeness with which an instrument reading approaches to the true value of the quantity. Thus accuracy of the measurement means conformity to the truth. Deviation from the true value is the integration of how accurately a measurement has been done (The concept of accuracy usually refers to full scale reading unless specified.)

Q) If a voltmeter having accuracy of 1% & full scale reading 100V is used to measure:-

1) 80V
2) 12V

how accurate will the readings be. Comment on answer.

1) T.V = 80V.

$M.V = (80 \pm 1\% \text{ of } 100)\ V$

$= 81V$ or $79V$

$$\% \text{ Error} = \frac{T.V - M.V}{T.V} \times 100\%$$

$$= \frac{80 - 79}{80} \times 100\% = 1.25\%$$

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(11) T.V = 12V

M.V = (12 ± 1% of 100) V

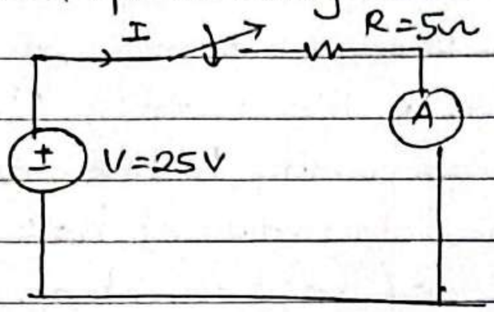
= 13 V or 11 V

$$\% \, \varepsilon = \frac{12-11}{12} \times 100\%$$

= 8.33%

→ Range selection plays a great role in the reduction of error. The range of the meters should be very close to the true value.

(11) Precision or Repeatability :-



T.V = 5A

A_1 → 4.6A, 4.5A, 4.7, 4.4 - - (Precise & accurate)

A_2 → 3, 3, 3, 3 (Precise only)

Precision specify the repeatability of a set of reading each made independently with the same instrument.
To differentiate betⁿ the accuracy & precision, consider an instrument that has defect in it's operation. The instrument may be giving a result that is highly repetable from measurement but far from true value. The data obtained from these instrument would be highly precised but inaccurate. Therefore precision doesn't guarantee the accuracy although accuracy requires precision.

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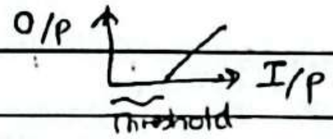
Threshold:- If the i/p of the instrument is ~~vera~~ increased ...y gradually from zero, there will be some minimum value below which no o/p can be detected.



(iii) Resolution :-

If the input to an instrument is slowly increased from an arbitary input value, it will be observe that o/p doesn't change until a certain increament is exceeded. This increment is termed as resolution. Thus resolution is defined as smallest change in input which results in a detectable output. In case of analog instrument, it is the significance of the smallest division on the scale whereas in the case of digital meter, it is significance of LSB.

iv) Sensitivity:-

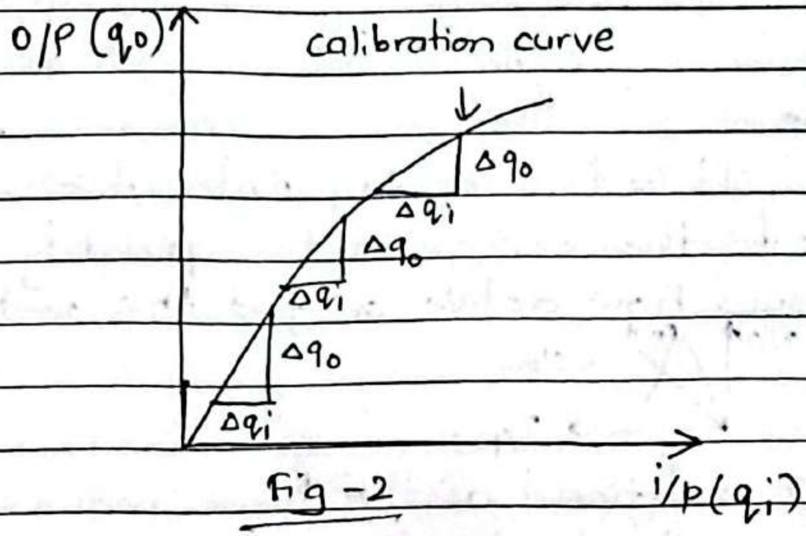


Fig -2

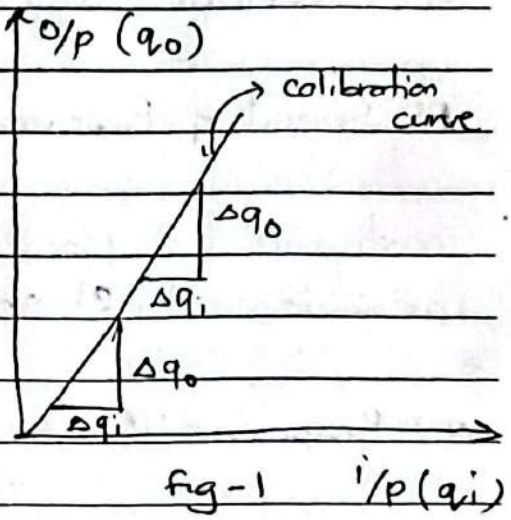


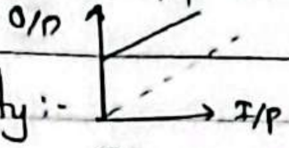
fig-1

Sensitivity of an instrument is the ratio of the magnitude of the output to the mag. of input. If the calibration curve has constant slope, then the sensitivity of the device remains constant throughout the entire range of the instrument as shown in fig (1). However if the calibration curve is not a straight line as shown in fig 2, then the sensitivity varies from range to range.

In general,

$$\text{Sensitivity}\ (S) = \frac{\text{small change in o/p}}{\text{small change in i/p}} \qquad \therefore S = \frac{\Delta q_o}{\Delta q_i}$$

Gradual shift in the indication(O/p) over a period of time where in the i/p variable doesn't change. (environmental factors)



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V) Linearity:-

It means constant sensitivity throughout the entire measurement range. It is defined as the ability to reproduce the input characteristic symmetrically & this can be expressed by the equation.

$$y = mx + c$$

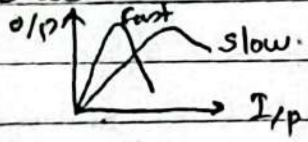
$y \rightarrow$ output, $x \rightarrow$ input, $m \rightarrow$ slope, $c \rightarrow$ intercept

The dynamic characteristics of measurement system are:- (A set of criteria defined for the instruments which are changing with time)

1) Speed of Response
2) Response time
3) Measuring lag

1) Speed of Response:-

It is the rapidity with which an instrument responds to the change in the quantity under measurement. It shows how active or fast the instrument is.



2) Response Time:-

It is defined as the time required by the instrument to settle to it's final steady position after the application of input.

3) Measuring Lag:-

An instrument doesn't react the immediately to a change. It is defined as the delay in the response of an instrument to a change in the measured quantity.

4) Fidelity. degree of closeness with which the system indicates or records the signal which is impressed upon it. ... to reproduce o/p in the same way as i/p.

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Meaasurement of R, L & C with Bridge Circuit:-

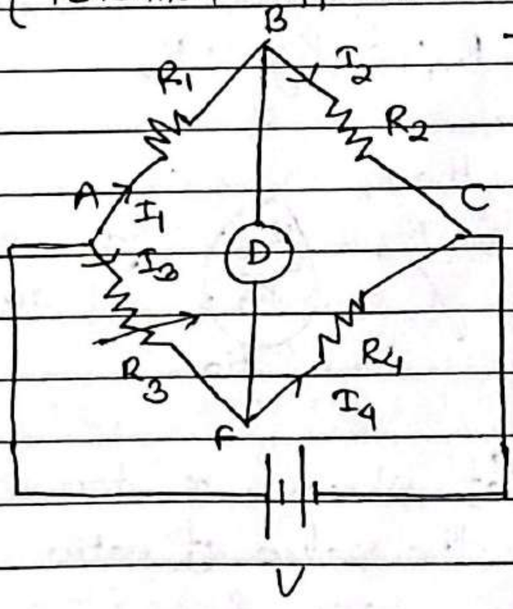
Resistance can be measured with the help of dc bridge whereas for the measurement of inductance & capacitance ac bridge is used.

Measurement of Resistance:-

Resistance can be measured with the help of Wheatstone Bridge. It neither measures low resistance nor high resistance. Only medium resistance from 1Ω to few $M\Omega$. ($1\Omega \rightarrow 0.1 M\Omega$) ($100k\Omega$)

It consists of four arms known as resistors & the ratio of the two of these resistors is kept at a fixed value. The remaining two arms are balanced, one of them is an unknown resistor while the other resistance of the other arm can be varied. The unknown resistance is then computed through balancing or null condition. (Potential difference betn. points B & F is zero)

→ no current passes through M (balanced condition)



When the bridge is balanced:-

$V_{AB} = V_{AF}$ (B & F points are in equal potential)

$\Rightarrow I_1R_1 = I_3R_3$ — (1)

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As no current flows through the detector,

$$I_1 = I_2 = \frac{V}{R_1 + R_2} \quad — (2)$$

$$I_3 = I_4 = \frac{V}{R_3 + R_4} \quad — (3)$$

From (1), (2) & (3)

$$\frac{\cancel{V}}{R_1 + R_2} \cdot R_1 = \frac{\cancel{V}}{R_3 + R_4} \cdot R_3$$

$$\Rightarrow \cancel{R_1 R_3} + R_1 R_4 = \cancel{R_1 R_3} + R_2 R_3$$

$$\Rightarrow R_1 R_4 = R_2 R_3 \quad — (4)$$

Eq^n IV gives the necessary condition for balanced. i.e. If the bridge is balanced then the product of resistances of any two opposite arms must be equal to that of other two

From eq^n (4),

$$R_4 = \frac{R_2}{R_1} \cdot R_3$$

If $R_4 = R_x$ = unknown, then,

$$\therefore R_x = \frac{R_2}{R_1} \cdot R_3 \quad — (4a)$$

unknown arm Ratio arms standard arm

For the measurement of unknown resistance R_x, it is placed in Arm-4 & then the value of ratio arms resistance R_1 & R_2 are same. Say $R_1 = 100\Omega$ & $R_2 = 50\Omega$. Thus, the ratio $R_2/R_1 = \frac{50}{100} = 0.5$.

Now we go on changing the value of R_3 until the detector current is zero.

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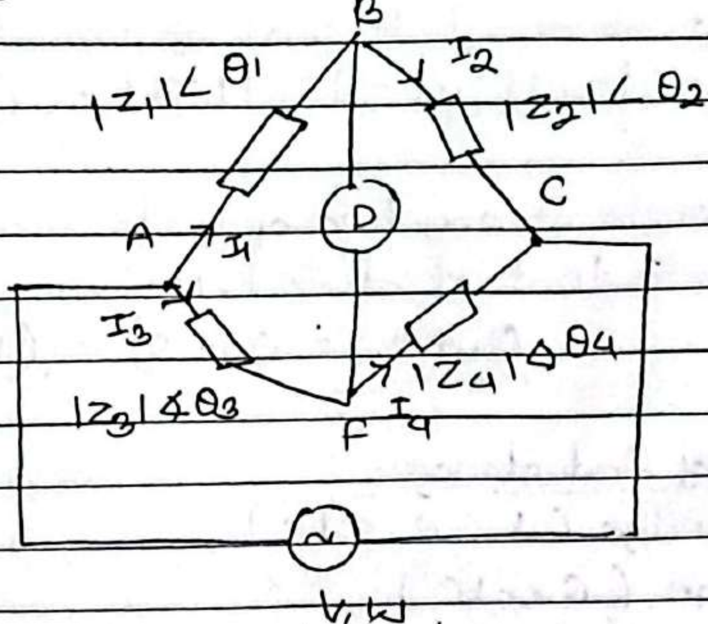
Suppose for $R_3 = 1000\Omega$, current becomes zero. So unknown resistance

$R_a = \frac{1}{2} \times 1000 = 500\Omega$. In this way we can

measure unknown value of resistances.

Balance condn. for AC bridge:-



When the bridge is balanced,

$$V_{AB} = V_{AF}$$

$$\Rightarrow I_1 |Z_1| \angle \theta_1 = I_3 |Z_3| \angle \theta_3 \quad \text{— (1)}$$

As no current flows through the detector (in balanced condn.)

$$I_1 = I_2 = \frac{V}{|Z_1| \angle \theta_1 + |Z_2| \angle \theta_2} \quad \text{— (2)}$$

$$I_3 = I_4 = \frac{V}{|Z_3| \angle \theta_3 + |Z_4| \angle \theta_4} \quad \text{— (3)}$$

From (1), (2), (3)

$$\frac{V}{|Z_1| \angle \theta_1 + |Z_2| \angle \theta_2} \cdot |Z_1| \angle \theta_1 = \frac{V}{|Z_3| \angle \theta_3 + |Z_4| \angle \theta_4} \cdot |Z_3| \angle \theta_3$$

$\Rightarrow |Z_1||Z_3|\angle(\theta_1+\theta_3)+|Z_1||Z_4|\angle(\theta_1+\theta_4) = |Z_1||Z_3|\angle(\theta_1+\theta_3)$
$+|Z_2||Z_3|\angle(\theta_2+\theta_3)$

$$\therefore \quad |Z_1||Z_4|\angle(\theta_1+\theta_4) = |Z_2||Z_3|\angle(\theta_2+\theta_3) \quad -(4)$$

Eq^n (4) gives the necessary condition for ac bridge to be balanced.

Thus, there are two conditions

1) The product of magnitude of any two opposite arms impedance must be equal to that of other two.

$$\text{i.e.} \quad |Z_1||Z_4| = |Z_2||Z_3| \quad -(4a)$$

2) Sum of the angle of any two opposite arms impedance must be equal to that of other two.

$$\theta_1+\theta_4 = \theta_2+\theta_3 \quad -(4b)$$

Measurement of Inductance:-

I) Maxwell's Bridge $(1 \le Q \le 10)$

II) Hay's Bridge $(Q \geq 10)$

Maxwell bridge is used for measurement of inductance of the coil having moderate quality factor $(1 \le Q < 10)$. It is neither used for measurement of high quality factor nor low quality factor.

Hay's Bridge is used for measurement of inductance of coil having ~~low~~ high quality factor $(Q \geq 10)$.

The quality factor of a coil is given by.

$$Q\text{-factor} = \tan\theta = \frac{X_L}{R} = \frac{\omega L}{R}$$

For high quality factor,

$X_L >> R$ i.e. R is negligible. Hence for a high

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quality factor coil,

$$\tan\theta \approx \frac{X_L}{R} = \frac{X_L}{0}$$

$$\therefore \theta \approx +90^\circ$$

Thus for high quality factor coil, angle betn voltage & current is nearly equal to 90°.

Impedance diagram of RL series ckt is given by:-

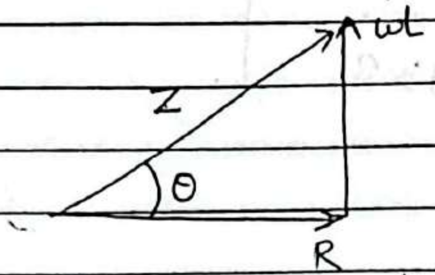


Fig- Impedance diagram of RL series ckt

For RC series ckt:-

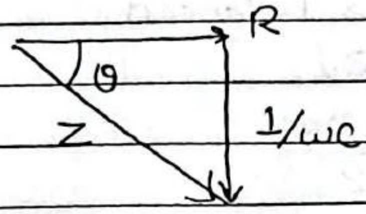
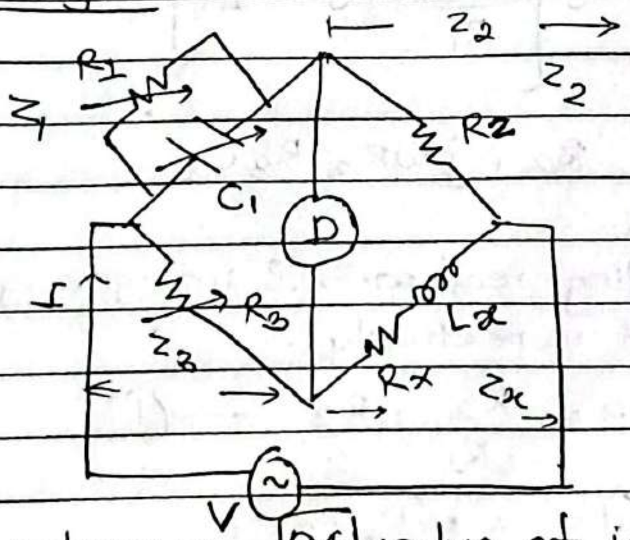


fig. 2. Imp. diagram for RC series ckt.

Maxwell's Bridge:-



→ Measures an unknown ~~or~~ value of inductance by comparing it with a known inductance or standard inductance.

* Four arms → four impedances (three known, one - - -)
* Detector → for balancing circuit

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$$Z_2 = R_2, Z_x = R_x + jX_{Lx} \qquad Z = R + jX_L$$

$$= R_x + j\omega L_x, \quad Z_3 = R_3$$

Note:
$$Z_{C_1} = -jX_{C_1}$$
$$= \frac{-j}{\omega C_1} * j/j$$
$$= \frac{1}{j\omega C_1}$$

$$Y_{C_1} = \frac{1}{Z_{C_1}} = j\omega C_1$$

$$Y_1 = \frac{1}{R_1} + j\omega C_1$$

$$Z_1 = R_1 // \frac{1}{j\omega C_1}$$
$$\Rightarrow \frac{R_1 * \frac{1}{j\omega C_1}}{R_1 + 1/j\omega C_1}$$
$$= \frac{R_1}{1 + j\omega R_1 C_1}$$

When the bridge is balanced,

$$Z_1 Z_x = Z_2 Z_3$$
$$\Rightarrow Z_x = \frac{Z_2 Z_3}{Z_1}$$
$$= Z_2 Z_3 Y_1$$

$$\Rightarrow (R_x + j\omega L_x) = R_2 R_3 \left[\frac{1}{R_1} + j\omega C_1\right]$$

$$\Rightarrow R_x + j\omega L_x = \frac{R_2}{R_1} R_3 + j\omega R_2 R_3 C_1.$$

Equating corresponding real parts & imaginary parts on both sides, we get respectively,

$$R_x = \frac{R_2}{R_1} \cdot R_3 \quad \text{—— (1)}$$

$$\& \ \cancel{\omega} L_x = \cancel{\omega} R_2 R_3 C_1$$

$$\therefore L_x = R_2 R_3 C_1$$

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In this way, we can measure unknown value of inductance.

Explanation:-

The sum of the angle of arm-2 & arm-3 is 0, so the sum of angle of R_1 and R_x must be zero. If the high quality factor coil is taken, then the angle of R_x is nearly equal to 90°. So the angle of arm 1 must be nearly equal to -90°. For this the resistance value of resistor R_1 must be big as much higher as possible & as we know the cost of decayed resistance box increases with the increase in the resistance value. Hence this bridge become impractical (uneconomical) for the measurement of inductance of the coil having high quality factor.

Maxwell bridge is also not used for measurement of inductance of the coil having low quality factor due to the convergence problem, i.e. it becomes very difficult to obtain balanced condition as balance condition go on shifting.

Advantages of Maxwell's Bridge:-

1) The frequency doesnot appear in the final expression, hence it is independent of frequency.

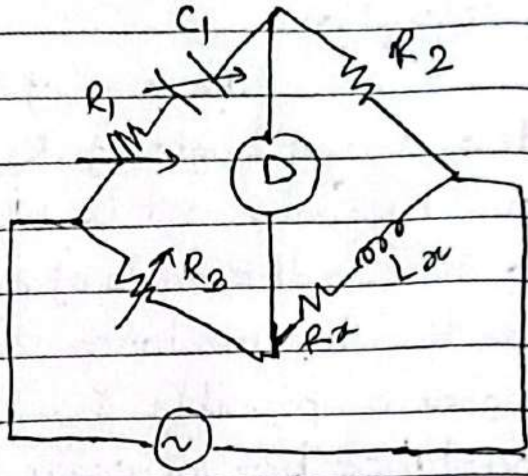
2) It is very useful for the wide range of measurement of inductor value at audio & power frequency.

Disadvantages → Very expensive
→ Limited to measurement of low quality coils.

Application → communication systems, electronic circuits, measure medium quality coils, power conversion circuits, filter circuits, instrumentation.

Hay's Bridge:- [$Q \geq 10$]:-

It is used for measurement of inductance of coil having high quality factor.



$Z_x = R_x + j\omega L_x$, $Z_2 = R_2$

$Z_1 = R_1 - jX_{C_1} = R_1 - \dfrac{1}{j\omega C_1}$

$Z_3 = R_3$

When the bridge is balanced,

$$Z_1 Z_x = Z_2 R_3$$

$$\Rightarrow \left(R_1 - \frac{1}{j\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

$$\Rightarrow R_1 R_x + j\omega L_x R_1 - j\frac{R_x}{\omega C_1} + \frac{L_x}{C_1} = R_2 R_3$$

$$= \left[R_1 R_x + \frac{L_x}{C_1}\right] + j\left[\omega L_x R_1 - \frac{R_x}{\omega C_1}\right] = R_2 R_3$$

Equating corresponding real parts & imaginary parts from both sides respectively,

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad \text{--- (1)}$$

$$\omega L_x R_1 - \frac{R_x}{\omega C_1} = 0 \quad \text{--- (2)}$$

From (2),

$$R_x = \omega^2 L_x R_1 C_1 \quad \text{--- (3)}$$

From (1) & (3)

$$\omega^2 R_1^2 C_1 L_x + \frac{L_x}{C_1} = R_2 R_3$$

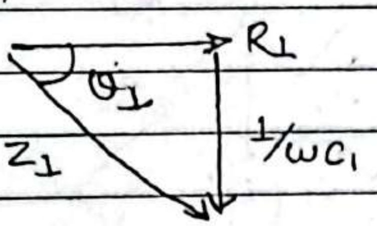
$$\Rightarrow L_x\left[\frac{1+\omega^2 R_1^2 C_1^2}{C_1}\right] = R_2 R_3$$

$$\therefore L_x = \frac{R_2 R_3 C_1}{1+\omega^2 R_1^2 C_1^2} \quad \text{— (4)}$$

From (4) & (3),

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 R_1^2 C_1^2}$$ gives unknown value of reactance.

The impedance diagram of R_1 & R_x respectively are given in 1 & 2.



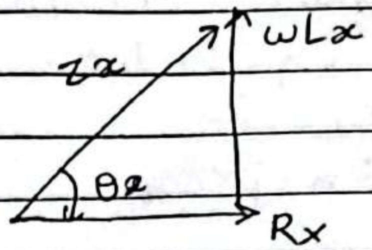


Fig -(1) Imp. diagram of Arm-1   Fig-2 Imp diag. of Arm-x

According to second balanced cond^n in term of angle, we have,

$$\theta_x = \theta_1$$

$$\Rightarrow \tan\theta_x = \tan\theta_1$$

$$\Rightarrow Q = \tan\theta_1$$

$$\Rightarrow Q = \frac{\frac{1}{\omega C_1}}{R_1}$$

$$\therefore Q = \frac{1}{\omega C_1 R_1} \quad \text{— (6)}$$

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From eq^n 4 & 3,

$$L_x = \frac{R_2 R_3 C_1}{1+\left(\frac{1}{Q}\right)^2}$$

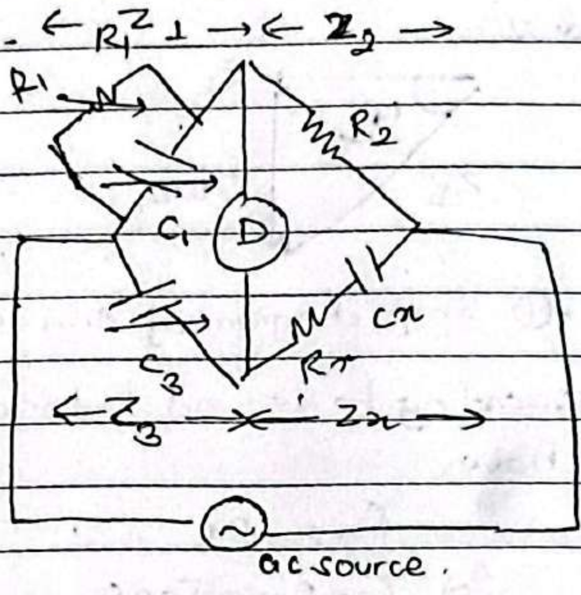
As ---- $Q \geq 10$

Thus, $\frac{1}{Q^2}$ is negligible & hence above eq^n becomes,

$\therefore L_x = R_2 R_3 C_1$ — (7) which is same as Maxwell Bridge.

Measurement of Capacitance:-

Schering's Bridge:-



Let, Z_1

$Z_x = R_x - jX_{C_x}$

$= R_x - j\frac{1}{\omega C_x}$

$Z_2 = R_2$

$Z_3 = -jX_{C_3} = -j\frac{1}{\omega C_3}$

$$Z_1 = \frac{R_1}{1+j\omega R_1 C_1}$$

When the bridge is balanced,

$Z_1 Z_x = Z_2 Z_3$

$\Rightarrow Z_x = Z_2 Z_3 / Z_1$

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$$\Rightarrow R_x - j\frac{1}{\omega C_x} = R_2\left(\frac{-j}{\omega C_3}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$\Rightarrow R_x - j\frac{1}{\omega C_x} = -j\frac{R_2}{\omega C_3 R_1} + \frac{C_1}{C_3}R_2$$

Equal real & imaginary parts from above expression:-

$$\therefore R_x = \frac{C_1}{C_3}R_2 \quad \text{—— } (1)$$

$$\& \; -\frac{1}{\omega C_x} = -\frac{R_2}{\omega C_3 R_1}.$$

$$\therefore \boxed{C_x = \frac{R_1}{R_2}C_3} \quad \text{—— } (2).$$

~~Q) A 1000 Hz bridge has follo~~

Capacitance Bridge:-

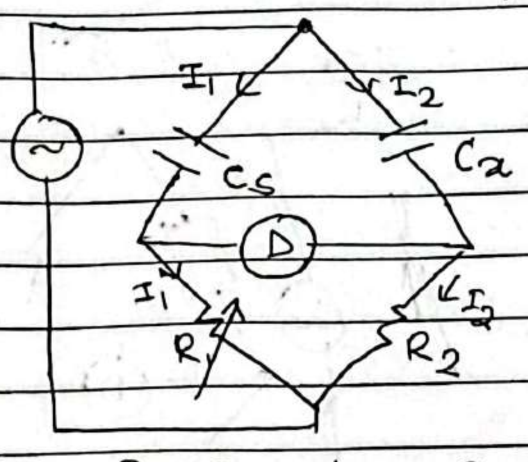

Fig. Capacitance Bridge

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Fig shows the circuit of a simple capacitance bridge. C_s is the Standard resistor, C_x is an unknown capacitance & R_1 & R_2 are standard resistors, one of both which is adjustable. An ac supply is used and the null detector. ~~Q~~ R_1 is adjusted until the null detector indicates zero & when this is obtained, the bridge is said to be balanced.

Working Principle:-

When the detector indicates null, the voltage drop across C_s must be equal that across C_x. & simillarlly, the voltage across Q must be equal to the voltage across P. Therefore,

$$V_{CS} = V_{Ca}$$

$$\Rightarrow i_1 X_{CS} = i_2 X_{C\cancel{x}} \quad — (1)$$

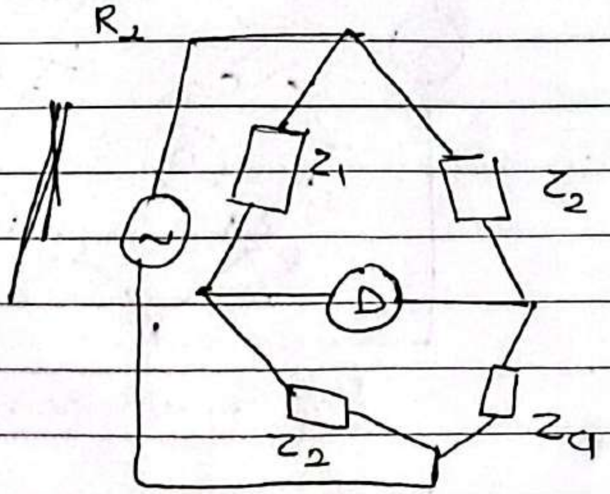
And,

$$V_Q = V_P \quad \text{or}, \; V_{R1} = V_{R_2}$$

$$\Rightarrow i_1 R_1 = i_2 R_2 \quad — (2)$$

Dividing eqⁿ. (1) by (2),

$$\frac{X_{CS}}{R_1} = \frac{X_{CX}}{R_2} \quad — (3)$$



Generalized circuit diagram:

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$$Z_1 Z_4 = Z_3 Z_2$$

$$\Rightarrow \frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} \quad — (4)$$

Substituting $\frac{1}{\omega C_s}$ for X_{CS} & $\frac{1}{\omega C_x}$ for X_{Cx} in (3)

$$\frac{1}{\omega C_S R_1} = \frac{1}{\omega C_x R_2}$$

$$\text{or, } C_x = \frac{R_1 \omega C_S}{R_2 \omega}$$

$$\therefore C_x = \frac{R_1 C_S}{R_2} \quad — (5)$$

It is seen that unknown capacitance C_x now be calculated from the known values of C_S, R_1 & R_2.

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Wien's Bridge

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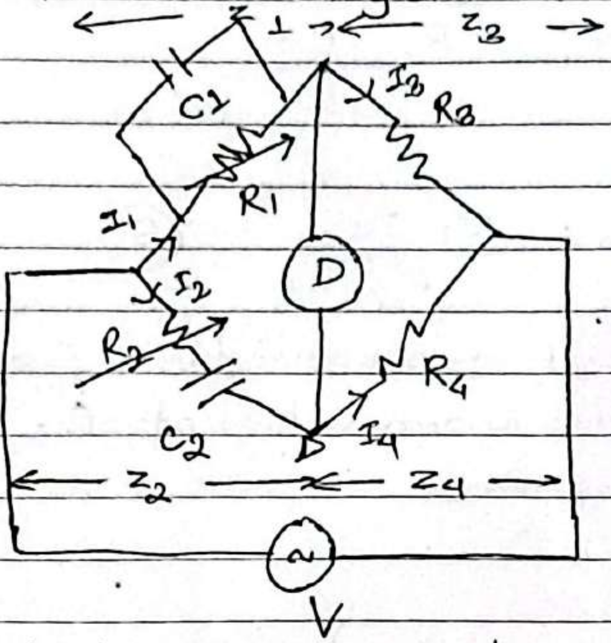
The Wien's bridge is an ac electrical circuit widely used for measuring frequency and can also be used for the measurement of capacitance with high accuracy. The bridge can be used at high voltages but the circuit is sensitive to frequency.

It consists of four arms, one arm with a series combination of resistor & capacitor & another with a parallel combination resistor & capacitor. The other two arms comprises a resistance. The circuit diagram is:-



A balance detector or null indicator is connected across two junctions B & D. The indicator shows null deflection when the bridge is balanced i.e. when the junctions B & D will be at same potential.

Let, Z_1, Z_2, Z_3 & Z_4 be the impedances of arms AB, AD, BC & CD respectively & given as:-

$$Z_1 = \frac{R_1}{1 + j\omega C_1 R_1}$$

$$Z_2 = R_2 - \frac{j}{j\omega C_2}, \quad Z_3 = R_3 \;\&\; Z_4 = R_4$$

When the bridge is balanced, we have,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\Rightarrow \left(\frac{R_1}{1+j\omega C_1 R_1}\right) R_4 = \left(R_2 - \frac{j}{j\omega C_2}\right) \times R_3$$

$$\Rightarrow \frac{R_4}{R_3} = \left(\frac{1+j\omega C_2 R_2}{j\omega C_2}\right)\left(\frac{1+j\omega C_1 R_1}{R_1}\right)$$

$$\Rightarrow \frac{R_4}{R_3} = \frac{1}{j\omega C_2 R_1} + \frac{C_1}{C_2} + \frac{R_2}{R_1} + j\omega C_2 R_2$$

$$\frac{R_4}{R_3} =$$

Equating the real terms,

$$\frac{R_4}{R_3} = \frac{C_1}{C_2} + \frac{R_2}{R_1}$$

The above eq^n. is used to determine the resistance ratio (R_4/R_3)

Now, Equating the imaginary terms,

$$\frac{1}{j\omega C_2 R_1} + j\omega C_2 R_2 = 0$$

$$\Rightarrow -\omega C_2 R_1 + \omega C_2 R_2 = 0$$

$$\Rightarrow \omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\Rightarrow 2\pi f = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad \Rightarrow f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

If suppose the bridge components are chosen such that, $R_1 = R_2 = R$ & $C_1 = C_2 = C$, then the above eq^n is given as:-

$$f = \frac{1}{2\pi RC} \text{ Hz}$$

Application:-

→ Used in determination of the frequency of the applied voltage as well as capacitance.

→ It can be used in frequency determining element in audio & radio frequency oscillators & harmonics distortion analyzer.

Numericals

Static Error ⇒ $\delta A = A_m - A_t$

δA = error,

A_m = measured value of quantity

A_t = true value of quantity

Relative Error =

$$\varepsilon_r = \frac{\overset{\text{Static Error}}{\text{absolute Error}}}{\text{True value}} = \frac{\delta A}{A_t} = \frac{\varepsilon_0}{A_t}$$

Relative error expressed in % i.e. multiplied by 100%.

$$\% \varepsilon_r = \frac{|A_m - A_t|}{A_t} \times 100\%$$

Static Correction ⇒ $\delta C = -\delta A$

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Q) A voltage has a true value of 1.50V. An analog indicating instrument with a scale range of 0–2.5V shows a voltage of 1.46V. What are the values of absolute error & correction. Express the error as a fraction of the true value & full scale deflection (fsd).

Sol^n Absolute Error $\delta A = A_m - A_t = 1.46 - 1.50 = -0.04V$

Absolute Correction $\Rightarrow \delta C = -\delta A = +0.04V$.

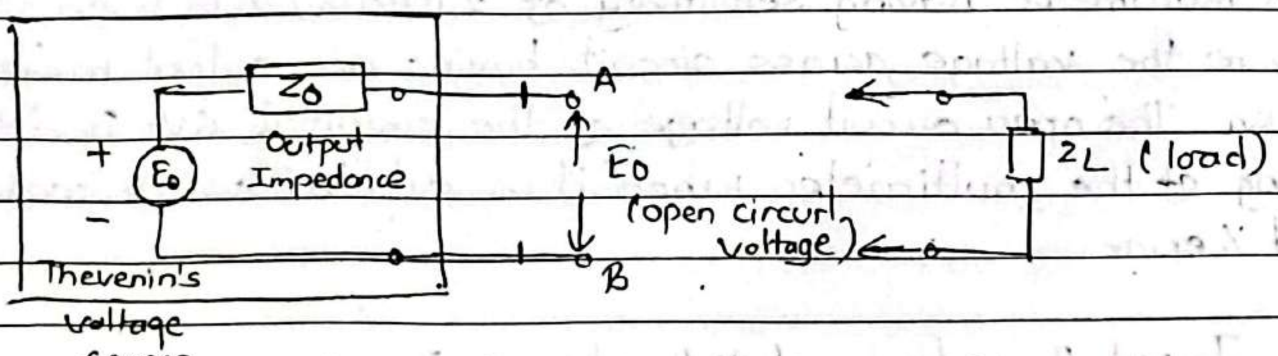
Relative Error $\Rightarrow \epsilon_r = \frac{\delta A}{A_t} = \frac{-0.04}{1.50} \times 100 = -2.66\%$

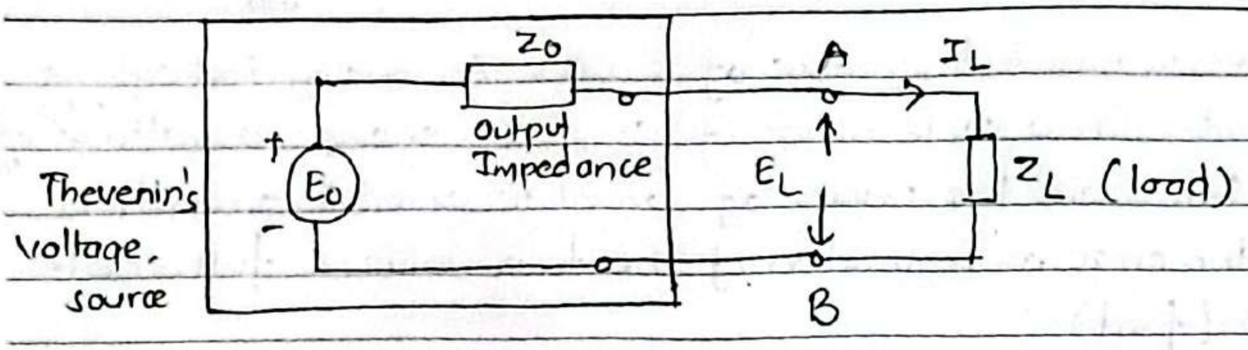
Relative error (expressed as percentage of fsd).

$$= \frac{-0.04}{2.5} \times 100 = -1.60\%$$

Loading Effects due to Shunt Connected Instruments

In measurement systems, voltage measuring, displaying & recording instruments like voltmeters, oscilloscopes etc. are connected across the circuit in shunt (parallel) with the circuit.





When a voltmeter with an input impedance Z_L is connected across A & B, a current I_L flows. This caused a voltage drop $I_L Z_0$.

∴ Output voltage under loaded conditions is:-

$$E_L = E_0 - I_L Z_0 = I_L Z_L$$

$$\text{or, } E_0 = I_L (Z_L + Z_0)$$

∴ Ratio of actual voltage appearing across load to voltage under no load:-

$$\frac{E_L}{E_0} = \frac{I_L Z_L}{I_L (Z_L + Z_0)} = \frac{1}{1 + \frac{Z_0}{Z_L}}$$

∴ Actual voltage measured, $E_L = \dfrac{E_0}{1 + \frac{Z_0}{Z_L}}$

Q) A multimeter having sensitivity of 2,000 Ω/V is used to measure the voltage across circuit having an output resistance of 10 kΩ. The open circuit voltage of the circuit is 6V. Find the reading of the multimeter when it is set to it's 10V scale. Find % error.

Sol

Input Impedance of Voltmeter → $Z_L = 20{,}000 \times 10\,\Omega$
$= 20\,k\Omega$

Output impendance $Z_0 = 10\,K\Omega$

Open circuit voltage of circuit under measurement $E_0 = 6V$

We have,

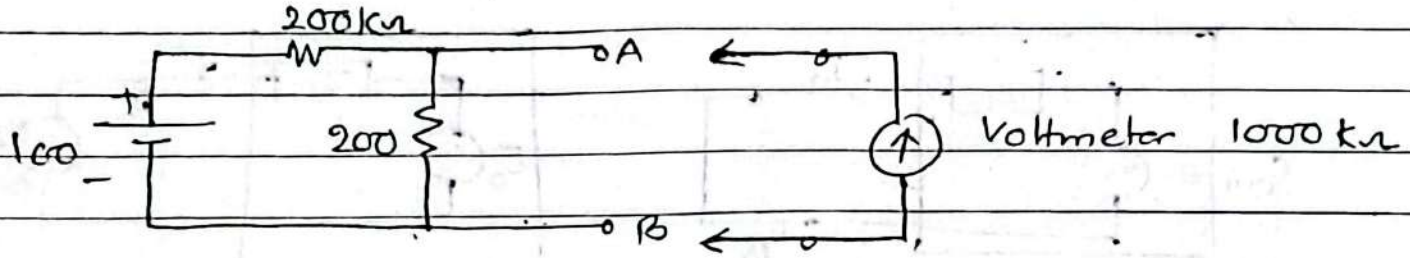
Reading of valtmeter,

$$E_L = \frac{E_0}{1+\frac{Z_0}{Z_L}} = \frac{6}{1+\frac{10}{20}} = 4V$$

$$\text{\% error in voltage reading} = \frac{4-6}{6} * 100$$

$$= -33\% \text{ or } 33\% \text{ low.}$$

Q) A 50V range valtmeter is connected across the terminals A & B of the circuit. Find the reading of the voltmeter under open circuit & loaded conditions. Find the accuracy & the loading error. The valtmeter has resistance of 1000 kΩ.



The open circuit voltage E_0 appearing across terminals A & B is:-

$$E_0 = 100 * \frac{200}{400} = 50V.$$

The output Impedance (resistance) in this case is:-

$$Z_0 = \frac{200 \times 200}{200+200} = 100 k\Omega$$

The equivalent circuit under loaded condition is :- (Zin 1000 KΩ i.e. $Z_L = 1000\ K\Omega$)

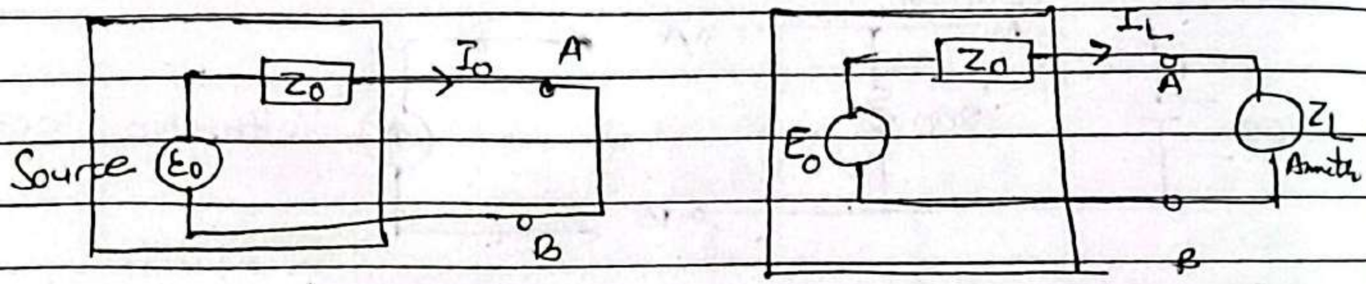
Voltage appearing across terminals A & B under loading condition is:-

$$E_L = \frac{E_0}{1+\frac{Z_0}{Z_L}} = \frac{50}{1+\frac{100}{1000}} = 45.5V$$

$$\text{Loading Error} = \frac{45.5-50}{50} = -9\% = 9\%\ \text{low}$$

$$\text{Accuracy} = 100 - \%\ \text{loading error}$$
$$= 100-9\% = 91\%$$

Loading Effects due to Series Connected Instruments :-



The value of current flowing betn terminals A & B is I_0.

$$I_0 = \frac{E_0}{Z_0}$$

$$E_0 = I_0 Z_0$$

Z_L = input impedance of ammeter

Then, $$I_L = \frac{E_0}{Z_0+Z_L} = \frac{I_0 Z_0}{Z_0+Z_L} = \frac{I_0}{1+Z_L/Z_0}$$

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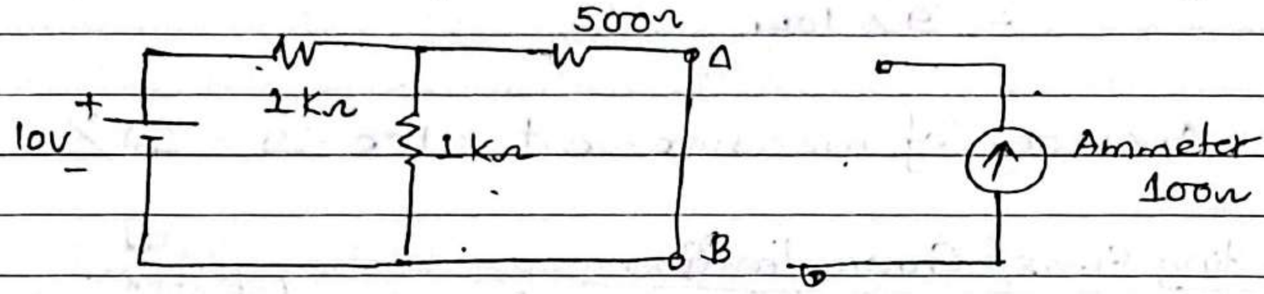
In order that measured value of current, be equal to the actual value of current, I_0, the value of $Z_0 > Z_L$. This means the input impedance of ammeter should be very small as compared with the output impedance of source.

In terms of admittance,

$$I_L = \frac{I_0}{1 + \frac{Y_0}{Y_L}}$$

Q) It is desired to measure value of current in the 500Ω resistor as shown in fig. by connecting a 100 Ω ammeter. Find:

a) actual value of current
b) measured value of current
c) percentage error in measurement & the accuracy.



Let, us reduce the actual circuit to an equivalent Thevenin's source.

Open circuit voltage as applying at terminals A & B is:

$$E_0 = 10 - \frac{10}{2000} \times 1000 = 5V$$

Replacing the sources by their internal resistances.

O/p Impedance of source looking into terminals A & B is

$$Z_0 = \frac{1000 \times 1000}{1000 + 1000} + 500 = 1000\Omega$$

The equivalent ckt (combine 100Ω)

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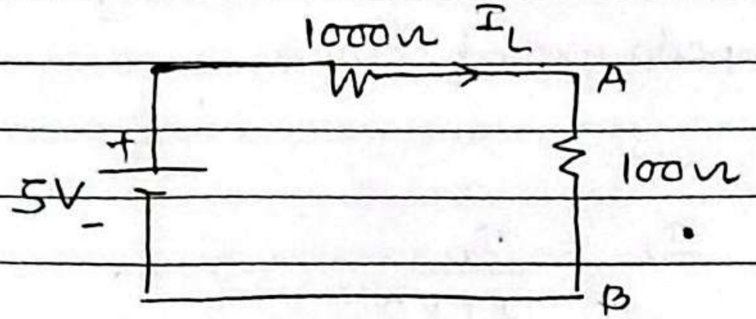
∴ Actual value of current ⇒

$$I_0 = \frac{E_0}{Z_0} = \frac{5}{1000}\ A = 5mA$$



b) When the ammeter is introduced,

$$I_L = \frac{E_0}{Z_0 + Z_L} = \frac{5}{1000+100}\ A = 4.55mA$$

c) $$\text{Error} = \frac{4.55-5}{5} \times 100\%$$

$$= -9\%$$

$$= 9\%\ \text{low}$$

d) Accuracy of measurement $= 100 - 9 = 91\%$

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Unit-3 Physical variable & transducers

Sensor:-

Sensor is a energy converting device. It converts one form into another form. If a sensor converts non-electrical energy, then the sensor is known as transducers.

There are many ways on which the sensor may be classified:

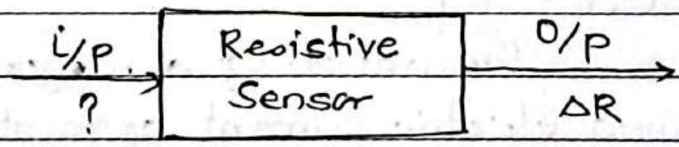
On the basis of requirement of external power supply, sensor may be classified as passive sensor & active sensor. Passive sensor are those which require external power supply whereas active sensor doent need such any external power supply. Thermocouple is one of the example of active sensor whereas potentiometer is the passive sensor.

On the basis of application, sensor may be classified as temperature sensor, pressure sensor, humidity sensor, displacement sensor, optical sensor.

On the basis of physical principle involved, sensor may be classified as:-

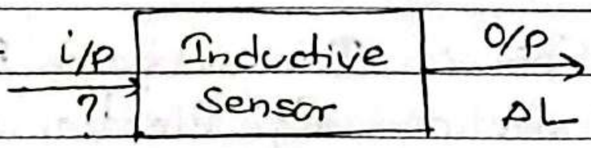
1) Resistive Sensor:-

The input being measured is transformed into change in resistance.



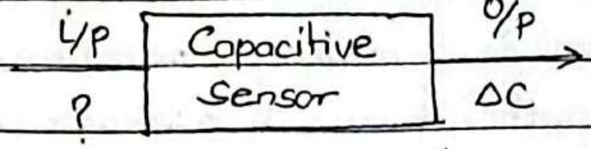
Example are potentiometer, strainguage, resistance thermometer, photoconductive cells.

2) Inductive Sensor



Examples are Linear Variable differential transformer (LVDT)

3) Capacitive Sensor :-

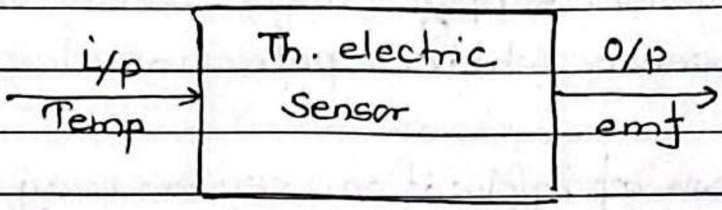


Examples are capacitive displacement sensor, capacitive liquid level sensor.

4) Thermoelectric Sensor :-

For the thermoelectric sensor, the input being heat or temp^r & the output is emf.

Example is thermocouple



5) Peizoelectric Sensor :-

The force applied on a crystal displaces the atoms in the crystal & results in acquring a surface charge. These sensor are used for measurement of transient pressure.

6) Hall Effect Sensor :-

The action of a magnetic field on a flat plate carrying electric current generate a potential difference which will be the measure of strength of the magnetic field.

7) Electromagnetic Sensor :- This works on Faraday's law of Electromagnetic induction. Eg- Electromagnetic flowmeter, hot wire Anemometer.

Physical Variable & their types :-

The measurement of any instrumentation system makes it's first contact with the primary sensing element or an input device.

The physical variables are the quantities required to be measured. All these quantities require primary detection element to be converted into another analogous form which is acceptable by later stage of measurement system.

There are various types of physical variables such as electrical variable, mechanical variable, bio-physical, process variable etc.

The measurand including electrical quantity like current, voltage, resistance, inductance, capacitance etc. are electrical variables.

The mechanical variable include force, pressure, displacement etc. Bio physical variable are included in human beings such as blood pressure, heartbeat

Transducer

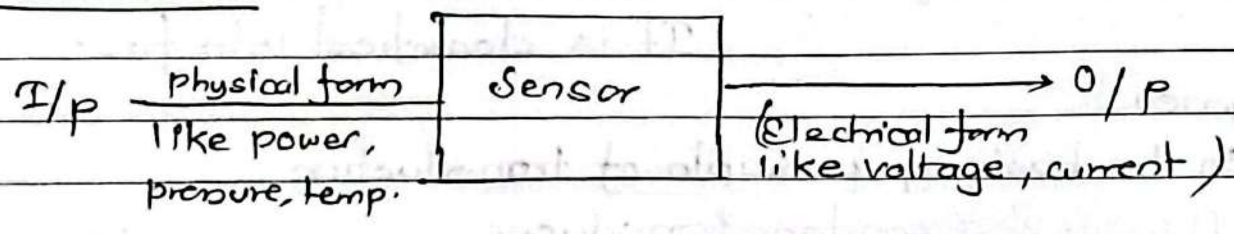


Fig - Transducer

A transducer is defined as the device which converts energy from one form to another form. In electrical measurement system transducer is defined as the device which converts a physical quantity into electrical quantity.

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The input quantity for must instrumentation system is non-electrical quantity. In order to use electrical methods & techniques for measurement, manipulation & control, non-electrical quantity is generally converted into the electrical form by a transducer.

Many physical variable such as light intensity, humidity, heat, liquid level & PH value may also be converted into electrical form by a transducer.

The transducer may consists of two important parts:-

- Sensing element
- transduction element

- A sensing element is that part of transducer which respond to a physical phenomenon.

- The transduction element transforms the o/p of a sensing element to an electrical output.

IMP
Classification of Transducer:-

It is classified into four categories:

I) On the basis of principle of transduction.
II) Primary & secondary transducer
III) Active & Passive transducer
IV) Transducer & inverse transducer

I) On the basis of principle of transduction:-

Transducer can be classified on the basis of principle of transduction as resistive, capacitive, inductive etc.

Depending upon how they convert the input quantity into resistance, capacitance or inductance respectively.

For example → thermistor works on the principle that the resistance vary with the temperature hence used in tempr. measurement.

The capacitor microphone is a capacitive transducer which works on the principle of sound, pressure varies the capacitance betn. fixed plate & diaphgram.

(11) Primary & Secondary Transducer :-

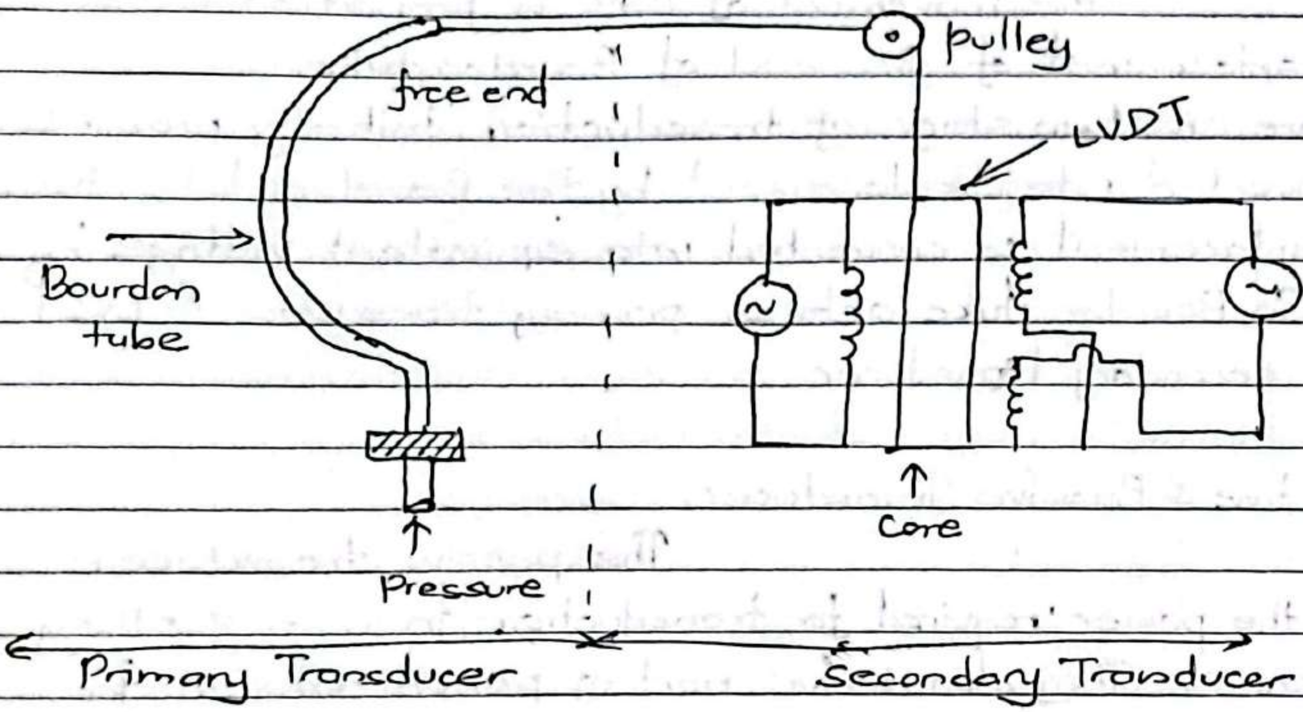

fig- Measurement of Pressure.

A sensing element converts physical variable into it's equivalent electrical o/p. This response depends upon the sensing element. Such type of transducer which converts physical variable into equivalent electrical quantity is

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known as primary transducer.

· In most of the measurement system there is a suitable working combination where a mechanical device acts as a primary transducer & electrical device acts as a secondary transducer.

The figure shown above is the measurement of pressure.

The Bourdon tube acts as a primary sense the pressure & convert it into displacement of it's free end. The displacement of free end moves the core of LVDT which produces an o/p voltage proportional to the movement of core of LVDT.

The movement of core is proportional to the displacement of free end of Bourdon tube.

There are two stages of transduction, initially pressure is converted into displacement by the Bourdon tube, then the displacement is converted into equivalent voltage by LVDT. So, Bourdon tube acts as primary transducer & LVDT acts as secondary transducer.

III) Active & Passive Transducer:-

The passive transducer derive the power required for transduction from an auxillary power source. They also derive part of power required for conversion from the physical quantity under measurement. They are also known as externally powered transducer. They example of passive transducer are resistive, inductive, capacitive transducer. Potentiometer is used for the measurement of displacement, it is resistive transducer powered by a external source. it is used for the measurement of linear

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displacement:

Active transducer are those which donot require auxillary power source. They are also known as self generating transducer since they develop their own voltage or current. The energy required for the production of o/p signal is obtained from the physical quantity they measure. Examples of active transducers are:-

thermocouples, photovoltaic cells, crystal oscillators.

iv) Transducer & Inverse Transducer:-

Transducer can be defined as the device which converts non electrical quantity into electrical form.

eg- microphone.

Inverse transducer is the device which converts electrical quantity into non-electrical form.

eg- loud speaker.

Characteristics of Transducer:-

It can be classified into two types:-

1) Input characteristics
2) Output characteristics

1) Input characteristics:-

It has following two types:-

a) Types of input & operating range
b) Load effect

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a) Type of input & operating range:-

Type of input which can be any physical quantity is generally determined. A physical quantity may be measured through the use of transducer i.e. selected. For the purpose depends upon the useful range of input quantity over which the transducer can be used.

The upper limit is decided by the transducer capability while the lower limit of range is normally decided by the transduction error.

b) Load Effect:-

Ideally the transducer have no loading effect in the input quantity being measured but in the actual practice, it is impossible.

The magnitude of loading effect can be expressed in terms of force power or energy extracted from the quantity under measurement, for the working of transducer.

Therefore the transducer that is selected for the particular application should ideally abstract no force, power or energy from the quantity under measurement.

ii) Output Characteristics:-

It has following three characteristics

a) Type of output
b) output Impedance
c) Useful output range.

a) Type of output:-

The type of o/p may be available from

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the transducer is in the form of voltage, current, impedance etc.

These output quantities may or may not be the latter stage of instrumentation system. They have to be manipulated, calculated i.e. their magnitude changes or they have to be change in their format by signal conditioning equipment.

b) Output Impedance:-

It determines the amount of power that can be transferred to the successive stage for the given output signal level.

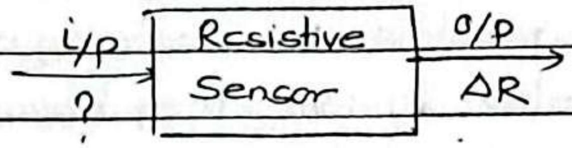
If the output impedance of a transducer is low as compared to the load impedance of successive stage, it has the characteristics of constant voltage.

On the other hand if the o/p impedance of the transducer is higher than that of the load impedance, it has the characteristics of constant current source. When the o/p impedance of transducer is equal to that of following stage, matching takes place & maximum power transfer from the next stage.

c) Useful output range:-

The output range of a transducer is limited at the lower end by noise signal. The upper limit is set by the maximum useful input level.

1) Resistive Sensor:-



eg-

a) Potentiometer (POT):-

It is one of the example of resistive sensor & it is used for the measurement of the displacement. The displacement may be either linear or rotatory. Hence there are two types of potentiometer.

1) Linear potentiometer
2) Rotatory potentiometer

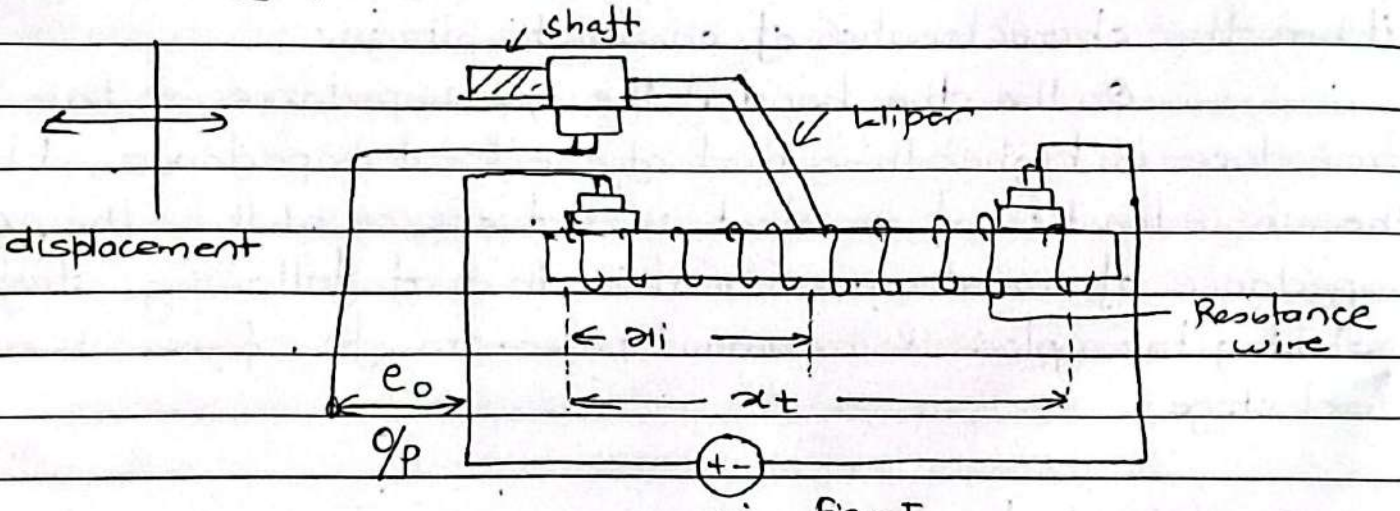


fig-I

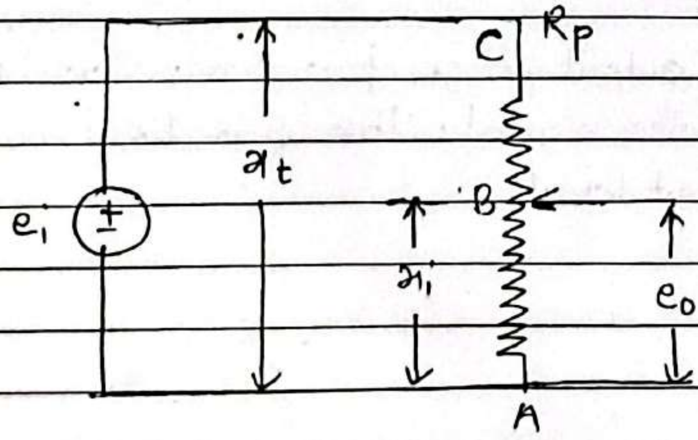


fig-1 (a) Electrical equivalent of fig-I.

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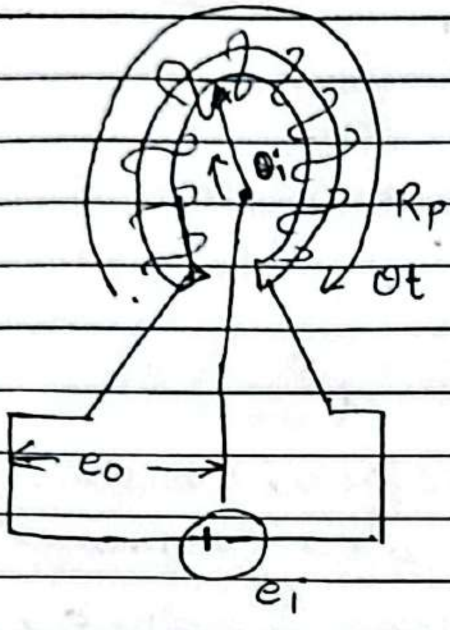

Fig- 2. Rotatory POT

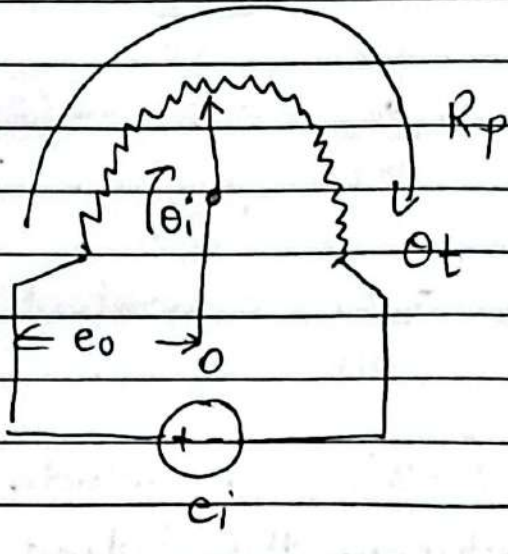

Fig- 2a Electrical equt of fig 2.

Let,

Input excitation voltage $= e_i$

Resistance of the potentiometer $= R_p$

Length " " " $= x_t$

Therefore, resistance per unit length $= \frac{R_p}{x_t}$

Displacement at any moment $= AB = x_i$

$\therefore$ Resistance of the displacement $= R_{AB} = \frac{R_p}{x_t} x_i$

$$= \frac{x_i}{x_t} \cdot R_p$$

$$= KR_p \text{ where}$$

$k = \frac{x_i}{x_t}$; $0 \leq K \leq 1$.

The ideal o/p voltage across the combination is given by:-

$$e_0 = \frac{R_{AB}}{R_{AB}+R_{BC}} \cdot e_i \quad \left(\frac{\text{Resistance at o/p terminals}}{\text{Resistance at i/p "}} \right) \times e_i$$

$$\Rightarrow e_0 = \frac{KR_p}{R_p} \cdot e_i$$

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$$\Rightarrow e_0 = Ke_i = \frac{x_i}{x_t} \cdot e_i \quad — (1)$$

$$\Rightarrow \frac{e_0}{e_i} = \frac{x_i}{x_t} = K \quad — (2)$$

$$\Rightarrow \frac{e_0}{x_i} = \frac{e_i}{x_t} = \text{constant} \quad — (3) \qquad \Rightarrow e_0 = \left(\frac{x_i}{x_t}\right) e_i$$

$$\Rightarrow e_0 \propto x_i \text{ (displacement)}$$

From 1-3, we conclude that there exists a linear relⁿ between the output & input voltage ideally as shown in fig-3.

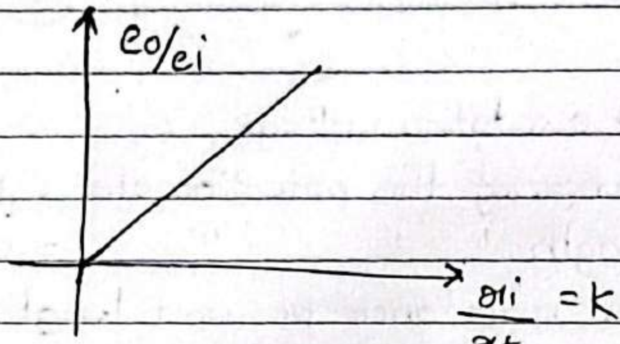

fig-3

The sensitivity of the potentiometer is given by:

$$S = \frac{o/p}{i/p}$$

$$S = \frac{e_0}{x_i} = \frac{e_i}{x_t} = \text{const} \quad — (4)$$

Thus, the sensitivity of the potentiometer is constant.

Eqⁿ 1-4 are also applicable for rotatory part, if we replace x_i with θ_i & x_t with θ_t. As we have, for rotatory part:-

$$e_0 = Ke_i = \frac{\theta_i}{\theta_t} \cdot e_i \quad — (1a)$$

$$e_0/e_i = \frac{\theta_i}{\theta_t} = K \quad — (2a)$$

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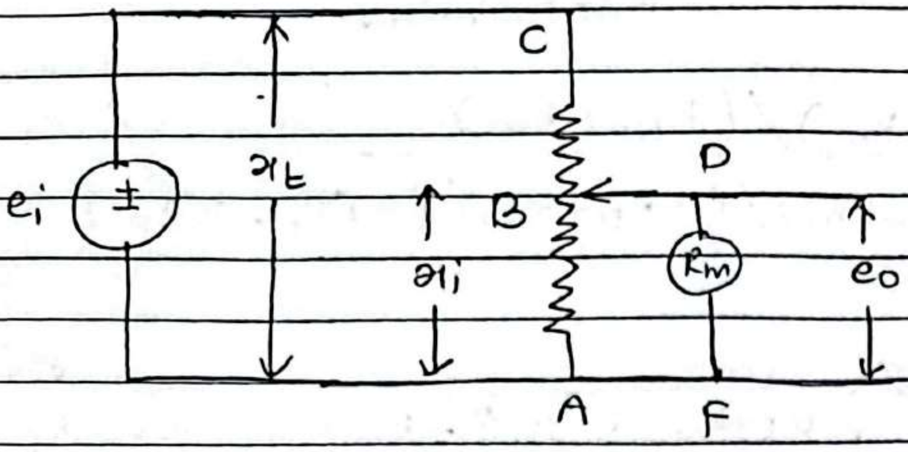
or, $\frac{e_o}{\theta_i} = \frac{e_i}{\theta t} = \text{const}$ — (3a)

$S = \frac{e_o}{\theta_i} = \frac{e_i}{\theta t}$ — (4a)

Loading Effect of the Potentiometer:-



If the resistance across the o/p terminals is infinite, we get a linear relation between the output & input voltage given by:-

$$e_o = Ke_i = \frac{x_i}{x_t} \cdot e_i$$

However, the output terminal of the potentiometer is connected to a device which impedance (R_m) is finite. Thus when an electrical instrument is connected across the output terminals, the indicated voltage is less than that given by the above equation. This effect is known as loading effect. This loading effect causes a non-linear relation between the output & input voltages. The loading effect is mainly caused by the input resistance of the output device.

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$$R_{BDFA} = K_{Rp} // R_m$$
$$= \frac{K_{Rp} R_m}{K_{Rp} + R_m}$$

Actual output voltage indicated by the voltmeter due to loading affect is:-

$$e_0 = \frac{R_{BDFA}}{R_{BDFA} + R_{BC}} \cdot e_i$$

$$\Rightarrow e_0 = \frac{(KR_pR_m)/(KR_p + R_m)}{\left[(KR_pR_m)/(KR_p+R_m)\right] + (R_p - KR_p)} \cdot e_i$$

$$\Rightarrow e_0 = \frac{KR_pR_m}{\cancel{KR_pR_m} + KR_p^2 - K^2R_p^2 + R_mR_p - \cancel{KR_mR_p}} \cdot e_i$$

Dividing num. & denominator with $R_m \cdot R_p$

$$e_0 = \frac{K}{K\frac{R_p}{R_m} - K^2\frac{R_p}{R_m} + 1} \cdot e_i \quad \text{--- (2)}$$

Let, $\alpha = \frac{R_m}{R_p}$

$$\therefore e_0 = \frac{K}{\frac{K}{\alpha} - \frac{K^2}{\alpha} + 1} \cdot e_i \quad \text{--- (3)}$$

$$\Rightarrow e_0 = \frac{\alpha K}{K - K^2 + \alpha} \cdot e_i = \frac{\alpha K}{\alpha + K(1-K)} * e_i \quad \text{--- (4)}$$

Eq^n ④ gives the output voltage with loading effect.

If $R_m \to \infty$ or $\alpha \to \infty$, then from eq^n 2-4, we get,

$$e_{oi} = K e_i \quad — ⑤$$

From eq^n ④ we conclude that there exists a non-linear relation bet^n output & input.

Loading Error in Potentiometer:-

1) Relative Error

2) Absolute Error

1) Relative Error (ε_r)

$$\varepsilon_r = \frac{\text{o/p voltage without loading effect} - \text{o/p voltage with loading effect}}{\text{o/p voltage without loading effect}}$$

$$\Rightarrow \varepsilon_r = \frac{K e_i - \dfrac{\alpha K}{\alpha + K(1-K)} e_i}{K e_i}$$

$$\Rightarrow \varepsilon_r = 1 - \frac{\alpha}{\alpha + K(1-K)}$$

$$\Rightarrow \varepsilon_r = \frac{\cancel{\alpha} + K(1-K) - \cancel{\alpha}}{\alpha + K(1-K)}$$

$$\therefore \varepsilon_r = \frac{K(1-K)}{\alpha + K(1-K)}$$

For particular value of α, relative error depends upon value of K.

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ii) Absolute Error ($\mathcal{E}_a$):-

$$\mathcal{E}_a = \frac{\text{o/p voltage without loading effect} - \text{o/p voltage with loading effect}}{\text{i/p voltage}}$$

$$\Rightarrow \mathcal{E}_a = \frac{Ke_i - \frac{\alpha K}{\alpha + K(1-K)} e_i}{e_i}$$

$$\Rightarrow \mathcal{E}_a = \frac{K^2(1-K)}{\alpha + K(1-K)}$$

$$\therefore \mathcal{E}_a = K \mathcal{E}_r$$

To find out the value of K, at which the error is max^m. we have,

$$\frac{d\mathcal{E}_r}{dK} = 0$$

$$\Rightarrow \frac{d}{dK}\left[\frac{K(1-K)}{\alpha + K(1-K)}\right] = 0$$

$$\Rightarrow [\alpha + K(1-K)](1-2K) - [K(1-K)][0+1-2K] = 0$$

$$\Rightarrow (1-2K)[\alpha + \cancel{K} - \cancel{K^2} - \cancel{K} + \cancel{K^2}] = 0$$

$$\Rightarrow \alpha(1-2K) = 0$$

$$\therefore 1 - 2K = 0 \quad [\because \alpha \neq 0]$$

$$\therefore \boxed{K = 0.5}$$

Thus, the error is maximum at $K = 0.5$ i.e. when the wiper is at the midpoint of the potentiometer.

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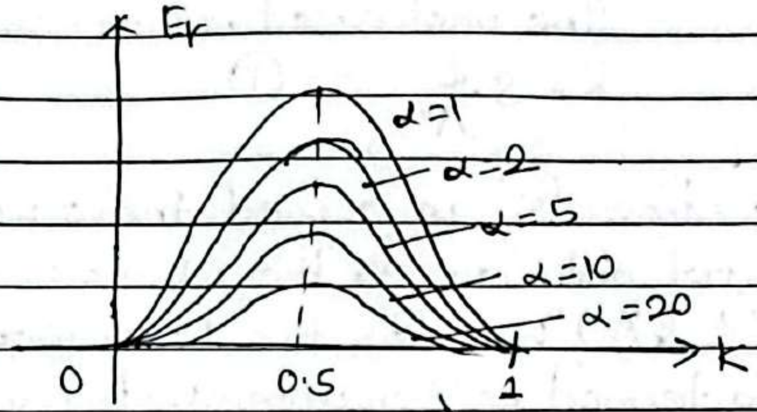
Effect of α on error of a Potentiometer:-

$$\varepsilon_r = \frac{K(1-K)}{\alpha + K(1-K)}$$

1) $\alpha = 1$

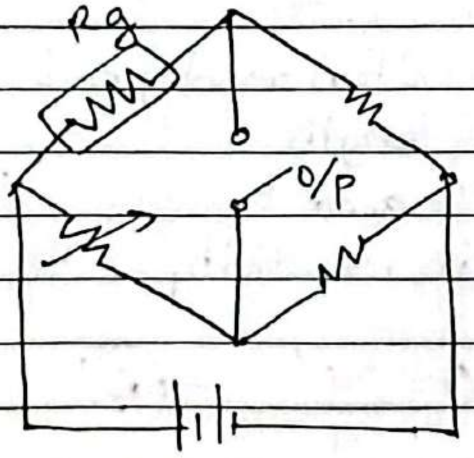


From the graph, we conclude that as the value of α increases loading error decreases that is linearity increases.

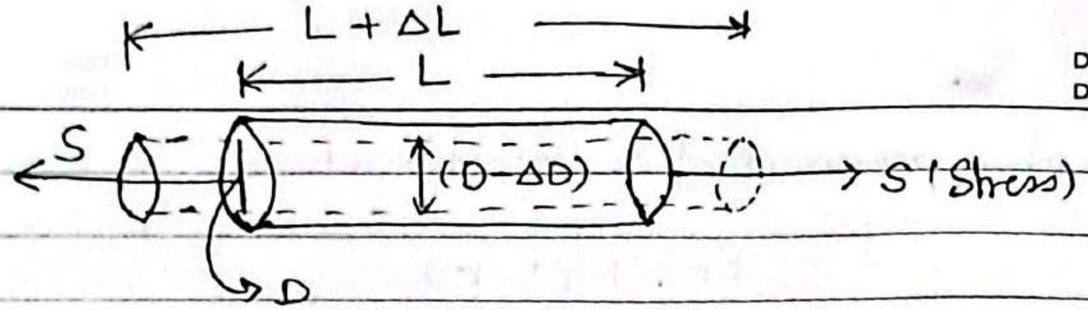
Method to reduce loading effect:-
1) Always use digital meter instead of analog meter
2) Use buffer amplifier in combination with voltmeter
3) Modify the construction of potentiometer.

b) Strain Guage

It is one of the example of resistive sensor & is used for the measurement of mechanical strength.



Strain → change in resistance
Mechanical signal → E. Signal



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Unstrained Resistance:-

The unstrained resistance is given by:-

$$R = \rho \frac{L}{A} \quad -(1)$$

If a tensile stress 'S' is applied, there will be the change in resistance not only due to the change in physical dimension (L & A) but also due to change in resistivity. This effect is termed as peizoresistivity. Thus change in resistance due to the change in resistivity is known as peizoresistive effect.

Diff. eqⁿ (1) partially with respect to S,

$$\frac{\partial R}{\partial S} = \frac{\rho}{A} \cdot \frac{\partial L}{\partial S} - \frac{\rho L}{A^2} \frac{\partial A}{\partial S} + \frac{L}{A} \frac{\partial \rho}{\partial S} \quad -(2)$$

(2) ÷ (1)

$$\frac{1}{R} \frac{\partial R}{\partial S} = \frac{1}{L} \frac{\partial L}{\partial S} - \frac{1}{A} \frac{\partial A}{\partial S} + \frac{1}{\rho} \frac{\partial \rho}{\partial S} \quad -(3)$$

If the variation is very very small, then,

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho} \quad -(4)$$

Per unit change in resistance depends upon:-

1) Per unit change in length
2) " " " " area
3) " " " in resistivity.

$$A = \frac{\pi D^2}{4} \quad -(5)$$

$$\therefore \frac{\partial A}{\partial s} = \frac{\pi}{4} \cdot 2D \cdot \frac{\partial D}{\partial s} \quad \text{— (6)}$$

(6) ÷ (5)
$$\frac{1}{A} \cdot \frac{\partial A}{\partial s} = \frac{2}{D} \cdot \frac{\partial D}{\partial s} \quad \text{— (7)}$$

From (3) & (7),
$$\frac{1}{R}\frac{\partial R}{\partial s} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{2}{D}\frac{\partial D}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s} \quad \text{— (8)}$$

If the variation is very very small,
$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{2\Delta D}{D} + \frac{\Delta \rho}{\rho} \quad \text{— (9)}$$

Poisson's ratio is defined as:-
$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\Rightarrow \mu = \frac{-(\Delta D/D)}{(\Delta L/L)}$$

$$\Rightarrow -\frac{\Delta D}{D} = \mu \frac{\Delta L}{L} = \mu \varepsilon \quad \text{— (10)}$$

where $\varepsilon = \frac{\Delta L}{L}$ = Strain

From (9) & (10),
$$\frac{\Delta R}{R} = \varepsilon + 2\mu\varepsilon + \frac{\Delta \rho}{\rho} \quad \text{— (11)}$$

Guage Factor:- (कति strain develop भाको ε, measure गर्छ)

It is defined as the ratio of per unit change in the resistance to per unit change in length.

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$$G = \frac{(\Delta R/R)}{(\Delta L/L)}$$

$$\Rightarrow \frac{\Delta R}{R} = G\frac{\Delta L}{L}$$

$$\Rightarrow \boxed{\frac{\Delta R}{R} = G\varepsilon} \quad \text{— (12)}$$

From (11) & (12)

$$G\varepsilon = \varepsilon + 2\mu\varepsilon + \frac{\Delta S}{S}$$

small

$$\therefore G = 1 + 2\mu + \frac{\Delta S/S}{\varepsilon} \quad \text{— (13)}$$

If the effect of resistivity is neglected then,

$$\therefore \boxed{G = 1 + 2\mu} \quad \text{— (14)}$$

Through strain is a unitless quantity but in measurement system it is expressed in microstrain.

$$\varepsilon = \frac{\Delta L}{L}$$

$$1 \text{ microstrain} = \frac{1\,\mu m}{1\,m}$$

Eg. $\varepsilon = 5$ microstrain

$\therefore \varepsilon = 5 \times 10^{-6}$

In measurement system, tensile strain is taken as positive strain whereas compressive strain is negative strain.

Eg- Compressive Strain = 3 microstrain

$$\varepsilon = -3 \times 10^{-6}$$

↳ Compressive.

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c) Resistance Thermometer (Resistance Temp^r Detector)

It is one of the example of resistive sensor & it is used for the measurement of temperature. These are two types of resistance thermometer.

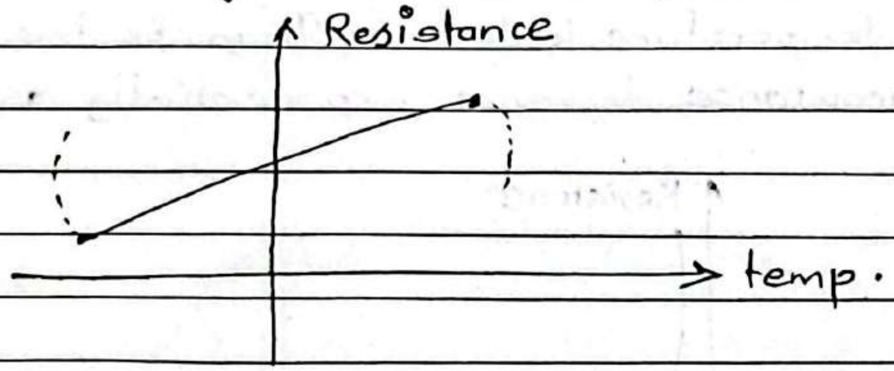
1) Metal resistance thermometer
2) Semiconductor thermometer (Thermistor)

1) Metal resistance thermometer :-

The resistance of the metal increases with the increase in temperature. The relation bet^n resistance & temperature is given by :-

(Resistance of metal) $R_t = R_0 [1 + \alpha t + \beta t^2 + \gamma t^3 + ---]$ where α, β & γ are temp^r coefficient of resistances (TCR) & the magnitude having $\alpha > \beta > \gamma$.

The variation of resistance & temp^r is shown in fig-①.



For the linear rel^n between resistance & temp^r we have

$$R_t = R_0 [1 + \alpha t]$$

The metal resistance thermometer are made by winding platinum wire or nickel wire or copper wire on a ceramics. Platinum has a closely rel^n between resistance & temp^r, gives good repeatability, has long term stability, has a temperature range of $-200°C$ to $850°C$, is relatively inert. Though platinum is more expensive than others metals, but it is most widely used. Ni & Cu are cheaper but have less stability & cannot be used over such large range of temperature having a range −80°C to 300°C & copper has −200°C to +250°C.

11) **Thermister:-**

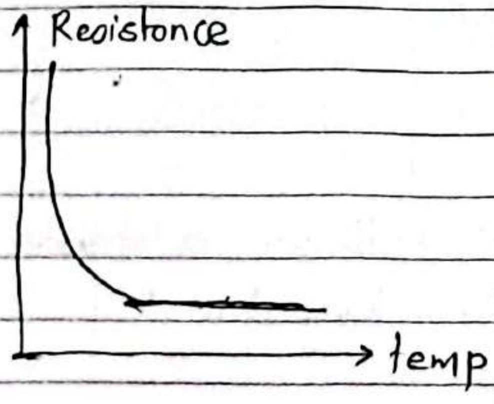
They are also known as thermally sensitive resistor. They are made from oxide of chromium, cobalt, manganese. These oxides are semiconductor in nature so as the temperature increases, resistance decreases. The relation between the resistance & temp^n. is given by:-

$$R_T = R_0 \, e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)}$$

where, initial final

β = constant depends upon types of material used. (3500 − 4500)

T & T_0 are temperature in Kelvin. Thus, as the temperature increases, resistance decreases exponentially as shown in the figure.



* Thermistor is a special type of resistor whose resistance changes with the change in temperature.

* ~~temp ↑~~ semiconductor/conductor → temp ↑ → Negative
resistance ↓ temp^n. coeff.

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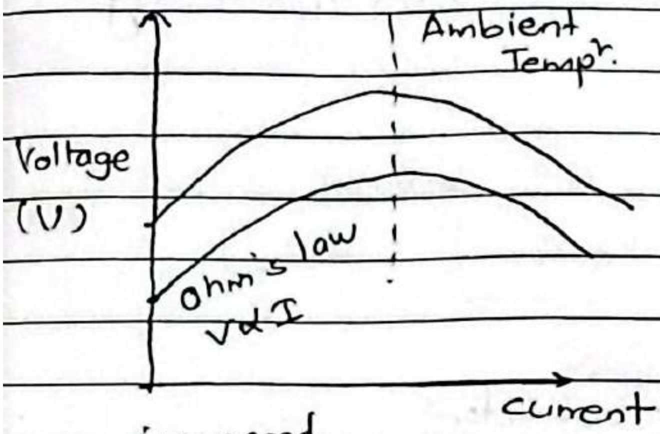
1° rise in $temp^r$.
↓
5% decrease in ~~tem~~ resistance (highly sensitive)

* Used for precision measurements, control & compensatic.

* $Temp^r$. range → $-60^\circ C$ to $15^\circ C$
  Resistance → 0.5Ω to $0.75M\Omega$

Characteristics of Thermistors:-
1) Resistance - $temp^r$.
2) Voltage - current.
3) Current - time

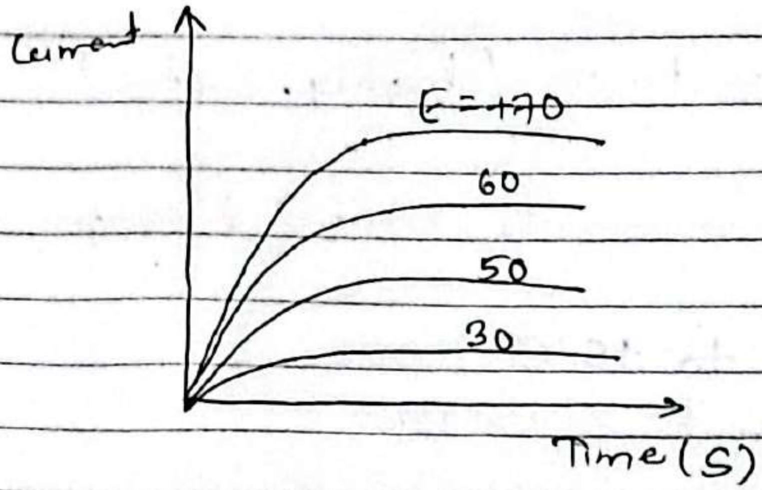


increased
The $temp^r$ is not so enough, it is able to produce heat in the thermistor or change in resistance of thermistor

* The voltage drop across a thermistor increases with the increasing current when it reaches a peak value.

* After peak value, the voltage drop decreases with increase in current
  ↳ negative resistance resistance characteristics.

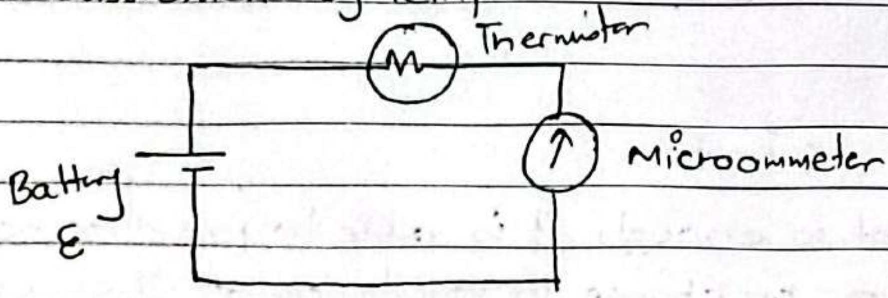
b) Current-time Characteristics:



* The time delay to reach maximum current is a function of applied voltage.
* When the heating effect occurs in a thermistor, a certain finite time is required for the thermistor to heat & the current to build up to a maximum steady state value.

Applications of Thermistors:-

1) Measurement of tempʳ.



temp↑, Res↓, → current↑

2) Control of Tempʳ.
3) Temperature Compensation :- -
4) Measurement of power at high frequencies
5) Vaccum measurement
6) Providing time delay

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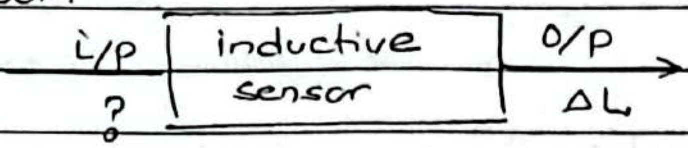
Advantages

* compact, rugged & inexpensive
* good stability, highly sensitive
* response time is fast
* ~~not affected~~

2) Inductive Sensor:-

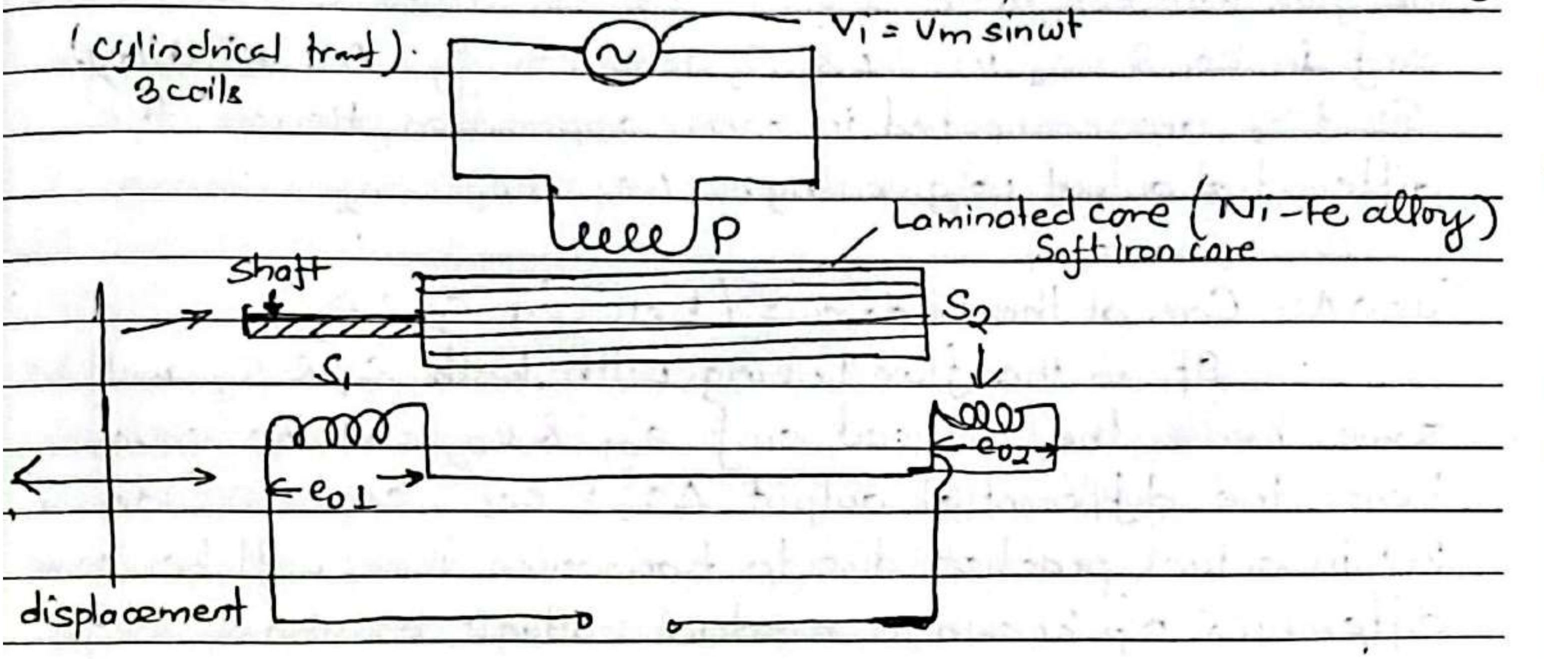


eg-

a) Linear Variable differential transformer [LVDT]

It is one of the example of inductive sensor & it is used for the measurement of displacement.

Displacement → Electrical signal



$$\Delta e_0 = e_{01} - e_{02}$$

→ Output vary linearly with displacement.

→ o/p two windings का o/p का difference है so differential.

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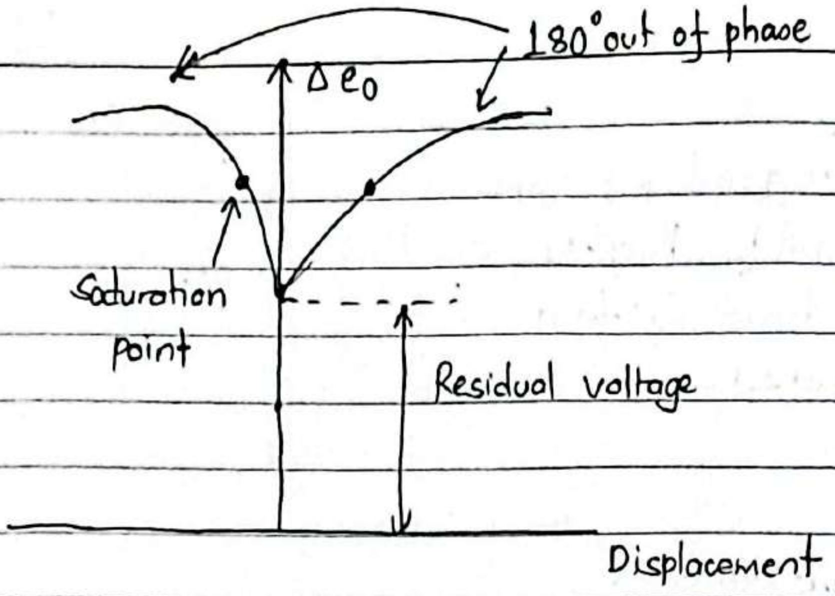

Construction:-

Working Principle:- If an input $V_i = V_m \sin\omega t$ is applied to the primary winding P, a sinusoidal flux will link with the primary winding. Due to the high permeability of the core, this flux will also link with both S_1 & S_2. As the windings S_1 & S_2 are stationary & the flux is rotating, so the flux will cut both S_1 & S_2. Hence, there will be induced emf ~~in~~ e_{o_1} & e_{o_2} in S_1 & S_2 respectively. As winding S_1 & S_2 are connected in series opposition. Hence, the differential output is given by:- $\Delta e_0 = e_{o_1} - e_{o_2}$

Case A:- Core at the midpoint (between S_1 & S_2):

If so the flux linking with both S_1 & S_2 will be same, hence the induced emf e_{o_1} & e_{o_2} will be equal. Hence, the differential output $\Delta e_0 = e_{o_1} - e_{o_2} = 0$ (ideally). But in actual practice due to harmonics there will be some differential o/p known as residual voltage as shown in fig.

Case B:- Core moving left:-

If so the flux linking with S_1 increases & that with S_2 decreases. Hence, Induced emf e_{o_1} increases

$\varepsilon_{s1} > \varepsilon_{s2}$ $\varepsilon_0 = +ve$

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& e_{o2} decreases. There exists a linear relation between the differential output Δe_0 & the displacement upto the saturation point. Beyond that the relation will be non-linear as shown in fig ( ). Suppose in this direction of displacement, the differential output is in phase with the input voltage V_i.

Case C: Core moving right:- S_2 by S_1

Replace S_1 by S_2, in phase by outphase, e_{o1} by e_{o2}, e_{o2} by e_{o1}

$\varepsilon_{s2} > \varepsilon_{s1} \Rightarrow \varepsilon_0 = -ve$

The magnitude of the differential output will be the measure of the displacement & phase angle of the differential output is the measure of the dirn. of displacement. In this way, we can measure both direction & magnitude of the displacement using LVDT.

Advantages :-

1) Linearity (i/p & o/p varies linearly) displacement की magnitude & direction
2) Hysteresis & eddy current loss low, friction, low
3) Resolution high
4) Output high

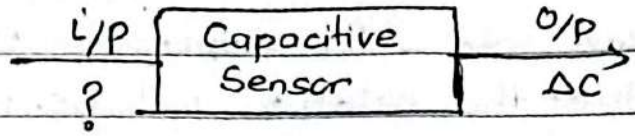
Disadvantages

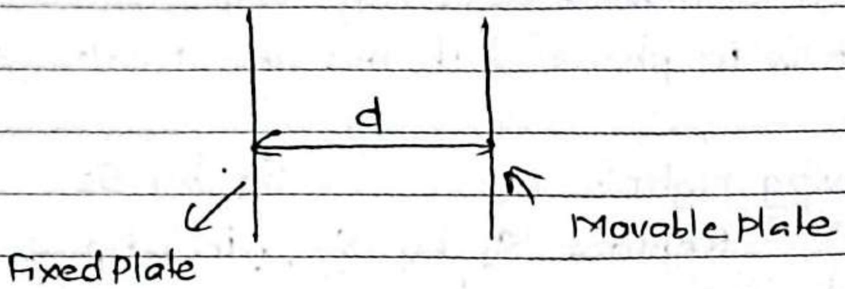
→ Whenever require large appreciable o/p, large displacement is necessary affected by
→ It is ~~sensitive to~~ vibration & temperature.
→ Sensitive to stray magnetic field

Application → Used as secondary transducer for measuring force, weight, pressure

→ To provide displacement feedback for hydraulic systems.
→ Brain probing in medical applications.

3) Capacitive Sensor:-





It consists of two parallel plates one is fixed & another movable. The object which displacement is to be measured is coupled with the movable plate. The capacitance of the parallel plate capacitor is given by:-

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad — \quad (1)$$

where $A \rightarrow$ overlapping area of the plate.

$d \rightarrow$ sepn. distance of the plates.

$\varepsilon = \varepsilon_0 \varepsilon_r$ = permitivity of the medium or dielectric between the plates. $\varepsilon_r = \varepsilon/\varepsilon_0$ (Relative permittivity)

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m (Free space permittivity)

As per capacitive sensor, the i/p must change the capacitance, so to make a change in the capacitance, the i/p must change either any one of the free quantity on the right hand side of eqn. (1).

Thus, there are 3 principles on which capacitive sensor works:

1) Principle of change in overlapping area (A)
2) " " " " Sepn. distance (d)
3) " " " " permittivity (ε)

Input - Displacement, Pressure, Force.

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1) Principle of change in overlapping area (A)

It consists of two parallel plates, one is fixed & another is movable. The object which displacement is to be measured is coupled with the movable plate. The capacitance of the parallel plate capacitor is given by:-

$$C = \varepsilon A/d = \varepsilon_0 \varepsilon_r A/d$$

This principle is used for measurement of displacement. The displacement may be either linear or angular.

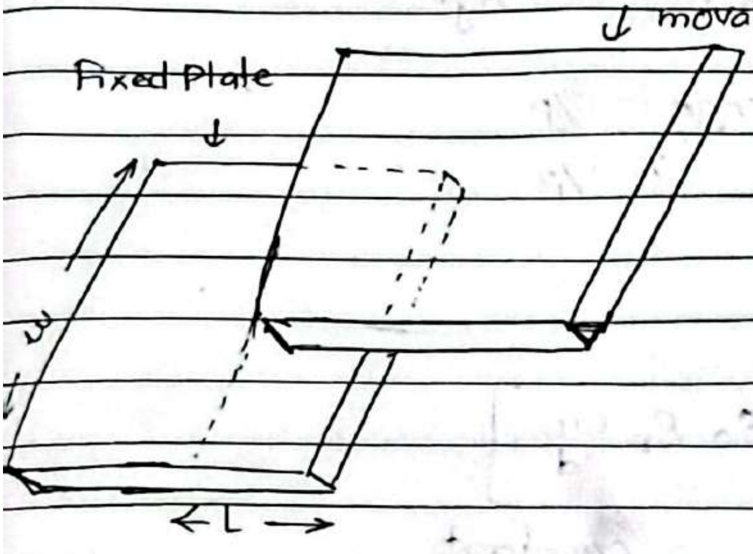

Fig-1 Linear Displacement

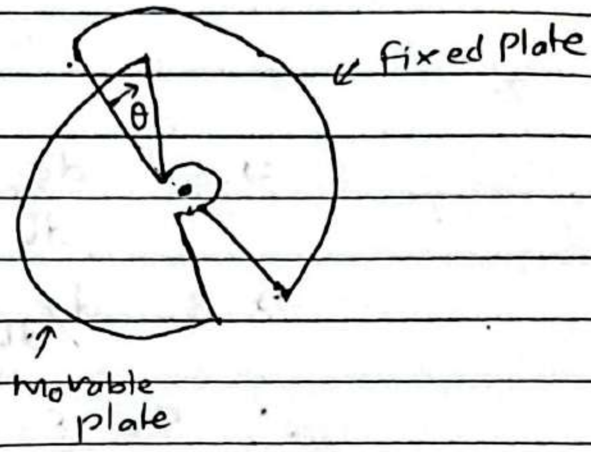

Fig 2> Angular displacement

Consider fig-1 for the linear displacement, the capacitance is given by:-

$$C = \varepsilon A/d = \varepsilon_0 \varepsilon_r \cdot \frac{w \cdot L}{d}$$

where, $w \rightarrow$ width of the plate

$L \rightarrow$ overlapping length

$\therefore \quad C \propto l.$

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Thus, the capacitance varies linearly with the overlapping length(L) as shown in fig. 3.

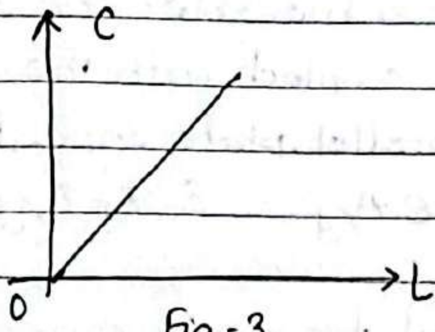

Fig-3

The sensitivity of the device is given by:-

$$S = \frac{\text{small change in o/p}}{\text{'' '' '' in i/p}}$$

$$\Rightarrow S = \frac{dc}{dL}$$

$$\Rightarrow S = \frac{d}{dL}\left[\varepsilon_0 \varepsilon_r \frac{wL}{d}\right]$$

$$\therefore S = \frac{\varepsilon_0 \varepsilon_r w}{d} = \text{Constant}$$

Thus, the sensitivity of the device is constant.

Consider fig-2 for angular displacement

The maximum overlapping area is given by:

$$A_{max} = \frac{\pi r^2}{2}$$

The corresponding max. capacitance is given by:-

$$C_{max} = \varepsilon \cdot \frac{A_{max}}{d} = \varepsilon_0 \varepsilon_r \frac{\pi r^2}{2d}$$

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The area corresponding to an overlapping angle θ radian is given by:-

$$A_\theta = \frac{\theta r^2}{2}$$

The corresponding capacitance is given by:-

$$C_\theta = \frac{\varepsilon A_\theta}{d} = \varepsilon_0 \varepsilon_r \cdot \frac{\theta r^2}{2d}$$

$$\therefore C_\theta \propto \theta$$

Thus, the capacitance varies linearly with the angular displacement θ.

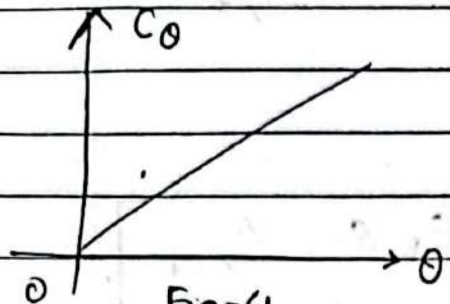


Fig-4

The sensitivity of the device is given by:-

$$S = \frac{\text{small change in o/p}}{\text{" " " i/p}}$$

$$\Rightarrow S = \frac{dC_\theta}{d\theta}$$

$$\Rightarrow S = d/d\theta \left[\varepsilon_0 \varepsilon_r \frac{\theta r^2}{2d} \right]$$

$$\therefore S = \frac{\varepsilon_0 \varepsilon_r r^2}{2d} = \text{constant}$$

Hence, the sensitivity of the device is constant.

4) Peizo-electric Sensor:-

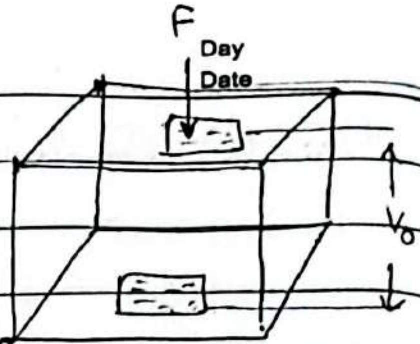


fig - I

In certain crystals, an electric field appears across certain surface of the crystal. If the dimension of the crystal is deformed by the application of a mechanical force, thus potential is produced as these materials generate an electric charge within them when deformed. This effect is reversible, if a varying potential is applied to the proper axis of the crystal, it will change the dimension of the crystal, thereby deforming it. This effect is known as peizo-electric effect.

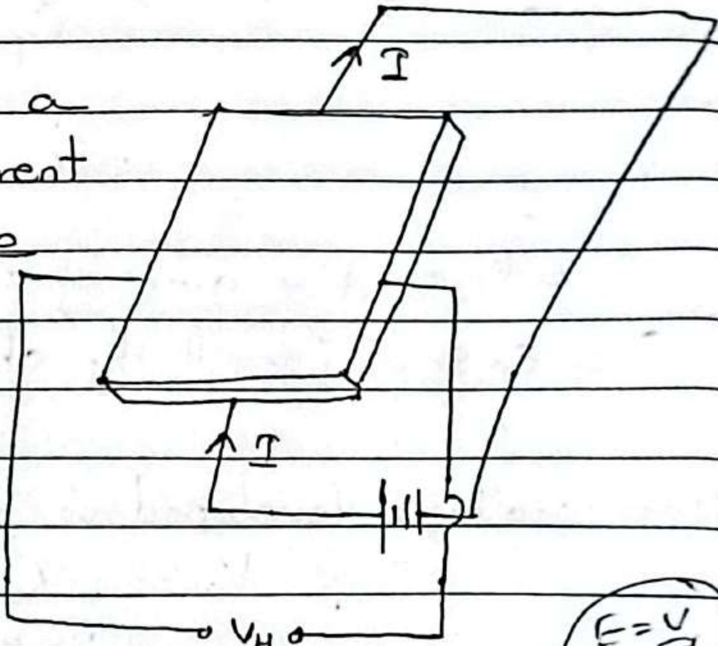
There are 3 types of peizoelectric materials

1) Natural eg- Quartz & Rochelle Salt
2) Synthetic eg- Lithium Sulphate, Ammonium dihydrogen Phosphate
3) Ceramics:- eg- Barium tartarate

Metal electrods are placed onto selected face of the piezoelectric crystal so that the lead wires can be connected for bringing in out the electric charge. The peizoelectric materials are insulators so that the electrods become the plates of the capacitor. Thus a peizoelectric sensor may be thought of as a charged generator or a capacitor. Peizoelectric effect is direction sensitive i.e. or tensile force produces a voltage of one polarity while a Compression force produces a voltage of opposite polarity

Hall Effect Sensor:-

The action of magnetic field on a flat plate carrying electric current generate a potential difference which will be the measure of strength of the magnetic field.



$= K_H \frac{BI}{t}$

Fig - I

$E = \frac{V}{d}$

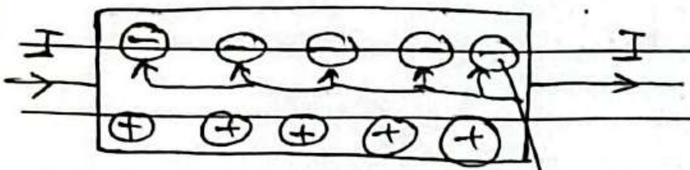


Fig - 2

deflection of e^-'s in presence of magnetic field

$$V_{Hm} = K_{HM} \frac{BI}{t}$$

$$V_{HS} = K_{HS} \frac{BI}{t}$$

To prove $V_{HS} > V_{HM}$

Proof:

$$K_H \propto \frac{1}{\rho}$$

$$\therefore K_{HM} \propto \frac{1}{\rho_m}$$

$$K_{HS} \propto \frac{1}{\rho_m}$$

As we know,

$$\rho_m > \rho_s$$

$$\therefore K_{HS} > K_{HM}$$

$$\therefore V_{HS} > V_{HM}$$

A beam of charge particles can be deflected by a magnetic field. A current flowing in a conductor is a beam of moving charges & thus can be deflected by a magnetic field as shown in fig-2. This effect was discovered by E.R. Hall & is now referred to as the Hall Effect.

Consider a plate of metal or semiconductor with the magnetic field applied at the right angle to the plane of plate as shown in fig-1. As the consequence of magnetic field, the moving charge particles in the plate are acted by a force which cause them to be deflected to one side of the plate. As a consequence one side of the plate becomes -vely charged & other +vely charged. This charge sep^n. produced an electric field in a material. The charge sep^n. growth until the force on the charge particle from the electric field just balanced the force produced by the magnetic field. Due to the electric field, there exists a p.d. bet^n. the two faces given by:-

$$V_H = K_H \frac{BI}{t}$$

$V_H \rightarrow$ Hall Voltage

$B \rightarrow$ magnetic flux density

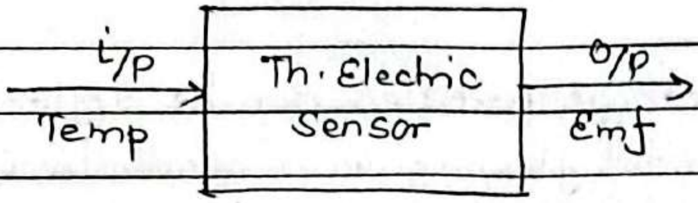
$I \rightarrow$ current

$t \rightarrow$ thickness of plate

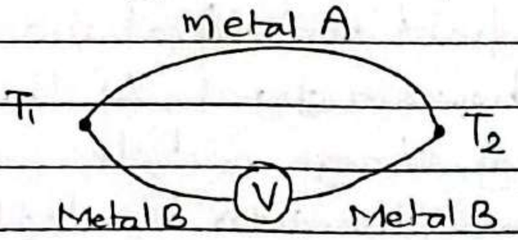
$K_H \rightarrow$ Hall's coefficient & it is inversely proportional to the charge density. i.e. $K_H \propto \frac{1}{\rho}$

This effect is more pronounced in the case of semiconductor than in metal.

Thermo- Electric Sensor :-



For thermoelectric sensor, i/p is the thermal energy or temp^r. & o/p is the emf or electrical energy. Extensively used for measurement of temp^r. variations. For eg- Thermocouple



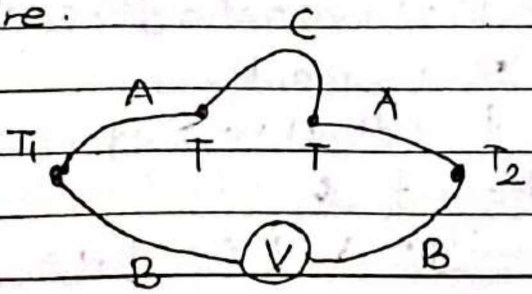
$V \propto (T_1 - T_2)$

$V = S(T_1 - T_2)$

$S \rightarrow$ Seebeck Coefficient

The thermocouple circuit is formed by connecting two metal wires end to end so that two functions are formed. If the two junctions are at the same temp^r, i.e. if $T_1 = T_2$, then there will be no any emf developed, but if there is some temperature differenences bet^n two junction temperature i.e. $T_1 \neq T_2$, then some emf will be developed. There are 5 basic laws of thermocouple.

1) The emf ~~total~~ developed totally depends upon the temperature difference of the two junction not on the length of the wire.



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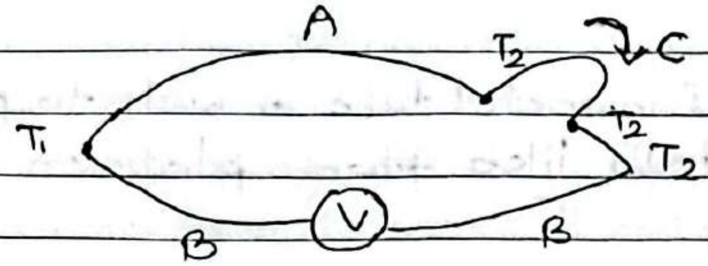
2) If a third metal C is connected either in metal A or in B, then emf developed will be unchanged provided the two newly formed junctions are at the some temperature. This means that connection of the valtmeter doesn't affect the emf developed.

3) 

If a third metal C is connected either at junction T_1 or T_2 then emf developed will be unchanged, this means that shouldering doesn't affect the emf developed.

4) Law of intermediate metal :-

emf of A w.r.t B

$$AB \rightarrow T_1, T_2 \rightarrow E_{AB} \text{ or } E_A^B$$

$$BC \rightarrow T_1, T_2 \rightarrow E_{BC} \text{ or } E_B^C$$

$$AC \rightarrow T_1, T_2 \rightarrow E_{AC} \text{ or } E_A^C$$

Then, we will find,

$$E_{AC} = E_{AB} + E_{BC}$$

$$\Rightarrow E_A^C = E_A^B + E_B^C$$

Note:-

$$E_{AB} = -E_{BA}$$

5) Law of intermediate temp^r.

$$AB \rightarrow T_1, T_2 \rightarrow E_{1,2}$$

$$AB \rightarrow T_2, T_3 \rightarrow E_{2,3}$$

$$AB \rightarrow T_1, T_3 \rightarrow E_{1,3}$$

Then we find, $E_{1,3} = E_{1,2} + E_{2,3}$

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Advantages

→ Low cost than resistance thermometer
→ Operating range of temp^r is wide
→ Less time lag & therefore can record rapid changes in temp^r.

Limitations

→ Low accuracy
→ Should be protected in metal tube or well to prevent contamination of metals like ~~plani~~ platinum.