Simulation - Analytical Decision Making

Problem 1: Wastewater Problem

You manage a seaside resort complex consisting of 143 time-share condominiums. On any given day, each condominium unit in the complex has an 87% chance of being occupied, independent of all other units. Each occupied unit produces a random amount of wastewater with a mean of 22.3 gallons and a standard deviation of 19.2 gallons, independent of all the other condominiums units. Vacant units do not produce any wastewater.

You process the wastewater from condominiums in treatment devices, each of which leases for $33.40 per day and can process up to 500 gallons of water per day. You are considering leasing 4, 5, 6, or 7 treatment devices. If the condominiums produce more wastewater than you are able to process, you must pay the municipal government a pollution fee of 10.5 cents for each gallon you do not process.

Which number of treatment devices will give you the lowest daily cost?

Use a sample size of 1000.

Problem 2: Heating Element Problem

You have a business heat-treating specialty industrial castings. The number of castings you receive for treatment each day is a Poisson random variable with a mean value of 4.1. You process the castings in a super-high-temperature oven that can hold up to 5 castings. This oven uses a heating element that sometimes fails; the probability of failure is as follows:

Day of Use	Failure Probability
1	1%
2	7%
3	9%
4	15%
5	25%

After the fifth day of use, the safety regulations for the oven require that the heating element be replaced even if it is still functioning.

On days that the heating element fails, you must wait until tomorrow to reprocess all the castings for that day. Thus, on days that the heating element is working, you have a total processing capacity of up to 5 castings, but on days that it fails, your capacity is effectively 0 castings.

You process the castings on a first-come, first-served basis -- if you cannot finish all the castings waiting to be processed on a given day, you save them in a queue and try to process as many as possible the next day.

You are considering 5 possible policies, parameterized by a number $d = 1, 2, 3, 4,$ or 5. At the *end* of the day, if the heating element has been in use for d days and has not failed, you replace it. On days when the element fails, you also replace it at the end of the day.

The economics of the operation are as follows:

- The heating element costs $800 to replace if it did not fail

- When the element fails, it costs $1500 to replace

- You receive $200 in revenue each time you finish processing a casting

- You estimate that each day that each casting spends waiting to be processed costs you $40 in loss of goodwill, storage costs, etc.

- You may assume all other costs and revenues to be negligible.

Determine by simulation which value of d gives you the highest expected profit over a 60-day period. You may ignore any costs and revenues from castings left in queue at the end of the period. Use a sample size of at least 500, and assume that you start with a new heating element on the first day.

You are also interested in whether the queue of unprocessed castings left at the end of the day exceeds 10 at any time during the 60-day period. **With the optimal value of d, what is the probability of this event?**

Problem 3: Maintenance Shop Operation

You operate a maintenance shop that repairs aircraft engine compressors for a large airline. The number of compressors you receive to be repaired each day is Poisson with a mean value of 8.1. The crucial step in each repair operation is realigning the compressor's blades. The machine has a part that wears out rapidly, as follows:

Day of Use	Chance of Part Failure
1	2%
2	4%
3	10%
4	12%
5	15%
6	20%
7 or higher	25%

On days when the part does not fail, you can process 10 compressors. On days when it does fail, you are equally likely to be able to process 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9 compressors. After a failure, you must replace the part and wait until the next day to resume work.

On any given day, if you have more compressors to repair than you are able to fix, you set the leftover compressors aside in a queue, and try to fix them the next day. You estimate that each

day each compressor waits in this queue costs you $80.

Your policy is to replace the alignment machine part after n days of use or when it fails, whichever comes first. You perform the replacement just before the start of each day's work. Replacement costs $500, whether done intentionally, or because of failure. (Occasionally, you will have to replace a part twice in one day: intentionally at the start of the day, and then later in the day because of a part failure.)

What value of n gives you the lowest total cost? Simulate a 100-day period with a sample size of 500, and try $n = 2, 3, 4, 5$, and 6. For the best choice of n, estimate the expected value of the following over the 100-day period: (a) the average number of compressors in the queue at the end of each day, (b) the number of scheduled part replacements (that is, the number of times you replace a part that has not failed), and (c) the number of times the part fails.

Problem 4: Computer Support Problem

Your company has 200 personal computers. Each computer has a 5% probability, independent of all other computers, of encountering a hardware problem on any given day. Most hardware problems require 3/4 of an hour of attention from a computer support technician (CST). Each problem that occurs, however, has a 30% chance (independent of all other problems) of developing into a more serious problem requiring an extra 2 hours of attention from a CST.

Each CST can spend up to 6 hours per day fixing problems, and costs the company $300 per day. Every CST hired must be paid their full salary each day, whether or not they have sufficient work to keep them busy. If the company does not have enough CST's to solve all the problems occurring on a given day, it hires an outside firm that charges $95 per hour.

You are trying to decide whether to hire 1, 2, 3, or 4 CST's.

Which option gives the lowest average cost per day? With this number, what is the average number of hours of support per day you must buy from the outside firm?

Use a sample size of 1000.