

Sampling

(a) Sampling of a sinusoidal waveform

Theory: The sampling theorem says that an analog signal bandlimited to B Hz, if sampled at $F_s > 2B$, the analog signal can be completely reconstructed from its samples. To compute the spectrum of an analog signal numerically, the sampled waveform has to be truncated to apply DFT. This truncation or rectangular windowing in time domain causes spectrum spreading. The more the width of the window is chosen the less is spreading. Note also that the DFT involves a sampling in frequency domain.

1. Take an analog waveform $x(t) = 10\cos(2\pi \times 10^3 t) + 6\cos(2\pi \times 2 \times 10^3 t) + 2\cos(2\pi \times 4 \times 10^3 t)$.
 2. Sample it at $F_s = 12$ kHz.
 3. Obtain DFT of $x(t)$ with $N = 64, 128, 256$ points and plot the respective magnitude spectra (Fig.1).
- Note the change in spectrum as N is increased.

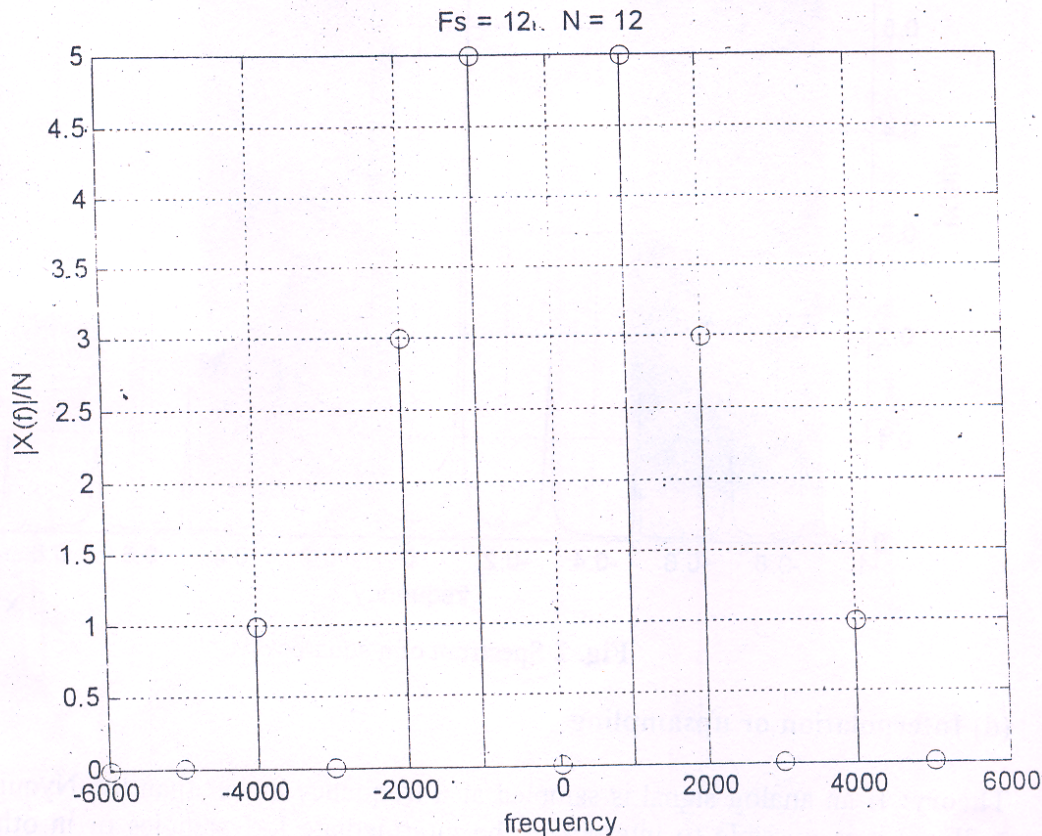


Fig. 1 Spectrum of $x(t)$

(b) Sampling at below Nyquist rate and effect of aliasing

1. Repeat above with $F_s = 8$ KHz, 5 kHz, 4 kHz.
- Note carefully, that sampling theorem requires $F_s > 2F_{\max}$ (without equality sign) for sinusoidal signal.

- Find out from the spectrum, what are the aliases of the original frequencies present in $x(t)$ when the sampling rate is below the Nyquist rate.

(c) Spectrum of a square wave

Theory: A square wave theoretically has an infinite bandwidth. For practical purposes, the spectrum beyond the 10th harmonic can be neglected.

- Take a square wave with time period $T = 1\text{ms}$ ($F = 1\text{ kHz}$).
- Sample it at $F_s = 20\text{ kHz}$.
- Obtain DFT of the sampled square wave with $N = 256$ points and plot the results. (Fig. 2)

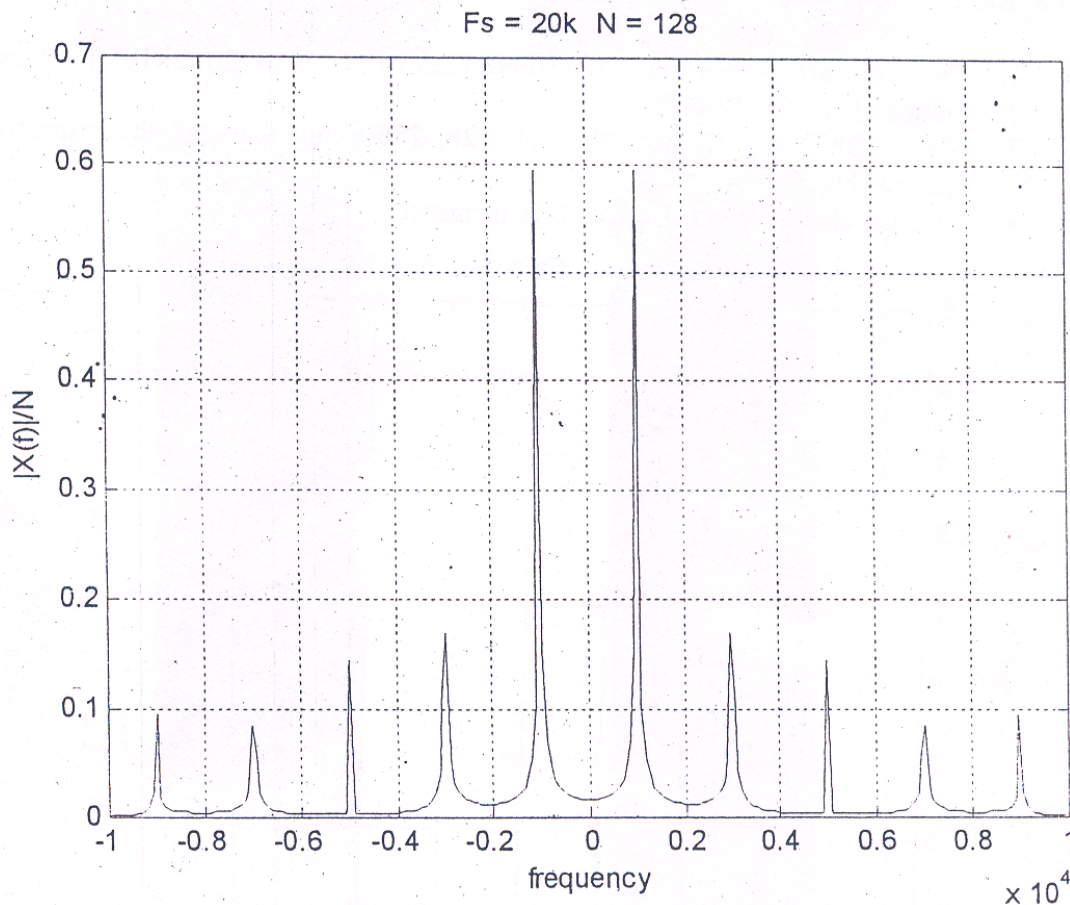


Fig. 2 Spectrum of a square wave

(d) Interpolation or upsampling

Theory: If an analog signal is sampled at a frequency higher than the Nyquist rate, ($F_{s1} > 2F_{\max}$) it is possible to interpolate the intermediate $L-1$ samples or in other words to obtain the samples at $F_{s2} = LF_{s1}$ frequency. This can be simply done by passing the sampled signals through an ideal lowpass filter of cutoff frequency F_{\max} and sampling it again at a higher rate. But in discrete time domain this is achieved as shown in Fig.3. Here, $F_{s1} = 12\text{ kHz}$ & $F_{s2} = 24\text{ kHz}$ ($L=2$).

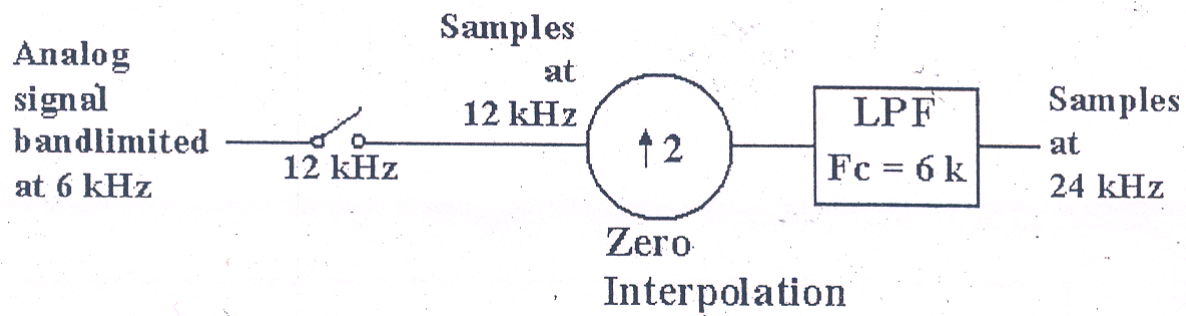


Fig.3 An example of upsampling

1. Take a lowpass signal of bandwidth 6 kHz.
2. Sample it at $F_{s1} = 12$ kHz.
3. Insert one zero in between every two samples.
4. Pass it through a lowpass filter of cutoff frequency 6 kHz.
 - Note that at step 4 the LPF is a digital filter & the sampling frequency you have to use is $F_{s2} = 24$ kHz.
5. Plot the output of the lowpass filter and compare it with the original signal sampled at $F_s = 24$ kHz.
 - The output of the lowpass filter may differ from the original one by certain delay and scaling factor.
- **MATLAB functions that you may need:**
`fft, fftshift, conv, square, firl, fir2, butter, filter, cheby1, cheby2, ellip`