Experiment No: 5

ADAPTIVE LINE ENHANCER

Theory: An adaptive line enhancer (ALE) is used to detect a low-level sine wave of unknown frequency in presence of noise. If the input frequency changes the filter adapts itself to be a bandpass filter centered at the input frequency. The ALE is usually realized by using the so-called adaptive filter. In a general adaptive filter, the filter coefficients are updated in time by an adaptation algorithm during an initial training phase, so that filter output $y(n)$ becomes a better and better estimate of the desired response $d(n)$ given during this phase. This is shown in Figure 1. The most widely used adaptation algorithm is the Least Mean Square (LMS) algorithm, which uses the error signal $e(n)$ in a feedback loop (not shown) for coefficient adaptation.
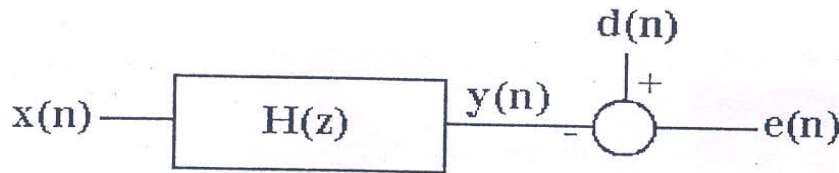

Fig.1: $d(n)$ is the desired response, $e(n)$ is the error.

The transfer function, $H(z) = w_0 + w_1 z^{-1} + w_2 z^{-2} + \ldots + w_p z^{-p}$

The filter coefficients are updated in time in the LMS algorithm, following a *steepest descent* along the negative direction of the *gradient of the mean-squared error*. The ideal steepest descent procedure leads to,

$$w(n+1) = w(n) - \frac{\mu}{2} \nabla_w \varepsilon^2 \Big|_{w=w(n)} \text{,where } \varepsilon^2 = E[e^2(n)],$$

$$\nabla_w \varepsilon^2 = \left[\frac{\partial \varepsilon^2}{\partial w_0} \quad \frac{\partial \varepsilon^2}{\partial w_1} \quad \ldots \quad \ldots \quad \frac{\partial \varepsilon^2}{\partial w_p} \right]^T \text{ and } \mu \text{ is a constant controlling the convergence rate.}$$

This can be equivalently written as,

$$w(n+1) = w(n) + \mu(p - Rw)\Big|_{w=w(n)}$$

where, $R = E[x(n)x(n)^T]$, $p = E[x(n)d(n)]$,

$x(n) = [x(n) \quad x(n-1) \quad x(n-2) \quad \ldots \quad x(n-p)]^T$ and $w(n) = [w_0 \quad w_1 \quad \ldots \quad \ldots \quad w_p]^T$.

However, in practice, R and p are not known *a priori* and are estimated from the data online, thus making the weight update recursion adaptive. In the case the LMS algorithm, R and p are replaced by the estimates : $x(n) x^T(n)$ and $x(n) d(n)$ respectively. This results in the celebrated LMS filter coefficient update formula :

$$w(n+1) = w(n) + \mu x(n) e(n).$$

It can be shown that the algorithm converges for $0 < \mu < \dfrac{2}{trace(R)}$.

In an adaptive line enhancer (Fig.2) the desired response is simply the input $x(n)$.
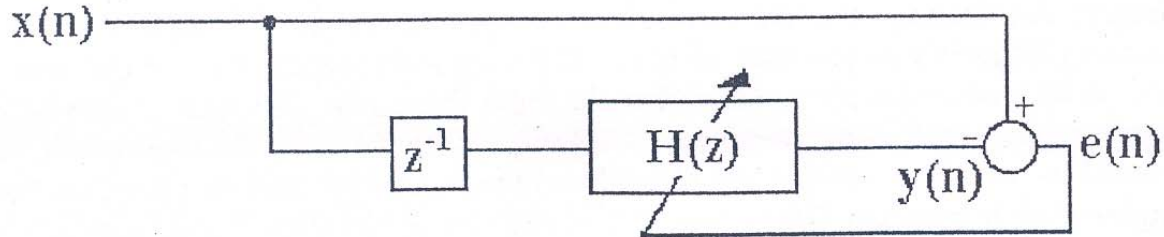

Fig.2: An adaptive line enhancer

Step:

1. Take a sinusoidal message waveform $m(t) = A\sin(2\pi \times F_0 t)$ (Take $A = 2$ and $F_0 = 1$ kHz).

2. Add white Gaussian noise $n(t)$ of zero mean and unity variance to $m(t)$ and obtain the input signal $x(t)$.

3. Sample it properly to generate the discrete input signal $x(n)$.

4. Pass $x(n)$ through the system as shown below (Fig.2)

5. Adapt the filter coefficients as
$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\, \mathbf{x}(n)\, e(n)$$
Take $\mu = 10^{-4}$.

6. Continue the iteration in step 5. until the relative change $\dfrac{\|\mathbf{w}(n+1) - \mathbf{w}(n)\|^2}{\|\mathbf{w}(n)\|^2} < \varepsilon'$

Take $\varepsilon' = 10^{-3}$.

7. Plot the latest transfer function $\left|H\left(e^{j2\pi}\right)\right|^2$ of the filter (Fig.3).

8. Repeat the above with $F_0 = 2, 3, 10$ kHz and observe the change in $\left|H\left(e^{j2\pi}\right)\right|^2$.

Latest tranfer function of the adaptive filter
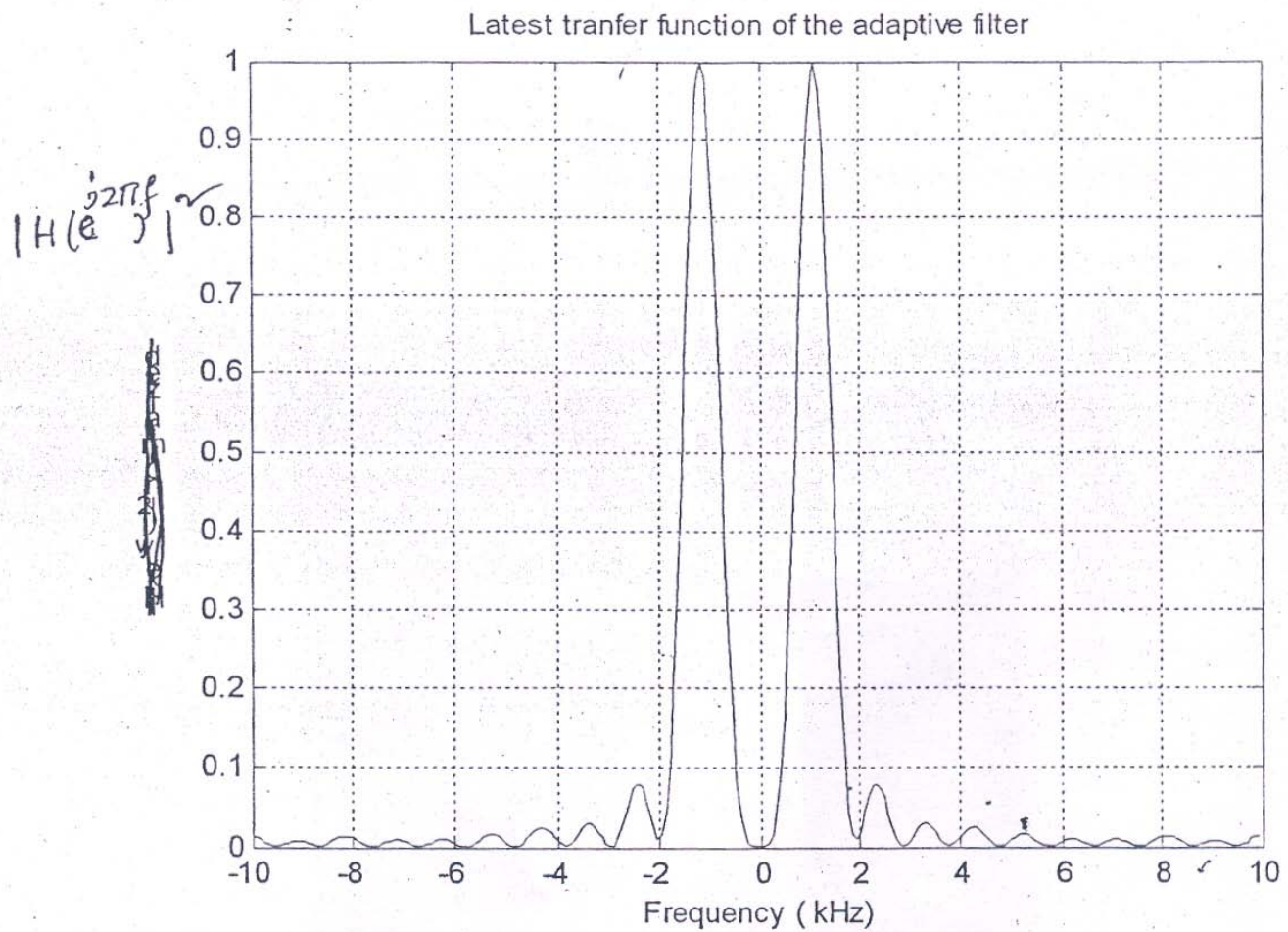
$|H(e^{j2\pi f})|$

Fig.3: The transfer function of the adaptive filter

- MATLAB functions that you may require: *randn, freqz.*

Reference:

[1] S.Haykin, *Adaptive Filter Theory*, Prentice-Hall, 1985

[2] J.Makhoul, "Linear prediction: A tutorial review," Proc. IEEE, vol. 63, pp. 649-661, 1975.