TITLE: DESIGNING LOW PASS FILTERS BY WINDOWING METHOD

OBJECTIVE: At the end of this experiment, you will be able to learn about :

(1) How to design FIR filters for various orders and cut-off Frequencies
(2) Whether pass band frequencies are attenuated and stop band frequencies are attenuated by the filter designed
(3) How your designed FIR filters are responding to the input signals contaminated by noise.

THEORY :

In digital signal processing, ideal filters having sharp cut-off are not realizable in practice since their impulse responses extend up to infinity.In order to realize filters practically ,we need to truncate the impulse response after a few samples.But this abrupt transition introduces many undesired features in the frequency domain(e.g. a large amount of power lies in the sidelobes). By windowing method, the truncated filter is modified to satisfy certain frequency domain requirements and to perform its task as well.

If $h_d(n)$ is the desired impulse response and if $H_d(w)$ is the corresponding Fourier Transform, it follows,

$$Hd(\omega) = \sum_{n=0}^{\infty} hd(n)\exp(-j\omega n)$$

And $$hd(n) = (1/2\pi)\int_{-\pi}^{\pi} Hd(\omega).\exp(j\omega n)d\omega$$

Let w(n) be the window sequence and $W(\omega)$ its F.T.

So, $$W(\omega) = \sum_{n=0}^{N-1} w(n)\exp(-j\omega n)$$

$$w(n) = (1/2\pi)\int_{-\pi}^{\pi} W(\omega).\exp(j\omega n)d\omega$$

[NOTE: w(n) is non-zero in the interval 0 to N-1,otherwise zero]

Now if we multiply w(n) and $h_d(n)$ to get

$h(n) = w(n)*h_d(n)$

$$H(\omega) = (1/2\pi)\int_{-\pi}^{\pi} Hd(v)W(\omega - v)dv$$

The window function is properly selected to achieve desired $H(\omega)$ specifications.

A few window functions

(1) Rectangular window : $w(n) = 1$; $n=0,1,\ldots,N-1$

$= 0$ otherwise

(2) Triangular window : $w(n) = 1 - 2*[n - (N-1)/2]\ /(N-1)$; $n=0,1,\ldots,N-1$

$= 0$ otherwise

(3) Hanning window: : $w(n) = 0.5 - 0.5 * \cos[2\pi n/(N-1)]$; $n=0,1,\ldots N-1$

$= 0$ otherwise

(4) Hamming window : $w(n) = 0.54 - 0.46 * \cos[2\pi n/(N-1)]$; $n=0,1,\ldots N-1$

$= 0$ otherwise

(5) Blackmnan window : $w(n) = 0.42 - 0.5 * \cos[2\pi n/(N-1)]$
$+ 0.08 * \cos[4\pi n/(N-1)]$; $n=0,1,\ldots N-1$

$=0$ otherwise

PROCEDURE:-

(1) In time domain, an LPF is defined as

$$h_d(n) = \omega_c/\pi \qquad \text{for } n=k$$
$$= (\sin(\omega_c*[n-k]))/(\pi(n-k)) \qquad \text{for } n \neq k$$

where N = no of samples
k=(N-1)/2

(2) Choose an window function w(n)

(3)Compute $h(n) = h_d(n) * w(n)$

(4)Use **freqz**(B,A,ω) function where B=h(n),A=1 and vary ω from $-\pi$ to π

(5)Plot the output of **freqz** function (This is your designed FIR filters response)

(6)Do these steps with all five windows as mentioned above.

(7) Now change N to a higher value and repeat the above [e.g. you may take N=8,64 and 512.]

(8) At each time, note the following
  (a) approximate transition width of the main lobe
  (b) peak of the first side lobe
  (c) maximum stop-band attenuation

NOTE : In order to test the performance of the filter in the time domain, do the following.

(a) Construct a signal x(n) containing two sinusoids, one having frequency in the pass band ,the other in the stop band

(b) Pass x(n) through the filter to get output y(n) [use filtfilt() function to do the same]. Observe input and output spectra(in dB) by using fft() function.

(c) Produce white noise using **rand**() function. Add this noise to x(n). Now repeat step (b) .

(d) Find the corresponding SNR [find the relative amplitudes of the sinusoid and that of other unwanted frequencies (in dB) and take the difference].

RESULTS:-

TABLE -I

N	RECTANGULAR WINDOW		
	TRANSITION WIDTH	PEAK OF FIRST LOBE	MAXIMUM STOPBAND ATTENUATION
8			
64			
512			

Repeat the above table for all the windows.

TABLE-II

N	RECTANGULAR WINDOW		
	SIGNAL AMPLITUDE	NOISE AMPLITUDE	SNR
8			
64			
512			

Repeat the above table for all the windows.

COMMENT :

REFERENCE : (1) Digital signal processing: principles,algorithms and applications : Proakis & Manolakis
(2) Digital signal processing : Oppenheim & Schafer