Today's agenda
- Binary number system
- Operators
- Problems
- Constraints → Time complexity

↳ decimal no. system
     ↳ {0 - 9} digits

000   010   020   30   91   100
001   011   021
         012

009   019   029   39   99

↳ Binary number system
     ↳ {0 1}

000 010   100
001 011   101

4, Conversion

→ convert decimal nos to binary:

30 →

$$\begin{array}{r|l} 2 & 30 \\ \hline 2 & 15 \\ \hline 2 & 7 \\ \hline 2 & 3 \\ \hline 2 & 1 \\ \hline & 0 \end{array}$$

2 | 30 — 0
2 | 15 — 1
2 | 7 — 1
2 | 3 — 1
2 | 1 — 1
0

→ 11110

Quiz

2 | 45 — 1
2 | 22 — 0
2 | 11 — 1
2 | 5 — 1
2 | 2 — 0
2 | 1 — 1
0

→ 101101

binary no. to decimal no. → bit index

$$0*2^3 \quad 0*2^1$$

$$\overset{4 \ 3 \ 2 \ 1 \ 0}{(\textcircled{1}\ 0\ \textcircled{1}\ 0\ \textcircled{1})_2:} \qquad \rightarrow \quad no. * 2^{bit\ index}$$

$$1*2^4 \quad 1*2^2 \quad 1*2^0$$

add all

$$\downarrow 1*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 1*2^0 = 2^0 + 2^2 + 2^4 = 21$$

$$0: \quad \text{unset bit / off bit}$$

$$1: \quad \text{set bit / on bit}$$

Quiz:

$$\overset{6\ 5\ 4\ 3\ 2\ 1\ 0}{1\ 0\ 1\ 1\ 0\ 1\ 0}$$

$$\downarrow 2^1 + 2^3 + 2^4 + 2^6 = 2 + 8 + 16 + 64 = 90$$

$$\downarrow (1\ 0\ 2\ 0\ 1\ 0)_2: \rightarrow \quad \text{invalid input}$$

Add binary number

$$\overset{+1\;+1}{3\;6}\,8$$
$$4\;5\;4$$
$$\overline{8\;2\;2}$$

$0 + 0 \rightarrow 0$ $0 + 1 \rightarrow 1$

$1 + 0 \rightarrow 1$ $1 + 1 + 1 \rightarrow 11$

$1 + 1 \rightarrow 10$

$\rightarrow \{0, 1\}$

①
$$\overset{1}{0}\;\overset{1}{1}\;\overset{+1}{0}\;\overset{+1}{1}\;1\;1$$
$$0\;1\;1\;1\;1\;0$$
$$\overline{1\;1\;0\;1\;0\;1}$$

②
$$\overset{1\;1}{}\;\overset{0}{}$$
$$1\;0\;1\;1\;0$$
$$0\;0\;1\;1\;1$$
$$\overline{1\;1\;1\;0\;1}$$

Quiz

$$\overset{1}{0}\overset{0}{1}\;\overset{1}{0}\;\overset{1}{1}\;\overset{1}{0}\overset{1}{1}\;1$$
$$0\;1\;0\;0\;1\;1\;1$$
$$\overline{1\;0\;1\;0\;0\;1\;0}$$

Bitwise operators: → {and, or, xor, left shift, right Shift}

binary no. ←→ binary no. → (binary answer)
↳ decimal no.

(26) and (35) ——— final answer ———→
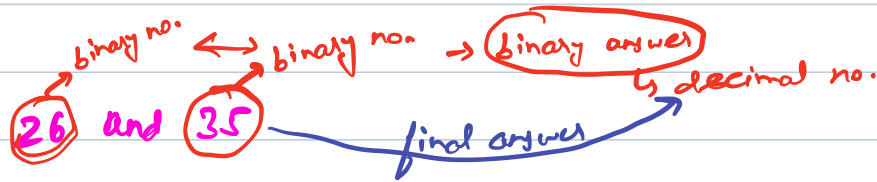
A	B	A & B	A | B	A ^ B
		0 is dominating	1 dominating	Same Same Puppy Shame
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

Symbol →	&&	&
	↓ relational operator (To combine multiple conditions)	↓ **Bitwise operator**
→	‖ ↙ true dominating ↓ relational operator	| ↓ Bitwise operator

① 23 & 10

$$23: \quad 1 \ 0 \ 1 \ 1 \ 1$$
$$10: \quad 0 \ 1 \ 0 \ 1 \ 0$$
$$\overline{\quad 0 \ 0 \ 0 \ 1 \ 0 \quad} \to 2^1 = 2$$
$$\qquad\quad 4 \ 3 \ 2 \ 1 \ 0$$

Quiz!

$$20: \quad 1 \ 0 \ 1 \ 0 \ 0$$
$$10: \quad 0 \ 1 \ 0 \ 1 \ 0$$
$$\overline{\quad 0 \ 0 \ 0 \ 0 \ 0 \quad} \to 0$$

② 23 | 10

$$23: \quad 1 \ 0 \ 1 \ 1 \ 1$$
$$10: \quad 0 \ 1 \ 0 \ 1 \ 0$$
$$\overline{\quad 1 \ 1 \ 1 \ 1 \ 1 \quad} \Rightarrow 2^0 + 2^1 + 2^2 + 2^3 + 2^4 = 31$$
$$\qquad\quad 4 \ 3 \ 2 \ 1 \ 0$$

Quiz:

$$20: \quad 1 \ 0 \ 1 \ 0 \ 0$$
$$10: \quad 0 \ 1 \ 0 \ 1 \ 0$$
$$\overline{\quad 1 \ 1 \ 1 \ 1 \ 0 \quad} \Rightarrow 2^1 + 2^2 + 2^3 + 2^4 = 30$$

③ 23 ^ 10

$$23: \quad 1 \ 0 \ 1 \ 1 \ 1$$
$$10: \quad 0 \ 1 \ 0 \ 1 \ 0$$
$$\overline{\quad 1 \ 1 \ 1 \ 0 \ 1 \quad} \Rightarrow 2^0 + 2^2 + 2^3 + 2^4 = 29$$

Quiz: 20 ^ 15

$$20: \quad 1 \ 0 \ 1 \ 0 \ 0$$
$$15: \quad 0 \ 1 \ 1 \ 1 \ 1$$
$$\overline{\quad 1 \ 1 \ 0 \ 1 \ 1 \quad} \Rightarrow 2^0 + 2^1 + 2^3 + 2^4 = 27$$
$$\qquad\quad 4 \ 3 \ 2 \ 1 \ 0$$

Q) you have been given a Positive no., identify whether the number is even or odd.

Ex: N = 8 → even

N = 7 → odd

Note: Use of +, -, *, / or % is not allowed.

N = 10:   1 0 1 0
   +1   0 0 0 1

N = 11:   1 0 1 1
   +1   0 0 0 1

N = 12:   1 1 0 0
   +1   0 0 0 1

N = 13:   1 1 0 1

Obs: (I) rightmost bit for even no. is 0

(II) rightmost bit for odd na is 1

↳ To check rightmost bit is 0 or 1, use bitwise and operator.

N = 10:   1 0 1 0   N = 11:   1 0 1 1
   &
    1:     0 0 0 1   1:   0 0 0 1
        _____   _____
10&1:   0 0 0 0   0 0 0 1

$N \& 1$
$\rightarrow 0 \rightarrow$ even

$\rightarrow 1 \rightarrow$ odd

// Psuedo code
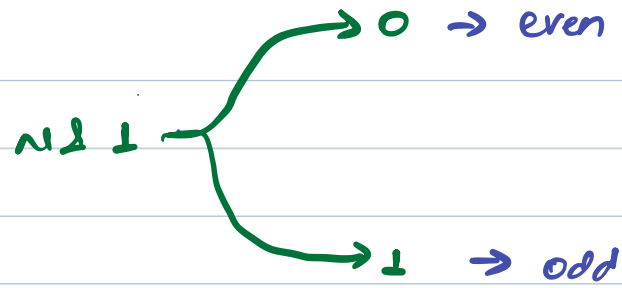
```
if (N & 1 == 0) {
    s.o.p ("even");
}
else {
    s.o.p ("odd");
}
```

Break till 9:33 Pm

|| Properties

↳ (i) Commutative Property

$$a \& b = b \& a$$
$$a | b = b | a$$
$$a \wedge b = b \wedge a$$

(II) Associative Property

$$a \& b \& c \rightarrow (a \& b) \& c = a \& (b \& c)$$
$$(a | b) | c = a | (b | c)$$
$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

(III)

(a) $N \& N = N$

$N \rightarrow$	1	1	0
$N \rightarrow$	1	1	0
$N \& N \rightarrow$	1	1	0

$N \rightarrow$	1	1	0
$0 \rightarrow$	0	0	0
$N \& 0 \rightarrow$	0	0	0

(b) $N \& 0 = 0$

(c) $N | 0 = N$

$N \rightarrow$	1	1	0
$0 \rightarrow$	0	0	0
$N | 0 \rightarrow$	1	1	0

$N \rightarrow$	1	1	0
$N \rightarrow$	1	1	0
$N | N \rightarrow$	1	1	0

(d) $N | N = N$

(e) $N \wedge 0 = N$

$N \rightarrow$	1	1	0
$0 \rightarrow$	0	0	0
$N \wedge 0 \rightarrow$	1	1	0

$N \rightarrow$	1	1	0
$N \rightarrow$	1	1	0
$N \wedge N \rightarrow$	0	0	0

(f) $N \wedge N = 0$

Q) Given arr[N], every element appears twice except for one element which appears once, find that unique element.

Ex: arr[7]: { 6 8 8 7 7 10 6 } → 10

arr[5]: { 2 2 1 9 9 } → 1

//idea
↳ Take xor of all array elements.

arr[7]: { 6 8 8 7 7 10 6 }

⇓

$6 \wedge 8 \wedge 8 \wedge 7 \wedge 7 \wedge 10 \wedge 6 \Rightarrow 6 \wedge 6 \wedge 7 \wedge 7 \wedge 8 \wedge 8 \wedge 10 = 10$

//Psuedo code

int ans = 0;

```
for (int i=0; i<n; i++) {
    ans = ans ^ arr[i];
}

S.O.p (ans);
```
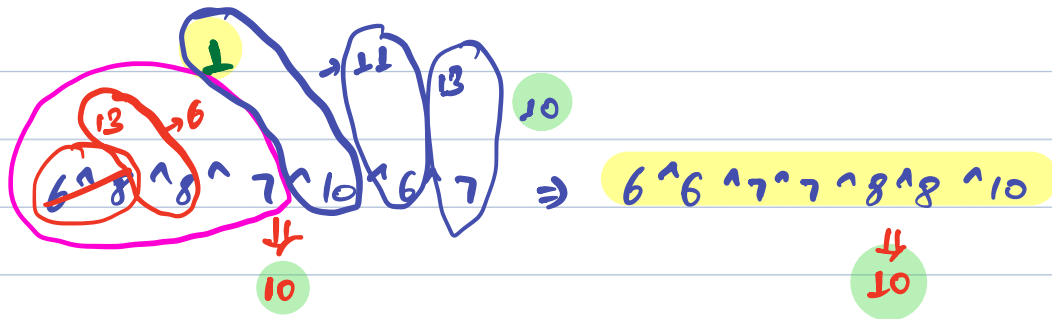
6 ^6 ^7 ^7 ^8 ^8 ^10

6: 0110   10:1110   6: 0110
8: 1000   8: 1000   7: 0111
6&8: 1110=14   8&8: 0110=6   0001=1

1: 0001   11: 1011   13: 1101
10: 1010   6: 0110   7: 0111
1011=11   1110=13   1010

10 + 12 + 13 + 15   =   12 + 15 + 10 + 13
Ans1   =   Ans2

↳ **left shift (<<)**

$$
\begin{array}{c c c c c c c}
6 & 5 & 4 & 3 & 2 & 1 & 0
\end{array}
$$

a : 45 :   1   0   1   1   0   1   → 45

a << 1 :   1   0   1   1   0   1   0   → 90

a << 2 : 1   0   1   1   0   1   0   0 → 180

*2 with every left shift
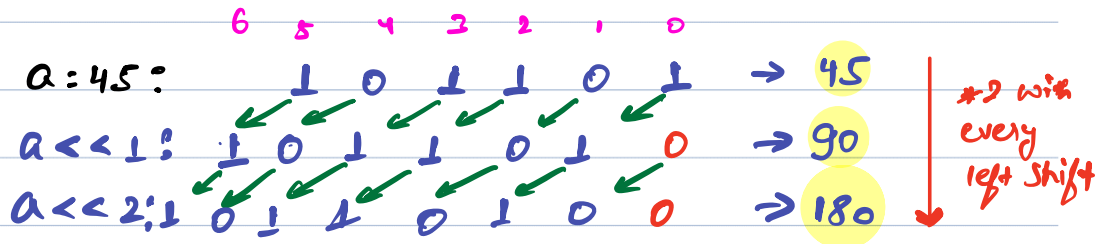
$$ a << N = a * 2^N $$

Quiz :   a == 1

$$ 1 << N = 1 * 2^N = 2^N $$

↳ for 2^N , you need to use **left shift operator.**

//right Shift (>>)
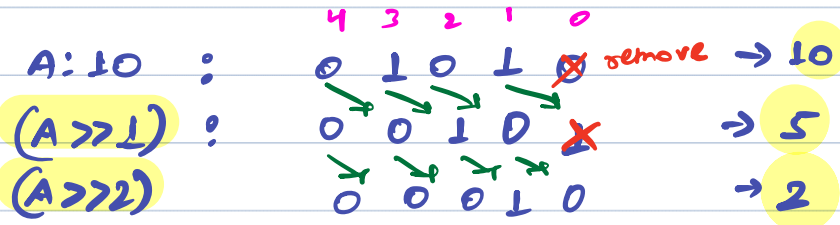
		4	3	2	1	0		
A: 10	:	0	1	0	1	0̶ remove	→	10
(A >> 1)	:	0	0	1	0	✗	→	5
(A >> 2)		0	0	1	0	0	→	2

$$A >> N : \frac{A}{2^N}$$

* Constraints

↳ **max allowed**

Array length

ex: $N = 10^5$

↳ $O(N^2)$ → Solution → x a

$$(10^5)^2 = 10^{10}$$

↳ $O(N\sqrt{N})$ →

$10^5 * \sqrt{10^5}$ ⇒ $10^5 * 10^{2.5} = 10^{7.5}$ ✓

vme → Satyam → iit delhi   dtu

↳

Maths → Problems memorize

understand   Understand
↓   +
                    memorize   45 mins
                                    ↓
                                    3 questions