Q1 Let,

G : Light is green
Y : Light is yellow
R : Light is red

i) now we need traffic light to be either of the colors but one state at a time

So, PL is

$$(G \vee Y \vee R) \wedge \neg(G \wedge Y) \wedge \neg(G \wedge R) \wedge \neg(Y \wedge R)$$

ii) Now traffic light goes from,

$G \rightarrow Y$, $Y \rightarrow R$, $R \rightarrow G$

Let us denote current state as (i) and next state as $(i+1)$

So, PL is

$$(G_i \wedge Y_{i+1}) \vee (Y_i \wedge R_{i+1}) \vee (R_i \wedge G_{i+1})$$

iii) Since traffic light can only remain in same state 3 times i.e $i, i+1, i+2$.

So, the PL is

$$\neg(G_i \wedge G_{i+1} \wedge G_{i+2}) \wedge \neg(Y_i \wedge Y_{i+1} \wedge Y_{i+2}) \wedge \neg(R_i \wedge R_{i+1} \wedge R_{i+2})$$

$$= (G \vee Y \vee R) \wedge \neg(G \wedge Y) \wedge \neg(G \wedge R) \wedge \neg(Y \wedge R)$$

----- so on expansion

Q2 $N \rightarrow$ non empty set of nodes
$R(n,m) \rightarrow$ edge from node n to m
$C(n,x) \rightarrow$ node n has color x

Axioms are :-

i) Connected nodes don't have same color

$$\forall n,m \in N, \forall x \left(R(n,m) \wedge C(n,x) \rightarrow \neg C(m,x) \right)$$

ii) exactly two nodes are allowed to wear yellow

$$\exists n_1, n_2 \in N \{ C(n_1, \text{yellow}) \wedge C(n_2, \text{yellow}) \wedge n_1 \neq n_2 \wedge \forall m \in N (m \neq n_1 \wedge m \neq n_2 \rightarrow \neg C(m, \text{yellow})) \}$$

iii) Starting from red node you can reach green in no more than 4 steps

$$\forall n \in N (C(n, \text{red}) \rightarrow (\exists n_1 \in N (R(n,n_1) \wedge C(n_1, \text{green})) \vee$$
$$\exists n_1, n_2 \in N (R(n,n_1) \wedge R(n_1,n_2) \wedge C(n_2, \text{green})) \vee$$
$$\exists n_1, n_2, n_3 \in N (R(n,n_1) \wedge R(n_1,n_2) \wedge R(n_2,n_3) \wedge C(n_3, \text{green})) \vee$$
$$\exists n_1, n_2, n_3, n_4 \in N (R(n,n_1) \wedge R(n_1,n_2) \wedge R(n_2,n_3) \wedge R(n_3,n_4) \wedge C(n_4, \text{green}))))$$

iv) For every color, there is atleast one node with this color

$$\forall x \in \{c_1 \dots c_k\}, \exists n \in N \; C(n,x)$$

v) The nodes are divided into exactly $|C|$ disjoint cliques of each color

$x \in \{c_1 \dots c_k\}$, $n \in N$

$$\forall x \exists n \, C(n,x) \wedge \forall n \exists x \, C(n,x) \wedge$$
$$\forall n \forall x (C(n,x) \rightarrow \neg \exists y (y \neq x \wedge C(n,y))) \wedge$$
$$\forall n \forall m \forall x (n \neq m \wedge C(n,x) \wedge C(m,x) \rightarrow$$
$$(R(n,m) \bigvee_{i=1}^{|N|} (\exists n_1 \dots n_i . (R(n,n_1) \bigwedge_{j=1}^{i-1} R(x_j, x_{j+1})$$
$$\wedge R(n_i, m))))))$$

Q3 Let us take predicates,

$R(x)$: x can read
$L(x)$: x is literate
$I(x)$: x is intelligent
$D(x)$: x is dolphin

i) PL : $R \rightarrow L$
FOL : $\forall x (R(x) \rightarrow L(x))$

ii) PL : $D \rightarrow \neg L$
FOL : $\forall x (D(x) \rightarrow \neg L(x))$

iii) PL : $D \wedge I$
FOL : $\exists x \; (D(x) \wedge I(x))$

iv) PL : $I \wedge \neg R$
FOL : $\exists x (I(x) \wedge \neg R(x))$

v) PL : $D \wedge I \wedge R \wedge ((D \wedge I \wedge R) \rightarrow \neg L)$
FOL : $\exists x (D(x) \wedge I(x) \wedge R(x)) \wedge$
$\forall y ((D(y) \wedge I(y) \wedge R(y) \rightarrow \neg L(y))$

for proving statement 4, we need to take negation of statement 4

i.e to prove $\neg (I \wedge \neg R)$
$= \neg I \vee R$

S1 : $R \rightarrow L = \neg R \vee L$
S2 : $D \rightarrow \neg L = \neg D \vee \neg L$
S3 : $D \wedge I$
S4 : $I \wedge \neg R$
S_5 : $\neg I \vee R$

We resolve I and S_5 to get

S_6 : R

resolve S_6 with S_1 to get

S_7 : L

resolve S_7 with C_2 to get contradiction as L and $\neg D \vee \neg L$ resolution leads to contradiction because both L and $\neg L$ can't be true at same time.

Hence, S_5 contradicts

So, S_4 i.e $I \wedge \neg R$ is True.

for proving statement 5

we negate statement 5 to get

$\neg D \vee \neg I \vee \neg R \vee L$

S_1 : $\neg R \vee L$
S_2 : $\neg D \vee \neg L$
S_3 : $D \wedge I$
S_4 : $I \wedge \neg R$
S_5 : $\neg D \vee \neg I \vee \neg R \vee L$

Since we already proved from S_3 that D and I are true

S_5 is reduced to

S_6 : $\neg R \vee L$

from S_1 we get $\neg R \vee L$ which contradicts S_6 as both $\neg L$ and L can't be true hence it is contradiction and

so, S_5 i.e $D \wedge I \wedge R \wedge (D \wedge I \wedge R \rightarrow \neg L)$ is True.
