# BTP-1
# Flash Drought Prediction

Mohammad Zaid (2021101004) and Pranjali Bishnoi (2021101038)

## Introduction

The term ‘flash droughts (FDs)’ is commonly used to describe droughts that develop quickly over a short time. These events typically occur when a combination of high temperatures, low humidity, and little to no precipitation leads to an abrupt depletion of soil moisture. They can be aggravated by factors such as heatwaves, strong winds, and evaporative demand, which increase the rate of water loss from the soil and vegetation.

To address the need for effective prediction strategies, this study explores the utility of two prominent drought prediction metrics, namely the Standardised Precipitation Index (SPI) and the Standardised Precipitation Evapotranspiration Index (SPEI). Leveraging the capabilities of standard machine learning models such as Artificial Neural Networks (ANN) and Recurrent Neural Networks (RNN), we aim to harness the potential of these indices in forecasting flash droughts with a high degree of accuracy.

## Drought Metrics - SPI and SPEI

We explored the following metrics for drought categorization :

SPI: The Standardised Precipitation Index (SPI), is used for detecting and characterising drought on the basis of a long term precipitation record.

SPEI: The Standardised Precipitation Evapotranspiration Index (SPEI) is an extension of the widely used Standardised Precipitation Index (SPI). The SPEI is designed to take into account both precipitation and potential evapotranspiration (PET) in determining drought.

We used SPI as the provided dataset for Hyderabad has precipitation value only.

## Metric Used for Drought Analysis - SPI

Gamma distribution is used to fit the frequency distribution of precipitation summed over individual months (for various time scales).

$$g(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{\frac{-x}{\beta}} \quad \text{for } x > 0$$ ; $$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y} dy$$

Two parameters : alpha (α) and beta (β).

Increasing α leads to a more "peaked" distribution, while increasing β increases the "spread" of the distribution.

For x<0 , g(x)=0

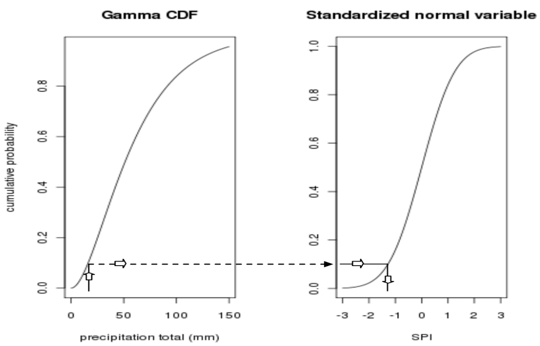
# Algorithm for SPI Calculation

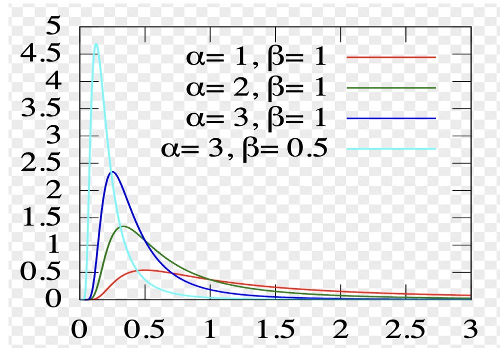
1. Define the time scale
2. Calculate the precipitation over the defined time scale
3. Calculate the precipitation sum over the chosen time scale - Sliding Window
4. Fit the sum data to a probability distribution - get the value of parameters
5. Now after fitting into probability distribution use the value of the parameters alpha and beta to deduce SPI
6. Calculate the cumulative probability for each data point from the gamma distribution (because we have the parameters now we can get the cumulative probability)
7. Transform the cumulative probability into the SPI value by using inverse

```
def calculate_precipitation_sum(data, time_scale):
    precipitation_sum = []
    for i in range(len(data) - time_scale + 1):
        sum_value = sum(data[i:i + time_scale])
        precipitation_sum.append(sum_value)
    return precipitation_sum
# Example usage:
precipitation_data = [10, 20, 15, 25, 30, 10, 5, 10, 15, 20]
precipitation_sum = calculate_precipitation_sum(precipitation_data, time_scale)
```

# Reasoning Behind SPI ( Standard precipitation Index)

1. Flexibility in parameters
2. Typically, rainfall has negatively skewed distribution i.e. smaller rainfall magnitudes occur with larger frequencies. Thus, instead of a common normal pdf, Gamma pdf provides better fit for the data.




# Interpretation of SPI values

From the precipitation data, we can have SPI values for each month (or 3 months, depending on time scale), which can provide categorization of drought.

Dataset : Daily rainfall data for 20 years spanning different Mandals of Hyderabad

Final Goal : Predict droughts based on past SPI values

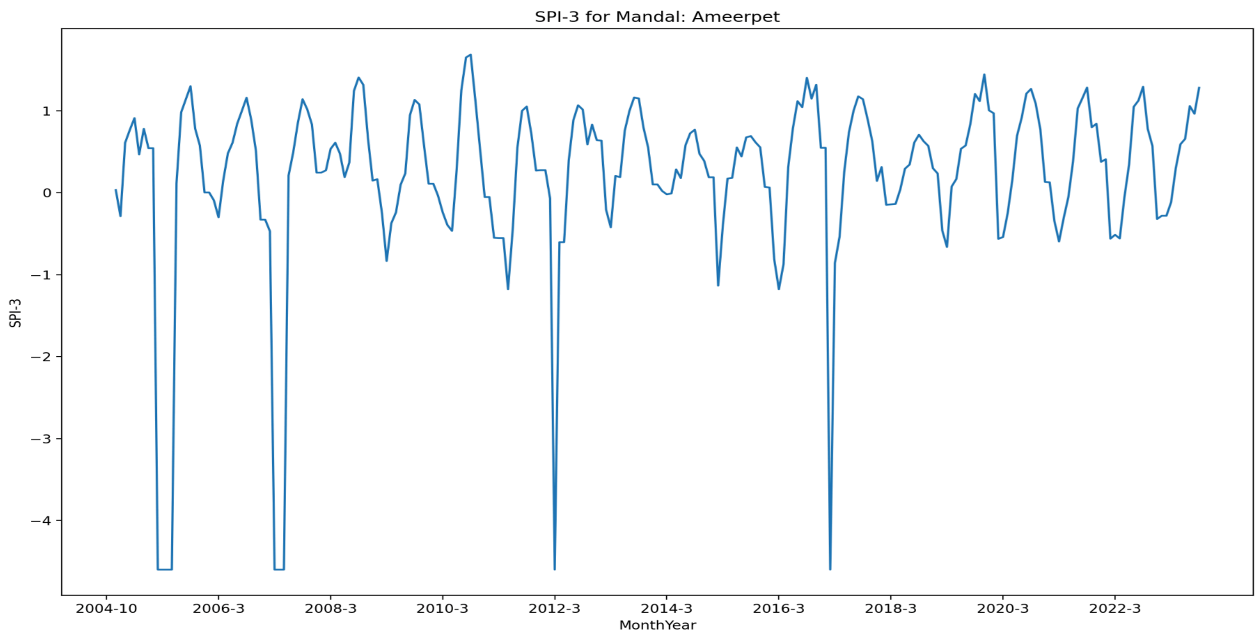
| SPI value | Category | Probability (%) |
|---|---|---|
| 2.00 or more | Extremely wet | 2.3 |
| 1.50 to 1.99 | Severely wet | 4.4 |
| 1.00 to 1.49 | Moderately wet | 9.2 |
| 0 to 0.99 | Mildly wet | 34.1 |
| 0 to −0.99 | Mild drought | 34.1 |
| −1.00 to −1.49 | Moderate drought | 9.2 |
| −1.50 to −1.99 | Severe drought | 4.4 |
| −2 or less | Extreme drought | 2.3 |

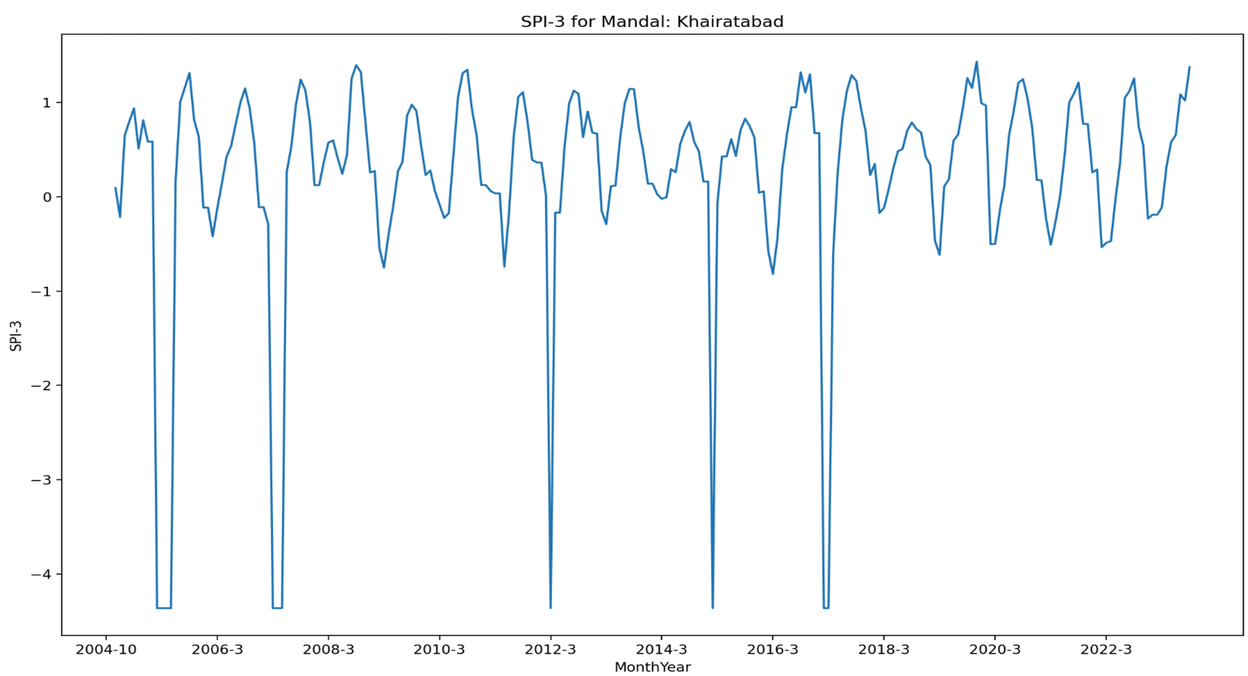
# Plots

The plots are showing the SPI-3 (Standardised Precipitation Index over a 3-month period) for a Mandal region.

The x-axis represents time in month-year format, spanning from 2004 to 2022. The y-axis displays the SPI-3 values, which oscillate between positive and negative numbers over time, indicating periods of wet and dry conditions.

There are 7 mandals given in the dataset, plots for any 2 of them are as follows:




# Drought Prediction

The following paper has been used as a reference:

*Article*

# Prediction of short and long-term droughts using artificial neural networks and hydro-meteorological variables

**Yousef Hassanzadeh[1], Mohammadvaghef Ghazvinian[2*], Amin Abdi[2], Saman Baharvand[3], Ali Jozaghi[3]**

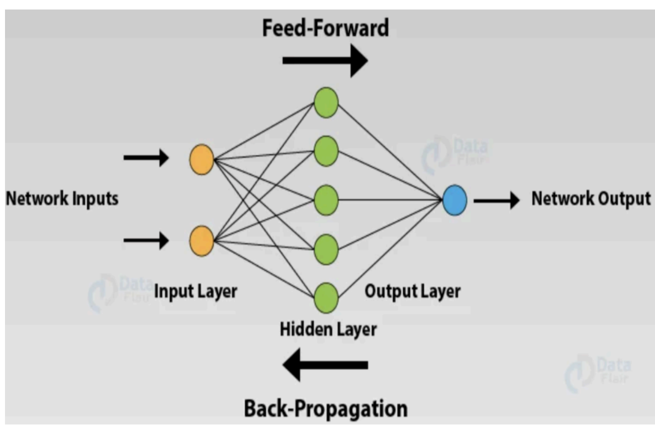
[1] Department of Water Engineering, Center of Excellence in Hydroinformatics, Faculty of Civil Engineering, University of Tabriz, Tabriz, Iran; yhassanzadeh@tabrizu.ac.ir

[2] Department of Water Engineering, Faculty of Civil Engineering, University of Tabriz, Tabriz, Iran; aabdi@tabrizu.ac.ir

[3] Department of Civil Engineering, University of Texas at Arlington, P.O. Box 19019, Arlington, TX 76019, USA; Saman.Baharvand@uta.edu, Ali.Jozaghi@mavs.uta.edu

* Corresponding author: mohammadvaghef.ghazvinian@mavs.uta.edu ; Tel.: +1-817-437-6440; Current affiliation: Department of Civil Engineering, University of Texas at Arlington, P.O. Box 19019, Arlington, TX 76019, USA

First off we used the ANN model

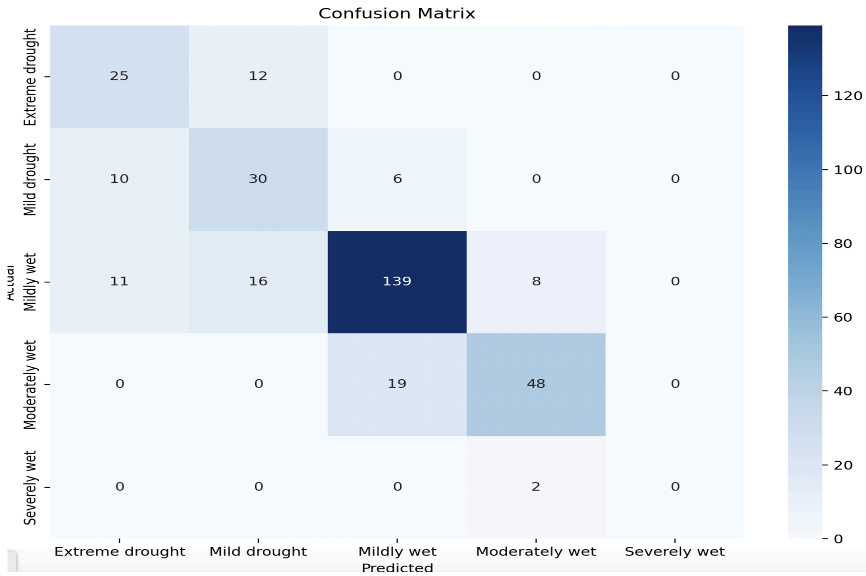



| Model No. | ANN Model |
|---|---|
| 1 | $SPI_t = f\left(SPI_{t-1}, SPI_{t-2}\right)$ |
| 2 | $SPI_t = f\left(SPI_{t-1}, SPI_{t-2}, P_{t-1}, P_{t-2}\right)$ |
| 3 | $SPI_t = f\left(SPI_{t-1}, SPI_{t-2}, V_{t-1}, V_{t-2}\right)$ |

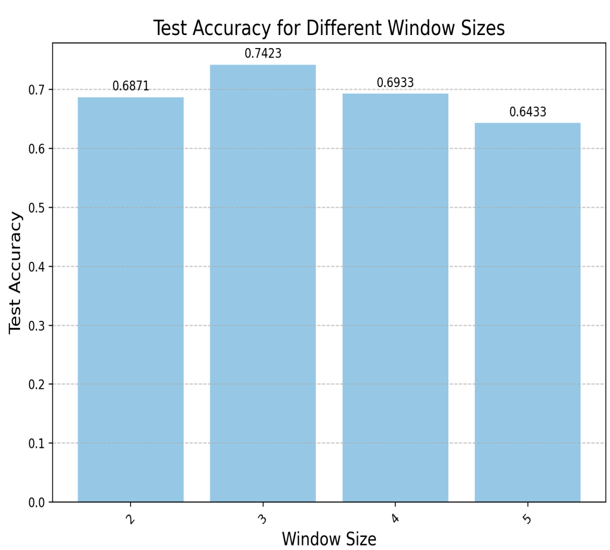
1. SPI as a time step t is predicted as a function of previous SPI and other input variables.
2. Model Needs to learn the relationship.
3. Looking too much into the past may degrade the performance.

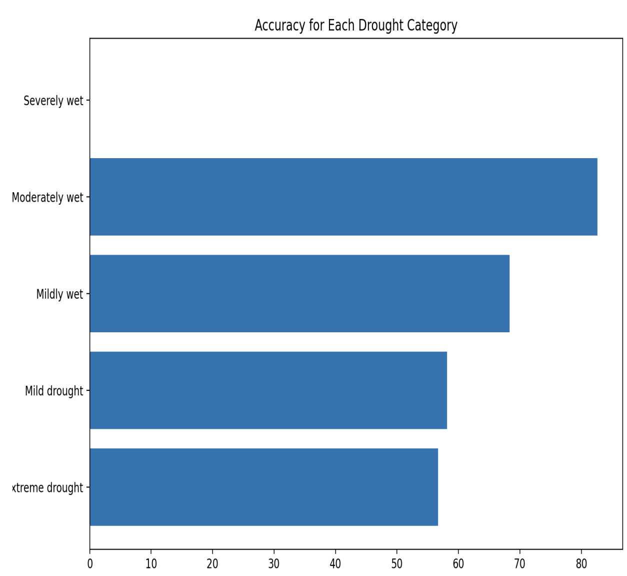
# Results

1. We applied the ANN model presented in the paper based on SPI values to the Hyderabad precipitation dataset.
2. Values are highest among diagonals.
3. Test Accuracy : 74 %
4. Structure of Dataset : Dataset contains 20 years of precipitation values for the 7 mandals of hyderabad.



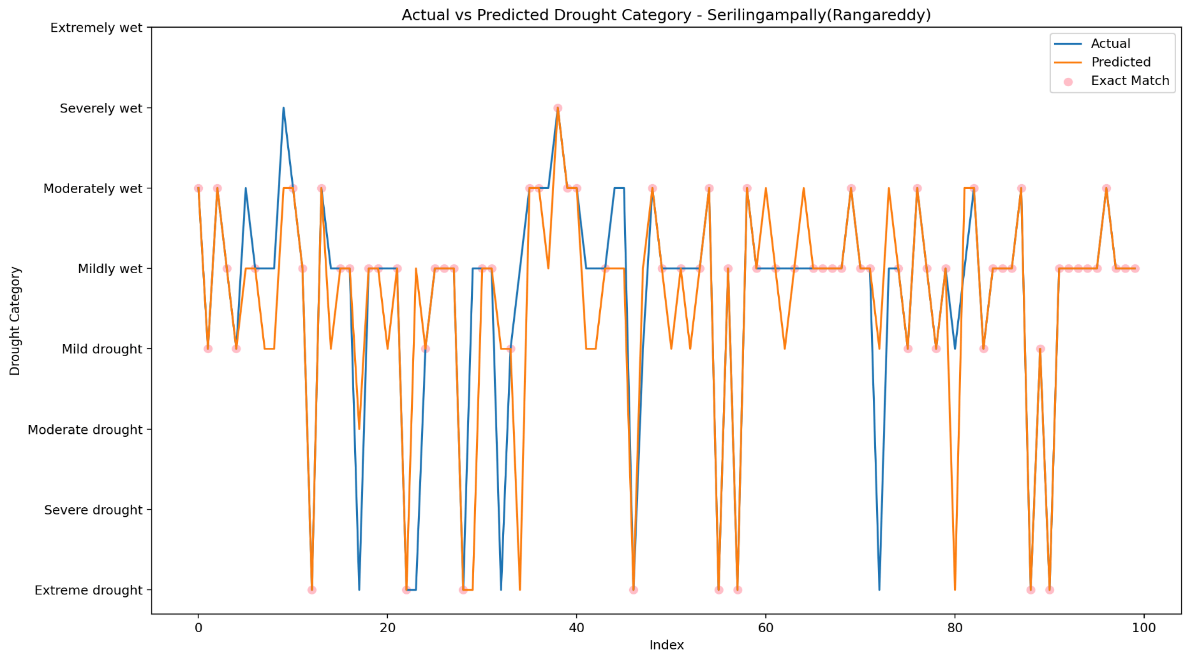

# Accuracy Plots:



# Actual vs Predicted Drought Category

The plot is for the Serilingampally mandal in Rangareddy district



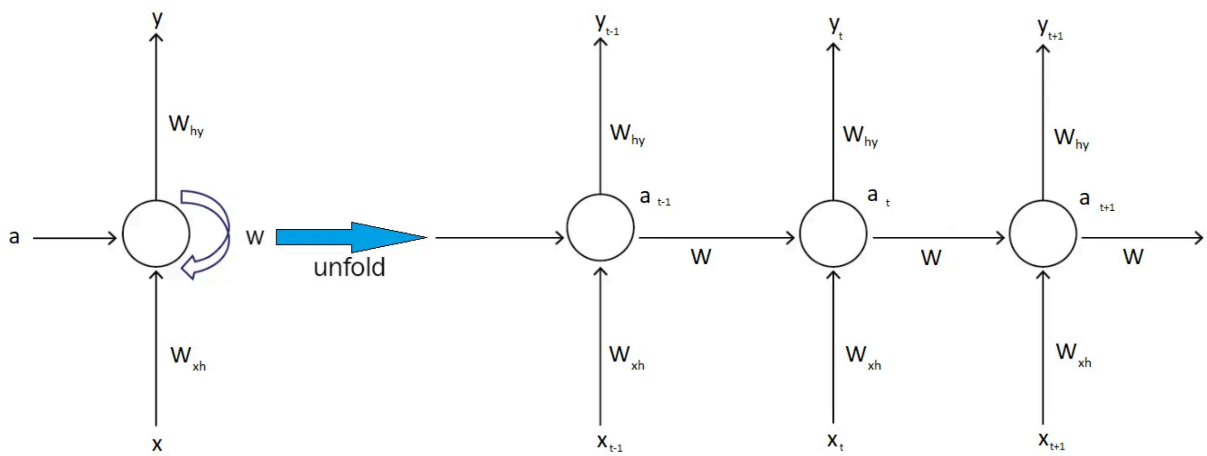

# Recurrent Neural Network

1. Accuracy : 82%
2. RNN performs better compared to ANN in terms of accuracy. However this is majorly because of the way training and testing data is split in RNN.
3. ANN splits data points into train and test randomly, hence they get more diverse points to test. However since RNN conventionally has the last set of data points as test data,the split is not so diverse. In our case , it's not tested on peaks(extreme drought).

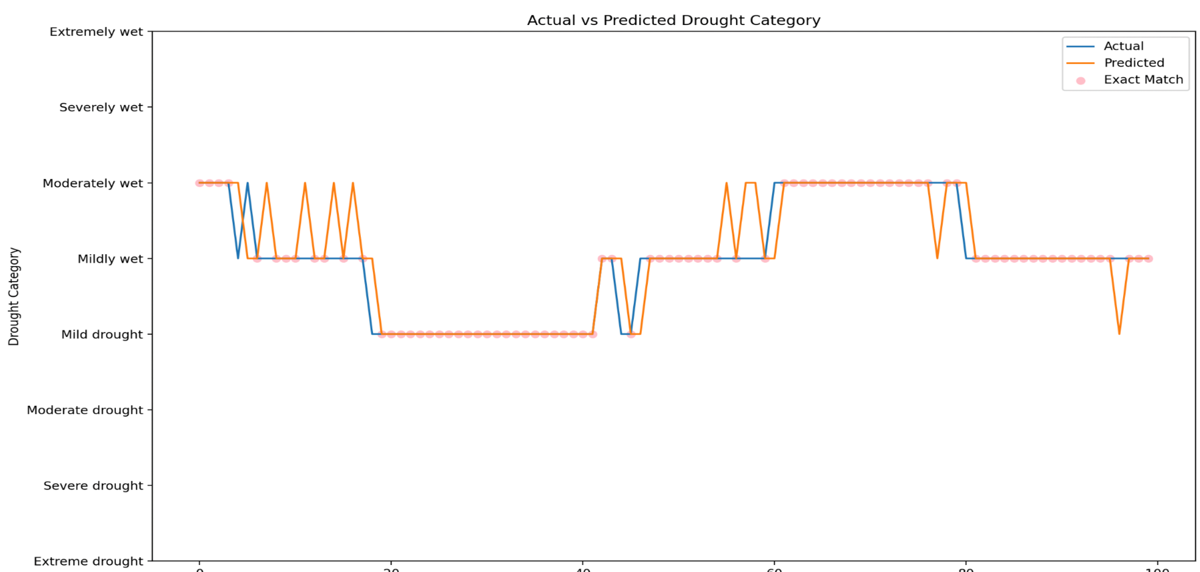
```

# Train Test Splitting - RNN
X_train, X_test, y_train, y_test = X[:int(0.8*len(X))], X[int(0.8*len(X)):], y[:int(0.8*len(y))], y[int(0.8*len(y)):]
```

```
## Train Test Split - ANN
X_train, X_test, y_train, y_test = train_test_split(X_window, y, test_size=0.2, random_state=42)
```

# RNN Architecture




# RNN Prediction

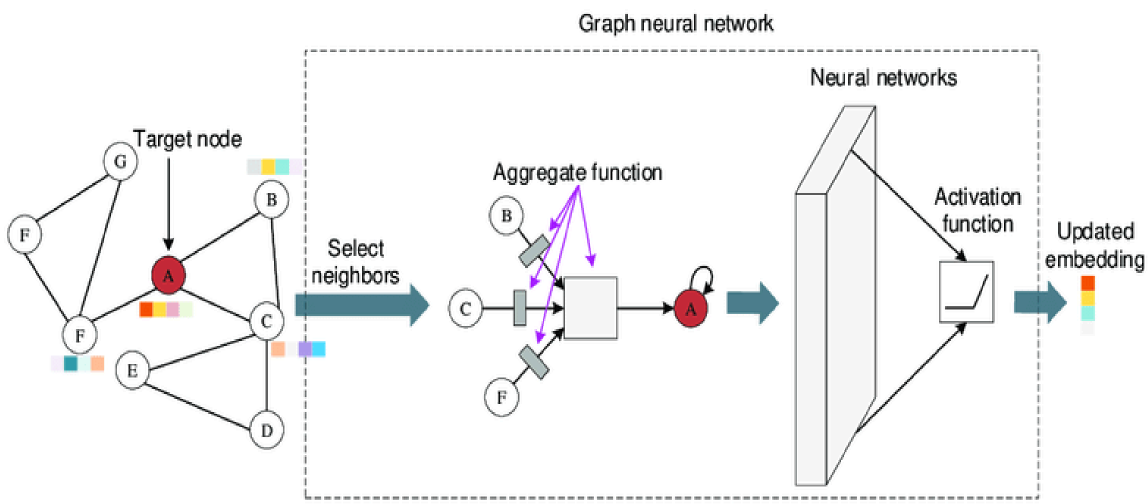



# Proposal : Graph Neural Network

1. Problem With ANN and RNN : Models only capturing temporal dependency and not the spatial dependency. We hypothesise that a model which takes advantage of both time as well as spatial dependency should work better.

2. GNN is a neural network that uses deep learning to analyse data structures that are represented as graphs.
3. Model Mandals as Nodes and make a fully connected graph,joining edges from every mandal to every other mandal.
4. However we need to formulate what the node embedding should capture and how should the Weight Matrix be defined.
5. Since GNN is a very recent development in Machine Learning, the architecture has not been analysed yet for Drought Prediction but GNN and ANN have been used in multiple research papers.

# GNN Architecture




# Conclusion

In our study of drought metrics, particularly the Standardised Precipitation Index (SPI), and its application in drought prediction using conventional Machine Learning Models, we have recognized certain limitations. Moving ahead with our project, our focus shifts towards devising improved algorithms and metrics for more accurate drought prediction.

Additionally, in the next part of the project, we aim to incorporate Graph Neural Networks into our framework to enhance the predictive capabilities and robustness of our models.

# BTP-2
# On Generalizing Descartes' Rule of Signs to Hypersurfaces

By Pranjali Bishnoi (2021101038)

Faculty Guide: Dr Abhishek Deshpande

## Descartes' rule of signs

**Descartes' rule of signs**: counts the roots of a polynomial by examining sign changes in its coefficients. **The number of positive real roots is at most the number of sign changes in the sequence of univariate polynomial's real coefficients** (omitting zero coefficients and the number of positive roots are counted with multiplicity).

Let $p(x) = a_0 + a_1x + \cdot \cdot \cdot + a_nx^n$ be a polynomial with real coefficients. The sign sequence of p(x) is the order of the signs of the nonzero coefficients $a_i$. A change in the sign sequence occurs any time the sign switches from + to −, or from − to +.

**Example 1.2.2.** *The polynomial* $p(x) = 3x^7 - 4x^6 - x^4 + 2x - 1$ *has sign sequence* $(+ - - + -)$. *The number of sign changes is three: from positive to negative, back to positive, then back to negative. Thus, Descartes' rule of signs tells us this polynomial has at most three positive roots, and the number is odd. So there are either one or three positive roots (counting multiplicity). In fact there is one, as illustrated in Figure 1.1.*

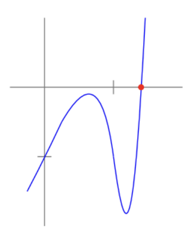


Figure 1.1: The graph of $p(x) = 3x^7 - 4x^6 - x^4 + 2x - 1$, with its one positive root emphasized.

## Generalizing descartes' rule of signs to Hypersurfaces

Descartes' rule of signs may be generalized to hypersurfaces in the following sense. Let $f : R^n_{>0} \rightarrow R$ be a signomial (a multivariate generalized polynomial, where we allow real exponents, restricted to the positive orthant), and consider the sets.

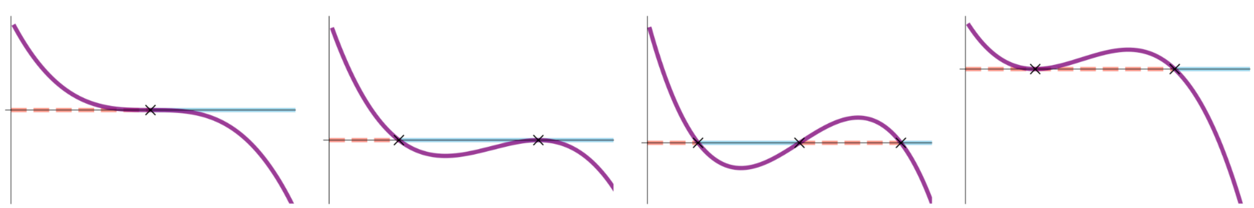
$$V_{>0}(f) := \{x \in R^n_{>0} \mid f(x)=0\} \text{ , } V^c_{>0}(f) := R^n_{>0} \backslash V_{>0}(f)$$

We aim at bounding the number of connected components of $V^c_{>0}(f)$ in terms of the relative position of the exponent vectors of each monomial of f in $R^n$, and the sign of the coefficients.

## Descartes' rule of signs "dual" presentation

- It gives an upper bound on the number of connected components of $R_{>0}$ minus the zero set of the polynomial.

- **If we write $f(x) = a_0 + a_1x + \cdots + a_nx^n$ with $a_n \neq 0$, and let ρ be the Descartes' bound on the number of positive roots, then there are at most ρ + 1 connected components. If ρ is odd, the upper bounds for the number of components where f is positive or negative agree, while if ρ is even, then there are at most ρ/2 + 1 connected components where f attains the sign of $a_n$.**



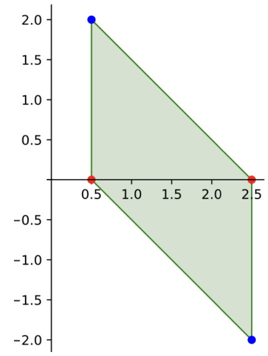
(a) (b) (c) (d)

FIGURE 1. Graphs of polynomials $p$ of degree three with coefficient sign sequence $(+-+-)$. In each figure, the connected components of $\mathbb{R}_{>0}$ minus the zero set of $p$, where $p$ evaluates positively or negatively, are shown in red and blue respectively. (a) $8 - 12x + 6x^2 - x^3$. (b) $9 - 15x + 7x^2 - x^3$. (c) $15 - 23x + 9x^2 - x^3$. (d) $3 - 7x + 5x^2 - x^3$.

## Terminologies

- The central object of study is a function:

$$f: \mathbb{R}^n_{>0} \to \mathbb{R}, \qquad f(x) = \sum_{\mu \in \sigma(f)} c_\mu x^\mu, \quad \text{with } c_\mu \in \mathbb{R} \setminus \{0\}$$




- $\sigma(f) \subseteq R_n$ is a finite set, called the support of f.

- $\sigma_+(f) := \{\alpha \in \sigma(f) | c_\alpha > 0\}$ and $\sigma_-(f) := \{\beta \in \sigma(f) | c_\beta < 0\}$.

- The support of the signomial $p_1(x_1,x_2) = x_1^{2.5} - 2x_1^{0.5}x_2^2 + x_1^{0.5} - x_1^{2.5}x_2^{-2}$ is $\sigma(p_1) = \{(2.5, 0), (0.5, 2), (0.5, 0), (2.5, -2)\}$. The points (2.5, 0), (0.5, 0) are positive, while the points (0.5, 2), (2.5, −2) are negative.

- For every $v \in R^n$ and $a \in R$, a hyperplane is defined $H_{v,a} := \{\mu \in R^n \mid v \cdot \mu = a\}$, and two half-spaces $H^+_{v,a} := \{\mu \in R^n \mid v \cdot \mu \geq a\}$ and $H^-_{v,a} := \{\mu \in R^n \mid v \cdot \mu \leq a\}$. $H^{+,\circ}_{v,a}$, $H^{-,\circ}_{v,a}$ denote the interior of $H^+_{v,a}$ and $H^-_{v,a}$ respectively.

- We say that v is a separating vector of σ(f) if for some $a \in R$ it holds

  $$\sigma_-(f) \subseteq H^+_{v,a}\ ,\ \sigma_+(f) \subseteq H^-_{v,a}$$

  The separating vector v is strict if $\sigma_-(f) \cap H^{+,\circ}_{v,a} \neq \varnothing$, and very strict if additionally $\sigma_-(f) \cap H_{v,a} = \varnothing$ for some $a \in R$. Let $S^-(f)$ denote the set of separating vectors of σ(f).

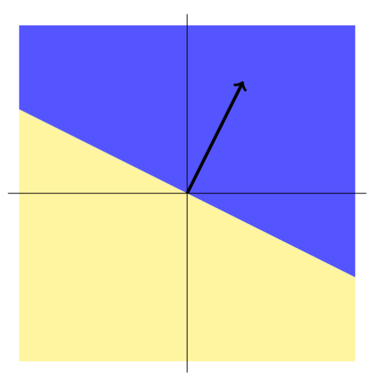


Figure 3.1: The half-spaces associated with $v = (1, 2)$ and $a = 0$, with $\mathcal{H}^+_{v,a}$ in blue (darker) and $\mathcal{H}^-_{v,a}$ in yellow (lighter)

- We say that v is an enclosing vector of σ(f) if for some a,b ∈ R, a ≤ b, it holds

$$\sigma_-(f) \subseteq H^+_{v,a} \cap H^-_{v,b} \text{ , } \sigma_+(f) \subseteq R^n \setminus (H^{+,\circ}_{v,a} \cap H^{-,\circ}_{v,b}).$$

- We denote by $E^-(f)$ the set of enclosing vectors of σ(f). For an enclosing vector v to be strict, there must be positive points on the side of the hyperplanes not containing the negative points

**Example 3.1.6.** *Consider the signomial*

$$f(x_1, x_2) = x_1^2 x_2 + 5x_1^{-2} x_2^{1.5} + 2x_2^{-2} - 2x_1^{.75} x_2^2 - 4x_2^{.3}$$

*where the support has partition* $\sigma_+ = \{(2,1), (-2,1.5), (0,-2)\}$ *and* $\sigma_- = \{(.75,2), (0,.3)\}$.

*Then* $v = (-2,1)$ *is an enclosing vector of* $f$ *with* $a = 0$ *and* $b = 1$*, as demonstrated in Figure 3.3.*

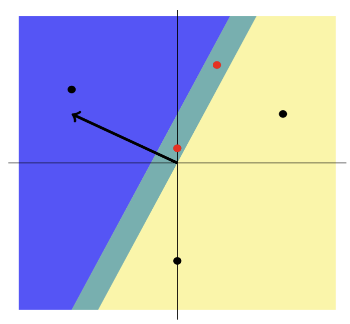
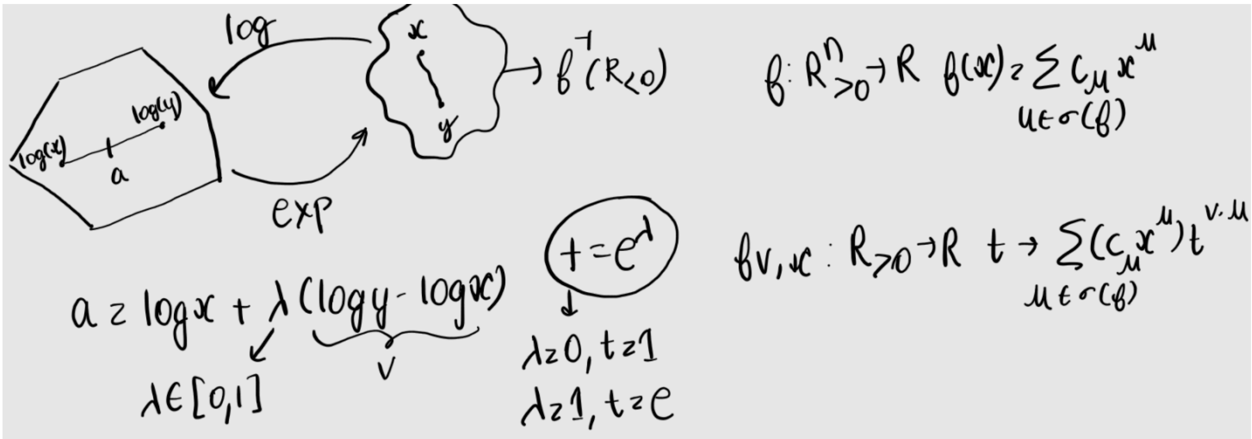


Figure 3.3: The enclosing vector $v = (-2,1)$, with $a = 0$ and $b = 1$. The half-plane $\mathcal{H}^+_{v,a}$ is shaded blue (darker) and $\mathcal{H}^-_{v,b}$ is shaded yellow (lighter). Their intersection is shaded green. Points in $\sigma_+$ are black, while points in $\sigma_-$ are red.

**Theorem 3.4.** *Let* $f \colon \mathbb{R}^n_{>0} \to \mathbb{R}$ *be a signomial. If at most one coefficient of* $f$ *is negative, then* $f^{-1}(\mathbb{R}_{<0})$ *is a logarithmically convex set. In particular,* $V^c_{>0}(f)$ *has at most one negative connected component.*

- The authors prove the stronger result that the negative connected component is log-convex, meaning there is a line connecting Log(x) and Log(y) in the logarithmic space $\mathrm{Log}(f_{-1}(R_{<0}))$.

- Let x, y ∈ $f_{-1}(R_{<0})$. To show connectedness, we need to find a path in $f_{-1}(R_{<0})$ which connects x and y. Define v = Log(y) − Log(x). It will be shown that the induced signomial in one variable, $f_{v,x}(t)$, gives our desired path on the interval [1, e]. Let e denote Euler's number.




- We know $f_{v,x}(t) < 0$ for all $t \in [1, \rho)$ for some $\rho \in (1, \infty]$. Observe $f_{v,x}(e) = f(y) < 0$ by assumption that $y \in f_{-1}(R<0)$; thus $e < \rho$. So $f_{v,x}(t) < 0$ on the interval $[1, e] \subset [1, \rho)$.

- Hence $\gamma_{v,x}(t) \in f_{-1}(R_{<0})$ for $t \in [1,e]$. Applying Log equality gives that $\tau_{v,\mathrm{Log}(x)}(\lambda) \in \mathrm{Log}(f_{-1}(R_{<0}))$ for all $\lambda \in [0,1]$. As $\tau_{v,\mathrm{Log}(x)}$ in the interval $[0,1]$ is simply the line segment joining Log(x) and Log(y), $\mathrm{Log}(f_{-1}(R_{<0}))$ is convex. This concludes the proof.

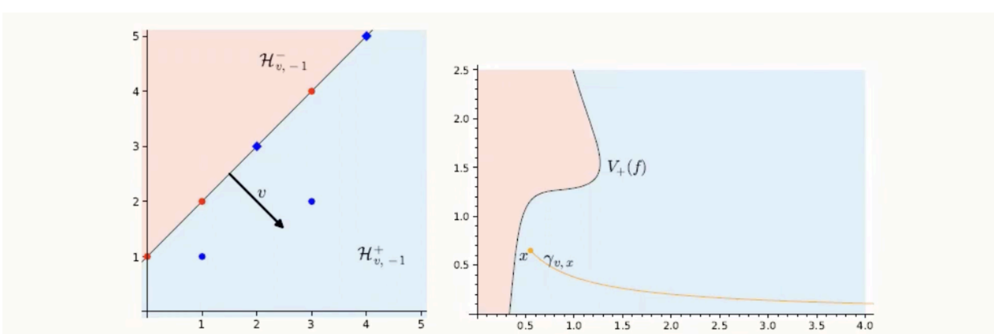
**Theorem 3.6.** *Let $f: \mathbb{R}^n_{>0} \to \mathbb{R}$ be a signomial. If there exists a strict separating vector of $\sigma(f)$, then*

(i) *$f^{-1}(\mathbb{R}_{<0})$ is non-empty and contractible.*
(ii) *The closure of $f^{-1}(\mathbb{R}_{<0})$ equals $f^{-1}(\mathbb{R}_{\leq 0})$.*

*In particular, $V^c_{>0}(f)$ has at most one negative connected component.*

$$p_2(x_1, x_2) = -x_1^4 x_2^5 + 3x_1^3 x_2^4 - x_1^3 x_2^2 - x_1^2 x_2^3 + x_1 x_2^2 - 3x_1 x_2 + x_2.$$

Then $v = (1, -1) \in S^-(p_2)$ is strict




$$3x_1^3x_2^4 + x_1x_2^2 + x_2 - x_1^3x_2^2 - x_1^2x_2^3 - 3x_1x_2 - x_1^4x_2^5$$