Assignment 2

CS9.312 Introduction to Quantum Information and Computation

Due date of submission: 20/01/2023

1. Consider the Pauli spin matrices $\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.
   Using the properties of Pauli matrices check if the matrix $\begin{pmatrix} \sigma_0 & \sigma_1 \\ -i\sigma_2 & \sigma_3 \end{pmatrix}$ is unitary .
2. Show that opposite points on the Bloch sphere are orthogonal to each other.
3. Consider the operator (4 x 4 matrix) in the Hilbert space $\mathbb{C}^4$

$$\rho = \frac{1}{4}(1-\epsilon)I_4 + \epsilon(|0\rangle \otimes |0\rangle)(\langle 0| \otimes \langle 0|)$$

   where ϵ is a real parameter with $\epsilon \in [0, 1]$ and $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Does ρ define a density matrix?
4. Consider the states $|0\rangle := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|1\rangle := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ in the Hilbert space $\mathbb{C}^2$ and the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle)$$

   in the Hilbert space $\mathbb{C}^4$. Let $(\phi, \theta \in \mathbb{R})$ and

$$|\alpha\rangle = \cos\phi\,|0\rangle + \sin\phi\,|1\rangle$$

$$|\beta\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$

   Find the probability $p(\phi, \theta) := |(\langle\alpha| \otimes \langle\beta|)\,|\psi\rangle\,|^2$
   Discuss p as a function of ϕ and θ.