

QUESTION 5(a) AND 5(b) REPORT

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SECTION: B

PART 5(a):

CASE 1:

We iterate from $i=1$ to $i=127$ and generate 0 or 1 randomly using the random function, the probability of getting a 0 or 1 at index i is obviously $1/2$ as both are equally likely.

CASE 2:

We iterate from $i=128$ to $i=1000000$, where $(x_i = x(i-1) \oplus x(i-127))$ for $i \geq 128$

Probability of getting 0 at index $i = [(probability\ of\ 0\ at\ index\ (i-1)) * (probability\ of\ 0\ at\ index\ (i-127))] + [(probability\ of\ 1\ at\ index\ (i-1)) * (probability\ of\ 1\ at\ index\ (i-127))]$

$$\begin{aligned} &= [(1/2) * (1/2)] + [(1/2) * (1/2)] \\ &= (1/4) + (1/4) \\ &= (1/2) \end{aligned}$$

Probability of getting 1 at index $i = [(probability\ of\ 0\ at\ index\ (i-1)) * (probability\ of\ 0\ at\ index\ (i-127))] + [(probability\ of\ 1\ at\ index\ (i-1)) * (probability\ of\ 1\ at\ index\ (i-127))]$

$$\begin{aligned} &= [(1/2) * (1/2)] + [(1/2) * (1/2)] \\ &= (1/4) + (1/4) \\ &= (1/2) \end{aligned}$$

Next, we generate 0 or 1 randomly using the random function (`rand()%2` approach), the probability of getting a 0 or 1 at index i is obviously $1/2$ as both are equally likely.

CODE:

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main()
{
    int count0=0, count1=0, rand0=0, rand1=0, array[1000000];
    srand(time(NULL));

    for (int x = 1; x <= 127; x++)
    {
        array[x] = (rand() % 2);
    }
    for (int x = 128; x <= 1000000; x++)
    {
        array[x] = array[x - 1] ^ array[x - 127];

        if (array[x] == 0)
        {
            count0++;
        }
        if (array[x] == 1)
        {
            count1++;
        }

        int random = rand() % 2;

        if (random == 0)
        {
            rand0++;
        }
        if (random == 1)
        {
            rand1++;
        }
    }
    printf("%d %d\n", count0, count1);
    printf("%d %d\n", rand0, rand1);
    return 0;
}
```

OUTPUT:

	Number of 0s	Number of 1s
0s and 1s in x_{128}, \dots, x_N for $N = 1e6$	499530	500343
0s and 1s using <code>rand() % 2</code> approach	500475	499398

PART 5(b):

First of all, we randomly generate bits for $1 \leq i \leq 127$

Then, we iterate from $128 \leq i \leq 1000000$

The possible cases are listed below:

$x[i-1]$	$x[i-127]$	$x[i]$
0	0	0
0	1	1
1	0	1
1	1	0

Picking the first and the last case from the truth table of an XOR gate as mentioned above, as we have to compute: $P(x_i = 0/x_{i-1} = 0)$ and $P(x_i = 0/x_{i-1} = 1)$

CODE:

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main()
{
    int count0 = 0, count1 = 0, array[1000000], array0 = 0, array1 = 0;
    srand(time(NULL));

    for (int i = 1; i <= 127; i++)
    {
        array[i] = (rand() % 2);
    }
    for (int i = 128; i <= 1000000; i++)
    {
        array[i] = array[i - 1] ^ array[i - 127];
    }
}
```

```

    if (array[i - 1] == 0)
    {
        count0++;
    }
    if (array[i - 1] == 1)
    {
        count1++;
    }
    if ((array[i] == 0) && (array[i - 1] == 0))
    {
        array0++;
    }
    if ((array[i] == 0) && (array[i - 1] == 1))
    {
        array1++;
    }
}

printf("P(xi = 0/x(i-1) = 0) = %lf\n", (double)array0 / (double)count0);
printf("P(xi = 0/x(i-1) = 1) = %lf\n", (double)array1 / (double)count1);
return 0;
}

```

OUTPUT:

we obtain the following output after running the code, 5 iterations are listed as follows:

$P(x_i = 0/x_{i-1} = 0)$	$P(x_i = 0/x_{i-1} = 1)$
0.499987	0.500010
0.502192	0.500024
0.501612	0.499989
0.500834	0.500008
0.500037	0.499986