25/1/2019

Notes on Joint probability: ①

Joint probability is simply the likelihood that two events will happen at the same time, P(A, B) expressed mathematically

but There are 2 conditions:

i) Event A and B must happen at the same time

ii) Event A and B must be independent.

Eg: Throwing two dice.

Eg: 1 What is the probability that number '5' will occur on both dice (twice), when two dice are rolled at the same time?

Ans: The probability of '5' occuring on each die is 1/6

$$P(A, B) = \underbrace{P(A)}_{\text{5 occur on first dice}} \cdot \underbrace{P(B)}_{\text{5 occur on second dice}}$$

$$= 1/6 \times 1/6$$

$$= 0.166 \times 0.166$$

$$= 0.027$$

Conditional probability = $P(A|B) = \frac{P(A \wedge B)}{P(B)}$.

Joint and conditional probability:

The joint and conditional probability bears some relations to each other.

$$P(A, B) = P(A|B) \cdot P(B)$$

$\Downarrow$

$P(A \cap B)$ (or)

$$P(A, B) = P(B/A) \cdot P(A)$$

Thus, conditional probability is a normalised version of a joint probability.

Eg 2: Bag 1 has 1 black ball and 2 white ba

Bag 2 has 1 black ball and 3 white ball

Bag 1 – 1 B, 2 W

Bag 2 – 1 B, 3 W

Suppose, we pick the bag at random and then selec a ball from that bag.

let

$P(A)$ = choosing the first bag = $1/2$.

$P(B)$ = Choosing a white ball, given that we have chosen the first bag.

$$P(B/A) = \frac{2}{3}$$

↳ bag 1

$$P(A \cap B) \Rightarrow P(A, B) = P(B/A) \cdot P(A)$$

$$= \frac{2}{3} \cdot \frac{1}{2} = 1/3.$$

(or)

$$P(A, B) = P(A/B) \cdot P(B)$$

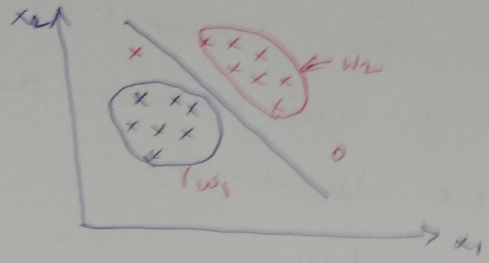
↳ Inter change the action

let $P(B) \Rightarrow$ choosing a first bag.

$P(A) \Rightarrow$ Choosing a white ball from first bag.

25/1/2019

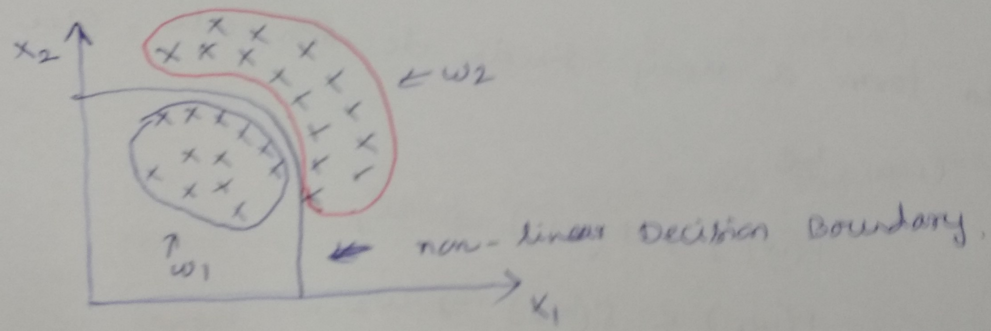
Bayes Decision Theory:



two classes: ω_1 & ω_2

→ In this case the two classes can be separated by linear boundary, This is also known as linearly separable classes

→ This is Supervised Learning



→ Among these non-linear separable classes, the most common one is 1. quadratic classifier.

2. Cubic classifier

Consider steel manufacturing department / steel plant

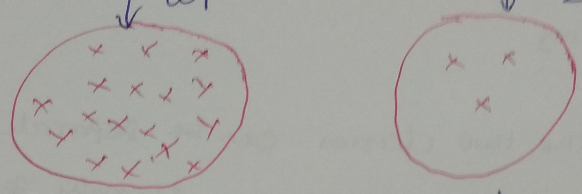
Two class problem:

$\omega_1 \downarrow$ Accept

$\omega_2 \downarrow$ Reject.

I have to take the previous history (i.e) how many objects are accepted and how many are rejected by the quality control department

$\downarrow \omega_1$ $\qquad\qquad$ $\downarrow \omega_2$



$\downarrow$ $P(\omega_1)$ $\qquad\qquad$ $\downarrow$ $P(\omega_2)$

I can form a very simple decision rule, The decision rule can be

$$P(\omega_1) > P(\omega_2) \Rightarrow \omega_1.$$
$$P(\omega_1) < P(\omega_2) \Rightarrow \omega_2$$

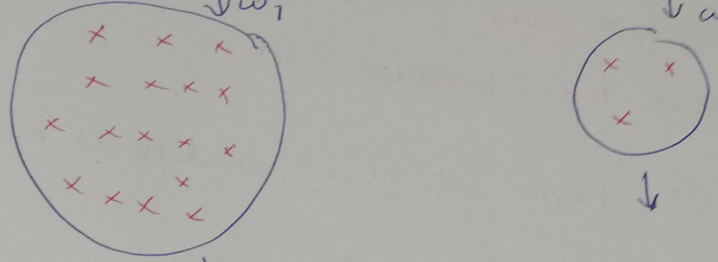
This is not really logical. The simple reason is If from the history, I have found out $P(\omega_1) > P(\omega_2)$. Then, for all the new incoming objects I always in favour of ω_1. even though it actully belongs to class ω_2.

$o(\omega_2) \Rightarrow \omega_1$

(i.e) The object is always accepted (or) always be (2) rejected based on a <u>Prior probability</u>. (i.e) $P(\omega_1)$ and $P(\omega_2)$.

To make our decision more logical, we have to combine some features, ~~letus~~ let us say (x).

$\downarrow \omega_1$ $\qquad\qquad\qquad \downarrow \omega_2$



$\downarrow$ $\qquad\qquad\qquad \downarrow$

I measure (x) $\qquad\qquad\qquad$ I measure (x)

I can find out $P(x/\omega_1)$ and $P(x/\omega_2)$

$\downarrow$

probability density function of (x) taking the objects ~~taken~~ from class ω_1.

$$\underbrace{P(x/\omega_1) \qquad P(x/\omega_2)}_{\text{Class Conditional PDF (probability density function)}}$$

<u>Now, The PR Problem is</u>

$P(\omega_1/x) > P(\omega_2/x) \Rightarrow \omega_1$ in favour of ω_1

$P(\omega_1/x) < P(\omega_2/x) \Rightarrow \omega_2$ in favour of ω_2.

$P(\omega_1/x)$ and $P(\omega_2/x)$

A more logical will be if this PDF can be combined with 'a priori probability' (i.e $P(\omega_1)$ and $P(\omega_2)$)

From the Preliminary probability Theory

The joint PDF (probability distribution fn)

An object belongs to class ω_i $(i=1,2)$ and at the same time have the feature x.

$P(\omega_i, x)$ = In terms of class conditional probability.

$$= P(\omega_i/x) \cdot P(x) \quad (\text{or})$$

$$= P(x|\omega_i) \cdot P(\omega_i)$$

$$\Downarrow$$

$$P(\omega_i/x) \cdot P(x) = P(x|\omega_i) \cdot P(\omega_i)$$

$$\boxed{P(\omega_i/x) = \frac{P(x|\omega_i) \cdot P(\omega_i)}{P(x)}}$$ Baye's rule.

Baye's classifier

$$\boxed{P(x|\omega_1) \cdot P(\omega_1) > P(x|\omega_2) \cdot P(\omega_2)} \Rightarrow \omega_1$$ for two classes.

$$P(x|\omega_i) \cdot P(\omega_i) > P(x|\omega_j) \cdot P(\omega_j) \quad i, j = 1, 2, 3 \ldots c$$

$\forall i \neq j$

and $P(\omega_2)$

$P(x|\omega_1) \cdot P(\omega_1) > P(x|\omega_2) \cdot P(\omega_2) \Rightarrow \omega_1$

(5)

case i) If $P(\omega_1) = P(\omega_2)$ objects is equally likely to be accepted (or) rejected.

then decision is based on class conditional probability

$P(x|\omega_1)$ and $P(x|\omega_2)$

case ii) If $P(x|\omega_1) = P(x|\omega_2)$ then decision is based on a priori probability $P(\omega_1)$ and $P(\omega_2)$.

case iii) In other cases, you consider both to make the decision.