nature of Discriminant function under different conditions:

i) Minimum - Risk - classifier

$$g_i(x) = -R(\alpha_i/x)$$

	$-(R_i/x)$
$(R_1/x) = 7$	-7
$(R_2/x) = 2$	-2
$(R_3/x) = 18$	-18

$g_2(x)$ is maximum

ii) Minimum Error-Rate classifier:

$\boxed{1 - P(\omega_i/x)} \Rightarrow$ is minimum.

$$g_i(x) = P(\omega_i/x)$$

Hence, which ever class the value of $g_i(x)$ is maximum, x can put in that class.

But the choice of D.F $g_i(x)$ is not unique. more generally, if we replace every $g_i(x)$ by $f(g_i(x))$ where $f(\cdot)$ is a monotonically increasing function, the resulting classification is uncharged.

This observation can lead to significant analytical and computational simplifications.

Minimum Error-Rate classifier

$$g_i(x) = P(\omega_i / x)$$

$$= \frac{P(x/\omega_i) \cdot P(\omega_i)}{P(x)}$$

$$g_i(x) = P(x/\omega_i) \cdot P(\omega_i)$$

$$g_i(x) = \ln P(x/\omega_i) + \ln P(\omega_i)$$

where ln - denotes natural logarithm.

⇒ The effect of any decision rule is to divide the feature space into 'c' decision regions. $R_1, \ldots R_c$

⇒ If $g_i(x) > g_j(x)$ for all $i \neq j$, then x is R_i.

and the decision rule is to assign X to ω_i.

ii) The two category case

Dichotomizer:

A classifier that places a pattern in one of any two categories has a special name - a dichotomizer

$g_1(x) = g_2(x)$ (3)

$g(x) = g_1(x) - g_2(x)$

A dichotomizer can be viewed as a machine that computes a single discriminant function $g(x)$, and classifies x according to algebraic sign of the result.

Minimum-Error-Rate discriminant function:

$$g_i(x) = P(\omega_i | x)$$

↓

two classes: $g_1(x) = P(\omega_1|x)$

two category case: $g_2(x) = P(\omega_2|x)$

$$g(x) = g_1(x) - g_2(x) = 0$$

$$g(x) = P(\omega_1|x) - P(\omega_2|x)$$

$$= P(x|\omega_1) \cdot P(\omega_1) - P(x|\omega_2) \cdot P(\omega_2)$$

$$= \ln P(x|\omega_1) + \ln P(\omega_1) - \ln P(x|\omega_2) + \ln P(\omega_2)$$

$$= \ln P(x|\omega_1) + \ln P(\omega_1) - \ln P(x|\omega_2) + \ln P(\omega_2)$$

$$\boxed{g(x) = \ln \frac{P(x|\omega_1)}{P(x|\omega_2)} + \ln \frac{P(\omega_1)}{P(\omega_2)}}$$