Histogram Quadratic Distance

10/1/12

Quadratic Form Distance:

→ It allows comparison of histograms across different bin locations.

→ Quadratic form metrics consider the cross-relation of the bins.

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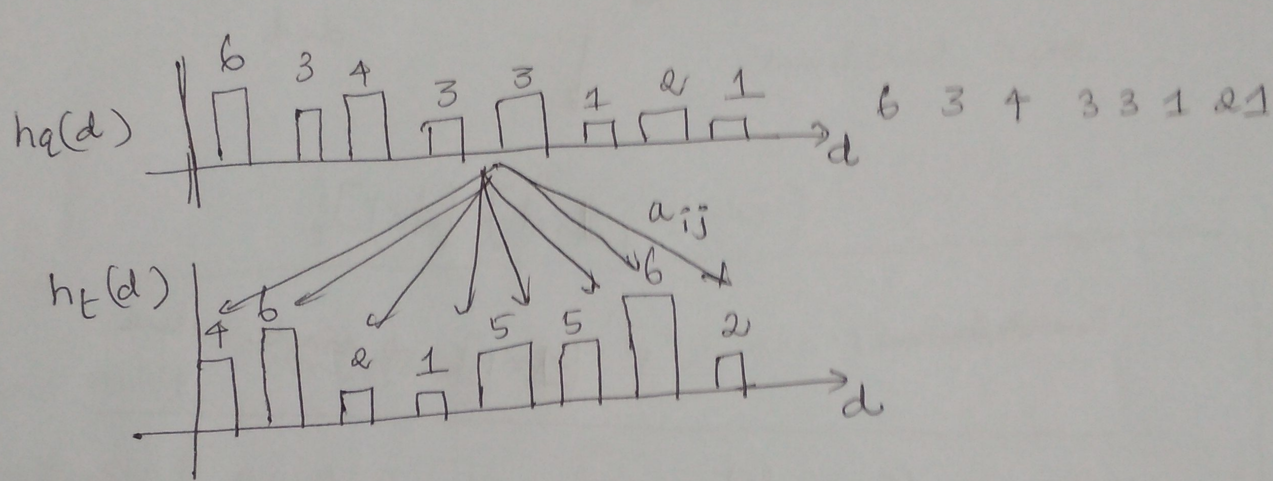

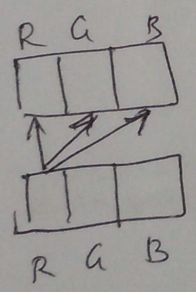

→ In a naive Implementation, the color histogram quadratic distance is computationally expensive.

→ The quadratic-form distance between color histograms hq and ht is given by

$$D(q,t) = (h_q - h_t)^T \underbrace{A}_{\text{transform matrix}} (h_q - h_t)$$

$A = [a_{ij}]$ and a_{ij} denotes the dissimilarity between histogram bins with indices i and j.

$$\underset{1 \times d}{\begin{bmatrix} 1, & 2, & \cdots & d \end{bmatrix}} \underset{d \times d}{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdot & a_{1d} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2d} \\ a_{31} & & & & \\ \vdots & & & & \\ a_{d1} & a_{d2} & a_{d3} & \cdot & a_{dd} \end{bmatrix}} \underset{d \times 1}{\begin{bmatrix} 1 \\ 2 \\ \vdots \\ d \end{bmatrix}} =$$

$$\downarrow$$

$$[1 \times d]\ [d \times 1]$$

$$[1 \times 1] \Rightarrow \text{single value}$$

10) Special cases of Quadratic form Distance:

$$D(q,t) = (h_q - h_t)^T A (h_q - h_t)$$

3) Histogram L_2 distance:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity matrix

$$D_2(q,t) = (h_q - h_t)^T A (h_q - h_t)$$

Across bins are not considered here.

$$= (h_q - h_t)^T (h_q - h_t)$$

since only corresponding bins (i, i) are ~~considered~~

Considers

$$\underset{M}{\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}} \Rightarrow \underset{M^T}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}} \Rightarrow \text{will result the same.}$$

Symmetry matrix

$$D_2(q,t) = (h_q - h_t)^2$$

$$D_2(q,t) = \sum_{i=1}^{d} [h_q(i) - h_t(i)]^2$$

Example is given in page 3, back side

3ii

iii) Histogram Mahalanobis Distance:

Another special case of the quadratic-form metric in which the transform matrix 'A' is given by the covariance matrix, that is $A = \Sigma^{-1}$.

$$D(q,t) = (h_q - h_t)^T \Sigma^{-1} (h_q - h_t)$$

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}$$

The value across bins are considered but the value are '0'.

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10/11

* Quadratic Form Distance between color histogram h_q & h_t is given by:

$$D(q,t) = (h_q - h_t)^T A (h_q - h_t)$$

where A is the transform matrix.

Eg : Consider the following example:

Given, the histogram of a pure red image:

h_q ~~x~~ $= [1, 0, 0]^T$ and a pure orange image:

h_t ~~y~~ $= [0, 1, 0]^T$

The transform matrix $A = \begin{bmatrix} 1 & 0.9 & 0 \\ 0.9 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

* Find the quadratic form distance & Euclidean distance.

Quadratic form Distance

~~D(h...)~~

$D(q,t) = (h_q - h_t)^T A (h_q - h_t)$ ——— Eqn 1

$h_q = x;$

Here, $h_q = [1, 0, 0]^T$ {ie, h_q is x}

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$h_t = y$

$$= [0 \quad 1 \quad 0]^T$$

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$h_q - h_t = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(h_q - h_t) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$ ——— Eqn 2

$$(h_q - h_t)^T = [1 \quad -1 \quad 0]$$ ——— Eqn 3

Substituting Eqns 2, 3 & $A = \begin{bmatrix} 1 & 0.9 & 0 \\ 0.9 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ in Eqn 1,

$\therefore$ Eqn ① becomes;

$$D(q,t) = \underset{1\times3}{\begin{bmatrix} 1 & -1 & 0 \end{bmatrix}} \underset{3\times3}{\begin{bmatrix} 1 & 0.9 & 0 \\ 0.9 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}$$

$$= \begin{bmatrix} (1-0.9) & (0.9-1) & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & -0.1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= (0.1) + (0.1) + 0$$

$$= \underline{\underline{0.2}}.$$

Euclidean Distance:

$$D_2^2(q,t) = (h_q - h_t)^2 \qquad \{ A = I : \text{Identity Matrix} \}$$

$$= \underset{1\times3}{\begin{bmatrix} 1 & -1 & 0 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 1 + 1 + 0 \end{bmatrix}$$

$$= \underline{\underline{2}}.$$

Minkowski distance.

$$ED = \sqrt{2} \text{ (By metric def.)}$$