

Pattern Recognition

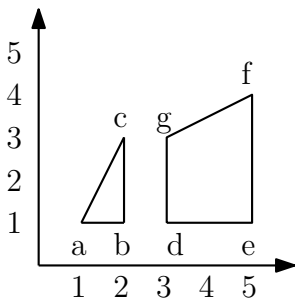
Hausdroff's Distance Example

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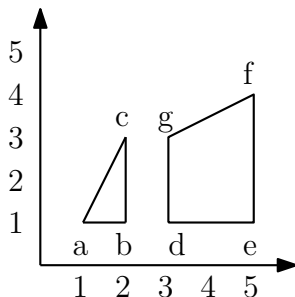
21th January 2019

Hausdorffs Distance Example



From the above figure, let the triangle be polygon A and the quadrilateral be polygon B. The vertex coordinates of polygon A are $a=(1,1)$, $b=(2,1)$ and $c=(2,3)$ and B are $d=(3,1)$, $e=(5,1)$, $f=(5,4)$ and $g=(3,3)$

Hausdorffs Distance Example



Bidirectional Hausdorffs distance

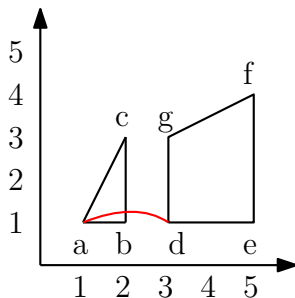
$$H(A,B) = \max(\hat{H}(A,B), \hat{H}(B,A))$$

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$d(a,b)$ = Euclidean Distance between
all points $a \in A$ and $b \in B$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Hausdorffs Distance Example



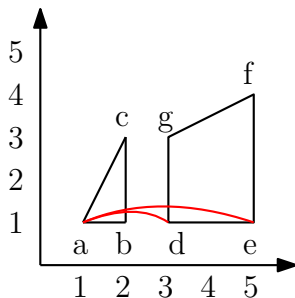
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$a=(1,1)$ and $d = (3,1)$

$$d(a,d) = \sqrt{(1-3)^2 + (1-1)^2} = \mathbf{2}$$

Hausdorffs Distance Example



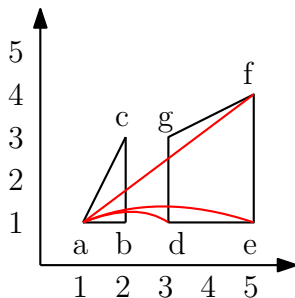
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$a=(1,1)$ and $e = (5,1)$

$$d(a,e) = \sqrt{(1-5)^2 + (1-1)^2} = 4$$

Hausdorffs Distance Example



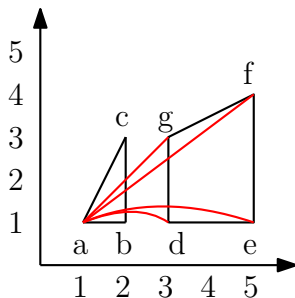
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$a=(1,1)$ and $f=(5,4)$

$$d(a,f) = \sqrt{(1-5)^2 + (1-4)^2} = 5$$

Hausdorffs Distance Example



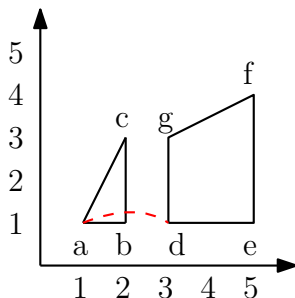
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$a=(1,1)$ and $g = (3,3)$

$$d(a,g) = \sqrt{(1-3)^2 + (1-3)^2} = \mathbf{2.83}$$

Hausdorffs Distance Example



$$d(a,d) = 2$$

$$d(a,e) = 4$$

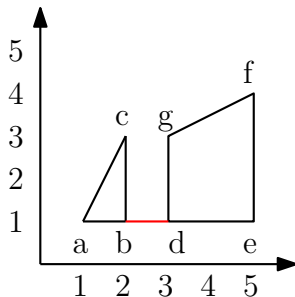
$$d(a,f) = 5$$

$$d(a,g) = 2.83$$

Minimum distance among the 4 is

$$d(a,d)=2$$

Hausdorffs Distance Example



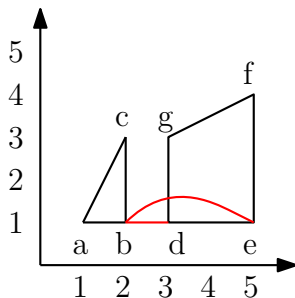
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$b = (2,1)$ and $d = (3,1)$

$$d(b,d) = \sqrt{(2-3)^2 + (1-1)^2} = 1$$

Hausdorffs Distance Example



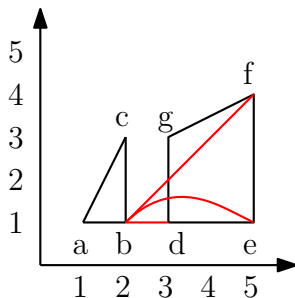
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$b = (2,1)$ and $e = (5,1)$

$$d(b,e) = \sqrt{(2-5)^2 + (1-1)^2} = \mathbf{3}$$

Hausdorffs Distance Example



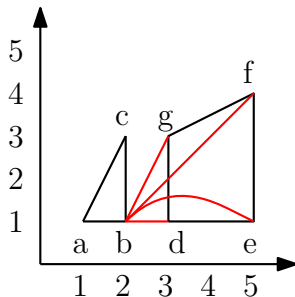
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$$b = (2,1) \text{ and } f = (5,4)$$

$$d(b,f) = \sqrt{(2-5)^2 + (1-4)^2} = \mathbf{4.24}$$

Hausdorffs Distance Example



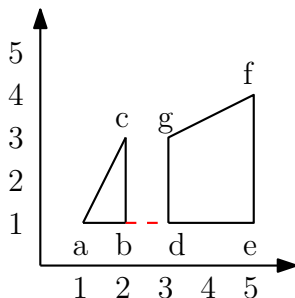
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$$b=(2,1) \text{ and } g = (3,3)$$

$$d(b,f) = \sqrt{(2-3)^2 + (1-3)^2} = \mathbf{2.24}$$

Hausdorffs Distance Example



$$d(b,d) = 1$$

$$d(b,e) = 3$$

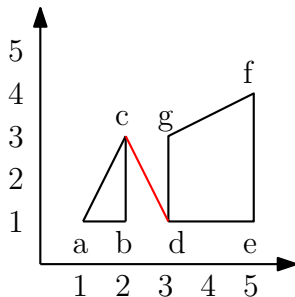
$$d(b,f) = 4.24$$

$$d(b,g) = 2.24$$

Minimum distance among the 4 is

$$d(b,d)=1$$

Hausdorffs Distance Example



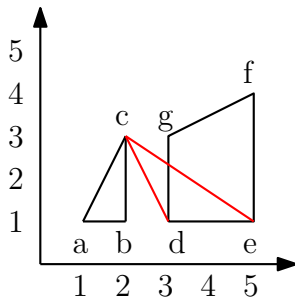
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$c=(2,3)$ and $d = (3,1)$

$$d(c,d) = \sqrt{(2-3)^2 + (3-1)^2} = \mathbf{2.24}$$

Hausdorffs Distance Example



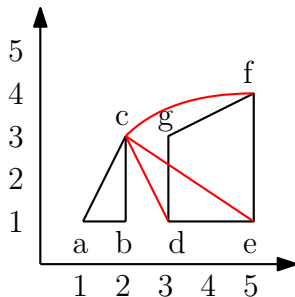
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$c=(2,3)$ and $e = (5,1)$

$$d(c,e) = \sqrt{(2-5)^2 + (3-1)^2} = \mathbf{3.61}$$

Hausdorffs Distance Example



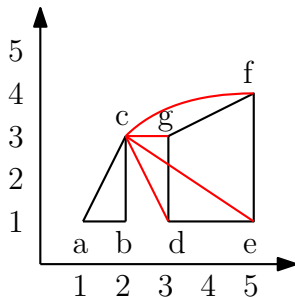
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$c = (2,3)$ and $f = (5,4)$

$$d(c,f) = \sqrt{(2-5)^2 + (3-4)^2} = \mathbf{3.16}$$

Hausdorffs Distance Example



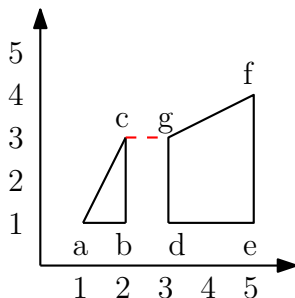
$\hat{H}(A,B)$ Calculation

$$\hat{H}(A,B) = \max(\min(d(a,b)))$$

$c=(2,3)$ and $g = (3,3)$

$$d(c,g) = \sqrt{(2-3)^2 + (3-3)^2} = 1$$

Hausdroffs Distance Example



$$d(c,d) = 2.24$$

$$d(c,e) = 3.61$$

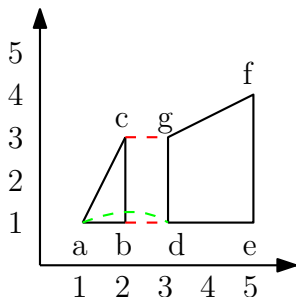
$$d(c,f) = 3.16$$

$$d(c,g) = 1$$

Minimum distance among the 4 is

$$d(c,g)=1$$

Hausdorffs Distance Example



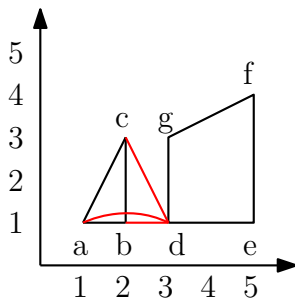
The minimum distance values from each point of polygon A are, $d(a,d)=2$, $d(b,d)=1$ and $d(c,g) = 1$

$$\hat{H}(A,B) = \max(d(a,d), d(b,d), d(c,g))$$

$$\hat{H}(A,B) = \max(2, 1, 1)$$

$$\hat{H}(A,B) = 2 \text{ (Shown in Green)}$$

Hausdorffs Distance Example



$$\hat{H}(B,A) = \max(\min(d(a,b)))$$

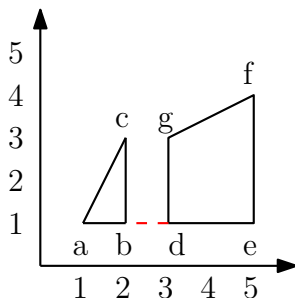
Euclidean distance is symmetric, so distance from point d to points in polygon A are,

$$d(d,a) = 2$$

$$d(d,b) = 1$$

$$d(d,c) = 2.24$$

Hausdorffs Distance Example



$$\hat{H}(B,A) = \max(\min(d(a,b)))$$

$$d(d,a) = 2$$

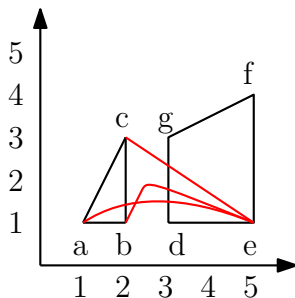
$$d(d,b) = 1$$

$$d(d,c) = 2.24$$

Minimum distance among the 3 is

$$d(d,b)=1$$

Hausdorffs Distance Example



$$\hat{H}(B,A) = \max(\min(d(a,b)))$$

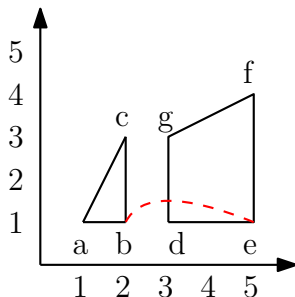
Euclidean distance is symmetric, so distance from point e to points in polygon A are,

$$d(e,a) = 4$$

$$d(e,b) = 3$$

$$d(e,c) = 3.61$$

Hausdorffs Distance Example



$$\hat{H}(B,A) = \max(\min(d(a,b)))$$

$$d(e,a) = 4$$

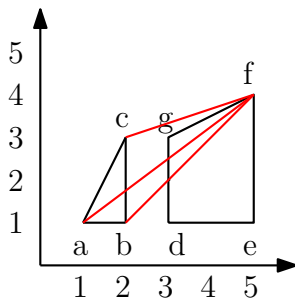
$$d(e,b) = 3$$

$$d(e,c) = 3.61$$

Minimum distance among the 3 is

$$d(e,b)=3$$

Hausdorffs Distance Example



$$\hat{H}(B,A) = \max(\min(d(a,b)))$$

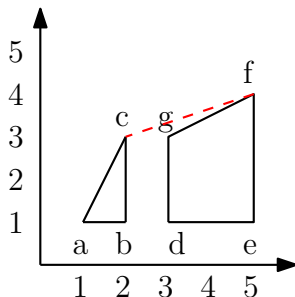
Euclidean distance is symmetric, so distance from point f to points in polygon A are,

$$d(f,a) = 5$$

$$d(f,b) = 4.24$$

$$d(f,c) = 3.16$$

Hausdorffs Distance Example



$$\hat{H}(B,A) = \max(\min(d(a,b)))$$

$$d(f,a) = 5$$

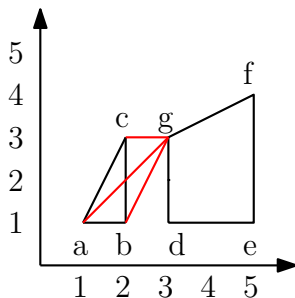
$$d(f,b) = 4.24$$

$$d(f,c) = 3.16$$

Minimum distance among the 3 is

$$d(f,c) = \mathbf{3.16}$$

Hausdorffs Distance Example



$$\hat{H}(B,A) = \max(\min(d(a,b)))$$

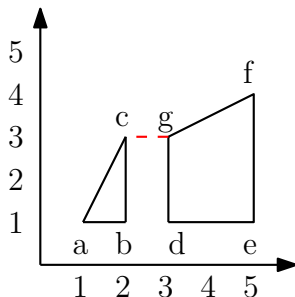
Euclidean distance is symmetric, so distance from point f to points in polygon A are,

$$d(g,a) = 2.83$$

$$d(g,b) = 2.24$$

$$d(g,c) = 1$$

Hausdorffs Distance Example



$$\hat{H}(B,A) = \max(\min(d(a,b)))$$

$$d(g,a) = 2.83$$

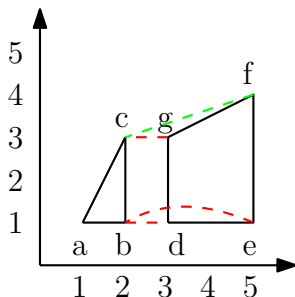
$$d(g,b) = 2.24$$

$$d(g,c) = 1$$

Minimum distance among the 3 is

$$d(g,c)=1$$

Hausdorffs Distance Example



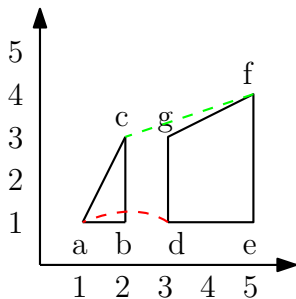
The minimum distance values from each point of polygon B are, $d(d,b)=1$, $d(e,b)=3$, $d(f,c)=3.16$ and $d(g,c)=1$

$$\hat{H}(B,A) = \max(d(d,b), d(e,b), d(f,c), d(g,c))$$

$$\hat{H}(B,A) = \max(1, 3, 3.16, 1)$$

$$\hat{H}(B,A) = 3.16 \text{ (Shown in Green)}$$

Hausdorffs Distance Example



The bidirectional Hausdorffs distance

$$H(A,B)=\max(\hat{H}(A,B),\hat{H}(B,A))$$

$$H(A,B)=\max(d(a,d),d(f,c))$$

$$H(A,B)=\max(2, 3.16)$$

$$H(A,B)=3.16 \text{ (Shown in Green)}$$

Distance Measure Summary

