14/2/2019

①

$\boxed{\text{Case 2:}}$

1. $\Sigma_i = \Sigma$

Σ is arbitrary. $\Rightarrow \Sigma_1 = \Sigma_2 = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$

x_1 and x_2 are not necessarily independent

2. Σ_i is same for all different classes

3. The samples are clustered in hyper ellipsoidal of same shape and size.

4. $\Sigma_1 = \Sigma_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$ $\Rightarrow$ σ_{12} and σ_{21} is same $cov(x_1, x_2) == cov(x_2, x_1)$ hence symmetry.

$$\boxed{g_i(x) = -\frac{1}{2}\left[(x-\mu_i)^t \Sigma_i^{-1} (x-\mu_i)\right] + \ln P(w_i)}$$

after ignoring constant $\frac{-d}{2}\ln(2\pi)$ and $-\frac{1}{2}\ln|\Sigma_i|$.

$\boxed{\text{By negating } g_i(x) \text{ is maximum}}$

$$g_i(x) = -\frac{1}{2}\underbrace{\left[(x-\mu_i)^t \Sigma_i^{-1} (x-\mu_i)\right]} + \ln P(w_i)$$

If all the cla[sses] are equ[ally] probab[le]

Minimum distance classifier

case 2: $\boxed{\text{squared Mahalanobis distance.}}$

case 1: $\boxed{\text{squared Euclidean distance}}$

Expansion of the quadratic form yields.

$$g_i(x) = \frac{-1}{2}\left[(x-\mu_i)^t \Sigma_i^{-1} (x-\mu_i)\right] + \ln P(\omega_i)$$

$$= \frac{-1}{2}\left[(x^t-\mu_i^t)\Sigma_i^{-1}(x-\mu_i)\right] + \ln P(\omega_i)$$

$$= \frac{-1}{2}\left[x^t\Sigma_i^{-1} - \mu_i^t\Sigma_i^{-1}(x-\mu_i)\right] + \ln P(\omega_i)$$

$$= \frac{-1}{2}\left[\underbrace{x^t \cancel{\Sigma_i^{-1}} x} - \mu_i^t\Sigma_i^{-1}x - x^t\Sigma_i^{-1}\mu_i + \mu_i^t\Sigma_i^{-1}\mu_i\right] + \ln P(\omega_i) \quad (A)$$

$x^t \Sigma_i^{-1} x$ is same for all classes ; hence ignored.

$$= -\frac{1}{2}\left(-2\mu_i^t\Sigma_i^{-1}x + \mu_i^t\Sigma_i^{-1}\mu_i\right) + \ln P(\omega_i)$$

$$\boxed{\mu_i^t\Sigma_i^{-1}x == \mu_i\Sigma_i^{-1}x^t}$$

$$= \mu_i^t\Sigma^{-1}x - \frac{1}{2}\mu_i^t\Sigma^{-1}\mu_i + \ln P(\omega_i)$$

w_i

$$\boxed{g_i(x) = w_i^t x + w_{i0}} \Rightarrow \text{linear equation/machine}$$

where

Case 2:

$w_i = \mu_i\Sigma^{-1}$

$w_{i0} = -\frac{1}{2}\mu_i^t\Sigma^{-1}\mu_i + \ln P(\omega_i)$

Case 1:

replace $\Sigma^{-1} = \frac{1}{\sigma^2}$

$w_i = \frac{1}{\sigma^2}\mu_i$

$w_{i0} = -\frac{1}{2\sigma^2}\mu_i^t\mu_i + \ln P(\omega_i$

Case 3:

What will be the nature of the decision boundary that separate the class w_i and w_j ?

$$g_i(x) - g_j(x) = 0$$

By deriving as like Previous [given in pag ④ ; case 1]

It turned to

$$w^t (x - x_0) = 0$$

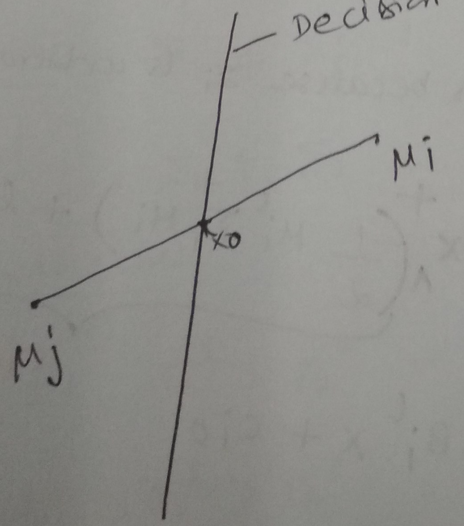
where

$$w = \Sigma^{-1} (\mu_i - \mu_j)$$

$$x_0 = \frac{1}{2} (\mu_i + \mu_j) - \frac{1}{(\mu_i - \mu_j)^t \Sigma^{-1} (\mu_i - \mu_j)} \ln \frac{P(\omega_i)}{P(\omega_j)} . (\mu_i - \mu_j)$$

can be written as

$$x_0 = \frac{1}{2} (\mu_i + \mu_j) - \frac{\ln [P(\omega_i) / P(\omega_j)]}{(\mu_i - \mu_j)^t \Sigma^{-1} (\mu_i - \mu_j)} . (\mu_i - \mu_j)$$



Decision surface of hyperplane but not orthogonal to the join joining μj and μi