24/4/2018

~~Linear~~ Multiple Discriminator functions. (MDA)

Every Class has discriminant function.

Design of Classifier: [class boundary] Example 1:

Assume ~~threre~~ there are '3' classes. ω_1 ω_2 ω_3.

$\omega_1 \downarrow g_1(x)$, $\omega_2 \downarrow g_2(x)$, $\omega_3 \downarrow g_3(x)$.

$$\begin{cases} g_1(x) > g_2(x) \text{ and} \\ g_1(x) > g_3(x) \Rightarrow \omega_1 \end{cases}$$

$$\begin{cases} g_2(x) > g_1(x) \text{ and} \\ g_2(x) > g_3(x) \Rightarrow \omega_2 \end{cases}$$

$$\begin{cases} g_3(x) > g_1(x) \text{ and} \\ g_3(x) > g_2(x) \Rightarrow \omega_3. \end{cases}$$

$g_1(x) = 10x_1 - x_2 - 10$

$g_2(x) = x_1 + 2x_2 - 10$

$g_3(x) = x_1 - 2x_2 - 10$

In order to ~~cat~~ classify a sample 'x'

what is the decision boundary b/w pair of classes?

$g_{12}(x) = g_1(x) - g_2(x)$

$= 9x_1 - 3x_2 = 0.$

$3x_1 - x_2 = 0.$

$g_{13}(x) = g_1(x) - g_3(x)$

$= 9x_1 + x_2 = 0$

$g_{23}(x) = g_2(x) - g_3(x)$

$= 4x_2 = 0;\ \boxed{x_2 = 0}$

which is nothing but x_1 - axis itself

$g_{12}(x) = 3x_1 - x_2 = 0$;

$\boxed{\text{Put } x_1 = 1}$ in $\Rightarrow 3 - x_2 = 0$

$\boxed{x_2 = 3}$

$\boxed{\text{Put } x_2 = 3}$ in $\Rightarrow 3x_1 - 3 = 0$

$\boxed{x_1 = 1}$

$g_{12}(x) = \boxed{\begin{matrix} x_1 = 1 \\ x_2 = 3 \end{matrix}}$

$\downarrow g_{12}(x)$

$9x_1 - 3x_2 = 0$

$9 - 3x_2 = 0$

$-3x_2 = -9$

$\boxed{x_2 = 3}$

$9x_1 - 9 = 0$

$9x_1 = 9$

$\boxed{x_1 = 1}$

$g_{13}(x) = 9x_1 + x_2 = 0$

$9^{(-1)} + x_2 = 0$

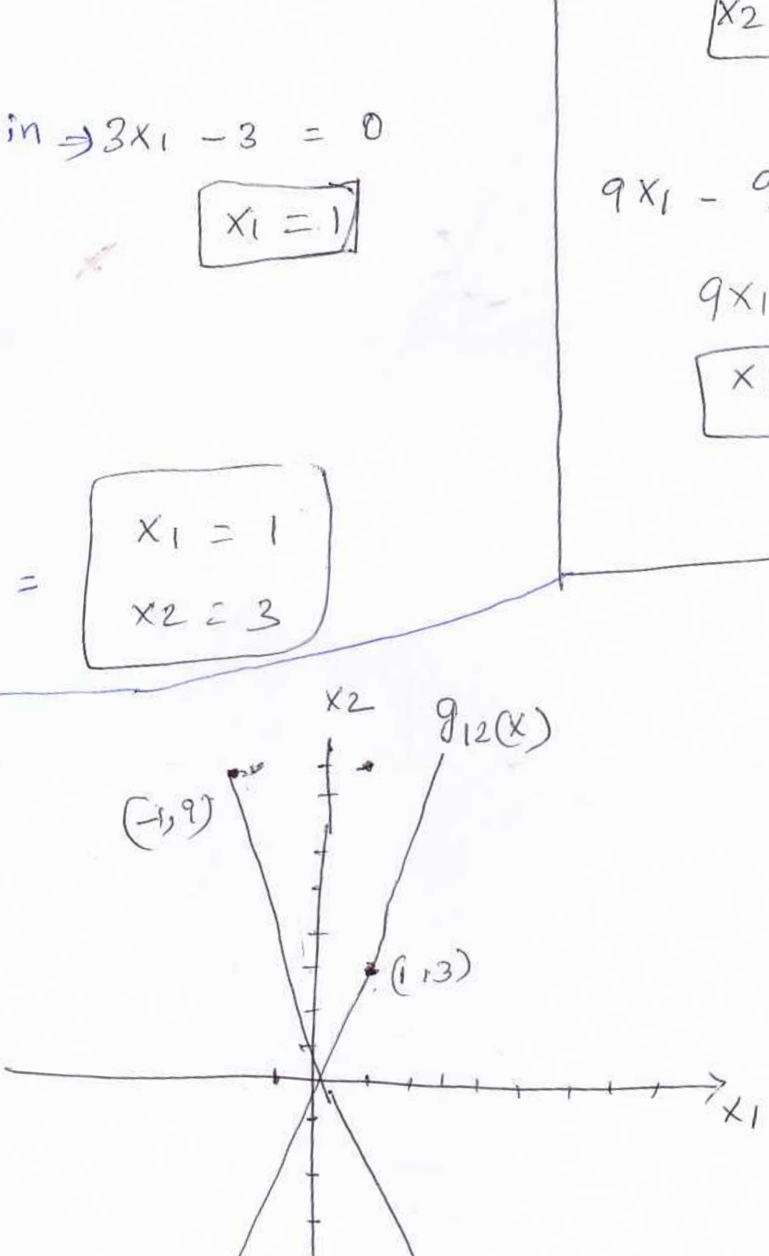
$\boxed{x_2 = +9}$ ✓

$9x_1 + x_2 = 0$

$9x_1 + 9 = 0$

$9x_1 = -9$

$\boxed{x_1 = -1}$ ✓



$9x_1 + x_2 = 0$

$-9 + x_2 = 0$

$\boxed{x_2 = 9}$

$g_{13}(x) = \boxed{\begin{matrix} x_1 = -1 \\ x_2 = 9 \end{matrix}}$

$9x_1 + x_2 = 0$

$\downarrow$

$9(1) + x_2 = 0$

$9 + x_2 = 0$

$\boxed{x_2 = -9}$

$9x_1 \cancel{+} \overset{-9}{9} = 0$

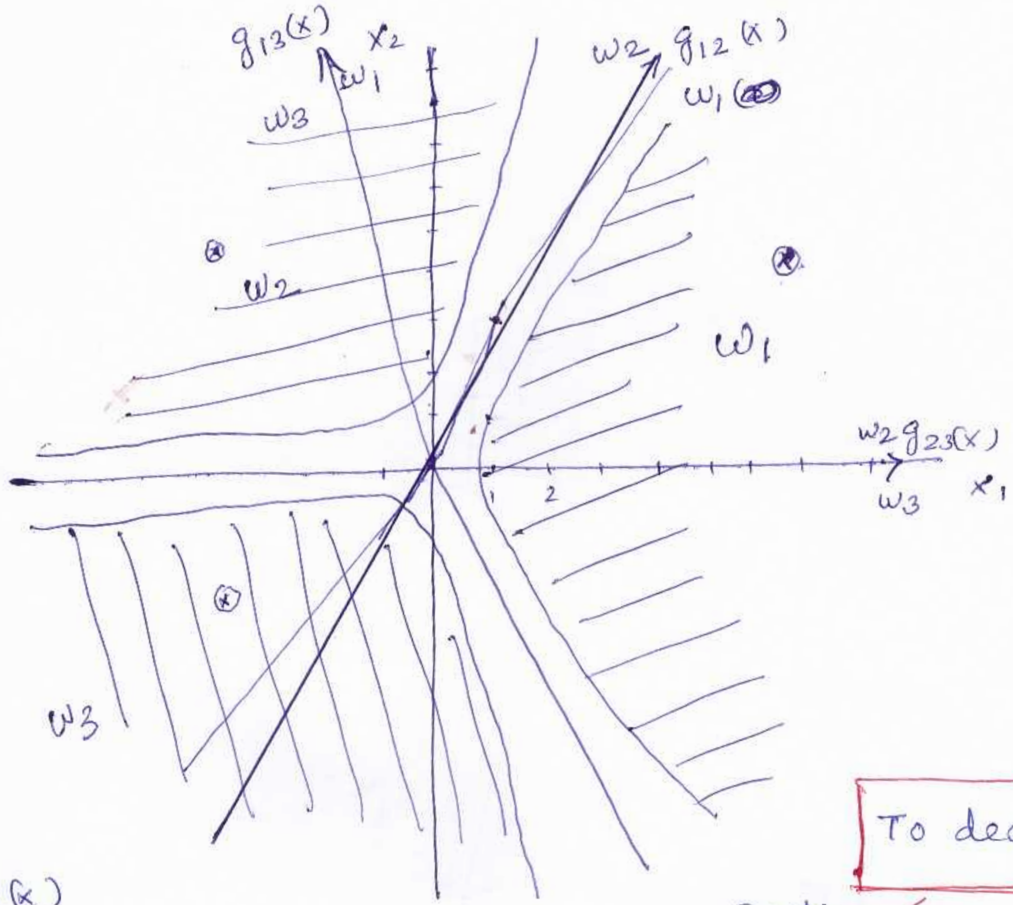
$9x_1 = +9$

$\boxed{x_1 = +1}$

$\boxed{\begin{matrix} x_1 = +1 \\ x_2 = -9 \end{matrix}}$ ✓ another soln.

2-D vectors, but '3' classes:

Plot these decision boundary:



To decide +ve / -ve

Consider $g_{12}(x)$

$3x_1 - x_2 = 0$

and plot

$$\boxed{\begin{array}{l} x_1 = 1 \;;\; \text{when } x_2 = 3 \\ x_2 = 3 \;;\; \text{when } x_1 = 1 \end{array}}$$

$9x_1 + x_2 = 0$

$$\boxed{\begin{array}{l} x_1 = -1 \;;\; \text{when } x_2 = 9 \\ x_2 = 9 \;;\; \text{when } x_1 = -1 \end{array}}$$

Testing:

Sample (1,1) put $g_{12}(x)$

$3x_1 - x_2 = 0$

$3 - 1 = 0$

$2 > 0 \Rightarrow \omega_1$

Similary = $g_{13}(x)$ (1,1)

$9x_1 + x_2 = 0$

$9 + 1 = 0$

$10 > 0$

Example 2:

$g_{12}(x) = 10x_1 - x_2 - 10 = 0$

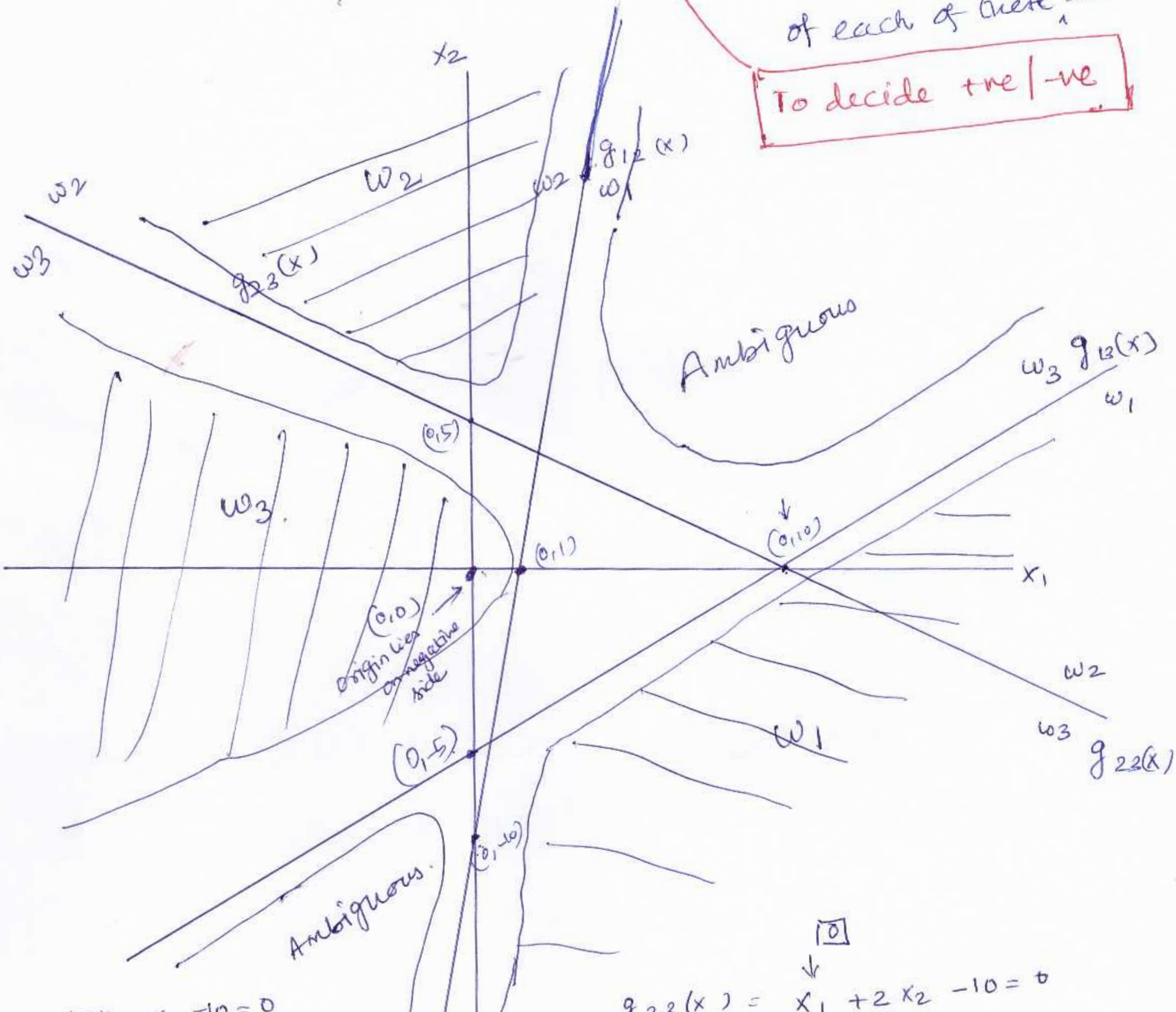
$g_{23}(x) = x_1 + 2x_2 - 10 = 0$

$g_{13}(x) = x_1 - 2x_2 - 10 = 0$

Put $x_1 = 0$; $x_2 = 0$

All are negative hence origin lies on negative side of each of these 3 lines

To decide +ve / -ve



$g_{12}(x) = 10x_1 - x_2 - 10 = 0$

$10\overset{\boxed{0}}{x_1} - x_2 - 10 = 0$

$-x_2 = 10$

$\boxed{x_2 = -10}$

$10x_1 - \overset{\boxed{0}}{x_2} - 10 = 0$

$10x_1 = 10$

$\boxed{x_1 = 1}$

$g_{23}(x) = \overset{\boxed{0}}{x_1} + 2x_2 - 10 = 0$

$2x_2 = 10$

$\boxed{x_2 = 5}$

$= x_1 + 2\overset{\boxed{0}}{x_2} - 10 = 0$

$\boxed{x_1 = 10}$

$g_{13}(x) = \overset{0}{x_1} - 2x_2 - 10 = 0$

$-2x_2 = 10$

$\boxed{x_2 = -5}$

$x_1 - 2\overset{\boxed{0}}{x_2} - 10 = 0$

$\boxed{x_1 = 10}$