— Mahalanobis Distance Example:

Suppose you have data for five people, and each person vector has a height, score on some test, and an age:

	X	Y	Z		$x-\bar{x}$	$y-\bar{y}$	$z-\bar{z}$
	height	score	Age				
	64	580	29		-4	-20	-11
	66	570	33		-2	-30	-7
	68	590	37		0	-10	-3
	69	660	46		1	60	6
	73	600	55		5	0	15
m	68	600	40				
	$\bar{x}$	$\bar{y}$	$\bar{z}$				

$n=5$

Question: Suppose you went to know how far another person $v=(66,640,44)$ is from this data. It turns out the Mahalonobis distance is **5.33** (no units)

$$Z = \begin{bmatrix} -4 & -20 & -11 \\ -2 & -30 & -7 \\ 0 & -10 & -3 \\ 1 & 60 & 6 \\ 5 & 0 & 15 \end{bmatrix} =$$

Find covariance matrix:

$$C = \frac{1}{N-1} Z^t Z$$

$$= \frac{1}{4} \begin{bmatrix} -4 & -2 & 0 & 1 & 5 \\ -20 & -30 & -10 & 60 & 0 \\ -11 & -7 & -3 & 6 & 15 \end{bmatrix} \begin{bmatrix} -4 & -20 & -11 \\ -2 & -30 & -7 \\ 0 & -10 & -3 \\ 1 & 60 & 6 \\ 5 & 0 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 16+4+0+1+25 & 80+60+0+60+0 & 44+14+0+6+75 \\ 80+60+0+60+0 & 400+900+100+3600+0 & 220+210+30+360+0 \\ 44+14+0+6+75 & 220+210+30+360+0 & 121+44+9+36+225 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 46 & 200 & 139 \\ 200 & 5000 & 820 \\ 139 & 820 & 440 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 11.50 & 50 & 34.75 \\ 50 & 1250 & 205 \\ 34.75 & 205 & 110 \end{bmatrix}$$

TO find Σ^{-1}

$$\Sigma^{-1} = \begin{bmatrix} 3.6885 & 0.0627 & -1.2821 \\ 0.0627 & 0.0022 & -0.0240 \\ -1.2821 & -0.0240 & 0.4578 \end{bmatrix}$$

$$\boxed{A^{-1} = \frac{adj(A)}{|A|}}$$

$= (h_q - h_t)^T A \cdot (h_q - h_t)$

$= (h_q - h_t)^T \Sigma^{-1} (h_q - h_t)$

$= (68-66 \quad 600-640 \quad 40-44)$

$$= (-2 \quad 40 \quad 4) \begin{bmatrix} 3.6885 & 0.0627 & -1.2821 \\ 0.0627 & 0.0022 & -0.0240 \\ -1.2821 & -0.0240 & 0.4578 \end{bmatrix} \begin{bmatrix} -2 \\ 40 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -9.9964 & -0.1825 & 3.4413 \end{bmatrix} \begin{bmatrix} -2 \\ 40 \\ 4 \end{bmatrix}$$

$= \sqrt{28.4573}$

$= 5.33.$