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Baye's classifier: Example for quadratic decision boundary.

$$\omega_1 = \begin{pmatrix}2\\6\end{pmatrix} \begin{pmatrix}3\\4\end{pmatrix} \begin{pmatrix}3\\8\end{pmatrix} \begin{pmatrix}4\\6\end{pmatrix}$$

$$\omega_2 = \begin{pmatrix}3\\0\end{pmatrix} \begin{pmatrix}1\\-2\end{pmatrix} \begin{pmatrix}3\\-4\end{pmatrix} \begin{pmatrix}5\\-2\end{pmatrix}$$

$$P(\omega_1) = P(\omega_2) = 0.5$$

Find the decision boundary between class ω_1 and ω_2

$$\boxed{g_1(x) - g_2(x) = 0}$$

Solution:

1) compute mean for class ω_1

	x_1	x_2	$x_1 - M_1$	$x_2 - M_2$
	2	6	-1	0
	3	4	0	-2
	3	8	0	2
	4	6	1	0
M_1	3	6		

$$M = (3 \ 6)$$

$$M_1^T = \begin{pmatrix}3\\6\end{pmatrix}$$

$$Z = \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$$

2) Find the covariance matrix Σ_1

$$C = \frac{1}{N} z^t z$$

$$= \frac{1}{4} \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -2 & 2 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1+0+0+1 & 0+0+0+0 \\ 0+0+0+0 & 0+4+4+0 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} = \Sigma_1^{-1} = \begin{bmatrix} \frac{1}{1/2} & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

Similarly find Σ_2:

x_1	x_2	$x_1 - \mu_1$ (-3)	$x_2 - \mu_2$ ($-(-2)$)
3	0	0	2
1	-2	-2	0
3	-4	0	-2
5	-2	2	0

$$\mu = \begin{pmatrix} 3 & -2 \end{pmatrix}$$

$$\mu^T = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$Z = \begin{bmatrix} 0 & 2 \\ -2 & 0 \\ 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$C = \frac{1}{N} Z^t Z$$

$$= \frac{1}{4}\begin{bmatrix} 0 & -2 & 0 & 2 \\ 2 & 0 & -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -2 & 0 \\ 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 0+4+0+4 & 0+0+0+0 \\ 0+0+0+0 & 4+0+4+0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad \Sigma_2^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Case 3: $\Sigma_i \neq \Sigma_j$ [covariance matrix of two classes are different]

As the covariance is different the classifier is not linear

$$g_1(x) = x^t A_1 x + B_1^t x + C_{10}$$

$$A_1 = -\frac{1}{2}\Sigma_1^{-1}$$

$$B_1 = \Sigma_1^{-1} \mu_1$$

$$C_{10} = -\frac{1}{2}\mu_1^t \Sigma_1^{-1} \mu_1 - \frac{1}{2}\ln|\Sigma_1| + \ln P(\omega_1)$$

$$\mu_1 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Sigma_1^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} \qquad \Sigma_2^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}.$$

$$g_1(x) = x^t A_1 x + B_1^t x + C_{10}$$

$$g_2(x) = x^t A_2 x + B_2^t x + C_{20}$$

$$A_1 = \frac{-1}{2} \Sigma_1^{-1} = -\frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1/4 \end{bmatrix}$$

$$A_2 = -\frac{1}{2} \Sigma_2^{-1} = -\frac{1}{2} \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} -1/4 & 0 \\ 0 & -1/4 \end{bmatrix}$$

$$B_1 = \Sigma_1^{-1} \mu_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 6+0 \\ 0+3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$B_2 = \Sigma_2^{-1} \mu_2 = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ -1 \end{bmatrix}$$

$$= -\frac{1}{2} M_1^t \Sigma_1^{-1} M_1 - \frac{1}{2} \ln|\Sigma_1| + \ln P(\omega_1)$$

$$= \frac{-1}{2} \underset{1\times 2}{\begin{bmatrix} 3 & 6 \end{bmatrix}} \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \underset{2\times 1}{\begin{bmatrix} 3 \\ 6 \end{bmatrix}} - \frac{1}{2} \ln \begin{vmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{vmatrix}$$

$$= \frac{-1}{2} \begin{bmatrix} 6 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \end{bmatrix} - \frac{1}{2} \ln \begin{vmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{vmatrix}$$

$$= \frac{-1}{2} [18 + 18] - \frac{1}{2} \ln 1$$

$$= -\frac{1}{2} (36) - \frac{1}{2} \underbrace{\ln(1)}_{0}$$

$$= -18 - \frac{1}{2}(0) = -18$$

$$C_{20} = -\frac{1}{2} M_2^t \Sigma_2^{-1} M_2 - \frac{1}{2} \ln|\Sigma_2| + \ln P(\omega_2)$$

$$= -\frac{1}{2} \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} - \frac{1}{2} \ln \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} \frac{3}{2} & -1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} - \frac{1}{2} \ln(4)$$

$$= -\frac{1}{2} \left[\frac{9}{2} + 2 \right] - \frac{1}{2} \ln(4)$$

$$= -\frac{1}{2} \left[\frac{9+4}{2} \right] - \frac{1}{2} \ln(4)$$

$$= \frac{-1}{2} \left[\frac{13}{2} \right] - 0.6931$$

$$= -\frac{13}{4} - 0.6931 = -3.25 - 0.6931 = -3.9431$$

$$g_1(x) - g_2(x) = 0$$

$$x^t (A_1 - A_2) x + (B_1 - B_2)^t x + c_{10} - c_{20} = 0$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1/4 \end{bmatrix} - \begin{bmatrix} -1/4 & 0 \\ 0 & -1/4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 6 - 3/2 \\ 3 - (-1) \end{bmatrix}^t X + -18 - (-3.943)$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} -3/4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 9/2 \\ 4 \end{bmatrix}^t X + \begin{bmatrix} -18 + 3.943 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} -3/4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 9/2 & 4 \end{bmatrix} X - 14.057$$

$$\begin{bmatrix} -3/4\, x_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 9/2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 14.057$$

$$-\frac{3}{4} x_1^2 + \frac{9}{2} x_1 + 4x_2 - 14.057 = 0$$

$$4x_2 = \frac{3}{4} x_1^2 - \frac{9}{2} x_1 + 14.057$$

$$x_2 = \frac{3}{16} x_1^2 - \frac{9}{8} x_1 + \frac{14.059}{4}$$

$$\boxed{x_2 = 0.1875\, x_1^2 - 1.125\, x_1 + 3.514}$$

$$\omega_1 = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 8 \end{pmatrix} \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

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$$\omega_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix} \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Assume $P(\omega_1) = P(\omega_2)$

$$\mu_1 = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \qquad \mu_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\Sigma_1 = \frac{1}{4}\left(\begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}^t \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \right)$$

$$= \frac{1}{4}\left(\begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -2 & 2 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \right)$$

$$= \frac{1}{4}\left(\begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix} \right) = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Sigma_2 = \frac{1}{4}\left(\begin{bmatrix} 0 & 2 \\ -2 & 0 \\ 0 & -2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 & 2 \\ 2 & 0 & -2 & 0 \end{bmatrix} \right) = \frac{1}{4}\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

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$$\Sigma_1^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$b = c = 0 \qquad \frac{1}{2}a = 1 \qquad 2d = 1$

$a = 2 \qquad d = 1/2$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$\Sigma_2^{-1} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \qquad \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$b = c = 0 \qquad 2a = 2d = 1$

$a = d = 1/2$

$$= \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}.$$

$$g_1(x) = x^t A_1 x + B_1^t x + C_{10}.$$

$$g_2(x) = x^t A_2 x + B_2^t x + C_{20}$$

$$A_1 = \frac{-1}{2}\Sigma_1^{-1} = \frac{-1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1/4 \end{bmatrix}$$

$$A_2 = \frac{-1}{2}\Sigma_2^{-1} = \frac{-1}{2}\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} -1/4 & 0 \\ 0 & -1/4 \end{bmatrix}$$

$$B_1 = \Sigma_1^{-1}\mu_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$B_2 = \Sigma_2^{-1}\mu_2 = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ -1 \end{bmatrix}$$

$$C_1 = \frac{-1}{2}\mu_1^t \Sigma_1^{-1}\mu_1 - \frac{1}{2}\ln|\Sigma_1| + \ln(P(\omega_1)).$$

$$= \frac{-1}{2}\begin{bmatrix} 3 & 6 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \end{bmatrix} - \frac{1}{2}\ln\begin{vmatrix} 1/2 & 0 \\ 0 & 2 \end{vmatrix} + \ln(1/2).$$

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$[6 \quad 3]\begin{bmatrix}3\\6\end{bmatrix} - \frac{1}{2} \times 0 + \ln(1/2)$ ①

$\frac{-1}{2}(36) + \ln(1/2) = -18 + \ln(1/2)$

$2d = 1 \qquad d = 1/2$

$= \frac{-1}{2}\mu_2^t \Sigma_2^{-1}\mu_2 - \frac{1}{2}\ln|\Sigma_2| + \ln(P(\omega_2))$

$= \frac{-1}{2}[3 \quad -2]\begin{bmatrix}1/2 & 0\\0 & 1/2\end{bmatrix}\begin{bmatrix}3\\-2\end{bmatrix} - \frac{1}{2}\ln\begin{vmatrix}2 & 0\\0 & 2\end{vmatrix} + \ln(1/2)$

$d = 1/2$

$= \frac{-1}{2}[3/2 \quad -1]\begin{bmatrix}3\\-2\end{bmatrix} + \ln(1/2) - \ln 2.$

$= -\frac{13}{4} + \ln(1/2) - \ln(2)$

$g_1(x) - g_2(x) = 0.$

$x^t(A_1 - A_2)x + (B_1 - B_2)^t x + C_{10} - C_{20} = 0$

$X = \begin{bmatrix}x_1\\x_2\end{bmatrix}$

$[x_1 \quad x_2]_{1\times 2}\begin{bmatrix}-3/4 & 0\\0 & 0\end{bmatrix}_{2\times 2}\begin{bmatrix}x_1\\x_2\end{bmatrix} + [9/2 \quad 4]\begin{bmatrix}x_1\\x_2\end{bmatrix} + (-18 + \ln(1/2))$

$- \left(\frac{-13}{4} + \ln(1/2) - \ln\dots\right.$

$\begin{bmatrix}\frac{-3x_1}{4} & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + 4.5x_1 + 4x_2 - 18 + \frac{13}{4} + \ln 2 = 0.$

$$-\frac{3x_1^2}{4} + \frac{9x_1}{2} + 4x_2 - 18 + \frac{13}{4} + \ln 2 = 0$$

$$4x_2 = \frac{3x_1^2}{4} - \frac{9x_1}{2} + \frac{59}{4} + \ln 2$$

$$x_2 = \frac{3}{16}x_1^2 - \frac{9}{8}x_1 + \frac{59}{16} + \frac{1}{4}.$$

$$= 0.1875\, x_1^2 - 1.125\, x_1 + 3.9375$$