Case 3: It is more general case.

1. Σ_i - arbitrary ; different classes have different co-variance matrix ; $\Sigma_i \neq \Sigma_j$

$$\Sigma_1 = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 2 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 1 & 1.2 \\ 1.2 & 3 \end{bmatrix}$$; x_1 and x_2 are not independent

2. The decision surface is hyper quadratic in nature

3. Covariance matrix is arbitrary.

From Ⓐ ; we can't ignore anything here because of Σ_i is arbitrary in nature.

From Ⓐ we can write:

$$-\frac{1}{2}\left[x^t \Sigma_i^{-1} x - \mu_i^t \Sigma_i^{-1} x - x^t \Sigma_i^{-1} \mu_i + \mu_i^t \Sigma^{-1} \mu_i\right] + \ln P(\omega_i) - \frac{1}{2} \ln |\Sigma_i|$$

$$= -\frac{1}{2}\left[\underset{\Downarrow}{x^t \Sigma_i^{-1} x} - 2\mu_i^t \Sigma^{-1} x + \mu_i^t \Sigma^{-1} \mu_i\right] + \ln P(\omega_i) - \frac{1}{2} \ln |\Sigma_i|$$

we can't ignore this because Σ_i is arbitrary

$$= x^t \underbrace{\left(-\frac{1}{2} \Sigma_i^{-1}\right)}_{A_i} x + \underbrace{\mu_i^t \Sigma_i^{-1}}_{B_i} x + \underbrace{\left(-\frac{1}{2} \mu_i^t \Sigma_i^{-1} \mu_i\right) + \ln P(\omega_i) - \frac{1}{2} \ln |\Sigma_i|}_{C_{i0}}$$

$$g_i(x) = x^t A_i x + B_i^t x + C_{i0}$$

$$\boxed{g_i(x) = x^t A_i x + B_i^t x + C_{io}}$$ → quadratic form

where

$$A_i = \frac{-1}{2} \Sigma_i^{-1}$$

$$B_i = \Sigma_i^{-1} M_i \text{ (or) } M_i \Sigma_i^{-1}$$

$$C_{io} = \frac{-1}{2} M_i^t \Sigma^{-1} M_i - \frac{1}{2} \ln|\Sigma_i| + \ln P(w_i).$$

The decision surface is quadratic hyperplane.

Summary: Multivariate case:

Case 1: $\Sigma_i = \sigma^2 I$; Same for all class

Case 2: $\Sigma_i = \Sigma$; same for all class

Case 3: $\Sigma_i \neq \Sigma_j$; different for different classes.

$\Sigma_1 \neq \Sigma_2$

Bivariate case:

Case 1: $\sigma_1^2 = \sigma_2^2 = \Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$

Case 2: $\sigma_1^2 > \sigma_2^2 = \Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$

Case 3: $\Sigma = \cancel{\text{arbitrary}} = \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$