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①

## Baye's classifier formula:

$$P(y|x) = \frac{P(x|y) \cdot P(y)}{P(x)}$$

$$P(w_i|x) = \frac{P(x|w_i) \cdot P(w_i)}{P(x)}$$

## Naive Bayes Assumptions:

$X = \langle x_1, \ldots x_n \rangle$ , $Y$ descrete valued.

$x_i$ and $x_j$ are conditionally independent given $Y$ for all ($i \neq j$)

## Naive Bayes formula:

$$P(x_1 \ldots x_n / y) = \Pi_i \, P(x_i/y)$$

## Solved in class: Example 1:

① Two boxes $B_1$ and $B_2$ contain 100 and 200 light bulbs respectively. The first box ($B_1$) has 15 defective bulbs and the second has 5 defective bulbs.

a) suppose a box is selected at random and one bulb is picked out. what is the probability it is defective?

b) suppose the bulb we tested was defective. what is the probability it came from box 1.

| | defective | not defective |
|---|---|---|
| $B_1$ | 15 | 85 |
| $B_2$ | 5 | 195 |

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$$P(D) = P(D/B_1) \cdot P(B_1) + P(D/B_2) \cdot P(B_2)$$

Since the box is selected at random, they are equally likely.

$$P(B_1) = P(B_2) = \frac{1}{2} = 0.5$$

$$P(D/B_1) = \frac{15}{100} = 0.15$$

$$P(D/B_2) = \frac{5}{200} = 0.025$$

$$P(D) = (0.15 \times 0.5) + (0.025 \times 0.5)$$

$$= 0.0875$$

Thus, There is about 9% probability a bulb is defective.

b) Suppose the bulb we tested was defective what is the probability it came from box 1. 2

$$P(B_1|D) = \frac{P(D/B_1) * P(B_1)}{P(D)}$$

$$= \frac{0.15 * 0.5}{0.0875}$$

$$= 0.8571 > 0.5$$

* Recall box 1 has three times more defective bulbs compared to box 2.

Example 2:

① * A new medical test is used to detect whether a patient has a certain cancer (or) not, whose test result is either + (positive) or − (negative)

* For patient with this cancer, the probability of returning positive test result is 0.98

* For patient without this cancer, the probability of returning negative test result is 0.97

* The probability for any person to have this cancer is 0.008

Question:

If positive test result is returned for some person, does he/she have this kind of cancer or not?

Idea:

$P(\omega_i | +) = ? \Rightarrow P(\omega_1 | +) = ?$

$P(\omega_2 | +) = ?$

If $P(\omega_1 | +) > P(\omega_2 | +) \Rightarrow \omega_1$

If $P(\omega_1 | +) < P(\omega_2 | +) \Rightarrow \omega_2$

(m)

Given

| | [illegible] x | |
|---|---|---|
| Class $w_i$ | + | − |
| $w_1$ cancer | 0.98 | |
| $w_2$ No cancer | | 0.97 |

Given

$P(+/w_1) = 0.98 \Rightarrow P(-/w_1) = 1 - 0.98 = 0.02$

$P(-/w_2) = 0.97 \Rightarrow P(+/w_2) = 1 - 0.97 = 0.03$

$P(w_1) = 0.008 \Rightarrow P(w_2) = 1 - 0.008 = 0.992$

$$P(w_1|+) = \frac{P(+/w_1) \cdot P(w_1)}{P(+)} = \frac{P(+/w_1) \cdot P(w_1)}{P(+|w_1) \cdot P(w_1) + P(+/w_2) \cdot P(w_2)}$$

$$= \frac{0.008 \times 0.98}{(0.98 * 0.008) + (0.03 * 0.992)} = 0.2085$$

$$P(w_2|+) = 1 - P(w_1|+)$$

$$= 1 - 0.2085 = 0.7915$$

Conclusion:

since $P(\omega_2|+) > P(\omega_1|+)$

NO cancer!