Managerial Economics
Unit 7: Game Theory

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Game theory

- Attempts to study decision making in situations where there is a mixture of conflict and cooperation

- A **game** is a competitive situation where two or more persons pursue their own interests and no person can dictate the outcome

Making strategy and game theory

- Strategic managerial decisions: Characterized by interactive payoffs in which managers must explicitly consider the actions likely to be taken by their rivals in response to their decisions.

- Nonstrategic managerial decisions: Do not involve other decision makers, so the reactions of other decision makers do not have to be considered.

Making strategy and game theory

- Interactive: When the consequence of a manager's decision depends on both the manager's own action and the actions of others

- There are no unconditional optimal strategies in game theory; the optimality of a strategy depends on the situation in which it is implemented

Basics

- **The Players:** who is part of the game and how do they interact?

- **Feasible strategy set:** specification of what a player will do under each feasible (i.e. possible) situation of the game

- **Outcomes:** defined by strategy choice of the players

- **Payoff matrix:** set of rewards to players depending upon the outcome of the game. Rationality assumption: players prefer higher payoff

- **Order of play:** simultaneous or sequential decisions by the players

Visual representation

- Matrix form: Form that summarizes all possible outcomes

- Extensive form: Form that provides a road map of player decisions

- Game trees: Game trees are another name for extensive form games and are similar to decision trees.

Examples

- Figure 14.1: A Two-Person Simultaneous Game

- Figure 14.2: Allied-Barkley Pricing: Sequential

- Figure 14.3: Allied-Barkley Pricing: Simultaneous

    ▸ Uses information sets to use the extensive form to represent
      simultaneous decisions

Matrix form: red payoff for Allied, blue payoff for Barkeley

A Two-Person Simultaneous Game

		Barkley's strategy	
		Spend at current level	Increase spending
Allied's strategy	Spend at current level	3, 4	2, 3
	Increase spending	4, 3	3, 2

Allied–Barkley Pricing: Sequential

Barkley

- $1.65 → **Allied**
  - $1.55 → 8, 5
  - $1.30 → 6, 2
  - $0.95 → 6, 0
- $1.35 → **Allied**
  - $1.55 → 14, 7
  - $1.30 → 8, 2
  - $0.95 → 5, 1
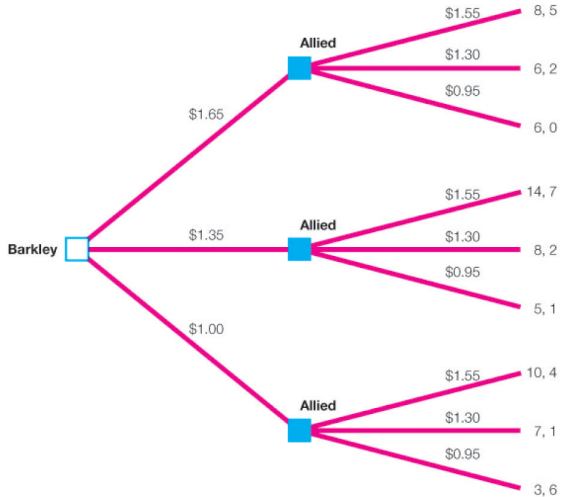- $1.00 → **Allied**
  - $1.55 → 10, 4
  - $1.30 → 7, 1
  - $0.95 → 3, 6

FIGURE 11-02

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Allied–Barkley Pricing: Simultaneous

FIGURE 11-03

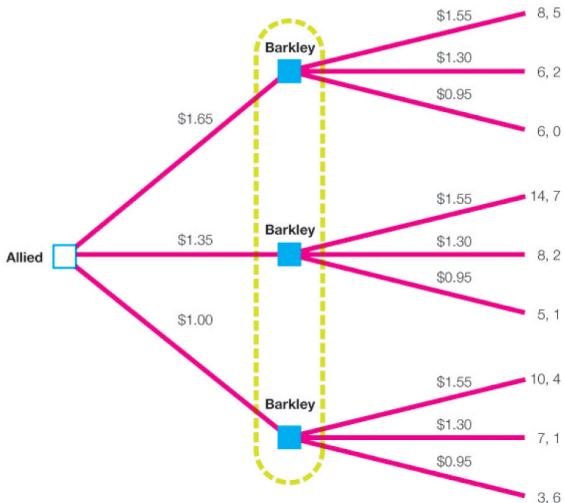# Solution concepts and equilibria

- Solution Concepts
    - Key to the solution of game theory problems is the **anticipation** of the behavior of others.

- Equilibria
    - equilibrium should be rational, optimal and stable
    - equilibrium: When no player has an incentive to unilaterally change his or her strategy
    - no player is able to improve his or her payoff by unilaterally changing strategy.
        - ★ (unilaterally = alone)

Dominant strategies

- Dominant strategies: A strategy whose payout in any outcome is higher relative to all other feasible strategies

  ▶ A strategy that is optimal regardless of the strategies selected by rivals

  ▶ Basically, any strategic element is taken out of the game (simplest possible form of a game)

- Example: Dominant strategy

  ▶ Figure 11.1: A Two-Person Simultaneous Game

  ▶ Barkley has a dominant strategy, which is to maintain the current spending level.

  ▶ Allied has a dominant strategy, which is to increase spending.

A Two-Person Simultaneous Game

FIGURE 11-01

Dominated strategies

- A dominated strategy is a strategy that is dominated by another strategy in all circumstances

- A dominated strategy will never be played

  - Barkley has a dominated strategy, which is to charge $1.00. There is no circumstance under which this strategy would yield a payoff greater than the other feasible strategies.

  - Allied has two dominated strategies, given the elimination of a Barkley price of $1.00. These are to price at $0.95 and to price at $1.30.

  - Figure 11.5: Iterative Dominance

    - This allows the elimination of dominated strategies from the feasible strategy set!

Matrix Form Representation of Figure 11.2

Allied's pricing strategies

Barkley's pricing strategies		$0.95	$1.30	$1.55
	$1.00	3, 6	7, 1	10, 4
	$1.35	5, 1	8, 2	14, 7
	$1.65	6, 0	6, 2	8, 5

FIGURE 11-04

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Iterative Dominance

A. Barkley's $1.00 strategy is eliminated.

Allied's pricing strategies

		$0.95	$1.30	$1.55
Barkley's pricing strategies	$1.35	5, 1	8, 2	14, 7
	$1.65	6, 0	6, 2	8, 5

B. Allied's $0.95 and $1.30 strategies are eliminated.

Allied's pricing strategies

		$1.55
Barkley's pricing strategies	$1.35	14, 7
	$1.65	8, 5

FIGURE 11-05

Dominant strategies

- Games with dominant strategies very simple, but rare
  - No interaction, so it is boring
- Iterative dominance helps to solve some games with some strategic interaction
- In most cases, no equilibrium in dominant strategies possible.

The Nash equilibrium

- Assuming that all players are rational, every player should choose the best strategy conditional on all other players doing the same.

- John Nash (Nobel Prize 1994)
  - http://nobelprize.org/economics/laureates/1994/nash-autobio.html
  - http://www.abeautifulmind.com

- How to find a Nash eq.: No player should have an incentive to deviate, then the strategy is a Nash-Equilibrium

- Note: all equilibria in dominant strategies are automatically also Nash-equilibria

Example: Nash equilibrium

- Figure 14.6: New Product Introduction

- Nash equilibrium is where Barkley produces product sigma and Allied produces product alpha.

New Product Introduction

		Allied		
		Product alpha	Product beta	Product zeta
Barkley	Product lambda	4, 6	9, 8	6, 10
	Product pi	6, 8	8, 9	7, 8
	Product sigma	9, 8	7, 7	5, 5

FIGURE 11-06

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Look for Barkley's optimal behavior vertically!

New Product Introduction with Other's Behavior

		Allied		
		Product alpha	Product beta	Product zeta
Barkley	Product lambda	4, 6	(B) 9, 8	6, 10 (A)
	Product pi	6, 8	8, 9 (A)	(B) 7, 8
	Product sigma	(B) 9, 8 (A)	7, 7	5, 5

FIGURE 11-07

Strategic foresight: the use of backward induction

- **Strategic foresight:** A manager's ability to make decisions today that are rational given what is anticipated in the future

- **Backward induction:** Used in game theory to solve games by looking to the future, determining what strategy players will choose (anticipation), and then choosing an action that is rational, based on those beliefs

  - In sequential games, backward induction involves starting with the last decisions in the sequence and then working backward to the first decisions, identifying all optimal decisions.
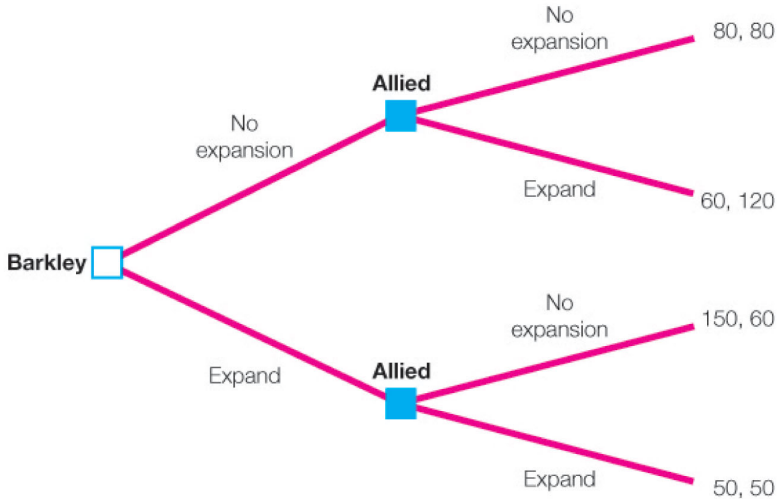
Allied–Barkley Expansion Decision

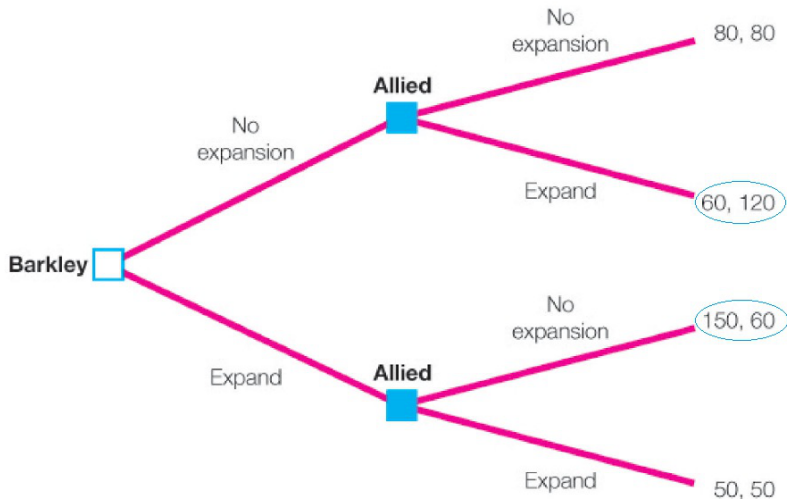| | | No expansion | 80, 80 |
Allied			
Barkley	No expansion	Expand	60, 120
	Expand	No expansion	150, 60
Allied			
		Expand	50, 50

FIGURE 11-08

Allied–Barkley Expansion Decision

FIGURE 11-08

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Strategic foresight: the use of backward induction

- Backward induction and the Centipede Game
- Centipede game: A sequential game involving a series of six decisions that shows the usefulness of backward induction in strategic thinking

  - Players A and B alternate in choosing R(r) or D (d)
  - Solve from the very end!
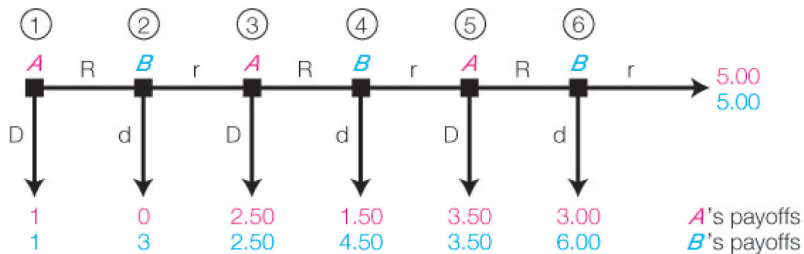 - Problem of efficiency!

The Centipede Game

FIGURE 11-09

Ultimatum Game

- two players have to decide how to divide a sum of money that is given to them, e.g. 100 Euro
- first player proposes how to divide the sum between the two players
- the second player can either accept or reject this proposal
  - If the second player rejects, neither player receives anything.
  - If the second player accepts, the money is split according to the proposal.
- The game is played only once so that reciprocation is not an issue.
- Equilibrium?

Ultimatum Game

- Equilibrium
  - Player one offers smallest unit, i.e. 1 cent
  - Why?
- Experimental evidence
  - Fairness does play a role
  - Offers below 20% are usually rejected
  - This results also comes out of high stake experiments

The Credibility of Commitments

- Credible: When the costs of falsely making a commitment are greater than the associated benefits

  - Figure 11.10: Does Barkley Have a Credible Threat?

  - It is not in Barkley's interest to drop price in response to Allied's price cut. The threat to do so is not credible.

- Subgame: A segment of a larger game

  - Figure 11.10 has three subgames and all are in equilibrium at the optimal solution, which implies that the equilibrium is a subgame perfect equilibrium.

Does Barkley Have a Credible Threat?

FIGURE 11-10

Does Barkley Have a Credible Threat?

Maintain price — 30, 50

Barkley 1

Drop price — 20, 70

Maintain price

Allied 3

Drop price

Maintain price — 40, 30

Barkley 2

Drop price — 15, 20
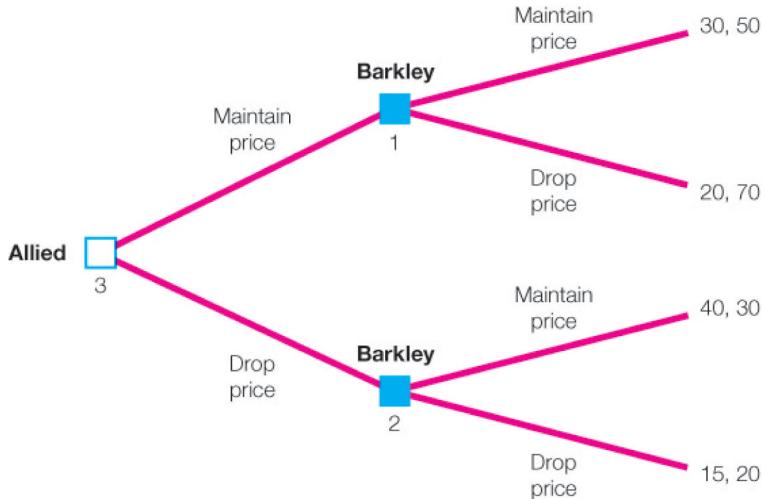
FIGURE 11-10

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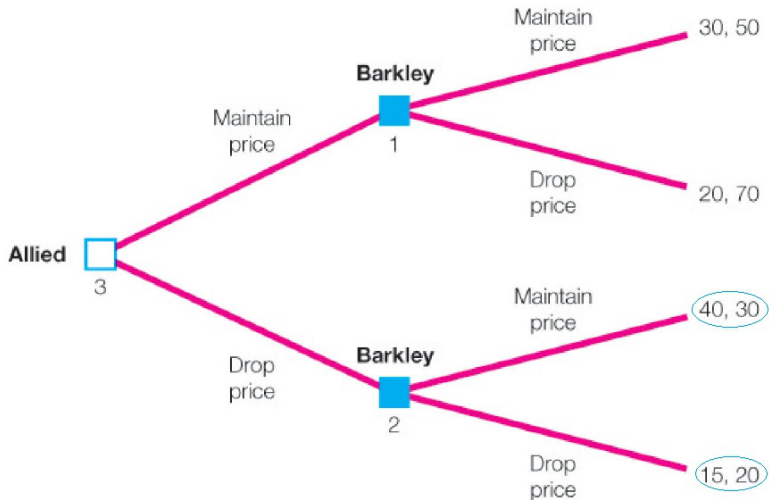Copyright © W.W. Norton & Company 2009

Cooperation and conflict

- Prisoner's dilemma
  - Allied and Barkley produce an identical product and have similar cost structures.
  - Each player must decide whether to price high or low.
  - Figure 11.11: Pricing as a Prisoner's Dilemma
    - Solution is for both to price low.
    - But . . .
    - both would be better off if both priced high.

Pricing as a Prisoner's Dilemma

Allied's pricing strategies

		Price high	Price low
Barkley's pricing strategies	Price high	5, 5	1, 20
	Price low	20, 1	3, 3

FIGURE 11-11

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Classical Prisoner's dilemma
2 prisoners are interrogated separately

	Possible strategies for Mulloy	
Possible strategies for Jones	*Confess*	*Do not confess*
Confess	Jones: 8 years Mulloy: 8 years	Jones: 2 years Mulloy: 10 years
Do not confess	Jones: 10 years Mulloy: 2 years	Jones: 4 years Mulloy: 4 years

Prisoner's dilemma

- Nash-equilibrium is inefficient

- Cooperation between the players would increase profits for both

- Why is it that cooperation is not possible?
  - Individual rationality (incentive to cheat) is the problem

- Note that Adam Smith's "invisible hand" goes in the wrong direction

Repeated games: prisoner's dilemma

- Repeated play can lead to cooperative behavior in a prisoner's dilemma game.

  ▶ Trust, reputation, promises, threats, and reciprocity are relevant only if there is repeated play.

  ▶ Cooperative behavior is more likely if there is an infinite time horizon than if there is a finite time horizon.

  ▶ If there is a finite time horizon, then the value of cooperation, and hence its likelihood, diminishes as the time horizon is approached. Backward induction implies that cooperation will not take place in this case.

Repeated games: cartel

- Assume a cartel game: 2 firms want to set the price high to maximize profits in the cartel.
  - But each firm has an incentive to cheat and reduce its price
  - Cooperation is very difficult to establish if players interact only once (one-shot game)
  - Only Nash-equilibrium is low/low.
- Why is it that you do observe cartels (cooperation) in real life???
  - Players in real life do not interact only once, they interact more often
  - Benefits of cooperation are higher if agents can interact more often

Repeated games

- If one player cheats (is undercutting the price), the other can punish him later (set also a low price)

- **One successful strategy:**
  - **Tit-for-tat:** start cooperative
  - then each player should do, what the other did in the previous round: solves cooperation problem, i.e. if the other firm cheated in last period, I should cheat now, . . .
  - Gain of cheating is 15 (=20-5), future loss is 2 (=5-3) in each consecutive period.
  - In 8 periods the gains from cheating are lost

- Does it work also, if there is a finite period (e.g. only 20 periods)?
  - Use backward induction (i.e. look at last period!)
  - End-game problem

Most-favored-customer clauses

- Version 1
  - If the firm reduces its price subsequent to a purchase, the early customer will get a rebate so that he or she will pay no more than those buying after the price reduction

- Version 2
  - You get a rebate, if you see the product cheaper somewhere else. ⇒ **Bestpreisgarantie**

- Looks like a very generous (consumer-friendly) device.

- It can also be a seen as a clever agreement to keep cartel discipline alive.

Payoff Matrix **before** Most-favored-customer clause

Farmer

	Set price at $2,000	Set price at $1,000
Acron Set price at $2,000	Farmer's profit: $5 million / Acron's profit: $5 million	Farmer's profit: $8 million / Acron's profit: –$2 million
Set price at $1,000	Farmer's profit: –$2 million / Acron's profit: $8 million	Farmer's profit: $2 million / Acron's profit: $2 million

Payoff Matrix **after** Most-favored-customer clause

Farmer

	Set price at $2,000	Set price at $1,000
Acron Set price at $2,000	Farmer's profit: $5 million / Acron's profit: $5 million	Farmer's profit: $4 million / Acron's profit: –$2 million
Set price at $1,000	Farmer's profit: –$2 million / Acron's profit: $4 million	Farmer's profit: $2 million / Acron's profit: $2 million

Incomplete information games

- A branch of game theory that loosens the restrictive assumption that all players have the same information

  - Asymmetric information is summarized in terms of player types. A type has characteristics unknown to other players that have different preference (payoff) functions.

    - Examples: low-cost type and high-cost type, tough type and soft type

  - Figure 11.12: Tough or Soft Barkley Managers

    - If Barkley managers are tough, they will fight, and Allied will not enter the market.

    - If Barkley managers are soft, they will not fight, and Allied will enter.

Tough or Soft Barkley Managers

A. Barkley managers are tough.

		Allied's strategies	
		Enter the market	Do not enter the market
Barkley's strategies	Fight (price low)	6, 2	8, 3
	No fight (price high)	5, 4	2, 3

B. Barkley managers are soft.

		Allied's strategies	
		Enter the market	Do not enter the market
Barkley's strategies	Fight (price low)	2, 2	3, 3
	No fight (price high)	7, 4	4, 3

FIGURE 11-12

Reputation building

- Even if Barkley would be soft, pretending to be tough in first periods would be good.

  - Would deter Allied from entering

- Reputation is very valuable.

  - Reputation requires a time horizon and incomplete information.

  - Reputation is based on a player's history of behavior and involves inferring future behavior based on past behavior.

Coordination games

- Coordination games have more than one Nash equilibrium and the players' problem is which one to select.

- Matching Games

  - ▶ Two Nash equilibria

  - ▶ Problems in coordination arise from players' inability to communicate, players with different strategic models, and asymmetric information.

- Matching Games

  - ▶ Figure 11.13: Product Coordination Game

    - ★ Nash equilibrium is for one firm to produce for the industrial market and the other to produce for the consumer market.

    - ★ Both firms would prefer the equilibrium with the higher payoff.

Product Coordination Game

	Allied's strategies	
Barkley's strategies	Produce for consumer market	Produce for industrial market
Produce for consumer market	0, 0	7, 7
Produce for industrial market	12, 12	0, 0

FIGURE 11-13

Coordination games: Battle of the Sexes

- Two Nash equilibria

- Players prefer different equilibria.

- Figure 11.14: Battle of the Sexes

  ▶ Nash equilibrium is for to one to produce high end and the other to produce low end.

  ▶ Both players prefer to produce the high-end product.

Battle of the Sexes

Allied's strategies

		High-end product	Low-end product
Barkley's strategies	High-end product	0, 0	11, 6
	Low-end product	6, 11	0, 0

Coordination games: First-Mover Games

- Two Nash equilibria

- Players prefer different equilibria.

- Figure 11.16 First-Mover Advantage

  - Nash equilibrium is for one firm to produce the superior product and the other to produce the inferior product.

  - Both firms want to produce the superior product, which yields the higher payoff, by moving first.

  - Barkley is predicted to move first because the payoff is higher for Barkley and therefore Barkley can afford to spend more to speed up development.

    $\Rightarrow$ Difference b/n good and bad outcome

First-Mover Advantage

	Allied's strategies	
	Produce superior product	Produce inferior product
Barkley's strategies — Produce superior product	25, 50	110, 70
Produce inferior product	30, 140	20, 30

FIGURE 11-16