4. SHEAR AND TORSION

4.1 DESIGN SHEAR STRENGTH OF CONCRETE

The design shear strength of concrete is given in Table 13 of the Code. The values given in the Code are based on the following equation:

$$\tau_c = \frac{0.85\ \sqrt{0.8 f_{ck}}\ (\sqrt{1+5\beta}-1)}{6\beta}$$

where

$\beta = 0.8\ f_{ck}/6.89\ p_t$, but not less than 1·0,

and $p_t = 100\ A_{st}/b_w d$.

The value of τ_c corresponding to p_t varying from 0·20 to 3·00 at intervals of 0·10 are given in Table 61 for different grades of concrete.

4.2 NOMINAL SHEAR STRESS

The nominal shear stress τ_v is calculated by the following equation:

$$\tau_v = \frac{V_u}{bd}$$

where

V_u is the shear force.

When τ_v exceeds τ_c, shear reinforcement should be provided for carrying a shear equal to $V_u - \tau_c\, bd$. The shear stress τ_v should not in any case exceed the values of $\tau_{c,max}$, given in Table J. (If $\tau_v > \tau_{c,max}$, the section is to be redesigned.)

TABLE J MAXIMUM SHEAR STRESS $\tau_{c,max}$

CONCRETE GRADE	M15	M20	M25	M30	M35	M40
$\tau_{c,max}$, N/mm²	2·5	2·8	3·1	3·5	3·7	4·0

4.3 SHEAR REINFORCEMENT

The design shear strength of vertical stirrups is given by the following equation:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

where

A_{sv} is the total cross sectional area of the vertical legs of the stirrups, and

s_v is the spacing (pitch) of the stirrups.

The shear strength expressed as V_{us}/d are given in Table 62 for different diameters and spacings of stirrups, for two grades of steel.

For a series of inclined stirrups, the value of V_{us}/d for vertical stirrups should be multiplied by $(\sin\alpha + \cos\alpha)$ where α is the angle between the inclined stirrups and the axis of the member. The multiplying factor works out to 1·41 and 1·37 for 45° and 60° angles respectively.

For a bent up bar,

$V_{us} = 0.87\, f_y\, A_{sv} \sin\alpha$

Values of V_{us} for different sizes of bars, bent up at 45° and 60° to the axis of the member are given in Table 63 for two grades of steel.

4.4 TORSION

Separate Charts or Tables are not given for torsion. The method of design for torsion is based on the calculation of an equivalent shear force and an equivalent bending moment. After determining these, some of the Charts and Tables for shear and flexure can be used. The method of design for torsion is illustrated in Example 11.

Example 10 Shear

Determine the shear reinforcement (vertical stirrups) required for a beam section with the following data:

Beam size	30 × 60 cm
Depth of beam	60 cm
Concrete grade	M 15
Characteristic strength of stirrup reinforcement	250 N/mm²
Tensile reinforcement percentage	0·8
Factored shear force, V_u	180 kN

Assuming 25 mm dia bars with 25 mm cover,

$$d = 60 - \frac{2.5}{2} - 2.5 = 56.25 \text{ cm}$$

$$\text{Shear stress, } \tau_v = \frac{V_u}{bd} = \frac{180 \times 10^3}{30 \times 56.25 \times 10^2} = 1.07 \text{ N/mm}^2$$

From *Table J* for M15, $\tau_{c,max} = 2.5$ N/mm²

τ_v is less than $\tau_{c,max}$

From *Table 61*, for $P_t = 0.8$, $\tau_c = 0.55$ N/mm²

Shear capacity of concrete section $= \tau_c\, bd$

$= 0.55 \times 30 \times 56.25 \times 10^2/10^3 = 92.8$ kN

Shear to be carried by stirrups, $V_{us} = V_s - \tau_c bd$

$$= 180 - 92{\cdot}8 = 87{\cdot}2 \text{ kN}$$

$$\frac{V_{us}}{d} = \frac{87{\cdot}2}{56{\cdot}25} = 1{\cdot}55 \text{ kN/cm}$$

Referring to *Table 62*, for steel $f_y = 250$ N/mm². Provide 8 mm diameter two legged vertical stirrups at 14 cm spacing.

Example 11 Torsion

Determine the reinforcements required for a rectangular beam section with the following data:

Size of the beam	30 × 60 cm
Concrete grade	M 15
Characteristic strength of steel	415 N/mm²
Factored shear force	95 kN
Factored torsional moment	45 kN.m
Factored bending moment	115 kN.m

Assuming 25 mm dia bars with 25 mm cover,

$$d = 60 - 2{\cdot}5 - \frac{2{\cdot}5}{2} = 56{\cdot}25 \text{ cm}$$

Equivalent shear,

$$V_e = V + 1{\cdot}6\left(\frac{T}{b}\right)$$

$$= 95 + 1{\cdot}6 \times \frac{45}{0{\cdot}3} = 95 + 240 = 335 \text{ kN}$$

Equivalent shear stress.

$$\tau_{ve} = \frac{V_e}{bd} = \frac{335 \times 10^3}{30 \times 56{\cdot}25 \times 10^2} = 1{\cdot}99 \text{ N/mm}^2$$

From *Table J*, for M15, $\tau_{c,max} = 2{\cdot}5$ N/mm²

τ_{ve} is less than $\tau_{c,max}$; hence the section does not require revision.

From *Table 61*, for an assumed value of $p_t = 0{\cdot}5$,

$$\tau_c = 0{\cdot}46 \text{ N/mm}^2 < \tau_{ve}.$$

Hence longitudinal and transverse reinforcements are to be designed Longitudinal reinforcement (*see 40.4.2* of the Code): Equivalent bending moment,

$$M_{e1} = M_u + M_t$$

$$= M_u + \frac{T_u(1 + D/b)}{1{\cdot}7}$$

$$= 115 + 45\left(1 + \frac{60}{30}\right)/1{\cdot}7$$

$$= 115 + 79{\cdot}4$$

$$= 194{\cdot}4 \text{ kN.m}$$

$$M_{e1}/bd^2 = \frac{194{\cdot}4 \times 10^6}{30 \times (56{\cdot}25)^2 \times 10^3} = 2{\cdot}05 \text{ N/mm}^2$$

Referring to *Table 1*, corresponding to $M_u/bd^2 = 2{\cdot}05$

$$p_t = 0{\cdot}708$$

$$A_{st} = 0{\cdot}708 \times 30 \times 56{\cdot}25/100 = 11{\cdot}95 \text{ cm}^2$$

Provide 4 bars of 20 mm dia ($A_{st} = 12{\cdot}56$ cm²) on the flexural tensile face. As M_t is less than M_u, we need not consider M_{e2} according to *40.4.2.1* of the Code. Therefore, provide only two bars of 12 mm dia on the compression face, one bar being at each corner.

As the depth of the beam is more than 45 cm, side face reinforcement of 0·05 percent on each side is to be provided (*see 25.5.1.7* and *25.5.1.3* of the Code). Providing one bar at the middle of each side,

Spacing of bar = 53·4/2 = 26·7 cm

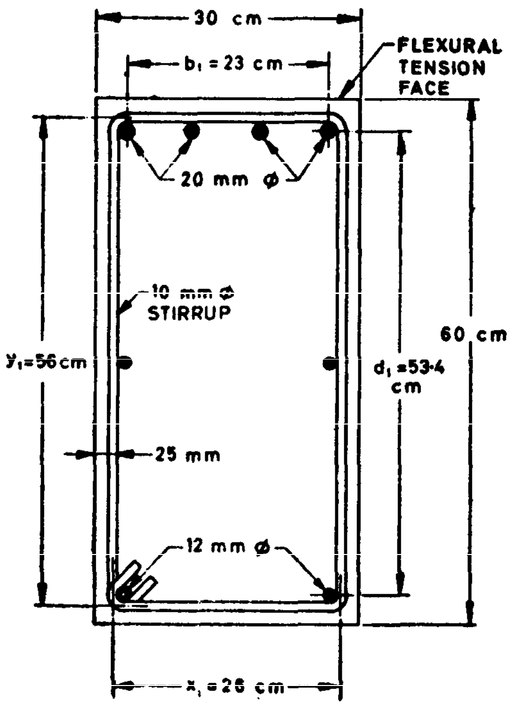
$$\text{Area required for each bar} = \frac{0{\cdot}05 \times 30 \times 26{\cdot}7}{100}$$

$$= 0{\cdot}40 \text{ cm}^2$$

Provide one bar of 12 mm dia on each side. Transverse reinforcement (*see 40.4.3* of the Code):

Area of two legs of the stirrup should satisfy the following:

$$A_{sv} = \frac{T_u S_v}{b_1 d_1 (0{\cdot}87 f_y)} + \frac{V_u S_v}{2{\cdot}5\, d_1 (0{\cdot}87 f_y)}$$



Assuming diameter of stirrups as 10 mm

$d_1 = 60 - (2{\cdot}5 + 1{\cdot}0) - (2{\cdot}5 + 0{\cdot}6) = 53{\cdot}4$ cm

$b_1 = 30 - 2\,(2{\cdot}5 + 1{\cdot}0) = 23$ cm

$$\frac{A_{sv}\,(0{\cdot}87 f_y)}{S_v} = \frac{45 \times 10^6}{23 \times 53{\cdot}4 \times 10^2} + \frac{95 \times 10^3}{2{\cdot}5 \times 53{\cdot}4 \times 10} = 366{\cdot}4 + 71{\cdot}2$$

$$= 437{\cdot}6 \text{ N/mm}$$

$$= 4{\cdot}38 \text{ kN/cm}$$

Area of all the legs of the stirrup should satisfy the condition that A_{sv}/S_v should not be less than $\frac{(\tau_{ve} - \tau_c)b}{0{\cdot}87 f_y}$

From *Table 61*, for tensile reinforcement percentage of 0·71, the value of τ_c is 0·53 N/mm^2

$$\frac{A_{sv}\,(0{\cdot}87 f_y)}{S_v} = (\tau_{ve} - \tau_c)b$$

$$= (1{\cdot}99 - 0{\cdot}53)$$

$$30 \times 10$$

$$= 438 \text{ N/mm} = 4{\cdot}38 \text{ kN/cm}$$

NOTE—It is only a coincidence that the values of $A_{sv}\,(0{\cdot}87 fy)/S_v$ calculated by the two equations are the same.

Referring *Table 62* (for $f_y = 415$ N/mm^2).

Provide 10 mm ϕ two legged stirrups at 12·5 cm spacing.

According to *25.5.1.7(a)* of the Code, the spacing of stirrups shall not exceed x_1, $(x_1 + y_1)/4$ and 300 mm, where x_1 and y_1 are the short and long dimensions of the stirrup.

$$x_1 = 30 - 2(2{\cdot}5 - 0{\cdot}5) = 26 \text{ cm}$$

$$y_1 = 60 - 2(2{\cdot}5 - 0{\cdot}5) = 56 \text{ cm}$$

$$(x_1 + y_1)/4 = (26 + 56)/4 = 20{\cdot}5 \text{ cm}$$

10 mm ϕ two legged stirrups at 12·5 cm spacing will satisfy all the codal requirements.