7. DEFLECTION CALCULATION

7.1 EFFECTIVE MOMENT OF INERTIA

A method of calculating the deflections is given in Appendix E of the Code. This method requires the use of an effective moment of inertia I_{eff} given by the following equation

$$I_{eff} = \frac{I_r}{1{\cdot}2 - \frac{M_r}{M}\frac{z}{d}\left(1 - \frac{x}{d}\right)\frac{b_w}{b}}$$

but, $I_r \leqslant I_{eff} \leqslant I_{gr}$

where

I_r is the moment of inertia of the cracked section;

M_r is the cracking moment, equal to $\frac{f_{cr} I_{gr}}{y_t}$

where

f_{cr} is the modulus of rupture of concrete, I_{gr} is the moment of inertia of the gross section neglecting the reinforcement and y_t is the distance from the centroidal axis of the gross section to the extreme fibre in tension;

M is the maximum moment under service loads;

z is the lever arm;

d is the effective depth;

x is the depth of neutral axis;

b_w is the breadth of the web; and

b is the breadth of the compression face.

The values of x and z are those obtained by elastic theory. Hence $z = d - x/3$ for rectangular sections; also $b = b_w$ for rectangular sections. For flanged sections where the flange is in compression, b will be equal to the flange width b_f. The value of z for flanged beams will depend on the flange dimensions, but in order to simplify the calculations it is conservatively assumed the value of z for flanged beam is also $d - x/3$. With this assumption, the expression effective moment of inertia may be written as follows:

$$\frac{I_{eff}}{I_r} = \frac{1}{1{\cdot}2 - \frac{M_r}{M}\left(1 - \frac{x}{3d}\right)\left(1 - \frac{x}{d}\right)\frac{b_w}{b_f}}$$

but, $\frac{I_{eff}}{I_r} > 1$

and $I_{eff} \leqslant I_{gr}$

Chart 89 can be used for finding the value of $\frac{I_{eff}}{I_r}$ in accordance with the above equation. The chart takes into account the condition $\frac{I_{eff}}{I_r} > 1$. After finding the value of I_{eff} it has to be compared with I_{gr} and the lower of the two values should be used for calculating the deflection.

For continuous beams, a weighted average value of I_{eff} should be used, as given in *B-2.1* of the Code.

7.2 SHRINKAGE AND CREEP DEFLECTIONS

Deflections due to shrinkage and creep can be calculated in accordance with clauses *B-3* and *B-4* of the Code. This is illustrated in Example 12.

Example 12 Check for deflection

Calculate the deflection of a cantilever beam of the section designed in Example 3, with further data as given below:

Span of cantilever	4·0 m
Bending moment at service loads	210 kN.m

Sixty percent of the above moment is due to permanent loads, the loading being distributed uniformly on the span.

$$I_{gr} = \frac{bD^3}{12} = \frac{300 \times (600)^3}{12} = 5{\cdot}4 \times 10^9\ \text{mm}^4$$

From clause *5.2.2* of the Code,

Flexural tensile strength,

$$f_{cr} = 0{\cdot}7\sqrt{f_{ck}}\ \text{N/mm}^2$$

$$f_{cr} = 0{\cdot}7\sqrt{15} = 2{\cdot}71\ \text{N/mm}^2$$

$$y_t = D/2 = \frac{600}{2} = 300\ \text{mm}$$

$$M_r = \frac{f_{cr} I_{gr}}{y_t} = \frac{2{\cdot}71 \times 5{\cdot}4 \times 10^9}{300}$$

$$= 4{\cdot}88 \times 10^7\ \text{N.mm}$$

$$d'/d = \left(\frac{3{\cdot}75}{56{\cdot}25}\right) = 0{\cdot}067$$

$d'/d = 0{\cdot}05$ will be used in referring to Tables.

From *5.2.3.1* of the Code,

$$E_c = 5700\sqrt{f_{ck}}\ \text{N/mm}^2$$

$$= 5\,700\sqrt{15} = 22{\cdot}1 \times 10^3\ \text{N/mm}^2$$

$$E_s = 200\ \text{kN/mm}^2 = 2 \times 10^5\ \text{N/mm}^2$$

$$m = E_s/E_c = \frac{2 \times 10^5}{22{\cdot}1 \times 10^3} = 9{\cdot}05$$

From Example 3,

$p_t = 1.117$, $p_c = 0.418$

$p_c(m-1)/(p_t m) = (0.418 \times 8.05)/(1.117 \times 9.05) = 0.333$

$p_t m = 1.117 \times 9.05 = 10.11$

Referring to *Table 87*,

$I_r/(bd^3/12) = 0.720$

$\therefore\ I_r = 0.720 \times 300 \times (562.5)^3/12$
$= 3.204 \times 10^9\ \text{mm}^4$

Referring to *Table 91*,

$$\frac{x}{d} = 0.338$$

Moment at service load, $M = 210$ kN.m $= 21.0 \times 10^7$ N.mm

$$M_r/M = \frac{4.88 \times 10^7}{21.0 \times 10^7} = 0.232$$

Referring to *Chart 89*.

$I_{eff}/I_r = 1.0$

$\therefore\ I_{eff} = I_r = 3.204 \times 10^9\ \text{mm}^4$

For a cantilever with uniformly distributed load,

$$\text{Elastic deflection} = \frac{1}{4}\frac{Ml^2}{EI_{eff}}$$

$$= \frac{21.0 \times 10^7 \times (4000)^2}{4 \times 22.1 \times 10^3 \times 3.204 \times 10^9}$$

$= 11.86$ mm ...(1)

Deflection due to shrinkage (*see* clause *B-3* of the Code):

$a_{cs} = k_3\ \Psi_{cs}\ l^2$

$k_3 = 0.5$ for cantilevers

$p_t = 1.117$, $p_c = 0.418$

$p_t - p_c = 1.117 - 0.418 = 0.699 < 1.0$

$$\therefore\ k_4 = 0.72 \times \frac{p_t - p_c}{\sqrt{p_t}}$$

$$= 0.72 \times \frac{(1.117 - 0.418)}{\sqrt{1.117}}$$

$= 0.476$

In the absence of data, the value of the ultimate shrinkage strain ξ_{cs} is taken as 0.000 3 as given in *5.2.4.1* of the Code.

$D = 600$ mm

$$\therefore\ \text{Shrinkage curvature}\ \Psi_{cs} = k_4\frac{\xi_{cs}}{D}$$

$$= \frac{0.476 \times 0.000\ 3}{600} = 2.38 \times 10^{-7}$$

$a_{cs} = 0.5 \times 2.38 \times 10^{-7} \times (4\ 000)^2$
$= 1.90$ mm ...(2)

Deflection due to creep,

$a_{cc\ (perm)} = a_{icc\ (perm)} - a_{i\ (perm)}$

In the absence of data, the age at loading is assumed to be 28 days and the value of creep coefficient, θ is taken as 1.6 from *5.2.5.1* of the Code.

$$E_{ce} = \frac{E_c}{1+\theta}$$

$$= \frac{22.1 \times 10^3}{1 + 1.6} = 8.5 \times 10^3\ \text{N/mm}^2$$

$$m = \frac{E_s}{E_{ce}} = \frac{2 \times 10^5}{8.5 \times 10^3} = 23.53$$

$p_t = 1.117$, $p_c = 0.418$

$p_c\ (m-1)/(p_t m) = 0.418(23.53 - 1)/(1.117 \times 23.53)$
$= 0.358$

Referring to *Table 87*,

$I_r/(bd^3/12) = 1.497$

$I_r = 1.497 \times 300\ (562.5)^3/12$
$= 6.66 \times 10^9\ \text{mm}^4$

$I_r \leqslant I_{eff} \leqslant I_{gr}$

$6.66 \times 10^9 \leqslant I_{eff} \leqslant 5.4 \times 10^9$

$\therefore\ I_{eff} = 5.4 \times 10^9\ \text{mm}^4$

$a_{icc\ (perm)}$ = Initial plus creep deflection due to permanent loads obtained using the above modulus of elasticity

$$= \frac{1}{4}\frac{Ml^2}{E_{cc}I_{eff}}$$

$$= \frac{1}{4} \times \frac{(0.6 \times 21 \times 10^7)\ (4\ 000)^2}{8.5 \times 10^3 \times 5.4 \times 10^9}$$

$= 10.98$ mm

$a_{i\ (perm)}$ = Short term deflection due to permanent load obtained using E_c

$$= \frac{1}{4} \times \frac{(0.6 \times 21 \times 10^7)\ (4\ 000)^2}{22.1 \times 10^3 \times 3.204 \times 10^9}$$

$= 7.12$ mm

$\therefore\ a_{cc(perm)} = 10.98 - 7.12 = 3.86$ ...(3)

$\therefore$ Total deflection (long term) due to initial load, shrinkage and creep

$= 11.86 + 1.90 + 3.86 = 17.62$ mm.

According to *22.2(a)* of the Code the final deflection should not exceed span/250.

$$\text{Permissible deflection} = \frac{4\ 000}{250} = 16\ \text{mm}.$$

The calculated deflection is only slightly greater than the permissible value and hence the section may not be revised.