3. COMPRESSION MEMBERS

3.1 AXIALLY LOADED COMPRESSION MEMBERS

All compression members are to be designed for a minimum eccentricity of load in two principal directions. Clause *24.4* of the Code specifies the following minimum eccentricity, e_{min} for the design of columns:

$e_{min} = \frac{l}{500} + \frac{D}{30}$, subject to a minimum of 2 cm.

where

l is the unsupported length of the column (*see 24.1.3* of the Code for definition of unsupported length), and

D is the lateral dimension of the column in the direction under consideration.

After determining the eccentricity, the section should be designed for combined axial load and bending (*see 3.2*). However, as a simplification, when the value of the minimum eccentricity calculated as above is less than or equal to $0.05D$, *38.3* of the Code permits the design of short axially loaded compression members by the following equation:

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

where

P_u is the axial load (ultimate),
A_c is the area of concrete, and
A_{sc} is the area of reinforcement.

The above equation can be written as

$$P_u = 0.4 f_{ck} \left(A_g - \frac{pA_g}{100} \right) + 0.67 f_y \frac{pA_g}{100}$$

where

A_g is the gross area of cross section, and
p is the percentage of reinforcement.

Dividing both sides by A_g,

$$\frac{P_u}{A_g} = 0.4 f_{ck} \left(1 - \frac{p}{100} \right) + 0.67 f_y \frac{p}{100}$$

$$= 0.4 f_{ck} + \frac{p}{100} (0.67 f_y - 0.4 f_{ck})$$

Charts 24 to *26* can be used for designing short columns in accordance with the above equations. In the lower section of these charts, P_u/A_g has been plotted against reinforcement percentage p for different grades of concrete. If the cross section of the column is known, P_u/A_g can be calculated and the reinforcement percentage read from the chart. In the upper section of the charts, P_u/A_g is plotted against P_u for various values of A_g. The combined use of the upper and lower sections would eliminate the need for any calculation. ***This is particularly useful as an aid for deciding the sizes of columns at the preliminary design stage of multi-storeyed buildings.***

Example 5 Axially Loaded Column

Determine the cross section and the reinforcement required for an axially loaded column with the following data:

Factored load	3 000 kN
Concrete grade	M20
Characteristic strength of reinforcement	415 N/mm²
Unsupported length of column	3·0 m

The cross-sectional dimensions required will depend on the percentage of reinforcement. Assuming 1·0 percent reinforcement and referring to *Chart 25*,

Required cross-sectional area of column, $A_g = 2\,700$ cm²

Provide a section of 60 × 45 cm.

Area of reinforcement, $A_s = 1.0 \times \frac{60 \times 45}{100}$
$= 27$ cm²

We have to check whether the minimum eccentricity to be considered is within 0·05 times the lateral dimensions of the column. In the direction of longer dimension,

$$e_{min} = \frac{l}{500} + \frac{D}{30}$$

$$= \frac{3.0 \times 10^2}{500} + \frac{60}{30} = 0.6 + 2.0 = 2.6 \text{ cm}$$

or, $e_{min}/D = 2.6/60 = 0.043$

In the direction of the shorter dimension,

$$e_{min} = \frac{3.0 \times 10^2}{500} + \frac{45}{30} = 0.6 + 1.5$$

$$= 2.1 \text{ cm}$$

or, $e_{min}/b = 2.1/45 = 0.047$

The minimum eccentricity ratio is less than 0·05 in both directions. Hence the design of the section by the simplified method of *38.3* of the Code is valid.

3.2 COMBINED AXIAL LOAD AND UNIAXIAL BENDING

As already mentioned in *3.1*, all compression members should be designed for minimum eccentricity of load. It should always be ensured that the section is designed for a moment which is not less than that due to the prescribed minimum eccentricity.

3.2.1 Assumptions—Assumptions (a), (c), (d) and (e) for flexural members (*see 2.1*) are also applicable to members subjected to combined axial load and bending. The assumption (b) that the maximum strain in concrete at the outermost compression fibre is 0·003 5 is also applicable when the neutral axis lies within the section and in the limiting case when the neutral axis lies along one edge of the section; in the latter case the strain varies from 0·003 5 at the highly compressed edge to zero at the opposite edge. For purely axial compression, the strain is assumed to be uniformly equal to 0·002 across the section [*see 38.1*(a) of the Code]. The strain distribution lines for these two cases intersect each other at a depth of $\frac{3D}{7}$ from the highly compressed edge. This point is assumed to act as a fulcrum for the strain distribution line when the neutral axis lies outside the section (*see* Fig. 7). This leads to the assumption that the strain at the highly compressed edge is 0·003 5 minus 0·75 times the strain at the least compressed edge [*see 38.1*(b) of the Code].

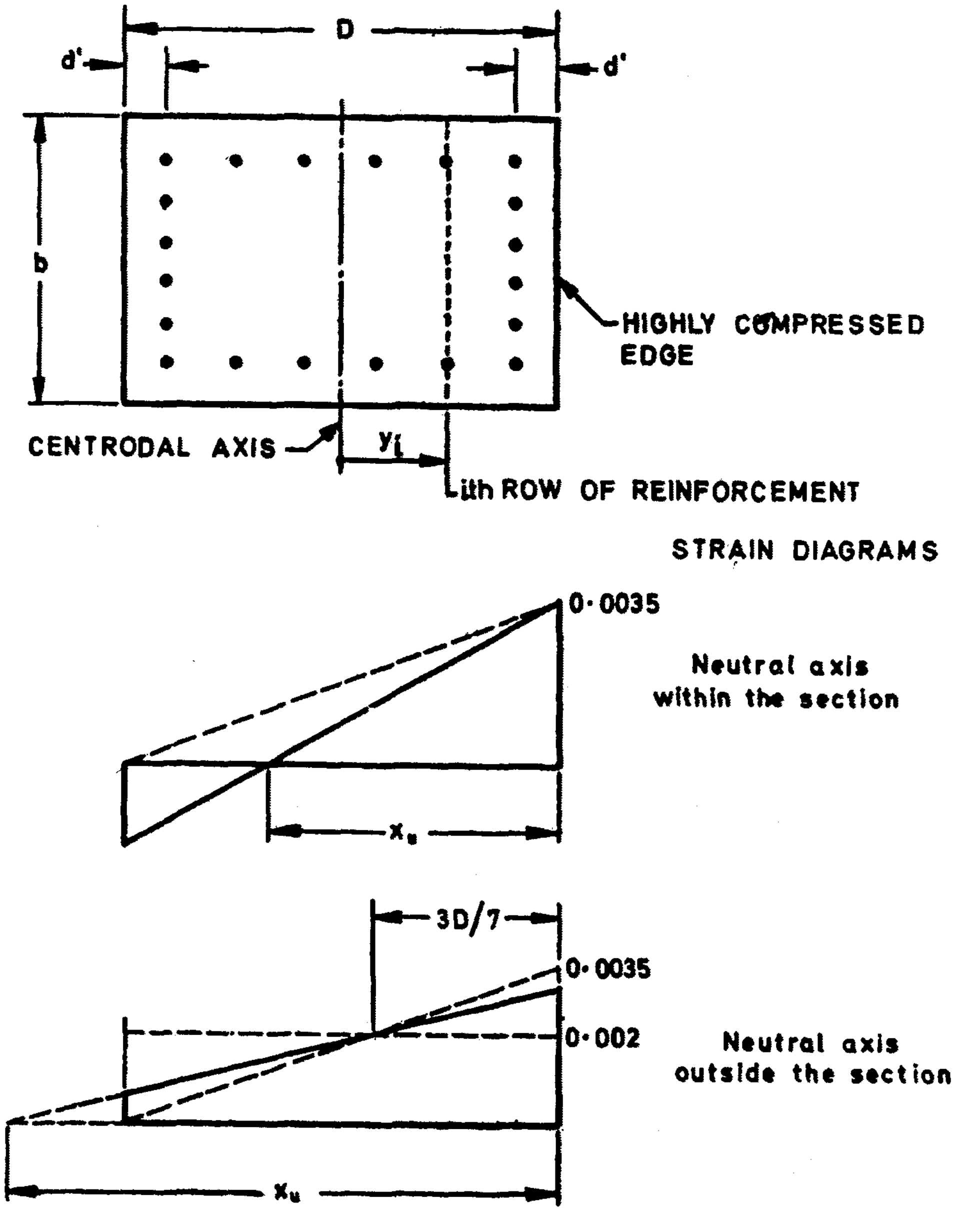

FIG. 7 COMBINED AXIAL LOAD AND UNIAXIAL BENDING

3.2.2 Stress Block Parameters When the Neutral Axis Lies Outside the Section — When the neutral axis lies outside the section, the shape of the stress block will be as indicated in Fig. 8. The stress is uniformly $0{\cdot}446\,f_{ck}$ for a distance of $\frac{3D}{7}$ from the highly compressed edge because the strain is more than 0·002 and thereafter the stress diagram is parabolic.

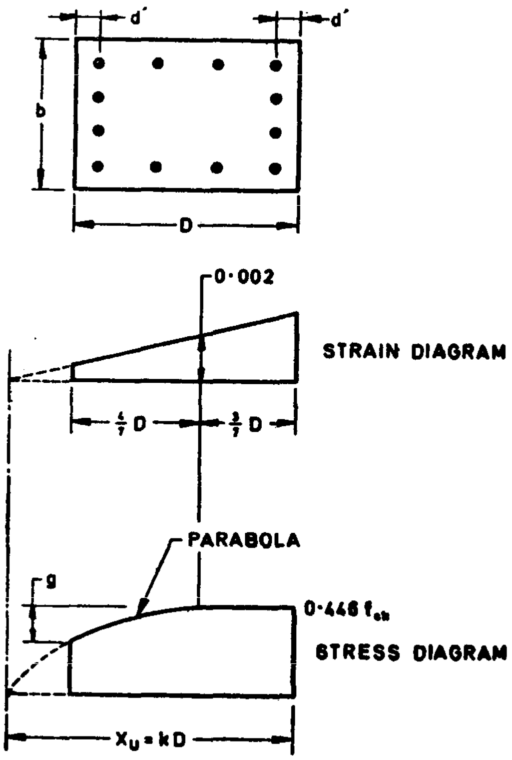

FIG. 8 STRESS BLOCK WHEN THE NEUTRAL AXIS LIES OUTSIDE THE SECTION

Let $x_u = kD$ and let g be the difference between the stress at the highly compressed edge and the stress at the least compressed edge. Considering the geometric properties of a parabola,

$$g = 0{\cdot}446\,f_{ck}\left[\frac{\frac{4}{7}D}{kD-\frac{3}{7}D}\right]^2$$

$$= 0{\cdot}446\,f_{ck}\left(\frac{4}{7k-3}\right)^2$$

Area of stress block

$$= 0{\cdot}446\,f_{ck}\,D - \frac{g}{3}\left(\frac{4}{7}D\right)$$

$$= 0{\cdot}446\,f_{ck}\,D - \frac{4}{21}gD$$

$$= 0{\cdot}446\,f_{ck}\,D\left[1-\frac{4}{21}\left(\frac{4}{7k-3}\right)^2\right]$$

The centroid of the stress block will be found by taking moments about the highly compressed edge.

Moment about the highly compressed edge

$$= 0{\cdot}446\,f_{ck}\,D\left(\frac{D}{2}\right) - \frac{4}{21}gD\left[\frac{3}{7}D + \frac{3}{4}\left(\frac{4}{7}D\right)\right]$$

$$= 0{\cdot}446\,f_{ck}\,\frac{D^2}{2} - \frac{8}{49}gD^2$$

The position of the centroid is obtained by dividing the moment by the area. For different values of k, the area of stress block and the position of its centroid are given in Table *H*.

TABLE H STRESS BLOCK PARAMETERS WHEN THE NEUTRAL AXIS LIES OUTSIDE THE SECTION

(*Clause* 3.2.2)

$k = \frac{x_u}{D}$	AREA OF STRESS BLOCK	DISTANCE OF CENTROID FROM HIGHLY COMPRESSED EDGE
(1)	(2)	(3)
1·00	$0{\cdot}361\,f_{ck}\,D$	$0{\cdot}416\,D$
1·05	$0{\cdot}374\,f_{ck}\,D$	$0{\cdot}432\,D$
1·10	$0{\cdot}384\,f_{ck}\,D$	$0{\cdot}443\,D$
1·20	$0{\cdot}399\,f_{ck}\,D$	$0{\cdot}458\,D$
1·30	$0{\cdot}409\,f_{ck}\,D$	$0{\cdot}468\,D$
1·40	$0{\cdot}417\,f_{ck}\,D$	$0{\cdot}475\,D$
1·50	$0{\cdot}422\,f_{ck}\,D$	$0{\cdot}480\,D$
2·00	$0{\cdot}435\,f_{ck}\,D$	$0{\cdot}491\,D$
2·50	$0{\cdot}440\,f_{ck}\,D$	$0.495\,D$
3·00	$0{\cdot}442\,f_{ck}\,D$	$0{\cdot}497\,D$
4·00	$0{\cdot}444\,f_{ck}\,D$	$0{\cdot}499\,D$

NOTE — Values of stress block parameters have been tabulated for values of k up to 4·00 for information only. For construction of interaction diagrams it is generally adequate to consider values of k up to about 1·2.

3.2.3 Construction of Interaction Diagram — Design charts for combined axial compression and bending are given in the form of interaction diagrams in which curves for P_u/bDf_{ck} versus $M_u/bD^2\,f_{ck}$ are plotted for different values of p/f_{ck}, where p is the reinforcement percentage.

3.2.3.1 For the case of purely axial compression, the points plotted on the y-axis of the charts are obtained as follows:

$$P_u = 0{\cdot}446\, f_{ck} bd + \frac{pbD}{100}\,(f_{sc} - 0{\cdot}446\, f_{ck})$$

$$\frac{P_u}{f_{ck} bD} = 0{\cdot}446 + \frac{p}{100 f_{ck}}\,(f_{sc} - 0{\cdot}446\, f_{ck})$$

where

f_{sc} is the compressive stress in steel corresponding to a strain of 0·002.

The second term within parenthesis represents the deduction for the concrete replaced by the reinforcement bars. This term is usually neglected for convenience. However, as a better approximation, a constant value corresponding to concrete grade M20 has been used in the present work, so that the error is negligibly small over the range of concrete mixes normally used. An accurate consideration of this term will necessitate the preparation of separate Charts for each grade of concrete, which is not considered worthwhile.

3.2.3.2 When bending moments are also acting in addition to axial load, the points for plotting the Charts are obtained by assuming different positions of neutral axis. For each position of neutral axis, the strain distribution across the section and the stress block parameters are determined as explained earlier. The stresses in the reinforcement are also calculated from the known strains. Thereafter the resultant axial force and the moment about the centroid of the section are calculated as follows:

a) *When the neutral axis lies outside the section*

$$P_u = C_1 f_{ck} bD + \sum_{i=1}^{n} \frac{p_i bD}{100}\,(f_{si} - f_{ci})$$

where

C_1 = coefficient for the area of stress block to be taken from Table H (*see 3.2.2*);

p_i = $\frac{A_{si}}{bD}$ where A_{si} is the area of reinforcement in the ith row;

f_{si} = stress in the ith row of reinforcement, compression being positive and tension being negative;

f_{ci} = stress in concrete at the level of the ith row of reinforcement; and

n = number of rows of reinforcement.

The above expression can be written as

$$\frac{P_u}{f_{ck}\, bD} = C_1 + \sum_{i=1}^{n} \frac{p_i}{100 f_{ck}}\,(f_{si} - f_{ci})$$

Taking moment of the forces about the centroid of the section,

$$M_u = C_1 f_{ck}\, bD \left(\frac{D}{2} - C_2 D\,.\right) + \sum_{i=1}^{n} \frac{p_i\, bD}{100}\,(f_{si} - f_{ci}) y_i$$

where

C_2D is the distance of the centroid of the concrete stress block, measured from the highly compressed edge; and

y_i is the distance from the centroid of the section to the ith row of reinforcement; positive towards the highly compressed edge and negative towards the least compressed edge.

Dividing both sides of the equation by $f_{ck}\, bD^2$,

$$\frac{M_u}{f_{ck} bD^2} = C_1\,(0{\cdot}5 - C_2) + \sum_{i=1}^{n} \frac{p_i}{f_{ck}\, 100}\,(f_{si} - f_{ci}) \left(\frac{y_i}{D}\right)$$

b) *When the neutral axis lies within the section*

In this case, the stress block parameters are simpler and they can be directly incorporated into the expressions which are otherwise same as for the earlier case. Thus we get the following expressions:

$$\frac{P_u}{f_{ck}\, bD} = 0{\cdot}36\, k + \sum_{i=1}^{n} \frac{p_i}{100 f_{ck}}\,(f_{si} - f_{ci})$$

$$\frac{M_u}{f_{ck} bD^2} = 0{\cdot}36\, k\,(0{\cdot}5 - 0{\cdot}416\, k) + \sum_{i=1}^{n} \frac{p_i}{f_{ck}\, 100}\,(f_{si} - f_{ci}) \left(\frac{y_i}{D}\right)$$

where

$$k = \frac{\text{Depth of neutral axis}}{D}$$

An approximation is made for the value of f_{ci} for M20, as in the case of *3.2.3.1*. For circular sections the procedure is same as above, except that the stress block parameters given earlier are not applicable; hence the section is divided into strips and summation is done for determining the forces and moments due to the stresses in concrete.

3.2.3.3 Charts for compression with bending — Charts for rectangular sections have been given for reinforcement on two sides (*Charts 27* to *38*) and for reinforcement on four sides (*Charts 39* to *50*). The Charts for the latter case have been prepared for a section with 20 bars equally distributed on all sides, but they can be used without significant error for any other number of bars (greater than 8) provided the bars are distributed equally on the four sides. The Charts for circular section (*Charts 51* to *62*) have been prepared for a section with 8 bars, but they can generally be used for sections with any number of bars but not less than 6. Charts have been given for three grades of steel and four values of d'/D for each case mentioned above.

The dotted lines in these charts indicate the stress in the bars nearest to the tension face of the member. The line for $f_{st} = 0$ indicates that the neutral axis lies along the outermost row of reinforcement. For points lying above this line on the Chart, all the bars in the section will be in compression. The line for $f_{st} = f_{yd}$ indicates that the outermost tension reinforcement reaches the design yield strength. For points below this line, the outermost tension reinforcement undergoes inelastic deformation while successive inner rows may reach a stress of f_{yd}. It should be noted that all these stress values are at the failure condition corresponding to the limit state of collapse and not at working loads.

3.2.3.4 Charts for tension with bending — These Charts are extensions of the Charts for compression with bending. Points for plotting these Charts are obtained by assuming low values of k in the expressions given earlier. For the case of purely axial tension,

$$P_u = \frac{pbD}{100} \ (0{\cdot}87 f_y)$$

$$\frac{P_u}{f_{ck}\, bD} = \frac{p}{100 f_{ck}} \ (0{\cdot}87 f_y)$$

Charts 66 to *75* are given for rectangular sections with reinforcement on two sides and *Charts 76* to *85* are for reinforcement on four sides. It should be noted that these charts are meant for strength calculations only; they do not take into account crack control which may be important for tension members.

Example 6 Square Column with Uniaxial Bending

Determine the reinforcement to be provided in a square column subjected to uniaxial bending, with the following data:

Size of column	45 × 45 cm
Concrete mix	M 25
Characteristic strength of reinforcement	415 N/mm²
Factored load (characteristic load multiplied by γ_f)	2 500 kN
Factored moment	200 kN.m
Arrangement of reinforcement:	(a) On two sides (b) On four sides

(Assume moment due to minimum eccentricity to be less than the actual moment).

Assuming 25 mm bars with 40 mm cover,
$d' = 40 + 12{\cdot}5 = 52{\cdot}5$ mm $= 5{\cdot}25$ cm
$d'/D = 5{\cdot}25/45 = 0{\cdot}12$
Charts for $d'/D = 0{\cdot}15$ will be used

$$\frac{P_u}{f_{ck}\, bD} = \frac{2\,500 \times 10^3}{25 \times 45 \times 45 \times 10^2} = 0{\cdot}494$$

$$\frac{M_u}{f_{ck}\, bD^2} = \frac{200 \times 10^6}{25 \times 45 \times 45 \times 45 \times 10^3} = 0{\cdot}088$$

a) Reinforcement on two sides,
Referring to *Chart 33*,
$p/f_{ck} = 0{\cdot}09$
Percentage of reinforcement,
$p = 0{\cdot}09 \times 25 = 2{\cdot}25$
$A_s = p\, bD/100 = 2{\cdot}25 \times 45 \times 45/100$
$= 45{\cdot}56$ cm²

b) Reinforcement on four sides
from *Chart 45*,
$p/f_{ck} = 0{\cdot}10$
$p = 0{\cdot}10 \times 25 = 2{\cdot}5$
$A_s = 2{\cdot}5 \times 45 \times 45/100 = 50{\cdot}63$ cm²

Example 7 Circular Column with Uniaxial Bending

Determine the reinforcement to be provided in a circular column with the following data:

Diameter of column	50 cm
Grade of concrete	M 20
Characteristic strength of reinforcement	250 N/mm² for bars up to 20 mmϕ 240 N/mm² for bars over 20 mm ϕ

Factored load 1 600 kN
Factored moment 125 kN.m
Lateral reinforcement:

(a) Hoop reinforcement
(b) Helical reinforcement

(Assume moment due to minimum eccentricity to be less than the actual moment).

Assuming 25 mm bars with 40 mm cover,

$d' = 40 \times 12{\cdot}5 = 52{\cdot}5 \text{ mm} = 5{\cdot}25 \text{ cm}$

$d'/D = 5{\cdot}25/50 = 0{\cdot}105$

Charts for $d'/D = 0{\cdot}10$ will be used.

(a) *Column with hoop reinforcement*

$$\frac{P_u}{f_{ck}\, D^2} = \frac{1\,600 \times 10^3}{20 \times 50 \times 50 \times 10^2} = 0{\cdot}32$$

$$\frac{M_u}{f_{ck}\, D^3} = \frac{125 \times 10^6}{20 \times 50 \times 50 \times 50 \times 10^3} = 0{\cdot}05$$

Referring to *Chart 52*, for $f_y = 250$ N/mm²

$p/f_{ck} = 0{\cdot}87$

$p = 0{\cdot}87 \times 20 = 1{\cdot}74$

$A_s = p\pi D^2/400$

$= 1{\cdot}74 \times \pi \times 50 \times 50/400 = 34{\cdot}16 \text{ cm}^2$

For $f_y = 240$ N/mm²,

$A_s = 34{\cdot}16 \times 250/240 = 35{\cdot}58 \text{ cm}^2$

(b) *Column with Helical Reinforcement*

According to *38.4* of the Code, the strength of a compression member with helical reinforcement is 1·05 times the strength of a similar member with lateral ties. Therefore, the given load and moment should be divided by 1·05 before referring to the chart.

Hence,

$$\frac{P_u}{f_{ck}\, D^2} = \frac{0{\cdot}32}{1{\cdot}05} = 0{\cdot}305$$

$$\frac{M_u}{f_{ck}\, D^3} = \frac{0{\cdot}05}{1{\cdot}05} = 0{\cdot}048$$

From *Chart 52*, for $f_y = 250$ N/mm²,

$p/f_{ck} = 0{\cdot}078$

$p = 0{\cdot}078 \times 20 = 1{\cdot}56$

$A_s = 1{\cdot}56 \times \pi \times 50 \times 50/400$

$= 30{\cdot}63 \text{ cm}^2$

For $f_y = 240$ N/mm², $A_s = 30{\cdot}63 \times 250/240$

$= 31{\cdot}91 \text{ cm}^2$

According to *38.4.1* of the Code the ratio of the volume of helical reinforcement to the volume of the core shall not be less than $0{\cdot}36\,(A_g/A_c - 1)\, f_{ck}/f_y$ where A_g is the gross area of the section and A_c is the area of the core measured to the outside diameter of the helix. Assuming 8 mm dia bars for the helix,

Core diameter $= 50 - 2\,(4{\cdot}0 - 0{\cdot}8)$

$= 43{\cdot}6$ cm

$A_g/A_c = 50^2/43{\cdot}6^2 = 1{\cdot}315$

$0{\cdot}36\,(A_g/A_c - 1)\, f_{ck}/f_y$

$= 0{\cdot}36 \times 0{\cdot}315 \times 20/250$

$= 0{\cdot}0091$

$$\frac{\text{Volume of helical reinforcement}}{\text{Volume of core}} = \frac{A_{sh}\pi\,.(42{\cdot}8)}{\frac{\pi}{4}(43{\cdot}6^2)\, s_h} = \frac{0{\cdot}09\, A_{sh}}{s_h}$$

where, A_{sh} is the area of the bar forming the helix and s_h is the pitch of the helix. In order to satisfy the codal requirement,

$0{\cdot}09\, A_{sh}/s_h > 0{\cdot}0091$

For 8 mm dia bar, $A_{sh} = 0{\cdot}503 \text{ cm}^2$

$$s_h \leqslant \frac{0{\cdot}09 \times 0{\cdot}503}{0{\cdot}0091}$$

$\leqslant 4{\cdot}97$ cm

3.3 COMPRESSION MEMBERS SUBJECT TO BIAXIAL BENDING

Exact design of members subject to axial load and biaxial bending is extremely laborious. Therefore, the Code permits the design of such members by the following equation:

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \leqslant 1{\cdot}0$$

where

M_{ux}, M_{uy} are the moments about x and y axes respectively due to design loads,

M_{ux1}, M_{uy1} are the maximum uniaxial moment capacities with an axial load P_u, bending about x and y axes respectively, and

α_n is an exponent whose value depends on P_u/P_{uz} (*see* table below) where $P_{uz} = 0{\cdot}45\, f_{ck}\, A_c + 0{\cdot}75\, f_y\, A_s$:

P_u/P_{uz}	α_n
$\leqslant 0{\cdot}2$	1·0
$\geqslant 0{\cdot}8$	2·0

For intermediate values, linear interpolation may be done. *Chart 63* can be used for evaluating P_{uz}.

For different values of P_u/P_{uz}, the appropriate value of α_n has been taken and curves for the equation

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} = 1{\cdot}0$$ have been plotted in *Chart 64*.

Example 8 Rectangular Column with Biaxial Bending

Determine the reinforcement to be provided in a short column subjected to biaxial bending, with the following data:

Size of column	40 × 60 cm
Concrete mix	M 15
Characteristic strength of reinforcement	415 N/mm²
Factored load, P_u	1 600 kN
Factored moment acting parallel to the larger dimension, M_{ux}	120 kN
Factored moment acting parallel to the shorter dimension, M_{uy}	90 kN

Moments due to minimum eccentricity are less than the values given above.

Reinforcement is distributed equally on four sides.

As a first trial assume the reinforcement percentage, $p = 1.2$

$p/f_{ck} = 1.2/15 = 0.08$

Uniaxial moment capacity of the section about xx-axis:

$$d'/D = \frac{5.25}{60} = 0.087\,5$$

Chart for $d'/D = 0.1$ will be used.

$$P_u/f_{ck}\,bD = \frac{1\,600 \times 10^3}{15 \times 40 \times 60 \times 10^2} = 0.444$$

Referring to *Chart 44*,

$M_u/f_{ck}\,bD^2 = 0.09$

$\therefore\ M_{ux1} = 0.09 \times 15 \times 40 \times 60^2 \times 10^3/10^6$
$= 194.4$ kN.m

Uniaxial moment capacity of the section about yy-axis:

$$d'/D = \frac{5.25}{40} = 0.131$$

Chart for $d'/D = 0.15$ will be used.

Referring to *Chart 45*,

$M_u/f_{ck}\,bD^2 = 0.083$

$\therefore\ M_{uy1} = 0.083 \times 15 \times 60 \times 40^2 \times 10^3/10^6$
$= 119.52$ kN.m

Calculation of P_{uz}:

Referring to *Chart 63* corresponding to $p = 1.2, f_y = 415$ and $f_{ck} = 15$,

$$\frac{P_{uz}}{A_g} = 10.3 \text{ N/mm}^2$$

$\therefore\ P_{uz} = 10.3\,A_g = 10.3 \times 40 \times 60 \times 10^2/10^3$ kN
$= 2\,472$ kN

$$\frac{P_u}{P_{uz}} = \frac{1\,600}{2\,472} = 0.647$$

$$\frac{M_{ux}}{M_{ux1}} = \frac{120}{194.4} = 0.617$$

$$\frac{M_{uy}}{M_{uy1}} = \frac{90}{119.52} = 0.753$$

Referring to *Chart 64*, the permissible value of $\frac{M_{ux}}{M_{ux1}}$ corresponding to the above values of $\frac{M_{uy}}{M_{uy1}}$ and $\frac{P_u}{P_{uz}}$ is equal to 0.58.

The actual value of 0.617 is only slightly higher than the value read from the Chart. This can be made up by slight increase in reinforcement.

$$A_s = \frac{1.2 \times 40 \times 60}{100} = 28.8 \text{ cm}^2$$

12 bars of 18 mm will give $A_s = 30.53$ cm²

Reinforcement percentage provided,

$$p = \frac{30.53 \times 100}{60 \times 40} = 1.27$$

With this percentage, the section may be rechecked as follows:

$p/f_{ck} = 1.27/15 = 0.084\,7$

Referring to *Chart 44*,

$$\frac{M_u}{f_{ck}\,bD^2} = 0.095$$

$\therefore\ M_{ux1} = 0.095 \times 15 \times 40 \times 60^2 \times 10^3/10^6$
$= 205.2$ kN.m

Referring to *Chart 45*

$$\frac{M_u}{f_{ck}\,bD^2} = 0.085$$

$\therefore\ M_{uy1} = 0.085 \times 15 \times 60 \times 40^2 \times 10^3/10^6$
$= 122.4$ kN.m

Referring to *Chart 63*,

$$\frac{P_{uz}}{A_g} = 10.4 \text{ N/mm}^2$$

$\therefore\ P_{uz} = 10.4 \times 60 \times 40 \times 10^2/10^3$
$= 2\,496$ kN

$$P_u/P_{uz} = \frac{1\,600}{2\,496} = 0.641$$

$$M_{ux}/M_{ux1} = \frac{120}{205.2} = 0.585$$

$$M_{uy}/M_{uy1} = \frac{90}{122.4} = 0.735$$

Referring to *Chart 64*,

Corresponding to the above values of $\frac{M_{uy}}{M_{uy1}}$ and $\frac{P_u}{P_{uz}}$, the permissible value of $\frac{M_{ux}}{M_{ux1}}$ is 0.6.

Hence the section is O.K.

3.4 SLENDER COMPRESSION MEMBERS

When the slenderness ratio $\frac{l_{ex}}{D}$ or $\frac{l_{ey}}{b}$ of a compression member exceeds 12, it is considered to be a slender compression member (*see 24.1.2* of the Code); l_{ex} and l_{ey} being the effective lengths with respect to the major axis and minor axis respectively. When a compression member is slender with respect to the major axis, an additional moment M_{ax} given by the following equation (modified as indicated later) should be taken into account in the design (*see 38.7.1* of the Code):

$$M_{ax} = \frac{P_u D}{2\,000}\left(\frac{l_{ex}}{D}\right)^2$$

Similarly, if the column is slender about the minor axis an additional moment M_{ay} should be considered.

$$M_{ay} = \frac{P_u b}{2\,000}\left(\frac{l_{ey}}{b}\right)^2$$

The expressions for the additional moments can be written in the form of eccentricities of load, as follows:

$$M_{ax} = P_u\, e_{ax}$$

where

$$e_{ax} = \frac{D}{2\,000}\left(\frac{l_{ex}}{D}\right)^2$$

$$\frac{e_{ax}}{D} = \frac{1}{2\,000}\left(\frac{l_{ex}}{D}\right)^2$$

Table I gives the values $\frac{e_{ax}}{D}$ or $\frac{e_{ay}}{b}$ for different values of slenderness ratio.

TABLE I ADDITIONAL ECCENTRICITY FOR SLENDER COMPRESSION MEMBERS

(*Clause* 3.4)

l_{ex}/D or l_{ey}/b	e_{ax}/D or e_{ay}/b	l_{ex}/D or l_{ey}/b	e_{ax}/D or e_{ay}/b
(1)	(2)	(3)	(4)
12	0·072	25	0·313
13	0·085	30	0·450
14	0·098	35	0·613
15	0·113	40	0·800
16	0·128	45	1·013
17	0·145	50	1·250
18	0·162	55	1·513
19	0·181	60	1·800
20	0·200		

In accordance with *38.7.1.1* of the Code, the additional moments may be reduced by the multiplying factor k given below:

$$k = \frac{P_{uz} - P_u}{P_{uz} - P_b} \leqslant 1$$

where

$P_{uz} = 0{\cdot}45\, f_{ck}\, A_c + 0{\cdot}75\, f_y\, A_s$, which may be obtained from *Chart 63*, and P_b is the axial load corresponding to the condition of maximum compressive strain of 0·003 5 in concrete and tensile strain of 0·002 in outermost layer of tension steel.

Though this modification is optional according to the Code, it should always be taken advantage of, since the value of k could be substantially less than unity.

The value of P_b will depend on arrangement of reinforcement and the cover ratio d'/D, in addition to the grades of concrete and steel. The values of the coefficients required for evaluating P_b for various cases are given in *Table 60*. The values given in *Table 60* are based on the same assumptions as for members with axial load and uniaxial bending.

The expression for k can be written as follows:

$$k = \frac{1 - P_u/P_{uz}}{1 - P_b/P_{uz}} \leqslant 1$$

Chart 65 can be used for finding the ratio of k after calculating the ratios P_u/P_{uz} and P_b/P_{uz}.

Example 9 Slender Column (with biaxial bending)

Determine the reinforcement required for a column which is restrained against sway, with the following data:

Size of column	40 × 30 cm
Concrete grade	M 30
Characteristic strength of reinforcement	415 N/mm²
Effective length for bending parallel to larger dimension, l_{ex}	6·0 m
Effective length for bending parallel to shorter dimension, l_{ey}	5·0 m
Unsupported length	7·0 m
Factored load	1 500 kN
Factored moment in the direction of larger dimension	40 kN.m at top and 22·5 kN.m at bottom

Factored moment in the direction of shorter dimension	30 kN.m at top and 20 kN.m at bottom

The column is bent in double curvature. Reinforcement will be distributed equally on four sides.

$$\frac{l_{ex}}{D} = \frac{6\cdot0 \times 100}{40} = 15\cdot0 > 12$$

$$\frac{l_{ey}}{b} = \frac{5\cdot0 \times 100}{30} = 16\cdot7 > 12$$

Therefore the column is slender about both the axes.

From Table I,

For $\frac{l_{ex}}{D} = 15, e_x/D = 0\cdot113$

For $\frac{l_{ey}}{b} = 16\cdot7, e_y/b = 0\cdot140$

Additional moments:

$$M_{ax} = P_u e_x = 1\,500 \times 0\cdot113 \times \frac{40}{100} = 67\cdot8 \text{ kN.m}$$

$$M_{ay} = P_u e_y = 1\,500 \times 0\cdot14 \times \frac{30}{100} = 63\cdot0 \text{ kN.m}$$

The above moments will have to be reduced in accordance with *38.7.1.1* of the Code; but multiplication factors can be evaluated only if the reinforcement is known.

For first trial, assume $p = 3\cdot0$ (with reinforcement equally on all the four sides).

$A_g = 40 \times 30 = 1\,200 \text{ cm}^2$

From *Chart 63*, $P_{uz}/A_g = 22\cdot5 \text{ N/mm}^2$

$\therefore\ P_{uz} = 22\cdot5 \times 1\,200 \times 10^2/10^3 = 2\,700 \text{ kN}$

Calculation of P_b:

Assuming 25 mm dia bars with 40 mm cover

d'/D (about xx-axis) $= \frac{5\cdot25}{40} = 0\cdot13$

Chart or Table for $d'/d = 0\cdot15$ will be used.

d'/D (about yy-axis) $= \frac{5\cdot25}{30} = 0\cdot17$

Chart or Table for $d'/d = 0\cdot20$ will be used.

From *Table 60*,

$$P_b \text{ (about } xx\text{-axis)} = \left(k_1 + k_2 \frac{p}{f_{ck}}\right) f_{ck} bD$$

$$P_{bx} = \left(0\cdot196 + 0\cdot203 \times \frac{3}{30}\right) \times 30 \times 30 \times 40 \times 10^2/10^3 = 779 \text{ kN}$$

$$P_b \text{ (about } yy\text{-axis)} = \left(0\cdot184 + \frac{0\cdot028 \times 3}{30}\right) \times 40 \times 30 \times 30 \times 10^2/10^3$$

$P_{by} = 672$ kN

$$\therefore\ k_x = \frac{P_{uz} - P_u}{P_{uz} - P_{bx}} = \frac{2\,700 - 1\,500}{2\,700 - 779} = 0\cdot625$$

$$k_y = \frac{P_{uz} - P_u}{P_{uz} - P_{by}} = \frac{2\,700 - 1\,500}{2\,700 - 672} = 0\cdot592$$

The additional moments calculated earlier, will now be multiplied by the above values of k.

$M_{ax} = 67\cdot8 \times 0\cdot625 = 42\cdot4$ kN.m
$M_{ay} = 63\cdot0 \times 0\cdot592 = 37\cdot3$ kN.m

The additional moments due to slenderness effects should be added to the initial moments after modifying the initial moments as follows (*see* Note 1 under *38.7.1* of the Code):

$M_{ux} = (0\cdot6 \times 40 - 0\cdot4 \times 22\cdot5) = 15\cdot0$ kN.m
$M_{uy} = (0\cdot6 \times 30 - 0\cdot4 \times 20) = 10\cdot0$ kN.m

The above actual moments should be compared with those calculated from minimum eccentricity consideration (*see 24.4* of the Code) and greater value is to be taken as the initial moment for adding the additional moments.

$$e_x = \frac{l}{500} + \frac{D}{30} = \frac{700}{500} + \frac{40}{30} = 2\cdot73 \text{ cm}$$

$$e_y = \frac{l}{500} + \frac{b}{30} = \frac{700}{500} + \frac{30}{30} = 2\cdot4 \text{ cm}$$

Both e_x and e_y are greater than 2·0 cm.

Moments due to minimum eccentricity:

$$M_{ux} = 1\,500 \times \frac{2\cdot73}{100} = 41\cdot0 \text{ kN.m} > 15\cdot0 \text{ kN.m}$$

$$M_{uy} = 1\,500 \times \frac{2\cdot4}{100} = 36\cdot0 \text{ kN.m} > 10\cdot0 \text{ kN.m}$$

$\therefore$ Total moments for which the column is to be designed are:

$M_{ux} = 41\cdot0 + 42\cdot4 = 83\cdot4$ kN.m
$M_{uy} = 36\cdot0 + 37\cdot3 = 73\cdot3$ kN.m

The section is to be checked for biaxial bending.

$$P_u/f_{ck}\,bD = \frac{1\,500 \times 10^3}{30 \times 30 \times 40 \times 10^2} = 0\cdot417$$

$$p/f_{ck} = \frac{3 \cdot 0}{30} = 0 \cdot 10$$

Referring to *Chart 45* ($d'/D = 0 \cdot 15$),

$M_u/f_{ck}\, bD^2 = 0 \cdot 104$

$\therefore \quad M_{ux1} = 0 \cdot 104 \times 30 \times 30 \times 40 \times 40 \times 10^3/10^6$

$= 149 \cdot 8$ kN.m

Referring to *Chart 46* ($d'/D = 0 \cdot 20$),

$M_u/f_{ck}\, bD^2 = 0 \cdot 096$

$\therefore \quad M_{uy1} = 0 \cdot 096 \times 30 \times 40 \times 30 \times 30 \times 10^3/10^6$

$= 103 \cdot 7$ kN.m

$$\frac{M_{ux}}{M_{ux1}} = \frac{83 \cdot 4}{149 \cdot 8} = 0 \cdot 56$$

$$\frac{M_{uy}}{M_{uy1}} = \frac{73 \cdot 3}{103 \cdot 7} = 0 \cdot 71$$

$$P_u/P_{uz} = \frac{1\,500}{2\,700} = 0 \cdot 56$$

Referring to *Chart 64*, the maximum allowable value of M_{ux}/M_{ux1} corresponding to the above values of M_{uy}/M_{uy1} and P_u/P_{uz} is 0·58 which is slightly higher than the actual value of 0·56. The assumed reinforcement of 3·0 percent is therefore satisfactory.

$A_s = pbD/100 = 3 \cdot 0 \times 30 \times 40/100$

$= 36 \cdot 0 \text{ cm}^2$