# Recent Advances in Empirical Analysis of Financial Markets: Industrial Organization Meets Finance \*

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#### Abstract

In this article I provide a selective review of recent empirical work in the intersection of finance and industrial organization. I describe an estimation method, which can be applied quite widely in financial and other markets where a researcher needs to recover agents' beliefs. Using four applications I illustrate how combining this method with data from auctions and with a theoretical model can be used to answer various economic questions of interest. I start with the primary market for sovereign debt, focusing on treasury bill auctions of the US and Canada. I show how auction data together with standard tools from Industrial Organization can be used to shed light on issues involving market structure, market power and front-running. I continue by looking at the Main Refinancing Operations of the European Central Bank, the main channel of monetary policy implementation in the EURO-zone, and illustrate how auction data can be used to learn about typically opaque over-the-counter lending markets. I also discuss how to use these data indirectly to learn about dynamics of banks' financial health and of their balance sheets. I then turn to the discussion of recent progress on estimation of systemic risk. I finish with thoughts on how to estimate a whole demand system for financial assets.

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# 1 Introduction

The last three decades marked a remarkable success in the development of tools in structural economic modeling, predominantly within the field of industrial organization. These techniques and tools can be applied more widely and the recent developments in health economics and to some extent in finance suggest that indeed there are many promising directions in which the literature can go. In this article I will talk about applying various tools from Industrial Organization to financial markets. The markets I will focus on here are (i) market for sovereign debt, and (ii) market for liquidity (short-term loans). I will argue that these and similar markets provide us with abundance of data and studying them can allow us to gain insights into many other areas of the economy. I will point out some open questions as I move through the applications.

The analysis of market power and market structure has been in spotlight of research in industrial organization (IO) since its early days. In finance, the market microstructure literature originates with Kyle (1985) and Kyle (1989). There are many papers that try to estimate the price impact of individual market participants based on Kyle's model. Using a clever research design Shleifer (1986) shows that indeed demand curve for US equities seem to be downward sloping and thus there is potential for price-impact when large orders are executed. I will discuss in this paper how to try to assign this market power to individual market participants and how to quantify it.

In particular, since most countries, including the US and Canada, organize their primary market for debt around a few primary dealers, it is important to get a better understanding about the rents these primary dealers are accruing. Typically, these primary markets are organized as auctions. Since the number of primary dealers is typically fairly low, one may worry that their individual market power may cause prices (or yields) that clear the auction to be too low (too high). With the internet making it very easy to enter and participate in various remote markets, it may seem tempting to open up the auctions to new participants. Using data from treasury bill auctions I show how to evaluate market structure, how to quantify rents that primary dealers may be collecting from observing customers' order flow, or how to evaluate the impact of entry of direct bidders in the US Treasury auctions. Since new entry might dissipate the rents the primary dealers enjoy, quantifying the effect of entry on primary market performance is a first order question.

The second main theme discussed in this paper involves indirect analysis of over-the-counter markets. Some basic opportunity cost arguments together with standard IO tools may allow us to peak into otherwise opaque over-the-counter markets, for which there typically is very little data. To illustrate these ideas I will describe recent work analyzing auctions of the European Central Bank, in which the ECB offers short-term loans to European banks. The basic idea underlying this approach is that while typically bidders participate in an auction with some exogenously given "value" for the object being sold, in the markets studied here this value will be determined by the opportunity cost of this transaction. Typically, this may involve transacting in another financial market, which may have a very different organizational form. As a leading example, consider the

above mentioned auctions of short-term loans. A bidder may obtain a loan from the auctioneer (a central bank) or he may approach a counterparty and negotiate terms privately. In equilibrium, the bidder should be indifferent between these two options and hence if one were able to observe the value of the bidder for a loan in the auction, one would equivalently be able to say what loan terms this bidder would have been able to obtain on the private market. Applying the IO tools allows us to estimate this value (or cost of alternatives financing) based on the observed bid and hence we will be able to gain insight into what terms a particular bank might have faced in the private market. Moreover, since these auctions are run at fairly high frequency (typically weekly), the dynamics of these funding costs might be an important source of information about issues like bank health. I will also show how this dynamics may be used to study systemic risk.

While introducing the applications, I describe an estimation method, which can be used quite widely in financial and other markets. The basic idea is that an economic model provides a link between the primitives of interest and observables which is based on some equilibrium relationship. Since this equilibrium relationship typically depends on agents' beliefs about rivals' strategies, the described method involves simulating these beliefs using a resampling strategy.

Finally, in the last section I briefly lay out two ways how to one can go about estimating a full demand system for financial assets, which are viewed as differentiated products. The first is based on a product-space demand model building on Deaton and Muellbauer (1980). The second, which may substantially reduce the number of parameters and also allows for evaluating welfare from introduction of new goods, is based on the characteristics space demand model developed in Berry, Levinsohn and Pakes (1995).

One caveat to have in mind while reading this paper is that with the recent antitrust case involving LIBOR fixing and ongoing investigation of bankers due to manipulation of other key reference interest rates (EURIBOR, TIBOR), we should be aware of possible collusion which would typically alter the key relationship between the observable data and objects of interest. Various models of collusion available in the IO literature might be used to address this issue, but I will leave the discussion of collusion for future surveys.<sup>1</sup>

# 2 Using Auction Data

To begin, in this section I review the workhorse model, variants of which will be utilized in the applications. Readers not interested in technical details of the theoretical model and estimation can skip this section. However, the description of the estimation algorithm and the discussion of the general ideas underlying it may be useful in many other applications. The data examined in the applications described later as well as data in many other settings are generated from strategic behavior of agents participating in various environments that can be described as games. Here we will talk mostly about auctions, but in many other environments the games that the agents

<sup>&</sup>lt;sup>1</sup>See Snider and Youle (2014) for an example discussing the LIBOR fixing.

play might be different: they might be choosing prices or quantities, designing products, choosing whether and which markets to market etc. The important takeaway from the following discussion is that typically the variables or parameters of interest will be related to observables via some equilibrium relationship, which involves players' beliefs about rivals' strategies (and possibly beliefs about the distribution of some aggregate uncertainty) at the time the game is played. One of the goals of this section is to describe an estimation method that will allow for recovering these beliefs from the observed data. The second goal is to develop a general theoretical model through which the observed data will be linked to unobservables of interest.

The common feature of auctions in financial markets examined below is that they involve a one-time sale of large quantities of homogeneous good: Treasury Bills, Bonds, Notes, or short-term loans. It is convenient and without much loss to abstract from the discreteness of the quantities demanded as the quantities requested are typically quite large. Instead, it is useful to express bidders' demands in terms of the share of the total supply. Treating the quantity domain as a connected set allows for convenient and elegant analysis of equilibrium bidding behavior, which also clearly reveals the intuitive relationship between an oligopoly pricing problem and a uniform price auction.

Virtually for any auction mechanism, we can express the equilibrium bid as willingness-to-pay minus a shading factor:

$$BID = WTP - SHADING. (1)$$

The "SHADING" term is thus the confounding factor that makes the link between the observed data and the parameters of interest hard - it arises due to strategic considerations players face in the game. Therefore, to back out the WTP from observed bids, one needs to use the knowledge of the rules of the auction mechanism and the equilibrium assumption to express the "SHADING" terms as a function of objects that are estimable given the available data. The following subsection formalizes this in the context of an auction model. More generally, a theoretical model of how agents behave typically allows the researcher to express the unobservables as some function (or correspondence) of the observables. Based on properties of this function (e.g., whether it is one-to-one) we can then argue whether the objects of interest can be (uniquely or partially) identified from the data. Obtaining estimates of the WTP is important also in order to evaluate whether the auction mechanism performs well: how much surplus does it fail to extract and how inefficient is the allocation. The answers to these questions allow us to bound how much revenue is lost. Ausubel, Cramton, Pycia, Rostek and Weretka (2014) show that it is impossible to rank the different auction mechanisms based on either efficiency or revenue and hence each particular application needs to be empirically evaluated on a case-by-case basis.

#### 2.1 Theoretical Model

I begin with the basic share auction model that is based on Wilson (1979). Since in most real-world auctions strategies that bidders may employ are restricted to take form of step functions, Wilson's original model should be modified to incorporate these restrictions as in Kastl (2012) before taking it to the data. I start with the basic symmetric model with private information and private values and will introduce its variants and various generalizations when discussing the applications later. Unlike Kyle (1989), Vives (2011), Rostek and Weretka (2012) or Ausubel et al. (2014) who obtain closed from solutions for equilibrium demands after imposing appropriate assumptions on utilities and uncertainty distributions, we will work here under less restrictive assumptions. Let us assume there are N potential bidders at the cost of obtaining only implicit equilibrium characterization via a set of necessary conditions. Each bidder is endowed with (possibly multidimensional) private signal,  $S_i$ , which affects the underlying value of the auctioned goods.

**Assumption 1** Bidders signals,  $S_0, S_1, ..., S_N$ , are drawn from a common support  $[0, 1]^M$  according to an atomless joint d.f.  $F(S_0, S_1, ..., S_N)$  with density f.

For the characterization results below some conditions on the smoothness of the distribution of residual supplies is needed. This can come either from properties of  $F(\cdot)$  or one can assume uncertain continuously distributed supply as is common in electricity auctions (and which will also be required for consistency of one of the proposed estimators). I will for simplicity assume that there is exogenous uncertainty about the supply, Q.

**Assumption 2** Supply Q is a random variable distributed on  $[\underline{Q}, \overline{Q}]$  with strictly positive density conditional on  $S_i$   $\forall i$ .

Obtaining a share q of the supply Q is valued according to a marginal valuation function  $v_i(q, S_i)$ . We will impose following conditions on this function.

**Assumption 3**  $v_i(q, S_i)$  is non-negative, measurable and bounded, strictly increasing in each component of  $S_i \forall q$ , and weakly decreasing and continuous in  $q \forall S_i$ .

Note that under Assumption 3, values are assumed to be private since  $v_i(\cdot)$  does not depend on private information of the rivals. Since in many settings (and particularly in financial markets) we may worry that values are interdependent and marginal value of bidder i might depend on signal of bidder j, it is important to provide more discussion of this assumption. I will provide this discussion while discussing one of the applications in Section 3.4.1 below.

Furthermore, to be consistent with the rules governing the actual auction markets, I will restrict the strategy set available to each bidder to step functions with at most K steps. The index k will henceforth correspond to the ordering of bidpoints that is increasing in the quantity dimension and decreasing in price. A bidpoint  $(q_k, b_k)$  together with the preceding bidpoint  $(q_{k-1}, b_{k-1})$  thus specify the marginal quantity,  $q_k - q_{k-1}$ , that a bidder is bidding  $b_k$  for. I will also allow a bid l, which is sure to lose, and hence basically corresponds to not participating in the auction.

**Assumption 4** Each player i = 1, ..., N has an action set:

$$A_{i} = \left\{ \begin{array}{c} \left(\vec{b}, \vec{q}, K_{i}\right) : \dim\left(\vec{b}\right) = \dim\left(\vec{q}\right) = K_{i} \in \{1, ..., K\}, \\ b_{ik} \in B = \{l\} \cup \left[0, \bar{b}\right], \ q_{ik} \in \left[0, 1\right], \ b_{ik} > b_{ik+1}, q_{ik} < q_{ik+1} \end{array} \right\}.$$

Finally, I will assume that rationing is pro-rata on the margin. This means that if demand at the market clearing price were to exceed the supply, all bidders' bids submitted exactly at that price would be adjusted proportionally.

I will use the term K-step equilibrium to refer to a Bayesian Nash Equilibrium of the auction game where the bidders are restricted to use at most K steps. In other words, a K-step equilibrium is a profile of strategies, such that each strategy is a step function with at most K steps and it maximizes the expected payoff for (almost) every type  $s_i$  for all i.

# 2.1.1 Discriminatory Auction

I begin with laying out the equilibrium of a discriminatory auction: an auction in which every bidder has to pay his bid for all units he/she wins. Let  $V_i(q, S_i)$  denote the gross utility:  $V_i(q, S_i) = \int_0^q v_i(u, S_i) du$ . The expected utility of a bidder i of type  $s_i$  who is employing a strategy  $y_i(\cdot|s_i)$  in a discriminatory auction given that other bidders are using  $\{y_j(\cdot|\cdot)\}_{j\neq i}$  can be written as:

$$EU(s_{i}) = \sum_{k=1}^{K_{i}} \left[ \Pr(b_{ik} > P^{c} > b_{ik+1}|s_{i}) V(q_{ik}, s_{i}) - \Pr(b_{ik} > P^{c}|s_{i}) b_{ik} (q_{ik} - q_{ik-1}) \right]$$

$$+ \sum_{k=1}^{K_{i}} \Pr(b_{ik} = P^{c}|s_{i}) E_{Q,s_{-i}|s_{i}} \left[ V(Q_{i}^{c}(Q, \mathbf{S}, \mathbf{y}(\cdot|S)), s_{i}) - b_{ik} (Q_{i}^{c}(Q, \mathbf{S}, \mathbf{y}(\cdot|S)) - q_{ik-1}) |b_{ik} = P^{c}]$$

$$(2)$$

where I let  $q_{i0} = b_{iK_i+1} = 0$ . The random variable  $Q_i^c(Q, \mathbf{S}, \mathbf{y}(\cdot|S))$  is the (market clearing) quantity bidder i obtains if the state (bidders' private information and the supply quantity) is  $(Q, \mathbf{S})$  and bidders submit bids specified in the vector  $\mathbf{y}(\cdot|S) = [y_1(\cdot|S_1), ..., y_N(\cdot|S_N)]$ .  $P^c$  is the (random) market clearing price.

A Bayesian Nash Equilibrium in this setting is a collection of functions such that (almost) every type  $s_i$  of bidder i is choosing his bid function so as to maximize his expected utility:  $y_i(\cdot|s_i) \in \arg\max EU_i(s_i)$  for a.e.  $s_i$  and all bidders i. The system of necessary conditions implicitly characterizing such a BNE is the link between the observables and unobservables that we seek to establish. Using a local perturbation argument, for a discriminatory auction Kastl (2012) obtains the following necessary conditions that the quantity requested in any step of a pure strategy that is a part of a K-step equilibrium has to satisfy.

**Proposition 1** Under assumptions 1-4 in any K-step Equilibrium of a discriminatory auction, for almost all  $s_i$ , every step  $k < K_i$  in the equilibrium bid function  $y_i(\cdot|s_i)$  has to satisfy

$$\Pr(b_{ik} > P^c > b_{ik+1}|s_i) \left[ v\left(q_{ik}, s_i\right) - b_{ik} \right] = \Pr(b_{ik+1} \ge P^c|s_i) \left(b_{ik} - b_{ik+1}\right) \tag{3}$$

and at the last step  $K_{i}$  it has to satisfy  $v\left(\overline{q},s_{i}\right)=b_{iK_{i}}$  where  $\overline{q}=\sup_{q,s_{-i}}Q_{i}^{c}\left(q,s_{-i},s_{i},\mathbf{y}\left(\cdot|S\right)\right)$ .

Note that this condition is simply a multi-unit counterpart of the equilibrium condition for bidding in a first-price auction: g(b)(v-b) = G(b), where G(b) is the CDF (and g(b) is the PDF) of the distribution of the first order statistic of rival bids (i.e., of the highest of rival bids) (see Guerre, Perrigne and Vuong (2000)). The trade-off in the multiunit environment remains virtually the same. The bidder is simply trading off the expected surplus on the marginal (infinitessimal) unit versus the probability of winning it.

Condition (3) provides us with a mapping from observables (bids) to the object of interest: the willingness-to-pay  $v(q, s_i)$  in the spirit of the general equation (1). The estimation approach is similar in spirit to Laffont and Vuong (1996) and Guerre et al. (2000) and has been proposed for the multiunit auction environment first by Hortaçsu (2002). Note that (3) is simply a necessary condition for an optimal choice of  $q_k$ . The set of these conditions (one for each k) thus identifies K points of the function  $v(q, s_i)$ . The optimality condition with respect to the bid  $b_k$  can be derived in a straightforward manner by differentiating (2), but it cannot be simplified and interpreted as naturally as equation (3). Nevertheless, it may allow a researcher to obtain tighter bounds on the function  $v(q, s_i)$ . McAdams (2008) discusses identification of the primitives using other restrictions in more detail.

The question of equilibrium existence in the environment where bidders bid using step functions has been addressed in Kastl (2012). Proposition 2 in that paper establishes existence of an equilibrium in distributional strategies for a discriminatory auction with private values and supply uncertainty whenever the potential dependence between private signals is not "too large." Formally, it is required that the probability measure associated with the distribution function  $F(S_1, ..., S_N)$  is absolutely continuous with respect to  $\prod_{i=1}^{N} \mu_i$  where  $\mu_i$  is the marginal of  $\mu$  on  $S_i$ . For the case of symmetrically informed bidders, but uncertain supply, Pycia and Woodward (2015) show that there is a unique pure-strategy Baysian-Nash equilibrium.

# 2.1.2 Uniform Price Auction

Turning to the uniform price auction, the expected utility of a bidder i who is employing a strategy  $y_i(\cdot|s_i)$  given that other bidders are using  $\{y_j(\cdot|\cdot)\}_{j\neq i}$  can be written as:

$$EU_{i}(s_{i}) = E_{Q,S_{-i}|S_{i}=s_{i}}u(s_{i}, S_{-i})$$

$$= E_{Q,S_{-i}|S_{i}=s_{i}}\left[\int_{0}^{Q_{i}^{c}(Q,\mathbf{S},\mathbf{y}(\cdot|S))} v_{i}(u, s_{i}, S_{-i}) du - P^{c}(Q, \mathbf{S}, \mathbf{y}(\cdot|S)) Q_{i}^{c}(Q, \mathbf{S}, \mathbf{y}(\cdot|S))\right]$$

where as before  $Q_i^c(Q, \mathbf{S}, \mathbf{y}(\cdot|S))$  is the (market clearing) quantity bidder i obtains if the state (bidders' private information and the supply quantity) is  $(Q, \mathbf{S})$  and bidders bid according to strategies specified in the vector  $\mathbf{y}(\cdot|S) = [y_1(\cdot|S_1), ..., y_N(\cdot|S_N)]$ , and similarly  $P^c(Q, \mathbf{S}, \mathbf{y}(\cdot|S))$  is the market clearing price associated with state  $(Q, \mathbf{S})$ .

Once again, the system of necessary conditions implicitly characterizing a BNE in this environment is the link between the observables and unobservables that we seek to establish. Kastl (2011) provides the following characterization result linking the bids and the underlying marginal values.

**Proposition 2** Under assumptions 1-4 in any K-step equilibrium of a Uniform Price Auction, for almost all  $s_i$ , every step k in the equilibrium bid function  $y_i(\cdot|s_i)$  has to satisfy:

(i) If 
$$v(q_{ik}, s_i) > b_{ik}$$

$$\Pr(b_{ik} > P^c > b_{ik+1}|s_i) \left[ v(q_{ik}, s_i) - E(P^c|b_{ik} > P^c > b_{ik+1}, s_i) \right] = q_{ik} \frac{\partial E(P^c I[b_{ik} \ge P^c \ge b_{ik+1}]|s_i)}{\partial q_{ik}}$$
(4)

(ii) If 
$$v(q_{ik}, s_i) \leq b_{ik}$$

$$\Pr(b_{ik} > P^{c} > b_{ik+1}|s_{i}) \left[ v\left(q_{ik}, s_{i}\right) - E\left(P^{c}|b_{ik} > P^{c} > b_{ik+1}, s_{i}\right) \right] + \left(5\right)$$

$$+ \Pr\left(\left(b_{ik} = P^{c} \lor b_{ik+1} = P^{c}|s_{i}\right) \land Tie\right) E\left(\left(v\left(Q^{c}, s_{i}\right) - P^{c}\right) \frac{\partial Q^{c}}{\partial q_{ik}} \middle| \left(b_{ik} = P^{c} \lor b_{ik+1} = P^{c}\right) \land Tie, s_{i}\right)$$

$$= q_{ik} \frac{\partial E\left(P^{c}I\left[b_{ik} \ge P^{c} \ge b_{ik+1}\right] \middle| s_{i}\right)}{\partial q_{ik}}.$$

A technical detail that arises in the model with discrete bidding is that when bidding above her marginal valuation for some units, we can no longer be sure that a tie does not occur with positive probability in equilibrium. The cost of shading the demand captured on the LHS of (5) now also includes the effect on the surplus at the expected quantity after rationing in the event of a tie.

Equations (3), (4) and (5) are the counterparts of equation (1) for the specific auction models we will be discussing below. I will now move on to describing how to estimate the "SHADING" term in each case.

Equilibrium of a uniform price auction where bidders use step functions is more subtle as ties may occur in equilibrium with positive probability. Existence (in fact existence of a pure strategy equilibrium in non-decreasing strategies) obtains for the limiting case of no restriction on the number of steps as results from McAdams (2006) can be applied. With the restriction on the number of steps, one can only provide existence results for an  $\varepsilon$ -equilibrium. See Section 3.1.1 in Kastl (2012) for further discussion. Vives (2011) characterizes equilibria without any a priori restriction on strategies for a particular parametrization of the private information, but allowing for a rich structure of dependence between values of individual bidders.

#### 2.2 Estimation

The estimation approach described in what follows can be generalized to many other settings than just auctions. The main point is that the relationship between observables (bids in case of an auction, submitted student preferences in case of school matching mechanisms, buy-sell orders in case of data from equity markets etc) and objects of interest, i.e., preferences, valuations or willingness-to-pay, is given by an equilibrium relationship of an explicit economic model, in which each participant's behavior depends on his/her beliefs about rivals' behavior. Typically the researcher assumes a Nash Equilibrium (or its appropriate form such as a Bayesian Nash Equilibrium or a refinement such as a Perfect Bayesian Equilibrium when selection is necessary) which then restricts participants beliefs about rivals' play to be consistent with the actual strategies. In many environments or games, the equilibrium strategies depend on the rivals' actions only indirectly, the actual uncertainty is about some functional of these actions. For example, in the applications discussed here, the uncertainty is about where the market will clear, i.e., about the market clearing price. In case of school choice, the uncertainty is about whether a given school might have an open spot for the applicant. In case of equity markets, the uncertainty might be about whether a counterparty for a particular trade can be found. One way to account for this uncertainty in estimation non-parametrically is to assume that all participants agree on its distribution and hence any differences in their observed strategies are due to differences in their private information. In that case, one can account for the uncertainty about the object of interest and thus estimate its probability distribution by "resampling," i.e., by following a bootstrap-like procedure where by drawing repeated samples of strategies (by sampling with replacement from the observed data) one simulates different possible states of the world and thus eventually obtains an estimate of the distribution of the random variable(s) of interest. This idea appeared originally in Hortagsu (2002) and was later applied in Hortagsu and McAdams (2010) and Kastl (2011). Such an estimator is a particular form of a V-statistic, which has useful implications for characterizing its asymptotic behavior. It should be intuitive to see that as one constructs more and more samples (as the data set gets larger and thus the observed strategies span more and more of the type space), one obtains more and more simulated states of the world, where the probability of a type profile being in some subset of the type space  $[0,1]^M \times [Q,\overline{Q}]$  corresponds to the probability of that subset implied by the population distribution functions given in Assumptions 1 and 2. I will now describe this approach in the context of auctions in detail.

# 2.2.1 Typical Data Set

A typical auction data set  $\{B_{1t}, ..., B_{Nt}, Q_t, X_t\}_{t=1}^T$ , consists of T auctions and for each t, the set of bids  $B_{1t}, ..., B_{Nt}$ , the realized supply,  $Q_t$ , and some auction covariates,  $X_t$ , where N is the number of (potential) bidders. In the auctions we will talk about here,  $B_{it}$  is typically a function  $b_{it}(q)$  which maps quantity requested (e.g., the face value of treasury bills the bidder is bidding

for) into the bid, typically specified in terms of yield. In most applications,  $b_{it}(\cdot)$  takes form of the step function,  $\{(q_{itk}, b_{itk})\}_{k=1}^{K_{it}}$  where  $K_{it}$  number of steps bidder i uses in auction t. Supply may be deterministic and announced by the auctioneer prior to the auction (as is the case with US Treasury auctions) or may be stochastic since the auctioneer may adjust the preannounced quantity later (as is the case with the ECB auctions of short-term loans). In some applications, bidders might be of different types, which may either be observable or latent. For example, we might have primary dealers and other bidders and we may have a good reason to believe that the underlying distribution of their willingness-to-pay may differ across these groups. We may therefore have C bidder classes, and thus have  $N_{ct}$  potential bidders of class c in auction t.

#### 2.2.2 Estimating the Distribution of the Market Clearing Price

Examining equations (3) and (4), it is clear that to establish a link between the observed bids and the object of interest,  $v(\cdot)$ , one needs to estimate the distribution of  $P^c$ . With such an estimate in hand, one can utilize the usual plug-in approach of Guerre et al. (2000) and obtain an estimate of  $v(\cdot)$ . The distribution of  $P^c$  from perspective of a bidder with signal  $s_i$  is defined as

$$H(X) \equiv \Pr\left(X \ge P^{c}|s_{i}\right) = E_{\left\{Q, S_{j \neq i}\right\}} I\left(Q - \sum_{j \neq i} y\left(X|S_{j}\right) \ge y\left(X|s_{i}\right)\right)$$

$$\tag{6}$$

where  $I\left(\cdot\right)$  is the indicator function and  $E_{\left\{\cdot\right\}}$  is an expectation over the random supply and other bidders' private information. Essentially, this equation states that the probability that the market clearing price will be lower than X is equal to the probability that the set of types and the realization of the supply are such that the aggregate demand of those types falls short of the supply. Hortaçsu and Kastl (2012) show that one can estimate  $H\left(\cdot\right)$  by first defining an indicator of excess supply at price X (given bid functions  $\left\{y_{j}\left(X|s_{j}\right)\right\}_{j\neq i}$  and i's own bid  $y_{i}\left(X|s_{i}\right)$ ) as follows:

$$\Phi\left(\left\{y_{j}\left(X|s_{j}\right)\right\}_{j\neq i};X\right)=I\left(Q-\sum_{j\neq i}y_{j}\left(X|s_{j}\right)\geq y_{i}\left(X|s_{i}\right)\right)$$

An estimator of H(X) is based on a V-statistic, which is based on all subsamples (with replacement) of size (N-1) from the full sample of NT datapoints:

$$\xi\left(\hat{F};X,h_{T}\right) = \frac{1}{\left(NT\right)^{(N-1)}} \sum_{\alpha_{1}=(1,1)}^{(T,N)} \dots \sum_{\alpha_{N-1}=(1,1)}^{(T,N)} \Phi\left(y_{\alpha_{1}},\dots,y_{\alpha_{N-1}},X\right) \tag{7}$$

where  $\alpha_i \in \{(1,1),(1,2),\dots(1,N),\dots,(T,N)\}$  is the index of the bid in the subsample and  $\hat{F}$  is the empirical distribution of bids, i.e., the empirical probability distribution over points in 2K-dimensional space.

It is straightforward to see that an estimator defined as a simulator of  $\xi\left(\hat{F};X,h_T\right)$  by drawing only M subsamples rather than all  $(NT)^{(N-1)}$  is consistent as  $T\to\infty$  (and under appropriate conditions on the rate at which the number of simulations, M, increases - see Pakes and Pollard (1989) for more details). Cassola, Hortaçsu and Kastl (2013) establish that it is also consistent as  $N\to\infty$  provided some further technical conditions are satisfied. This latter result is particularly important when studying financial markets and auctions in general (but also in other applications), since it allows to use data from one auction at a time and thereby keep constant many unobservable characteristics. When the economic environment is changing and it is hard to capture these changes through covariates so that pooling data across auctions might be dubious, proceeding with estimation within an auction might be the preferred way to go. Hortaçsu and Kastl (2012) further show how to modify this estimator in order to allow for asymmetries and the presence of covariates by introducing weighting into the resampling procedure which is similar to a nonparametric regression.

Applying the estimation approach described above, we can thus obtain an estimate of the "Shading" factor in (1) that determines the wedge between the submitted bid and the actual willingness-to-pay. With an estimate of bidder's actual willingness-to-pay, which is of course ultimately equivalent to having an estimate of that bidder's demand curve, we can start evaluating that bidder's market power, which leads naturally to our first application.

# 3 Market Power and Market Structure in Treasury Securities

# 3.1 US Bond, Notes and Treasury Bill Auctions

To begin our discussion of applications of the previously described theory and estimation procedure, I start by focusing on one of the most important financial markets in the world: the market for US Debt Instruments (treasury bills, notes and bonds). In 2013 alone, the US Treasury auctioned \$7.9 trillion of debt. In 2007, the net interest amounted to \$238 billion and represented 8.7% of Government expenditures. The average daily trading volume exceed \$565 billion that same year (the global equity trading was under \$420 billion daily at that time). In mid 2008, there was \$4.6 trillion in outstanding marketable debt, which was approximately a quarter of the US credit markets. Based on its size, it is clear that this is one of the most important financial markets in the world.

#### 3.1.1 Market Rules

The US Treasury issues new debt via an auction. Historically, a discriminatory (or multiple price) auction format was used (for instruments of all maturities) until a switch to a uniform price (or single price) auction format occurred in the 90s following an experiment the Treasury conducted for few maturities. Under this format, the auctioneer announces the size of the issuance, bidders submit

(sealed) bids, currently via an electronic interface, and the market clearing price is determined as the highest price (or lowest yield) at which there is no excess supply, ie at which demand equals supply. There are two types of bids that bidders may submit: (i) noncompetitive (up to a limit of \$5 million), whereby a bidder guarantees himself an allocation of the requested amount at whatever the market clearing price will be and (ii) competitive, in which both the price and quantity can be freely specified. The market for new issuance of debt has been historically organized around a fairly small number of "primary dealers." In addition to primary dealers, there are also other participants in the treasury auctions: the direct and indirect bidders. Direct bidders, as the name suggests, participate on their own behalf (own trading accounts). As we will see below, these become more and more important in the treasury auctions since they purchase larger share over time. The indirect bidders are customers placing bids through a primary dealer. It is important to point out that some of these customers are very large players in the treasury markets themselves: various asset managers or large pension funds that simply choose not to participate directly. This list of customers may also include Foreign monetary authorities placing their bids through the New York Fed. One question we will address here is the extent to which observing such customer order flow contributes to PDs' profits.

# 3.1.2 Primary Dealer System

The PD system started in 1960 with 18 PDs, grew to 46 PDs by 1988 and declined again to the current 22 PDs. To be a primary dealer (as currently governed by the Primary Dealers Act of 1988), a bank or securities broker/dealer must commit to active participation in security auctions (i.e., to submit "reasonable bids" for a fraction of at least  $\frac{1}{N^{PD}}$  of the announced supply, where  $N^{PD}$  is the number of primary dealers. Primary dealers are also required to make bids when the Fed conducts open market operations and to report on their activity to the Fed's open market trading desk. There are, however, also many important benefits. For example, during the implementation of the Quantitative Easing (QE) program, only PDs were allowed to participate. Also, and perhaps more importantly, as mentioned above primary dealers act as intermediaries at the primary market: they submit bids on behalf of their clients who choose not to participate directly.

# 3.2 Market Power and Uniform Price Auction

When talking about market power, typically we have in mind a situation where a profit maximizing firm chooses price (or output) such that it exceeds (falls short of) the competitive price (quantity), i.e., p(q) > C'(q). In IO many researchers such as Bresnahan (1989), Porter (1983), Ellison (1994) or Genesove and Mullin (1998) estimated the extent of market power by introducing a parameter,  $\theta$  that spanned the price between the competitive equilibrium and the monopoly,  $p = mc + \theta p'(q)q$ . Typically, these studies would proceed by estimating the demand elasticity using appropriate instruments (supply shifters) from the observed (p,q) pairs and then estimating jointly the marginal

cost and the market power parameter  $\theta$ .<sup>2</sup>

Let us recall for the moment the equation for optimal pricing by a monopolist that faces an uncertain demand (satisfying appropriate regularity conditions):

$$\mathbb{E}p + \mathbb{E}p'(q)q = c'(q) \tag{8}$$

Rewriting this equation as

$$\mathbb{E}p = c'(q) - \theta \mathbb{E}p'(q)q \tag{9}$$

where (loosely speaking)  $\theta \in [0, 1]$  is a parameter indexing market power. For  $\theta = 0$  we have perfect competition and for  $\theta = 1$  we have a monopoly. Since the early days of the New Empirical Industrial Organization, parameter  $\theta$  was one of the central objects of interest. Assuming some behavioral model (such as Nash Equilibrium in prices), and given some useful variation in the data, this parameter can be estimated in a fairly straightforward manner.

Let us rewrite (4) as

$$\underbrace{E\left(P^{c}|b_{k} > P^{c} > b_{k+1}\right)}_{\text{BID BY A PRICETAKER}} = \underbrace{v(q_{k})}_{\text{WTP}} - \underbrace{\frac{q_{k}}{\Pr\left(b_{k} > P^{c} > b_{k+1}\right)}}_{\text{MARKET POWER (SHADING)}} \underbrace{\partial E\left(P^{c}; b_{k} \geq P^{c} \geq b_{k+1}\right)}_{\text{MARKET POWER (SHADING)}}$$
(10)

Comparing (9) and (10) it seems natural to estimate the extent of market power of individual bidders in a uniform price auction by evaluating the last term (10). The goal here is not to estimate  $\theta$  as in (9, but rather to evaluate the importance of the last term in (10) and study how it varies across bidders and/or auctions. In the context of treasury auctions, in the next subsection we will be interested whether the shading factor of primary dealers differs from that of other market participants.

# 3.3 Market Structure in the US Treasury Auctions

#### 3.3.1 Bidding Data and Market Overview

Hortaçsu, Kastl and Zhang (2015) analyze the detailed bidding data from July 2009 - Oct 2013. The securities in their sample range from 4 week bills to 10 year notes, with 822 auctions of 4-week, 13-week, 26-week, 52-week bills and cash management bills, and 153 auctions of 2-year, 5-year, and 10-year notes. The total volume of issuance through these auctions was 27.3 trillion US dollars, with the average issue size around 28 billion dollars. While anonymized, these data differentiate bidders by three classes: primary dealers, direct bidders and indirect bidders. The measures of market concentration typically employed in anti-trust or IO literature such as HHI or C4 are not alarmingly high in this market (HHI is 561 for bills and 450 for notes and bonds). Nevertheless,

<sup>&</sup>lt;sup>2</sup>See Bresnahan (1989) for a review.

we may worry that primary dealers may be able to significantly leverage their market power in this market - especially since they get to observe the customers' order flow. Primary dealers are, as a class of bidders, the largest purchasers of primary issuances. In terms of tendered quantities, primary dealer tenders comprise 69% to 88% of overall tendered quantities. Direct bidders tender 6% to 13% and Indirect bidders 6% to 18% of the tenders. In terms of actual purchases (or quantities awarded), the PDs obtain about 63% of the auction volume. Nevertheless, the share of direct bidders is rising over the past few years. There are several natural questions to ask with these data. First, we may want to verify that the message that the concentration measures tell (i.e., that the market seems fairly competitive) is indeed supported by the data. Second, we may be interested in digging deeper into the primary dealer system and try to quantify the rents that primary dealers enjoy due to observing their customers' bids before submitting their own.

To look at the first question, Hortaçsu et al. (2015) begin with investigating the bids directly. Table 3 in that paper summarizes how quantity-weighted bids compare to a US Treasury published benchmark yield on the day of the auction. Indirect bidders systematically bid lower yields than the market-level prevailing yield (and substantially so in auctions of 6-month Tbills), the direct bidders bid around the prevailing yields and the primary dealers bid on average above these yields. Of course these numbers per se are hard to interpret, since there might be other effects at play that are not visible when looking just at the quantity-weighted bids. For example, if primary dealers need to absorb much larger amounts of the auctioned instruments (recall that they have to bid for  $\frac{1}{N^{PD}}$  share of the supply), they will need to be compensated for this and hence their bids might reflect this compensation even in absence of any market power or direct price effect considerations.

Table 4 investigates the differentials in bid yields through regressions of the following form:

$$b_{it}^{qw} = \alpha_g + \beta Q_{it} + \xi_t + \varepsilon_{it} \tag{11}$$

where  $b_{it}^{qw}$  is the quantity-weighted average yield bid submitted by bidder i in auction t,  $\alpha_g$  is the intercept corresponding to group g (i.e., primary dealers, direct bidders and indirect bidders),  $Q_{it}$  is the total (or maximal) quantity demanded by i in auction t expressed as a share of the total issuance,  $\xi_t$  are auction fixed effects, and  $\varepsilon$  is the usual regression error term. The table illustrates the results separately for Treasury Bills (short-term maturities) and Treasury Notes (long-term), as it is possible that the market dynamics are very different across these different classes of securities. Since the regressions also include auction fixed effects, they provide within-auction comparisons that account for differing supply-demand conditions that affect the level of the bids.

The main findings are that indeed Primary Dealers systematically (and statistically significantly) bid higher yields than Direct and Indirect Bidders. Primary Dealer bids are 2 (4) basis points higher than Direct (Indirect) bids in the Bills sector, and 6 (11) basis points higher in the Notes sector. These results hold even when bidder size is controlled for using the bidder's share of the total tender size. There are two main ways through which bidder size may affect the bids:

bidders demanding larger quantity may have higher demand for the security, but they may also have higher market power. The regressions indicate that larger bidders systematically bid higher yields. The effect is quite large – the coefficient estimate indicates that a size increase of 10% of total issue size accounts for 1(6) basis point increase in the bid yield.

While accounting for bidder size appears to lower the differences in bid yields across bidder classes, Primary Dealers still appear to bid higher yields than Direct and Indirects even accounting for their offer share of the total issue.

As noted above, since bids reflect both differences in demand and also differences in market power, it is difficult to interpret these documented differences in bids. Even though larger bidders bid higher yields (i.e., lower prices), this is not prima facie evidence that large bidders exercise market power; it is possible that larger bidders also have lower willingness-to-pay. This is where the model comes in - to allow the researcher to decompose these effects.

# 3.3.2 Modification of the Model of Bidding

We now need to modify the model of bidding introduced earlier in order to separate out the market power and demand components of bid heterogeneity so that we explicitly incorporate that the three groups of bidders (primary dealers, and direct and indirect bidders) may potentially be exante heterogeneous. As mentioned previously, the model, and the measurement it will allow us to conduct, will rely on the assumption of bidder optimization. In essence, what we will end up doing is to measure the elasticity of (expected) residual supply faced by each bidder. This is directly observable in the data, and does not require behavioral assumptions. This elasticity will be our measure of the "potential market power" possessed by each bidder. Assuming that bidders are expected profit maximizers who will exercise their market power in a unilateral, noncooperative fashion, we can then estimate the willingness-to-pay/demand that rationalizes the observed bid. In order to do that I modify Assumption 1 as follows:

**Assumption 1'** Direct and indirect bidders' and dealers' private signals are independent and drawn from a common support  $[0,1]^M$  according to atomless distribution functions  $F^P(.)$ ,  $F^I(.)$ , and  $F^D(.)$  with strictly positive densities.

Similarly, Assumption 3 is replaced by one allowing for heterogeneous valuation function:  $v^g(q, S_i^g)$  for  $g \in \{P, I, D\}$  for "Primary," "Indirect," and "Direct." Note that, as mentioned before, Assumption 3 (and of course even its current modification) implies that learning other bidders' signals does not affect one's own valuation – i.e. we have a setting with *private*, not interdependent values. This assumption may be more palatable for certain securities (such as shorter term securities, which are essentially cash substitutes) than others, but is the most tractable one under which we can pursue the "demand heterogeneity" vs. "market power" decomposition. In Section 3.4.1 I will describe a formal test of this assumption for the case of the Canadian treasury

bill market, and in that context private values cannot be rejected. Note that under this assumption, the additional information that a primary dealer j possesses due to observing her customers' orders,  $Z_j^P$ , simply consists of those submitted orders. As will become clear below, this extra piece of information allows the primary dealer to update her beliefs about the competitiveness of the auction, or, somewhat more precisely, the distribution of the market clearing price. I will come back to this issue when discussing the application quantifying the value of the order flow.

Let us now define the probability distribution of the market clearing price from the perspective of a direct bidder j, who is preparing to make a bid  $y^D(p|s_j)$ . The probability distribution of the market clearing price from the perspective of direct bidder j will be:

$$\Pr\left(p \geq P^{c}|s_{j}\right) = E_{\left\{S_{k \in \mathcal{D} \cup \mathcal{P} \cup \mathcal{I} \setminus j}, Z_{l \in \mathcal{P}}\right\}} I\left(Q - \sum_{m \in \mathcal{P}} y^{P}\left(p|S_{m}, Z_{m}\right) - \sum_{l \in \mathcal{I}} y^{I}\left(p|S_{j}\right) - \sum_{k \in \mathcal{D} \setminus j} y^{D}\left(p|S_{k}\right) \geq y^{D}\left(p|S_{j}\right)\right)$$

$$(12)$$

where  $E_{\{\cdot\}}$  is an expectation over all other bidders' (including indirect bidders, primary dealers, and other direct bidders) private information, and  $I(\cdot)$  is the indicator function.

This appears to be a complicated expression, but it essentially says that the probability that the market clearing price  $P^c$  will be below a given price level p is the same as the probability that residual supply of the security at price p will be higher than the quantity demanded by bidder j at that price. In the expression inside the indicator is the residual supply function faced by bidder j. This residual supply function is uncertain from the perspective of the bidder, but its distribution is pinned down by the assumption that the bidder knows the distribution of its competitors' private information and the equilibrium strategies they employ. This assumption is also what makes the estimation technique based on resampling the observed strategies to simulate the perceived uncertainty work.

For a primary dealer, the distribution of the market clearing price is slightly different, since the dealer will condition on whatever information is observed in the indirect bidders' bids. In a (conditionally) independent private values environment, this information does not affect the primary dealer's own valuation, or her inference about other bidders' valuations. The distribution of the market clearing price from the perspective of primary dealer j, who observes the bids submitted by indirect bidders m in an index set  $\mathcal{M}_{\parallel}$ , is given by:

$$\Pr\left(p \geq P^{c}|s_{j}, z_{j}\right) = E_{\left\{S_{k \in \mathcal{I} \setminus \mathcal{M}_{|}}, S_{l \in \mathcal{D}}, S_{n \in \mathcal{P} \setminus j}|z_{j}\right\}} I\left(RS\left(Q, p\right) \geq y^{P}\left(p|s_{j}, z_{j}\right) + \sum_{m \in \mathcal{M}_{|}} y^{I}\left(p|s_{m}\right)\right)$$

$$(13)$$

where  $RS\left(p,Q,\vec{S},\vec{Z}\right) = Q - \sum_{k \in \mathcal{I} \setminus \mathcal{M}_{|}} y^{I}\left(p|S_{k}\right) - \sum_{l \in \mathcal{D}} y^{D}\left(p|S_{l}\right) - \sum_{n \in \mathcal{P} \setminus j} y^{P}\left(p|S_{n},Z_{n}\right)$ , i.e., the residual supply at price p given supply realization Q and realization of private information  $\left(\vec{S},\vec{Z}\right)$ . Note that the main difference in this equation compared to equation (12) is that the dealer conditions on all observed customers' bids, all bids in index set  $\mathcal{M}_{j}$ . This is exactly where

the dealer "learns about competition" – the primary dealer's expectations about the distribution of the market clearing price are altered once she observes a customer's bid.

Finally, the distribution of  $P^c$  from the perspective of an indirect bidder is very similar to a direct bidder, but with the additional twist that the indirect bidder recognizes that her bid will be observed by a primary dealer, m, and can condition on the information that she provides to this dealer. The distribution of the market clearing price from the perspective of an indirect bidder j, who submits her bid through a primary dealer m is given by:

$$\Pr\left(p \geq P^{c}|s_{j}\right) = E_{\left\{S_{k \in \mathcal{I}\backslash j}, S_{l \in \mathcal{D}}, S_{n \in \mathcal{P}} Z_{n \in \mathcal{P}}|s_{j}\right\}} I\left(Q - \sum_{k \in \mathcal{I}\backslash j} y^{I}\left(p|S_{k}\right) - \sum_{l \in \mathcal{D}} y^{D}\left(p|S_{l}\right) - \sum_{n \in \mathcal{P}} y^{P}\left(p|S_{n}, Z_{n}\right) \geq y^{I}\left(p|s_{j}\right)\right)$$
(14)

where  $y^{I}(p|s_{i}) \in Z_{m}$ .

Note that the probability distributions from perspective of a direct bidder or uninformed primary dealer, or that of an informed primary dealer (given his/her observation of customer order flow) and that of an indirect bidder can then be estimated using equations (12), (13) and (14) using the resampling technique described above. With the estimates of the probability distributions of market clearing price in hand, we can use equation (10) to estimate the willingness-to-pay (or, equivalently, the shading factor) at every observed bid. it is also worth pointing out the connection with the Kyle model (Kyle 1985). In Kyle's model there is an informed trader and uninformed liquidity traders and a market maker. In our current model, market participants can be heterogeneous in terms of their information sets and hence their price impact (or what is typically called in the context of Kyle-style models lambda) is also different.

Given equation (10) a natural definition of shading is as follows.

**Definition 1** The average shading factor is defined as: 
$$S(\theta_i) = \frac{\sum_{k=1}^{K_i} q_k [v_i(q_k, \theta_i) - \mathbb{E}(P^c | b_k > P^c > b_{k+1}, \theta_i)]}{\sum_{i=1}^{K_i} q_i}$$
.

This is a quantity-weighted measure of shading, where shading at step k is defined as the difference between the marginal value,  $v_i(q_k, \theta_i)$  and the expected market clearing price, conditional on  $k^{th}$  step being marginal,  $\mathbb{E}(P^c|b_k > P^c > b_{k+1}, \theta_i)$ . Another way to interpret this shading factor is to note that it corresponds to the weighted sum of the second term on the right-hand side of equation (10), which is essentially the expected inverse elasticity of the residual supply curve faced by the bidder - the object of interest in general studies of market power in IO.

Table 5 in Hortaçsu et al. (2015) reports the results of regressions similar to those for bids. The main results are that Primary Dealers shade their bids 1.9 basis points more than Direct Bidders, and 3.5 basis points higher than Indirect Bidders. Controlling for bidder size, the shading differentials decline slightly, to 1.4 basis point against Direct Bidders and 3 basis points against Indirect Bidders. The authors also find that larger bidders choose to shade their bids more, which is consistent with the usual economic intuition. The coefficient estimate suggests that going from

zero to 10% market share allows a dealer to shade her bids by 0.3 basis points. Lastly, the authors report that there are larger differentials in shading for Notes than for Bills, albeit the qualitative findings remain the same.

Putting the previous findings together, the results suggest that, under the assumption of expected profit maximization, the main reason why Primary Dealers bid higher yields than other bidder groups is not because they have lower valuation for the securities, but because they are able to exercise more market power: the size of their demands and their information about customer order flow allows them to leverage their impact on the market clearing price by "shading" their bids more.

#### 3.3.3 Quantifying Bidder Surplus

A question related to bid-shading that we can answer through the estimates obtained thus far is to quantify how much infra-marginal surplus bidders are getting from participating in these auctions. To compute ex-post surplus, we first obtain point estimates of the "rationalizing" marginal valuation function v(q, s) at the (observed) quantities that the bidders request as described above. We then compute the area under the upper envelope of the inframarginal portion of the marginal valuation function, and subtract the payment made by each bidder. To compute ex-ante (or rather interim) surplus, one can integrate the ex-post surpluses over the distribution of the market clearing prices.

I should provide here abundant caution regarding what "infra-marginal bidder surplus" means. Any counterfactual auction system would also have to allow bidders to retain some surplus. Indeed, in Figure 1, we see very clearly that even if bidders bid perfectly competitively, i.e. reveal their true marginal valuations without any bid shading, they would gain some surplus from the auction, just because they have downward sloping demand curves. Indeed, if there are any costs of participating in the auction, it would have to be justified by the expected surplus. In terms of assessing the cost effectiveness of the issuance mechanism, the most we can say is that the bidder surplus under an efficient allocation reflects a conservative upper bound to the amount of cost-saving that can be induced by a change in issuance mechanism.

The fact that bidders' surpluses are fairly small in these multiunit auctions has been documented fairly widely, in various markets and under both auction formats. Kastl (2011) reports that in the case of Czech treasury auctions, the participants enjoyed surplus on the order of few basis points (in terms of the annualized yield of the treasury bills). This suggests that the inverse elasticity of the (expected) residual demand curve is fairly high and hence the participants cannot excessively leverage their market power, in other words if they were to change their orders quite substantially, the resulting change in price would be small. Since that paper also finds that the allocation was close to efficient, it concludes that the mechanism used by the Czech treasury is working fine. Kang and Puller (2008) look at Korean treasury auctions, which have both a uniform price and discriminatory auction format and they argue that the auctions seem fairly competitive under both

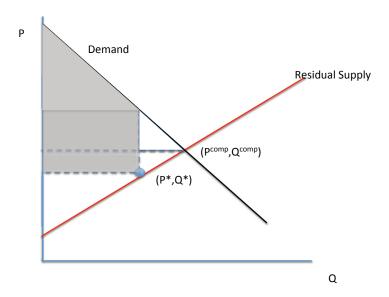


Figure 1: Illustration of Bid Shading when Residual Supply is Known

formats. Using auctions run by the Receiver General, the fiscal agent of the Canadian federal government, Chapman, McAdams and Paarsch (2007) use bounds to argue that surpluses bidders enjoy in the discriminatory auctions of term deposits are tiny. While in all these examples there are rents that accrue to the participants due to some market power they enjoy (ie the residual supplies faced by these participants are not completely flat), it seems that these rents are very small, especially if one were to take into account the typical cost of employing a number of skilled people to run a trading desk.

An interesting question for further research is evaluating the impact of competition on these rents. Elsinger, Schmidt-Dengler and Zulehner (2015) analyze the Austrian treasury auctions and, in particular, investigate the impact of competition on bidders' surpluses. They argue that evaluating the surpluses using a model similar to the one used here leads to them finding a smaller effect of competition on surplus than would be the case using a simple reduce-form measures.

Hortaçsu et al. (2015) report that Direct and Indirect bidder surpluses in the case of US Treasury auctions range from 0.002 to 3.58 basis points across the maturity spectrum, with the shorter end of the maturity spectrum generating very low surpluses in general. Once again, these surpluses may reflect the outside option of not buying these securities in auction and purchasing them in the when-

issued or resale markets. Aggregating the surpluses over the entire set of auctions in their data set (which amounted to about \$27 trillion in issue size), Direct and Indirect Bidders' aggregate surplus is estimated to be about \$1.6 billion, or about 0.6 basis points in terms of the annualized yield. Primary Dealers' infra-marginal surplus, however, appears to be significantly larger. For Primary Dealers, the derived surplus might not necessarily be in line with the differentials with the quoted secondary market prices of these securities. Primary Dealers' demand is typically quite large, and fulfilling such levels of demand is likely to have a price impact in the secondary markets. Moreover, retaining Primary Dealership status has a number of complementary value streams attached to it beyond the profits derived from reselling the new issues. For example, being a Primary Dealer allows firms access to open market operations and, especially in this period, the QE auction mechanism that is exclusive to primary dealerships. Between March 2008 and February 2010, Primary Dealers also had access to a special credit facility from the Fed to help alleviate liquidity constraints during the crisis. Quantifying these costs and benefits to being a primary dealer is an interesting avenue for future research.

Hortacsu et al. (2015) report that Primary Dealers derive most of their infra-marginal surplus from the longer end (2 to 10 year notes) of the maturity spectrum. There may be a number of reasons why demand for this part of the maturity spectrum is more heterogeneous across bidders. One possibility is the presence of different portfolio needs across dealers' clientele. Moreover, there are typically alternative uses for such securities beyond simple buy-and-hold – Duffie (1996) shows that this part of the spectrum can be particularly valuable for its use as collateral in repo transactions. Surpluses derived from the shorter end of the maturity spectrum, which may have fewer alternative uses, are much smaller. Overall, Hortagsu et al. (2015) find that Primary Dealers' derived surplus aggregated to \$6.3 billion during their sample period. Compared against the \$27 trillion in issuance, Primary Dealer surplus makes up for 2.3 basis points of the issuance. Along with the Direct Bidder and Indirect Bidder surpluses, bidder surplus added up to 3 basis points during this period. Once again, I should emphasize that any other issuance mechanism would have to provide bidders with surpluses to ensure participation and to reward them for their private information. Moreover, even if bidders are behaving in a perfectly competitive manner, without displaying any bid-shading, they would enjoy surpluses. However, we can conservatively estimate that revenue gains from further optimizing the issuance mechanism are bounded above by 3 basis points.

Using regression analysis of the estimated surpluses, Hortaçsu et al. (2015) find that Direct and Indirect bidders gain significantly lower surpluses than Primary Dealers, and that Indirect bidder surplus especially is not statistically different from zero. Furthermore, larger bidders gain higher surpluses. Fro example, an increase in market share from 0 to 10% of the issue size is correlated with a rise in surplus of \$830k in the Notes auctions. However, Direct and Indirect Bidders gain lower surpluses than Primary Dealers even when size is controlled for. They also report results of a regression with the number of Indirect Bidders whose bids a Primary Dealer routes in the auction

as another control variable. This variable serves a rough proxy for the order-flow information that the Primary Dealer is privy to. Indeed, the regression reveals a significant correlation between the number of Indirect Bidders who routed their bids through a Primary Dealer, and the surplus (controlling for the bidder's size). An additional Indirect Bidder going through a Primary Dealer would be correlated with about seven thousand dollars more in surplus for Bills and \$200K gain in Primary Dealer surplus for Notes. Since Primary Dealers on average route 2.5 Indirect Bids in Notes auctions, this estimate suggests that we can ascribe about \$500K or about 25% of their surplus in Notes auctions to information contained in Indirect bids. However, it should be noted that there are important caveats to interpreting this as the "value of order flow." It is possible that Primary Dealers who observe more Indirect bids may have systematically higher valuations for the securities, and hence may be getting higher surpluses due to this.

To estimate the value of order flow more convincingly, one would like to have a data set that would reveal a primary dealer's bid both before the customers' orders arrive and his/her updated bid after customers' bids arrive. Since the US Treasury data only contains final bids, it is not well suited to answering this question. In the section we turn to the next application where I will describe the analysis of Canadian Treasury auctions, which will provide us with an opportunity to quantify the value of customers' order flow and also to address the important question of what it is that primary dealers might be learning about when observing customers' bids.

#### 3.4 Value of Customer Order Flow

Hortaçsu and Kastl (2012) analyze data from Canadian treasury auctions, which are similar to the data from US Treasury auctions described in the previous section. The data set contains all individual bids submitted in these auctions. Canadian treasuries are sold via discriminatory auctions, hence the link between the data and the objects of interest, the willingness-to-pay measures, is provided by equation (2). As previously mentioned, a unique feature of the Canadian data set is that it contains timestamps of individual bids and the researcher can thus observe if and how bids change within an auction as the deadline approaches. In particular, primary dealers frequently submit bids only to subsequently being asked to submit bids on behalf of a client. Since orders of large clients, such as pension funds, are tracked separately by the Canadian Treasury, the timestamps reveal when such bids arrived and oftentimes the primary dealer subsequently decides to revise his/her original bid. Hortaçsu and Kastl (2012) build a simple model that rationalizes why a primary dealer might want to submit a bid before customers' orders arrive - basically introducing uncertainty in whether customers' bids arrive at all and if they do, whether the subsequently updated dealer bid will make it in time to make the deadline.

Figure 1 in Hortaçsu and Kastl (2012) shows that many customers wait literally until the very last minute before submitting their bids. Such "last minute" bidding behavior by customers can be rationalized as a strategic response by customers who do not want dealers to utilize the

information in their bids. There may be reasons for customers to voluntarily share their information with dealers as well. For example, Bloomberg Business published an article on 4/4/2013 where a representative of Blackrock, the world's largest asset manager, described why Blackrock chooses to participate indirectly by submitting bids through PDs: "While we can go direct, most of the time we don't. We feel that the dealers provide us with a lot of services. Our philosophy at this point is, to the extent we can share some of that information with trusted partners who won't misuse that information, we prefer to reward the primary dealers that provide us all that value." In previous work studying the Canadian treasury market, Hortaçsu and Sareen (2006) find that some dealers' modifications to their own bids in response to these late customer bids narrowly missed the bid submission deadline, and that such missed bid modification opportunities had a negative impact on dealers' ex-post profits.

With slight modifications on top of our auction model from Section 2.1 that allow for customers and primary dealers submitting bids in various times, we can use equation (2) and the method described in Section 2.2 to estimate willingness-to-pay that would rationalize each observed bid. With these estimates in hand, it is natural to proceed by comparing expected profits corresponding to a primary dealer's bid that was submitted absent any customer's order information to expected profits corresponding to the bid that was submitted after observing customer's order (both within the same auction). While intuitive, this approach would be premature. In Assumption 3, we have imposed that bidders' values are private - in particular, we assumed that if a primary dealer were to learn a customer's signal by observing that customer's bid, she would not update her estimate of the value. This assumption is often questioned and treasury bills are often mentioned as an example of objects that should be modeled using a model with interdependent values. Given the data available in the Canadian treasury auctions, we can formally test this assumption, at least as far interdependency of values between primary dealers and their customers is concerned.

# 3.4.1 Private versus Interdependent Values in Treasury Bill Auctions

Recall that in the Canadian Treasury Bill auctions the researcher observes instances where a bidder (primary dealer) submits two bids in the same auction, one before customer's order arrives and one after it arrives. This setting thus provides us with a natural laboratory where to investigate whether primary dealers are just learning about competition in the upcoming auction (and not updating their valuation estimates) or whether the PDs may also be learning about fundamentals, and hence update their value estimates after observing customers' bids. When deciding how to translate willingness-to-pay into the bid, the primary dealer must form beliefs about the distribution of the market clearing price (as equation (2) reveals). To form such beliefs, the primary dealer integrates over all uncertainty: the signals of rival primary dealers, the signals of all customers, which primary dealer a customer might route her bid through etc. By observing a customer's order, part of this

<sup>3</sup>http://www.bloomberg.com/news/articles/2013-04-04/bond-traders-club-loses-cachet-in-most-important-market.

uncertainty gets resolved: a primary dealer can therefore update her belief about the distribution of the market clearing price by evaluating (13). Since this updating can be easily replicated in the estimation by appropriately adapting the resampling technique, it allows us to form a formal hypothesis test about whether values are indeed private. In particular, let  $\left\{v_k^{BI}\left(q_k,\theta\right)\right\}_{k=1}^K$  denote the vector of the estimated willingness-to-pay that rationalizes the observed vector of bids before the customer's order arrives (hence, "BI"),  $\left\{b_k^{BI}\left(q_k,\theta\right)\right\}_{k=1}^{K}$ , and  $\left\{v_k^{AI}\left(q_k,\theta\right)\right\}_{k=1}^{K^{BI}}$  denote the vector of the estimated willingness-to-pay that rationalizes the observed vector of bids after the customer's bid arrives (hence, "AI"),  $\left\{b_k^{AI}\left(q_k,\theta\right)\right\}_{k=1}^{K^{AI}}$ . If we were to observe a bid for the same quantity being part of both the bid before customer's information arrive and the bid after its arrival, we can simply formulate a statistical test at that quantity. Let  $T_{i}(q) = v^{BI}(q,\theta) - v^{AI}(q,\theta)$  be the difference between the rationalizing willingness-to-pay for a given quantity, q, before and after information arrives (taking into account the updating about the distribution of the market clearing price during the estimation). Testing the null hypothesis (of no learning about fundamentals) then involves testing that  $T_i(q) = 0, \forall j, q$ . Of course, the null hypothesis is essentially a composite hypothesis of all modeling assumptions (e.g., in addition to private values, we also assume independence etc.). To test the hypotheses jointly, since there is no result on uniformly most powerful test, we can define several joint hypotheses tests and perform them concurrently. For example, the following test is motivated by the well-known  $\chi^2$ -test, which can be additionally standardized by the standard deviation of the asymptotic distribution of each individual test statistic.

$$T = \sum_{(i,t)=(1,1)}^{(N,T)} T_{i,t}(q)^2$$
(15)

Hortaçsu and Kastl (2012) report that the null hypothesis of no learning about fundamentals from customer's bids can be rejected neither based on (15), nor based on several alternative tests. While this still does not preclude interdependency of values between primary dealers themselves, given that many of the customers are large players (such as Blackrock) this evidence is at least suggestive that modeling information structure in treasury auctions as private values is reasonable. It is important to note that the crucial point about the information structure is not that the security of interest might have common unknown value at the secondary market after the auction, but rather whether or not the primary dealers have different information about this ex-post valuebefore the auction. If the information is symmetric, and potentially imperfect, the heterogeneity in values (and thus in bids) might still be attributable purely to heterogeneity in the private component of the values.

#### 3.4.2 Quantifying Order Flow

Having established that the private value assumption is a reasonable approximation in this environment, we can now start quantifying the value of customers' order flow information. Observing

the dynamics of bidding within an auctions is particularly useful for two reasons: First, since the bidding takes place typically within few minutes before the auctions deadline, it is quite reasonable to assume that nothing else is changing other than primary dealer's getting (or not) information about customers' orders. In language of empirical industrial organization, this allows the researcher to control unobserved heterogeneity at a very fine level: one needs to rely neither on variation across auctions, nor across bidders within an auction. This, therefore, allows the researcher to quantify the value of order flow by calculating the expected surplus (profits) of a primary dealer associated with the original bid (before customers' order information arrives) and compare it to the expected profits associated with a bid after the customer's information arrives. Of course, in doing so, one has to take into account that the beliefs about the distribution of the market clearing price will change when customers' order arrives as can be clearly seen by comparing equations (12) and (13). With these two distributions, in hand we can now estimate not only ex-post profits (given the realized demands of other primary dealers and other customers), but also the ex-ante profits given the appropriate distribution of the market clearing price. Hortaçsu and Kastl (2012) report that about one third of PDs' profits can be attributed to the customers' order flow information. Overall, however, the auctions seem fairly competitive and the profits thus do not seem excessive - recall that Hortacsu et al. (2015) reached a similar conclusion for the US Treasury auctions.

The main takeaway from the Canadian treasury auctions is thus that (i) customers' order flow is an important source of primary dealers' rents (up to one third in the Canadian auctions) and (ii) private values may be a reasonable approximation of the information structure in treasury bill auctions.

Another important question is why indirect bidders voluntarily choose to submit their bids through a primary dealer rather than to participate in the treasury auctions directly. This is an excellent direction for future research. For example, the quote from a Blackrock executive mentioned in footnote 3 above suggests that there might complementary services that PDs are providing to the indirect bidders. One example might be helping them to execute a large trade in an asset that otherwise quite illiquid, but there may be many others. Building a theory of these complementarities and quantifying their value would be an excellent research question to pursue.

# 3.5 Extracting Information about Bank's Balance Sheets and their Over-the-Counter Trading Opportunities

In this section I want to focus on a different kind of auctions, the Main Refinancing Operations (MRO) of the European Central Bank (ECB), analyzed in Cassola et al. (2013). MROs are the main tool of monetary policy implementation at ECB's disposal. Basically, these are discriminatory auctions of short-term (typically 1-weekl) repo loans.<sup>4</sup> Banks participate by offering bids for loans

<sup>&</sup>lt;sup>4</sup>Auctions were substituted by a full-allotment mechanism in October 2008 following the collapse of the auction market in the aftermath of the Lehman failure. Looking forward, however, it is likely that the ECB will go back to the auction mechanism as soon as possible.

at various yields and of various sizes. The loans are collateralized, but the quality requirements for collateral are typically much weaker than those in the private transactions. By varying the minimum bid rate (minimum yield) and the total available supply, the ECB is trying to target the overnight unsecured interest rate (EONIA) in the financial markets. There are four key facts about the raw data at the onset of the financial crisis in mid- to late 2007. First, the spreads between secured and unsecured interest rates (even short-term) diverged substantially starting in August 2007 for all maturities. Second, bids in the MROs became "more aggressive" - in the sense that the submitted yields became higher. Third, bids became quite dispersed suggesting an increased heterogeneity in demand (either within or across banks). Fourth, some and in few instances in fact most if not all banks submitted bids that exceeded the published EURIBOR (Euro Interbank Offer Rate) for loans with the same maturity, where bids are for collateralized loans whereas EURIBOR is supposed to represent the rate for an unsecured loan. This is clearly evidence that most if not all banks would not be able to secure funding at the EURIBOR

One important question to address is, therefore, whether the increased heterogeneity in bids is due to an increased heterogeneity in willingness-to-pay or whether the heterogeneity in bids could also be masking the strategic effect: some banks might simply be changing their bids in response to other banks' changing their bidding behavior rather than changes in their underlying willingness-to-pay.

There are two main goals of the analysis. The first goal is to illustrate that in order to interpret the bidding data in a meaningful way, one needs an economic model such as the one in Section 2.1. The reason is that the bids are strategic choice variables that the participants (banks) submit taking the strategic environment, i.e., the auction game being played, into account. This implies that one needs a model to decompose the changes in bids into those caused by changes the underlying objects of interest, such as the willingness-to-pay for a loan, and those caused by changes in the strategic environment. As before, I will proceed in a sequence of steps. First, I will illustrate that a natural interpretation of a bank's willingness-to-pay for a loan in the MROs is the cost of funding of that bank in that week. Second, I will talk about how the time series variation of these funding costs can be used to draw inference about dynamics of banks' balance sheets and hence to distinguish "healthy" banks and those that might be getting into financial difficulties.

# 3.5.1 Willingness-to-Pay and Funding Costs

In Assumption 3 I imposed some structure on the willingness-to-pay as one of the primitives of our model, but at this point it is worthwhile to discuss where this object may be coming from. Due to the substitutable nature of ECB and interbank loans, these demand functions are very much dependent on a bank's outside funding opportunities. Figure 2 depicts the willingness-to-pay function  $v(q, s_i)$  for some realization of the private signal  $s_i$ . In this figure, I assume that the bank's total collateralized borrowing capacity,  $K_{it}$ , is less than its liquidity need,  $R_{it}$ . Specically,

consider the following stylized model in which bank i has a liquidity need (possibly due to a reserve requirement, to improve its balance sheet, or to close a funding gap) of  $R_{it}$  at time t. This must be fulfilled through three alternative channels: (i) a loan from the ECB, (ii) unsecured interbank lending, which is done through over-the-counter deals, or (iii) secured interbank lending, which is also done over-the-counter. These channels are substitutes, but access to them is limited based on collateral availability. In particular, assume that bank i has  $L_{it}$  units of liquid (high-quality) collateral acceptable by secured interbank lending counterparties. The bank also has  $K_{it}L_{it}$  units of securities that are acceptable by the ECB and perhaps also by other counterparties as collateral, but are either subject to "haircuts." The haircuts applied to this set of securities effectively increase the interest rate at which the bank can borrow against these securities; these rates are bounded below by the "secured" interbank lending rate,  $s_{it}$ , that the bank faces, and bounded above by the "unsecured" interbank lending rate,  $u_{it}$ , which requires no collateral. The bank's willingness-topay for the rst  $R_{it} - K_{it}$  euros of funding, thus, is equal to its unsecured funding rate,  $u_{it}$ . Between the  $R_{it} - K_{it}$  and  $R_{it} - L_{it}$ , the bank faces different haircut rates depending on its portfolio of securities it can post as collateral. The last  $L_{it}$  euros of funding can be obtained from the "secured" interbank market.

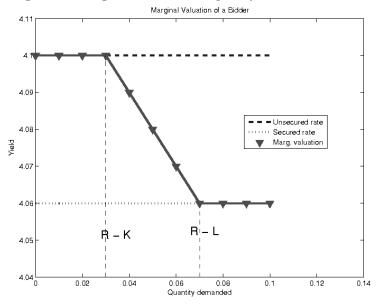


Figure 2: Marginal Value for Liquidity in ECB Auctions

We can now use the equilibrium relationship between bids and values described in Proposition 1 together with the methods described in Section 2.2.2 to obtain estimates of the willingness-to-pay of each bank in a given week or, equivalently (given the discussion above), of funding costs in that week that rationalize the observed bids. Since as mentioned above the MROs are the main tool of ECB's monetary policy implementation, virtually all financial institutions in the EURO-zone

participate in these auctions. A researcher can, therefore, recover a panel of funding costs: for each bank that participates, one recovers a rationalizing willingness-to-pay in a given week, which corresponds to the next-best alternative of funding for that bank. On the the other hand, this next-best alternative depends on the quality of that bank's available collateral and on its credibility or perceived riskiness. This suggests that looking at the dynamics of these funding costs can be informative about individual banks' financial health.

#### 3.5.2 Dynamics of Banks' Balance Sheets

Having obtained a time series of funding costs for each bank, we can ask whether there are significant patterns over time, and in particular during the onset of the financial crisis. Cassola et al. (2013) report (Table III and Figure 6) that for about two thirds of banks the funding costs indeed significantly increased, albeit for many of those banks this increase was not too big. However, there were few banks for whom the funding costs increased quite substantially - up to 66 basis points (in terms of the annualized yield), which is quite substantial given the short-term maturity of the loans.

## 3.5.3 Separating the Strategic Effect

As mentioned above, in order to meaningfully interpret the observed changes in bids, one needs to take into account the strategic effect. Cassola et al. (2013) provide a useful illustration of this important point. Figure 7 in Cassola et al. (2013) shows the bid distributions separately for banks that are identified as suffering relatively more from the financial crisis, in the sense that these banks' funding costs significantly increased during the early period of the crisis, and banks whose funding costs remained relatively unaffected by the crisis. While the two distributions are different, they have very similar means and virtually overlapping supports. Nevertheless, Figure 8 illustrates that the two distributions of willingness-to-pay (or funding costs) are quite different across the two groups. Naturally, the distribution of funding costs of the group of banks that were affected more by the crisis dominates that of the other group in the sense of first order stochastic dominance.

Perhaps more importantly, Table 1 illustrates this point further. The table depicts regressions of various performance measures taken from the balance sheets data on changes in bids, changes in funding costs or both. The regressions are of the following form:

$$P_{i,2007} = \beta_0 + \beta_1 \Delta b_i^{qw} + \beta_2 \Delta v_i^{qw} + \beta_3 P_{i,2006} + \varepsilon_i$$
(16)

where  $P_{i,t}$  is the value of an accounting performance measure in firm i and year t, a change  $\Delta b_i^{qw}$  ( $\Delta v_i^{qw}$ ) is defined as quantity-weighted mean bid (value) after the crisis (post August 2007) minus quantity-weighted mean bid (value) before crisis (pre August 2007). The table nicely illustrates that the changes in the estimated primitives (funding costs) are correlated in the expected way with

Table 1: Correlation between bank performance ratios and marginal values/bids

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ROA'07	ROA'07	ROA'07	ROE'07	ROE'07	ROE'07	CTI'07	CTI'07	CTI'07
0.0520		0.126	-0.00613		0.0156	0.0734		-0.0370
(0.133)		(0.140)	(0.0288)		(0.0302)	(0.150)		(0.157)
	-0.156	-0.188*		-0.0514**	-0.0553**		0.274**	0.283**
	(0.107)	(0.113)		(0.0230)	(0.0242)		(0.119)	(0.126)
0.561***	0.562***	0.564***		, ,	,		, ,	, ,
(0.0260)	(0.0259)	(0.0260)						
,	,	,	0.240***	0.241***	0.242***			
			(0.0299)	(0.0297)	(0.0298)			
			,	,	,	0.568***	0.568***	0.569***
							(0.0853)	(0.0855)
0.0782*	0.116***	0.0798*	0.0399***	0.0450***	0.0407***	,	0.284***	0.294***
(0.0457)	(0.0225)	(0.0456)	(0.00974)	(0.00474)		(0.0716)	(0.0585)	(0.0712)
(/	( -)	()	(	( )	()	( /	()	( ' )
390	390	390	390	390	390	386	386	386
0.546	0.549	0.550	0.143	0.154	0.155	0.104	0.116	0.116
	0.0520 (0.133) 0.561*** (0.0260) 0.0782* (0.0457) 390	ROA'07 ROA'07  0.0520 (0.133) -0.156 (0.107) 0.561*** 0.562*** (0.0260) (0.0259)  0.0782* 0.116*** (0.0457) (0.0225)  390 390	ROA'07         ROA'07         ROA'07           0.0520 (0.133)         0.126 (0.140)           -0.156 (0.107)         -0.188* (0.113)           0.561***         0.562*** (0.0260)         0.564*** (0.0259)           0.0782* (0.0457)         0.116*** (0.0225)         0.0798* (0.0456)           390         390         390	ROA'07         ROA'07         ROA'07         ROE'07           0.0520 (0.133)         0.126 (0.140) (0.0288)         -0.00613 (0.0288)           -0.156 (0.107)         -0.188* (0.113)         -0.00613 (0.0288)           0.561***         0.562*** (0.0260)         0.564*** (0.0260)         -0.0260)           0.240*** (0.0299)         0.0798* (0.0457)         0.0399*** (0.00974)           390         390         390         390	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

The dependent variables in this table are bank performance ratios (ROA, ROE, and CTI) reported year-end 2007. The independent variables are the pre- vs. post-crisis (Aug. 9, 2007) change in a bank's quantity-weighted average price bids ( $\Delta$  bids) and estimated marginal valuations ( $\Delta$  marginal values).

the performance measures. Banks whose funding costs increase suffer a larger drop in Return-on-Assets (or Return-on-Equity) or their Cost-to-Income ratio worsens. Perhaps surprisingly, similar correlations do not appear when one uses the bids directly, i.e., before "purging" them of the strategic effect. Such divergence in results would be impossible in a single-unit auction since a bid is a monotone function of value. There are two reasons for which such difference may arise in this environment. First, in multiunit auctions we may not have a monotone equilibrium. More plausibly though, some aggregation occurs before the regressions are run: bids and values are multidimensional objects - they are step functions - and they are aggregated to one-dimensional statistics by taking a quantity-weighted average and then a difference is taken between two such averages (pre and post crisis). This aggregation may thus also upset the monotonicity. In any case, Table 1 illustrates that taking the strategic effect out from the bids is a necessary step to obtain the quantities of interest. While we may find some of the assumptions underlying the model of equilibrium bidding behavior questionable or unsatisfactory, the model still delivers estimates that pass an out-of-sample test presented in this table.

# 4 Systemic Risk

The recent financial crisis highlighted the need provide measures of importance of individual financial institutions. Empirical measures of systemic risk that directly estimate the overall impact of individual institutions on the whole system, have been proposed in the financial economics literature. Acharya, Pedersen, Philippon and Richardson (2010) provide a simple theoretical framework,

We also control for the year-end 2006 performance ratios. Standard errors are in parentheses, with (\*\*\*) indicating p<0.01, (\*\*) p<0.05, and (\*) p<0.1.

in which they propose to measure systemic risk by an institution's marginal contribution to the shortfall of capital in the financial system that can be expected in a crisis. Instead, Brownlees and Engle (2010) measure it by the expected shortage of capital of an institution given its degree of leverage. Adrian and Brunnermeier (forthcoming) measure the marginal contribution of each institution to the "value at risk" of the whole system, conditional on that institution being under distress relative to its median state (hence the name CoVaR).

An alternative to these measures is to estimate the network of financial linkages and evaluate the extent to which shocks are likely to spill over from one institution to another. There are several ways to proceed: First method proposed in Bonaldi, Hortaçsu and Kastl (2013) recovers the financial network from the panel of estimated funding costs described in Section 3.5.1. Second method developed in the literature relies on estimating the financial network using equity prices (Billio, Getmansky, Lo and Pelizzon 2012). Third and probably the most preferred method would be to observe the financial network directly and evaluate systemicness of individual nodes as one of the measures taken from network theory. If such data were available (i.e., when regulators were to collect such information), one could thus compute for example the Katz centrality measure and rank banks according to their systemicness and vulnerability measures.

Let us consider a panel data set of banks' funding costs that would rationalize their bids in the Main Refinancing Operations of the European Central Bank that we obtain as in Section 3.5.1. Given this panel of banks' revealed short-term funding costs, the network structure can be estimated by looking at the covariation of a given bank's funding cost with other system banks' (lagged) funding costs, controlling for various sources of comovement due to common asset exposures, such as holdings of sovereign bonds. Implementing this through traditional linear regression, however, yields a dimensionality problem: the number of covariates in the regressions potentially exceeds the number of data points available for each bank in the data set. To combat this problem, which is commonly encountered in Genetics and Machine learning, Bonaldi et al. (2013) utilize the adaptive elastic net method. This is a modification of the Least Absolute Shrinkage and Selection Operator (LASSO) method, which has been shown to have several desirable properties. In particular, as opposed to other methods for model selection, like the standard LASSO, the elastic net performs well under the presence of highly correlated covariates, as measured by prediction accuracy, which is a feature that likely applies in the case of financial networks.

$$v_{i,t} = \beta_{0,i} + \sum_{j} \beta_{i,j} * v_{j,t-1} + X_t \gamma_i + \varepsilon_{i,t}$$

$$\begin{bmatrix} v_{1,t} \\ \vdots \\ v_{Q,t} \end{bmatrix} = \begin{bmatrix} \beta_{0,1} \\ \vdots \\ \beta_{0,Q} \end{bmatrix} + \underbrace{\begin{bmatrix} \beta_{1,1} & \cdots & \beta_{1,Q} \\ \vdots & \ddots & \vdots \\ \beta_{Q,1} & \cdots & \beta_{Q,Q} \end{bmatrix}}_{PT} \begin{bmatrix} v_{1,t-1} \\ \vdots \\ v_{Q,t-1} \end{bmatrix} + \eta_t.$$
 (17)

where  $\eta_t = X_t^T \gamma + \epsilon_t$ . The elastic net estimator is then defined as follows:

$$\hat{\beta}_{enet} = \underset{\beta}{\operatorname{argmin}} \|y - \beta Z\|_{2}^{2} \text{ s.t. } (1 - \alpha_{e}) \|\beta\|_{2}^{2} + \alpha_{e} \|\beta\|_{1} \le t$$
(18)

where  $Z = \begin{bmatrix} \vec{1} \ \vec{v}_{t-1} \ X \end{bmatrix}$ . There are two parameters that a researcher needs to choose value for when implementing the elastic net estimator. Typically, the upper bound for the constraint, t, is set following a rule-of-thumb approach developed in statistics (and available as an option in standard packages such as 'glmnet' in R). There is less guidance about choosing the weights on the two norms:  $\alpha$ . Two extreme cases of the elastic net estimator are LASSO ( $\alpha_e = 1$ ) and Ridge regression ( $\alpha_e = 0$ ). Bonaldi et al. (2013) report that the estimation results are not very sensitive to the particular value of  $\alpha$  as long as they are not too close to one of the extrema.

Given the estimates of interbank interactions based on the funding costs, the network of spillover effects can be viewed as a directed graph with banks in place of nodes. A link from i to j represents that an increase in the cost of funding of bank i is associated with a positive change in the cost of funding of bank j, one period ahead. Moreover, one can assign a weight to this link equal to the predicted response in j's cost of funding corresponding to a 100bp positive shock to i's cost of funding.

To characterize the systemicness of a bank Bonaldi et al. (2013) use a generalized version of Katz centrality, traditionally used in social network analysis to determine the influence of a node within the network. The generalization of Katz centrality for weighted networks is defined as:

$$k_i^c(\lambda) = \sum_{s=1}^{\infty} \sum_{j=1}^{Q} \lambda^s (B^s)_{i,j}$$

where B is the matrix of coefficients from (17). Such a measure takes into account the direct effect of a bank on its immediate neighbors (those banks that are directly affected by it) but also on the neighbors of its neighbors, as well as on their neighbors, and so on and so forth. Assuming stationarity, let  $K = \sum_{s=1}^{\infty} B^s = (I - B)^{-1} - I$ .  $K_{i,j}$  can then be interpreted as the sum across s of the responses in i's WTP at t + s to a 100bp temporary shock to j's WTP at t or as a the permanent response in i's WTP to a permanent shock of 100bp to j's WTP. It is then natural to define

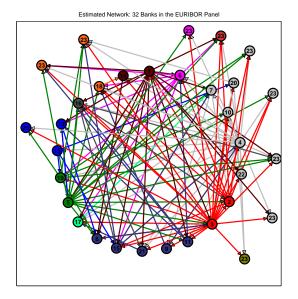


Figure 3: Financial Network

**Definition 2** Systemicness:  $k_j^s = \frac{1}{Q} \sum_i K_{i,j}$  measures the average effect of an exogenous shock to j's cost of funding on all other banks' funding costs.

**Definition 3** Vulnerability:  $k_i^v = \frac{1}{Q} \sum_j K_{i,j}$  measures the average effect of an exogenous aggregate shock to all bank's funding costs on i's cost of funding.

These measures coincide with Katz centrality for  $\lambda = 1$ , but a lower value of the parameter may be chosen to downweight more distant paths.

Bonaldi et al.'s (2013) results suggest that most of the banks are only weakly connected and, therefore, if one were to suffer an adverse shock there would likely be a rather limited effect on the other ones. On the other hand, they argue that there are a few banks that are quite central: an increase in their funding costs would result in a very significant increase (up to 95 basis points per 100 basis points shock) in the funding costs of the other banks.

There are two main advantages to using the elastic net estimator in these applications. First, it seems reasonable to assume that the network of (first-order) financial links should be sparse: there are great many financial institutions and any one of them is hardly exposed directly to cash flow (or funding costs etc) of all the other banks. The LASSO-type estimator "shrinks" the coefficients that contribute the least to the model fit towards zero and hence for sufficiently strict penalty parameter it makes many coefficients in the connectivity matrix B zero. Second, while a simple LASSO estimator (which only has the  $L^1$ -norm of the coefficients as a constraint) would achieve the first objective, the sparse connectivity matrix, it might fail badly if the right-hand side variables in (17) are correlated. The second part of the constraint (the  $L^2$ -norm) in (18) helps to cope with

this issue.

Bonaldi et al. (2013) find that banks' vulnerability scores (as measured on data from 2007 and 2008) contribute significantly to predict the probability that a bank would be subsequently bailed out. No such correlations are found for systemicness measures.

Several other empirical measures of network linkages among financial institutions have been developed in the literature, utilizing different sources of data and methods. Battiston, Puliga, Kaushik, Tasca and Caldarelli (2012) analyze the network of cross ownership relations (equity investment) among a small sample of banks that were among the top recipients of aid from the FED through its emergency loans program, during 2008-2010. They use a feedback centrality measure to rank banks according to their systemic impact. Greenwood, Landier and Thesmar (2015) rank banks according to how vulnerable they are to shock propagation. However, they focus exclusively on shocks to equity that force assets liquidation in order to meet a target leverage ratio. In their model, fire sales of assets decrease other banks' equity, providing a channel for propagation. Billio et al. (2012) have utilized lag relationships between stock returns (in particular bivariate Granger causality tests) to characterize the network structure of the largest financial institutions in the US. Their exercise relies on the assumption of informationally inefficient securities markets, so that predictive correlation patterns between publicly traded securities are possible. The data based bids from MROs described earlier, however, is not public information, and is observed by the ECB only. The banks only observe their own bids, their final allocation, the market clearing interest rate and the average interest paid in the auction. Diebold and Yilmaz (2011) also estimate a network of connections among financial institutions. Their reduced form model is a vector autoregression, but their variable of interest are stock return volatilities. Their network is defined as the (h-step) forecast error covariance matrix. From such a network, they derive measures of total connectedness to and from other banks, analogous to the systemicness and vulnerability measures of Bonaldi et al. (2013). One advantage of Bonaldi et al.'s (2013) measure is that the units in which it is reported can be directly interpreted as changes in funding costs.

The key issue that all above mentioned papers that are based on recovering a network from observed panel data face is identification of the causal effects. In order to interpret the coefficients from the VAR defined as in (17) as weighted edges of a graph of the financial network, the researcher needs to argue that the estimated coefficients have indeed been consistenly estimated: that there is no endogeneity problem plaguing the estimation, such as the one arising from omitted variables. For example, two banks may be exposed to the same asset that suffers a negative shock. If the researcher fails to control for this common exposure during the estimation, the estimated model might wrongly attribute such correlation in funding cost dynamics (or equity returns) to a link between these two institutions. Typically there are few candidate instrumental variables that one may be able to use such as bailouts, but it is unlikely that there will be sufficiently many. Therefore, researchers including all the above-mentioned papers typically analyze only correlation patterns. Bonaldi et al. (2013) probably go furthest in this respect as they attempt an instrumental variable

estimation where the endogenous lagged values of funding costs of rival banks are instrumented by CDS prices and bond prices. They report that the vulnerability and systemicness rankings remain qualitatively stable when the endogeneity is (at least partially) tackled in this way.

It is important to realize, however, that the empirical model given by (17) is still a reduced form. It does not attempt to capture the full network formation game: links may get formed or dissolved over time and the system given by (17) thus only approximates the "average" network. Nevertheless, one could envision estimating a rolling version of (17) and thus estimating how systemic risk of different institutions varies over time. Financial regulation is quickly evolving and the issue of systemically important financial institutions is still on the table. As of today, how to define and quantify what systemically important really means is far from settled. There are still many open questions and directions in which research involving financial networks can be going, but providing a framework in which systemic risk can be quantified seems to be a particularly useful one.

# 5 Further work

I conclude this paper by pointing to yet another toolbox from IO that is slowly gaining interest in empirical finance literature. This toolbox consists of the tools for estimation of demand systems for differentiated products. This has been an extremely active area in IO with many seminal papers that laid down foundations for modern merger analysis or calculation of welfare from new products. There are many applications in finance where similar types of questions can and should be addressed. In case of financial markets, a researcher may often be interested in studying (sub)markets for several substitutable or complementary assets such as different equities or government securities of different maturities etc. The objects of interest typically involve the matrix of own- and cross-price elasticities and the substitution matrix. These objects are typically derived from a fully specified demand model. While going into details is beyond the scope of this paper, there are two main directions that the literature pursues: (i) analyzing demand for differentiated products in a characteristic space where consumers preferences are modeled over characteristics of products rather than products themselves and (ii) analyzing demand systems in the product space where one approximates the underlying (Marshallian) demand functions by a flexible functional form that allows for rich dependence, potentially on the full vector of prices. Each approach has its benefits and costs and I refer the reader to the relevant IO literature for more details.<sup>5</sup>

The classical reference in finance is Markowitz (1952), which was the first paper to cast the portfolio choice problem in the mean-variance framework. Hence, Markowitz (1952) viewed different products (portfolios) as bundles consisting of mean-variance characteristics. For characterizing demand for individual securities rather than portfolios, one hurdle may be specifying a meaning-

<sup>&</sup>lt;sup>5</sup>For characteristic space demand models see for example Berry et al. (1995), Nevo (2001), Petrin (2002) or Berry, Levinsohn and Pakes (2004). For product space demand models see for example Deaton and Muellbauer (1980), Hausman, Leonard and Zona (1994) or Chaudhuri, Goldberg and Jia (2006).

ful characteristic space. Obvious choices of characteristics of securities to include in addition to mean returns and variances are maturity, likelihood of default etc. Koijen and Yogo (2014) write down such a model which allows them to analyze how the demand elasticity for various equities changed over time and what features of individual institutional investors could potentially explain the heterogeneity in their underlying asset demands. Two key advantages of the characteristic space approach is the reduction of the number of parameters that need to be estimated and the ability to evaluate potential introduction of new products.

In many interesting cases, including those involving financial markets, it may not be easy to define the relevant characteristics over which agents would have preferences and that would capture an important part of variation in demand for them. Mean return, return variance, maturity and perhaps an asset covariance with the market return (its beta) might be such relevant characteristics, but it might be hard to capture other important ones succinctly. When it is not clear how to define the characteristic space, the product space offers an alternative approach. As mentioned above this approach relies on specifying explicit dependence of demand for each good on all other prices, i.e., rather than modeling indirect utility from consuming a good, the researcher models the Marshallian demand. Hortaçsu and Kastl (2015) is a current working paper that takes this approach and aims to analyze dependence of demands for government securities of different maturities on each other. The downside of this approach is the inability to evaluate counterfactuals with new products and the relatively large number of parameters that need to be estimated.

In any case, I believe that it is important to bring the tools from demand estimation literature into finance. While it might not be as important to study welfare coming from the introduction of new products (even though it very well might be as new derivatives get introduced), it certainly is very important to understand the substitution patterns between products. For example, when the Office of Debt Management seeks to fulfill its goal of minimizing the cost of debt servicing over time, it ought to take into account the impact of increasing the supply of a security of a given maturity on demand for other maturities. Also, in order to steer interest rates along the yield curve, one needs to understand where the demand for the different maturities is coming from and how it is intertwined with the other securities. In order to investigate how liquidity and safety premia of US Treasuries (as studied in Krishnamurthy and Vissing-Jorgensen (2012) for the aggregate demand) vary across maturities, we also would need the full demand system. There are indeed many directions where more research is needed.

# 6 Conclusion

This paper offered a brief discussion of how methods from Industrial Organization can be used to analyze important questions in financial markets. Financial markets are an example of a setting where isolating a nice exogenous source of variation that could inform the researcher about various objects of interest (such as the elasticity of demand for treasury bills) is very scarce, since imple-

menting a proper experiment is typically infeasible. Moreover, there are many counterfactuals that data have very little to say directly to, or for which any inference can be performed only under fairly heroic assumptions (such as comparing revenues from auction formats across different time periods in order to evaluate the effect of the mechanism on expected revenues). The aim of this paper was to illustrate how models can help us evaluating such counterfactuals by providing a researcher with an equilibrium relationship that creates a link between observable data and primitive quantities of interest. If the model is specified in a way such that it is reasonable to assume that the primitives are invariant to the counterfactuals considered, it can then be used to evaluate such counterfactuals even in absence of such scenarios in the data. Since equilibrium relationships are often functions of agents' beliefs about rivals' strategies, I described here a resampling method, which under certain assumptions, yields consistent estimates of these beliefs.

The applications considered in this paper included evaluating performance of US Treasury auction system, estimating the value of customer order flow, addressing the issue of private versus common values in treasury auctions, learning indirectly about the evolution of a bank's financial health and an alternative definition of systemic risk.

Finally, I think that taking the by-now fairly widely accepted methods for demand estimation provided by the Industrial Organization literature to finance is a promising direction for future work. I believe that allowing for heterogeneity in households' preferences, heterogeneity in large institutional investors' preferences and also for market power on both sides of the equity markets (and other financial markets) is an important direction to take the empirical asset pricing literature. There are some very recent attempts to address these issues, but more needs to be done.

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