Combining Choice and Response Time Data: a Drift-Diffusion Model of Mobile Advertisements*

Khai Chiong
University of Texas at Dallas

Matthew Shum
California Institute of Technology

Ryan Webb
University of Toronto

Richard Chen
Happy Elements, Inc.

Abstract

Endogenous response time data is increasingly becoming available to applied researchers of economic choices. However, the usefulness of such data for preference estimation is unclear. Here, we adapt a sequential-sampling model — previously-validated to jointly explain subjects' choices and response times in laboratory experiments — to model users' responses to video advertisements on mobile devices in a field setting. Our estimates of utility correlate positively with out-of-sample measures of ad engagement, thus providing external validation of the value of incorporating endogenous response time information into a choice model. We then use the model estimates to assess the effectiveness of manipulating attention towards an advertisement. Counterfactual simulations predict that requiring users to watch some portion of the ad — as is the practice of some online platforms (e.g. YouTube) — generate only modest increases in click-through rates and revenue.

Keywords: Mobile advertising, Attention, Drift-diffusion model, Response times, Sequential sampling models

JEL codes: L81, M37, D83, D87, C15, C22

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1 Introduction

The principle of revealed preference — that agents' preferences can be recovered from the choices that they make — underlies large swaths of the empirical literature in economics. In many choice situations, however, the amount of time it takes decision-makers' to make a choice (*response times*) are also sharply informative about their preferences (Frydman & Krajbich, 2017; Konovalov & Krajbich, 2019; Alos-Ferrer, Fehr & Netzer, 2020).[1] A swift food choice at a restaurant may indicate strong preferences for the chosen dish relative to the other available options; the time an online shopper spends deciding between different clothing items may suggest each item is equally stylish in the shopper's mind. As Block & Marschak (1960) describe in their seminal formulation of the Random Utility model:

> "The shorter the delay, the larger is the difference between certain values said to be attached by the subject to two alternative responses. A very long delay reveals a state of (almost) indifference, the 'conflict-situation': Hamlet took a very long time to decide whether to kill his uncle."

Why do decisions take time? A recent theory literature has proposed models of *sequential information sampling*, in which the utilities of options are unknown and must be learned by the (costly) sampling of noisy utility information over time (Woodford, 2014; Fudenberg, Strack & Strzalecki, 2018; Gossner, Steiner & Stewart, 2020; Hebert & Woodford, 2019; Morris & Strack, 2019; Fudenberg, Newey, Strack & Strzalecki, 2020). The time it takes a decision-maker to accumulate enough information or evidence favoring a given option is modeled directly; therefore both the choice and response time are endogenously related as part of a dynamic structural equation. One of the earliest such models is the drift-diffusion model (DDM), originally developed in psychophysics and neuroeconomics for studying decisions made on relatively short time-scales (Ratcliff & McKoon, 2008; Fehr & Rangel, 2011). The DDM and its various extensions have been well-validated in laboratory experiments, where the specification of a dynamic choice model has been shown to improve the estimation of utility parameters in prediction tasks (see Clithero, 2018b, for a review). However the relevance of such a dynamic model paired with response time data for explaining real-world, consequential decisions has remained unexamined.

[1] Response times are referred to by a variety of names, including reaction times, decision times, latencies (in psychology), and dwell times (in tech companies).

Our contribution in this paper is two-fold. First, we apply the sequential-sampling framework in a field setting to assess the role of response times in utility estimation. We consider a two-stage extension of the DDM to analyze responses to advertisements on mobile devices, a decision scenario which in recent years has become the dominant component of internet advertising.[2] An important vehicle of mobile advertising are video trailers — in which users of an app are shown a short video advertisement before continuing to use the app. During the ad, users are prompted to make a binary choice to either "click-through" to the app-store and download the advertised app, or "click-back" and return to the originating app which they were using before the ad played. By combining response times with choice data, we can separately identify the utility parameters which make up the decision process both while the ad is playing and after it ends, as well as the users' prior inclination before the ad has even started.

An important message of our results is that incorporating response times with choice data using a DDM framework improves estimates of users' preferences. As a validation exercise, we correlate the estimated utility parameters across users with three out-of-sample measures of ad and app engagement. We find that those users who have *higher* estimated utility parameters, elicit higher payments from the advertisers, are more likely to install the advertised app, and are more likely to upgrade the app once installed. This provides external validation and highlights the methodological advantages from analyzing response time data in conjunction with choice data. By contrast, the standard random utility modelling approach ignores response time data altogether. When we fit a logit model to our data, we find that the implied utilities from this model correlate poorly with the three external measures: now, those with higher estimated utilities are those who receive *lower* bids from advertisers, and also have *lower* install and upgrade probabilities.

Our second contribution is to perform counter-factual assessments of the optimal design and format of advertisements. These counter-factuals address a long-running question of whether drawing people's attention towards an item increases its demand (Gossner et al., 2020). Our field setting directs users' attention towards the advertised app as the video plays, thus we can separately identify the change in the click-through probability from this manipulation of attention. This allows us to consider the benefits for advertisers in switching from a "skippable" ad (in

[2] In 2018, mobile advertising in the United States accounted for roughly 75% of all spending on digital ads, eclipsing TV advertising for the first time (https://www.forbes.com/sites/johnkoetsier/2018/02/23/mobile-advertising-will-drive-75-of-all-digital-ad-spend-in-2018-heres-whats-changing/).

which users are allowed to exit anytime by clicking) to a "non-skippable" format in which users are required to watch part of the ad before they can exit. For instance, the online video platform YouTube forces viewers to watch the first five seconds. Such non-skippable ads mimic the types of decision tasks previously used to calibrate the DDM and measure attentional manipulations in the laboratory (Mormann, Koch & Rangel, 2012; Gwinn, Leber & Krajbich, 2019). While we find that click-through rates would indeed be higher if the ad were made non-skippable for an initial 10-15 seconds, the overall benefits are modest. The marginal clicks gained by making the ad non-skippable are, overall, those who are less persuaded by the ad. Indeed, these marginal users elicit lower bids from potential advertisers, implying that, in aggregate, there is only a small revenue increase from requiring users to watch some portion of the ad.

More broadly, the DDM-based framework used in this paper provides a method for integrating *endogenous* response time data into an empirical analysis. Our results suggest that mobile advertisements have high frequency effects which vary substantially across users and over time. The use of response times in conjunction with choice data is critical for identifying these effects. As more "clickstream" datasets become available to researchers in economics and marketing — in which timestamps are recorded for all the choices that agents are observed to make — econometric methods for utilizing this additional data source are crucial.[3] These applications demonstrate the value of applying the DDM framework beyond the controlled laboratory settings in which it has previously been used.[4]

2 The Drift-Diffusion Model (DDM) Paradigm

Consider a binary choice situation, where the difference in the utilities of the two options (1 and 0) determines an optimizing agent's choice.[5] In a sequential sampling model, the utilities are assumed to be unknown to the agent who only learns about them gradually over time. Mathematically, the perceived utility difference between alternative 1 and 0 can be modeled as

[3] For an extended discussion of the endogeneity of response times in discrete choice settings, see Webb (2019).

[4] There are a few papers in the marketing literature which incorporate both choices and response times in a (non-DDM) structural choice model; see Otter, Allenby & Van Zandt (2008), Seiler & Pinna (2017) and Ursu, Wang & Chintagunta (2018).

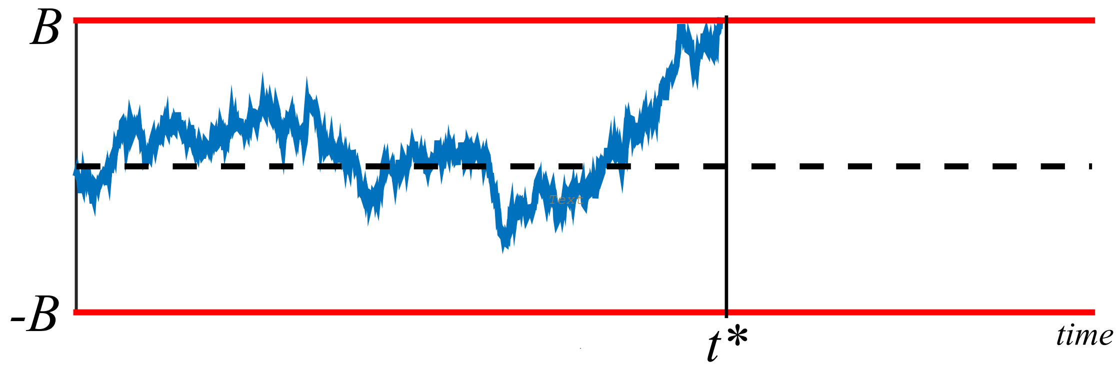
[5] We follow the presentation in Webb (2019). Book-length treatments of this modeling framework include Luce (1991) and Link (1992). Detailed mathematical derivations of accumulation models are presented in Smith (2000). Also see Busemeyer & Townsend (1992), Ratcliff & McKoon (2008), Krajbich, Armel & Rangel (2010).

a Gaussian process $Z(t)$ which evolves according to the stochastic differential equation

$$dZ(t) = (\mu + \gamma Z(t))\ dt + \sigma\, dW(t), \quad t > 0, \quad Z(0) = 0. \tag{1}$$

As noted by Fudenberg et al. (2018), this stochastic process can be interpreted as a process of "stochastic evidence accumulation" in favor of one or the other alternatives. Agents do not observe the drift term μ, which represents the true utility difference between alternatives; rather they only observe the noisy signal process $Z(t)$, where the noise follows a Wiener process W_t with standard deviation σ.[6]

Figure 1: A Sample Path of $Z(t)$ from the DDM.



In this example, the user chooses 1, with a response time of t^*.

The γ parameter in the diffusion process captures serial dependence in the diffusion process. In the sequential sampling literature, γ is called a "leakage" parameter (e.g. Usher & McClelland, 2001). When $\gamma = 0$ the stochastic process (1) is a continuous-time random walk with drift in which each sample is independent and weighted equally. When $\gamma \neq 0$, the information accumulated prior to t can affect the information perceived by the user at time t, thus accommodating carryover effects over time. As we will see below, the inclusion of these parameters is important for explaining observed patterns in the response time data.

[6]Indeed, Roitman & Shadlen (2002) have previously applied the DDM to model primates watching videos, specifically in the form of a binary motion discrimination task. They find that the accumulation of neural activity in the monkey parietal cortex varied with the informativeness of the visual stimuli.

Paired with this utility difference process is a decision rule. The most common decision rule is the symmetric "first passage" rule: for some threshold or barrier $B > 0$, the agent chooses 1 once $Z(t) > B$ and chooses 0 once $Z(t) < -B$. The response time is the first passage time to either B or $-B$. See Figure 1. For the purposes of specification and estimation, we will follow most applications of the DDM and take the threshold B to be exogenously given.[7] In this simple setting, the choice probabilities of the basic DDM (Equation (1) have the logistic form (like a standard logit model), however the additional information provided by the response time distribution reduces the variance of utility parameter estimates (Clithero, 2018a; Webb, 2019).

The DDM, and its various cousins, originated in psychology as a model of subjects' choices and reaction times; it has been successfully verified and calibrated in a wide variety of laboratory experiments and mapped to neural activity in the brain.[8] More recently, a number of researchers have estimated the structural parameters of the DDM using economic choices from laboratory experiments, including Frydman & Nave (2016), Clithero (2018a) and Webb (2019); see Clithero (2018b) for a review.[9]

Stochastic evidence accumulation is a natural interpretation for settings where information about utilities arrives sequentially and must be integrated by the decision-maker, as in a typical psychophysical experiment or the sequence of video images in an advertisement. Recent work also suggests that a stochastic accumulation process applies to decisions in which evidence is sampled (recalled) directly from the decision-maker's memory, rather than simply from sensory evidence (Shadlen & Shohamy, 2016). More broadly, the DDM choice framework resembles a "high-frequency" version of dynamic search or learning models which have been estimated structurally.[10]

[7]Optimal decision rules in the no-leakage ($\gamma = 0$) setting have been studied since Wald's (1973) sequential sampling model (see discussions in Lehmann (1959) and Bogacz, Brown, Moehlis, Holmes & Cohen (2006)), up to recent theoretical work by Fudenberg et al. (2018), Echenique & Saito (2017), Baldassi, Cerreia-Vioglio, Maccheroni, Marinacci & Pirazzini (2019) and Ke & Villas-Boas (2019). Particularly, Fudenberg et al. (2018) show that optimal decision rules in the DDM setting typically involve time-varying choice thresholds, and Fudenberg et al. (2020) discuss testing and identification of this model. As far as we are aware, there are no optimality results for the leakage ($\gamma \neq 0$) case.

[8]Gold & Shadlen (2002), Hare, Schultz, Camerer, O'Doherty & Rangel (2011)

[9]Relatedly, Krajbich, Bartling, Hare & Fehr (2015) critique the psychology literature which has used only reaction time data to infer agents' choice rules, without taking subjects' preferences or valuations of the choices into account.

[10]See, e.g., Erdem & Keane (1996), Crawford & Shum (2005), Hong & Shum (2006), Honka (2014), Lu & Hutchinson (2017)

3 Application: Video advertisements on mobile platforms

The number and variety of applications on mobile devices ("mobile apps") has soared in the last decade. A dominant revenue model for apps is the "freemium" model in which users download the app for free, but are subsequently monetized with periodic exposure to advertisements. As a result, mobile advertising has recently become the largest segment of digital advertising. We will refer to the originating mobile app with which the user was engaged as the "publisher", and the creator of the advertisement as the "advertiser". In most cases, both the advertiser as well as the publisher are apps – an example could be users playing the *Candy Crush* app being shown a video ad for the *Clash of Clans* app.[11] Because of this, the same app can be active on both the publisher and advertiser sides of the market, as they seek both to monetize existing users by showing them ads, as well as acquire new users by advertising on other apps.

A very common ad format in mobile advertising is the "skippable" video ad: users on a particular publisher app are served a 30 second video ad. While the ad is playing, and after it is finished, users are prompted to take one of two actions. The user can either close the ad and return to the publisher's originating app, or the user can click on the 'install' button which takes the user to an App Store (where the user has the opportunity to find out more about the advertised app and install it). We will use "click-back" to denote the first action, and "click-through" to denote the second.

Our data come from a mobile ad network, which acts as an intermediary between advertisers and publishers in this two-sided mobile advertising platform. We will focus on data from a single advertiser (a puzzle game) and we consider all ad impressions by this advertiser between October 1 through November 15, 2016, for a total of $N = 232,664$ ad servings.

Table 1 summarizes our dataset. There is substantial observed heterogeneity across users, and we include a number of user-specific covariates in our analysis. These covariates include features of the users' mobile devices, such as device language, brand, and operating system. We also include dummy variables for the five largest publishers in the data sample.

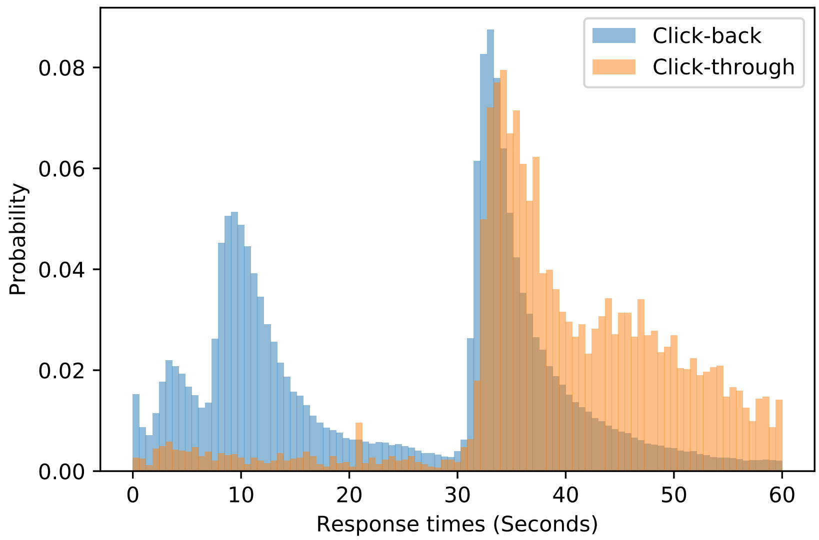
For the endogenous choice variable, 3.17% are click-throughs, and 96.83% are click-backs. This matches the average for ads on our platform, where click-through rates are typically between 1.5-3%. We define the endogenous response time as the time (in seconds) elapsed

[11] See Chen & Chiong (2016) for a study of a mobile advertising auction market in which advertising rates are set.

Table 1: Summary statistics.

Variables		Mean	Sd. Dev.
Conditioning covariates:			
iOS Dummy	$\in \{0, 1\}$	0.556	0.497
Samsung Dummy	$\in \{0, 1\}$	0.237	0.425
iOS Version	$\in [0, 10.3]$	5.366	4.818
Android Version	$\in [0, 8]$	2.205	2.510
Screen Resolution (millions of pixels)	$\in [0.137, 5.595]$	1.325	0.961
Connected via WiFi	$\in \{0, 1\}$	0.747	0.435
English language	$\in \{0, 1\}$	0.467	0.499
Publisher 1	$\in \{0, 1\}$	0.182	0.386
Publisher 2	$\in \{0, 1\}$	0.087	0.283
Publisher 3	$\in \{0, 1\}$	0.058	0.234
Publisher 4	$\in \{0, 1\}$	0.037	0.190
Publisher 5	$\in \{0, 1\}$	0.031	0.172
Choice variables:			
Click-through		0.0317	0.1751
Response time	(secs.)	25.11	14.43
Response time, for click-through	(secs.)	39.52	11.00
response time, for click-back	(secs.)	24.64	14.29
# observations (“impressions”)		232,664	

Figure 2: Empirical density of the observed response times



from the start of the ad until a decision has been made. We plot the normalized histograms of response time for click-backs ($y = 0$) and click-throughs ($y = 1$) in Figure 2.[12]

Several interesting patterns emerge. First, response times *matter*, in that the distributions of response time vary substantially across users' choices: the click-back distribution is multimodal, with two modes appearing while the ad is still playing, and another mode after the ad is over. Overall, the average time to click-back is 24.6 seconds (median=31.6). The click-through distribution, while also bimodal, has a prominent large mode after the ad is over, and another much smaller mode while the ad is playing; overall, the average time to click-through is 39.5 seconds (median=38.5).[13] Critically, in the post-ad stage, click-throughs tend to occur later then click-backs. This pattern is consistent with the well-known observation in perceptual psychology that response times tend to be longer for the less-frequent action (Luce, 1991; Woodford, 2016).

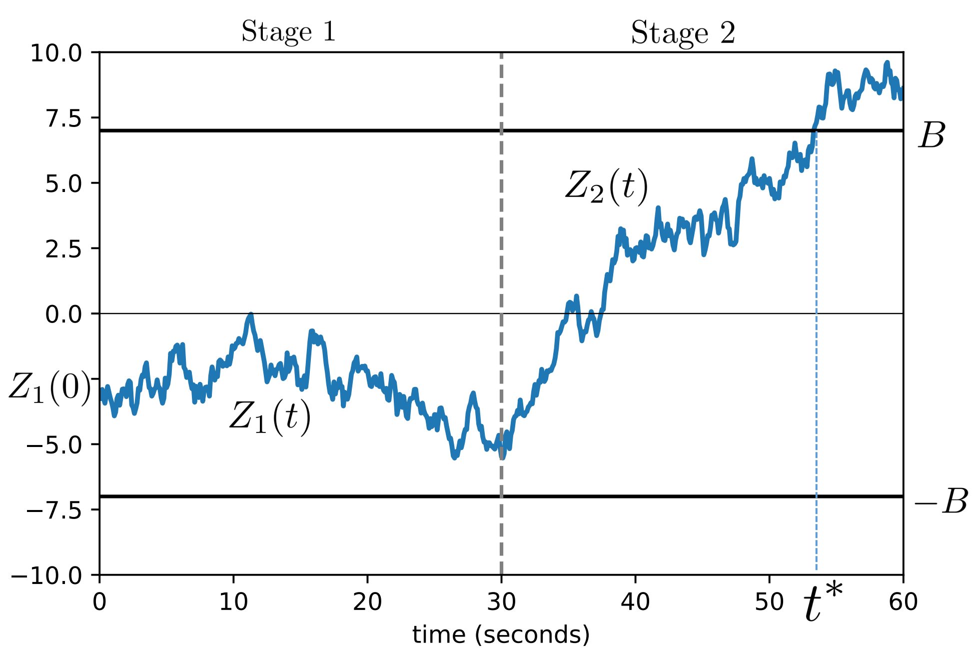
[12]We dropped users with response times exceeding 60 seconds; such excessive response times likely arise from users' inattention while the video ad was playing (e.g., looking away from the phone).

[13]These patterns in click-through rates and response times are typical within the platform: across 3,796 distinct 30-second ads, the average click-through is 2.95% and response times are 35.09s for click-through and 25.86s for click-back.

3.1 A two-stage DDM of mobile advertisements

In order to map the DDM framework to the mobile advertising data, we propose a two-stage extension of the DDM. In the first stage, which we call the *ad-play* stage, the video ad plays for up to 30 seconds. After the ad finishes, the second *post-ad* stage commences. Since the ad is skippable, users are able to actively make a choice (click-through or click-back) during both stages, but they will not be able to exit the ad until a choice is made. While we use a diffusion process to model both stages, we permit the parameters of the process to vary across the two stages, to reflect differences in the types and sources of information between the two stages: during the ad-play stage, the predominant source of information is the video ad itself, but in the post-ad stage users may be comparing the information conveyed in the ad with recollections of their previous game-play experiences on the originating app.

Figure 3: Two-stage Drift-Diffusion Model



Formally, during the initial *ad-play stage*, information accumulates as:

$$dZ_1(t) = \mu_1\, dt + \gamma_1\, Z_1(t)dt + \sigma\, dW(t) \quad \text{for } 0 \leq t \leq 30. \tag{2}$$

The drift for this process is denoted μ_1 and can readily be interpreted as a measure of the utility difference between the advertised and originating app conveyed by the video ad. If the process strikes the upper (resp. lower) barrier, the users will click-thru (resp. click-back). After the ad finishes playing, the *post-ad* stage commences and the information accumulation process is allowed to differ, instead governed by μ_2 and γ_2:

$$dZ_2(t) = \mu_2\, dt + \gamma_2 Z_2(t)\, dt + \sigma\, dW(t), \quad \text{for } t > 30. \tag{3}$$

Even though the parameters change between the two stages, the sample paths of the two-stage model will always be continuous even at $t = 30$ (when the ad ends); this is illustrated in Fig. 3, which shows a sample path for the two-stage model.

Finally, we allow the initial value of the diffusion process, at $t = 0$, to be nonzero and vary randomly across users. Specifically, we assume that the initial position $Z_1(0)$ is a Gaussian random variable:

$$Z_1(0) \sim \mathcal{N}\left(\mu_0, \sigma_0^2\right) \tag{4}$$

where $\mathcal{N}(a, b)$ denotes a Gaussian distribution with mean a and variance b. The initial position $Z_0(t = 0)$ has an intuitive interpretation as a user's prior belief regarding the utility difference μ, and our specification allows these beliefs to vary across users. Following Fudenberg et al. (2018), we interpret μ_1 as the mean of the utility signals regarding the advertised app conveyed by the video ad itself. By contrast, μ_2 represents the mean of post-ad recollections as users compare the advertised app with the originating app. In our empirical specifications, we also accommodate a substantial amount of user heterogeneity by allowing the parameters μ_0, μ_1, and μ_2 to depend on user-level characteristics. Thus it is ultimately an empirical question how much μ_1 differs from μ_2, and how much these parameters vary across different users.

To interpret the leakage parameter γ_1 (the same argument holds for γ_2), note that, from the mathematical properties of diffusions, the position of the first stage process $Z_1(t)$, at any point in time, is a Gaussian random variable:

$$Z_1(t) \sim \mathcal{N}\left(\frac{\mu_1}{\gamma_1}\left(e^{\gamma_1 t} - 1\right), \frac{\sigma^2}{2\gamma_1}\left(e^{2\gamma_1 t} - 1\right)\right), \text{ for } 0 \le t \le 30. \tag{5}$$

In this equation, the sign of the leakage parameter γ_1 determines whether the mean and variance of the sample path $Z_1(t)$ converges (for $\gamma_1 < 0$) or diverges (for $\gamma_1 > 0$) as $t \to \infty$; in the absence of γ_1, the sample paths likewise diverge with probability one. As we will see below, these parameters play an important role for the model to generate the observed pattern of response times in Figure 2 above.

3.1.1 Parameter identification

Before proceeding, we provide some discussion regarding identification of the structural model parameters. We start with the simple "non-leaky" single-stage DDM, corresponding to Eq. (1) with $\gamma = 0$. In this setting, the parameters of the model are identified up to σ, using only two moments of the data: the mean response time,[14] and the click-through probability.

Lemma 1. *In the simple DDM, the drift μ and the bound B are identified using only two moments of the data: $\bar{t} \equiv \mathbb{E}[t^*]$, the mean response time, and $\bar{y} \equiv \mathbb{E}[y]$, the probability of click-through. The variance of the diffusion, σ^2, is not separately identified and we normalize $\sigma = \bar{\sigma}$. Then, the estimates $\hat{\mu}$ and $\hat{B}$ are:*

$$\hat{\mu} = \begin{cases} -\sqrt{\frac{\bar{\sigma}^2(2\bar{y}-1)\log\left(\frac{\bar{y}}{1-\bar{y}}\right)}{2\bar{t}}} & \text{if } \bar{y} < 0.5 \\ \sqrt{\frac{\bar{\sigma}^2(2\bar{y}-1)\log\left(\frac{\bar{y}}{1-\bar{y}}\right)}{2\bar{t}}} & \text{if } \bar{y} \geq 0.5 \end{cases} \tag{6}$$

$$\hat{B} = \frac{\bar{\sigma}}{2}\sqrt{\frac{2\bar{t}\log\left(\frac{\bar{y}}{1-\bar{y}}\right)}{2\bar{y}-1}} \tag{7}$$

The expression for $\hat{\mu}$ in Eq. 6 has components familiar from the discrete-choice literature: in particular, the log odds-ratio transformation $\bar{\sigma}\log\left(\frac{\bar{y}}{1-\bar{y}}\right)$ is the inverse of the binary logit choice probability $\bar{y} = \exp(\mu)/[1 + \exp(\mu/\bar{\sigma})]$, in which μ is the latent utility difference between the binary choice options. Unlike the binary logit model, which is a model only of choice, here Eq. (6) demonstrates how the estimated $\hat{\mu}$ is affected by the response time – namely, it is larger when the average response time $\bar{t}$ is shorter. The intuition is clear: when a choice is made rather quickly, the preference of the individual must be more intense.[15] For instance, at the sample averages of $\bar{y} \approx 0.03$ and $\bar{t} \approx 25s$, the corresponding utility difference $\hat{\mu} = -0.25$, but changes to $\hat{\mu} = -0.19$ when the average response time is increased to 45s.

Moreover, the impact of response time on the estimated utilities is the greatest when the choice probability is further away from 0.5 (see also Clithero (2018a)). In other words, estimated utilities are much more sensitive to response times precisely when the two choice probabilities

[14] Moreover, the non-leaky simple DDM predicts the *same* response time distribution conditional on choice $y = 0$ or $y = 1$ (see Shadlen, Hanks, Churchland, Kiani & Yang (2006)). Thus the distributions of response times that we observe in Figure 2 are much richer than required for identification of the basic DDM.

[15] To our knowledge, such identification results are new in the primarily experimental literature where DDM is prevalent. See Fudenberg et al. (2020) for additional identification results for DDMs.

are very unequal, which is exactly the case in our dataset. In such situations, data on reaction times is particularly valuable for identification.

Compared to this setting, our two-stage model contains additionally the parameters μ_0, σ_0, γ_1, μ_2, and γ_2. These parameters are pinned down by the differences in the relative frequency of click-thrus vs. click-backs in the two stages. Figure 2 shows that click-backs are relatively more frequent in the first stage, which suggests that the first-stage drift parameter should be more negative than the second-stage drift.

In addition, the γ parameters affect the variance of the accumulation process (cf. Eq. (5)), so that the higher-order moments of the distribution of response times provide identification for these parameters. For instance, as discussed previously, positive values of γ_1 imply that the mean and variance of the first stage diffusion process will diverge which, for the DDM, implies that the barrier will likely be passed before the ad finishes. But from Fig. 2, we see that most click-thrus and a large number of click-backs are made in the second stage, after the ad finishes, which suggests that $\gamma_1 < 0$, as otherwise the variance of the first stage diffusion process diverges and we should see many more choices made while the ad is playing.

4 Estimation Results

For the two-stage DDM, the optimal choices and response times are not characterized in closed forms.[16] Therefore, we utilize a simulated nonlinear least squares for estimation, in which we find structural parameter values which minimize the sum of squared differences (across all observations) between the observed and predicted values of choices and response times. Details on the estimation procedure are contained in the Appendix, section B.

4.1 Preliminary specifications

We start with a series of simplified preliminary models which, while descriptively incomplete for our dataset, nevertheless highlight the benefits of using response time data in conjunction with

[16] In the simple DDM model, however, closed-form expressions are available for the optimal choice probabilities and associated response times (Shadlen et al. (2006)), and software packages (Wiecki, Sofer & Frank (2013) or Drugowitsch (2016)) are available which exploit these closed forms for fitting the model to data. For the standard Ornstein-Uhlenbeck process proposed in Equation 1, a closed-form density of the first-passage time is only available when there is a single absorbing barrier, as opposed to two absorbing barriers here (Alili, Patie & Pedersen (2005)).

choice data in estimation. The results are reported in Table 2.

In Column (1), the parameters of the simple DDM are recovered using the equations in Lemma 1, which only requires two moments from the data — the mean response times and the overall click-through rate. The drift parameter of the simple DDM (column 1) is $\mu_1 = -0.253$, the negative value of which indicates a utility difference favoring the more frequent choice of $y = 0$ (click-back). The bound B is estimated to be 6.771, which is many times the standard deviation σ, which (as per Lemma 1) is fixed at 1 in estimation.

Table 2: Estimates from preliminary model specifications

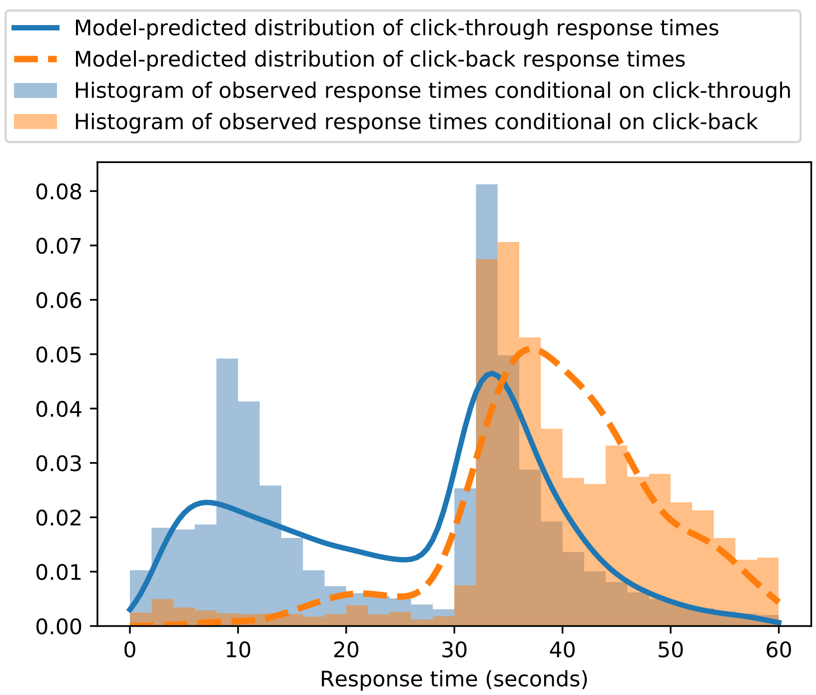
	(1) Simple DDM		(2) Binary Logit		(3) Simple 2-stage DDM (no covariates)	
	Est.	*(Std err)*	*Est.*	*(Std err)*	*Est.*	*(Std err)*
B, *Bound*	6.771	(0.0093)	—		7.003	(0.2390)
μ_0	—		—		-3.208	(0.1440)
μ_1	-0.253	(0.0133)	—		-0.169	(0.0081)
μ_2	—		—		-0.345	(0.0143)
σ_0	—		—		0.050	(0.0016)
σ	1.000	(fixed)	1.000	(fixed)	1.000	(fixed)
γ_1	—		—		-0.045	(0.0021)
γ_2	—		—		0.099	(0.0037)
u_1, *"utility diff"*	—		-3.419	(0.0118)	—	

In column (1), estimates are recovered using the Method of Moments estimators in Equations 6 and 7. Standard errors (enclosed in parenthesis) computed using the Delta Method. Columns (3) estimates obtained via Simulated MLE, with bootstrapped standard errors.

As counterpoint, in Column 2 we naïvely disregard the response time data and fit a binary logit model $P(y = 1) = [1 + \exp(-u_1/\sigma)]^{-1}$ using only the users' choice data. Comparing the estimated utility difference between the DDM and the binary Logit, we see that on the basis of choice data alone, the logit model requires a substantially larger utility difference (-3.419 $>>$ -0.253) to explain the asymmetric choice probabilities observed in the data. In the DDM, asymmetric choice probabilities such as in our data (3% click-throughs) need not imply a large utility difference – with long response times, very unequal choice probabilities can still be consistent with modest utility differences.[17]

[17] cf. Eq. (6) above, where longer response times is indicative of indifference of preferences.

Figure 4: Homogeneous User Estimates: Response Time Distributions.



Finally, in column 3, we consider a simplified specification of our two-stage DDM omitting all user-specific covariates (model parameters are assumed to be identical across users). The model fits both the choice and response time data well: the predicted click-through rate is 3.10%, which closely matches the average CTR in the data (3.17%). The model also provides a good fit of the response time distributions, matching both the bi-modal distribution of click-back response times and the uni-modal distribution of click-through times (Figure 4). Finally, the model also captures the finding that click-throughs tend to occur later than click-backs in the post-ad stage. We now proceed to our preferred specification, which incorporates user-specific covariates.

4.2 Full model results

Table 3 contains estimates from a specification in which μ_1 and μ_2, the drift terms in the model, as well as μ_0, the mean of the initial position of the diffusion process, are parameterized as linear indices in the observed covariates. We highlight several findings here.

The estimate of μ_0, the constant term in the mean of the distribution of users' initial values is roughly halfway between 0 and the lower boundary $-B$. This indicates that overall, users' prior inclinations towards the video ad are quite pessimistic. The leakage parameters γ_1 (-0.0643) and γ_2 (0.0421) are opposite in sign, which is expected given the patterns in response times conditional on choice presented in Figure 2 above. The negative value for γ_1 tends to dampen the diffusion process in the ad-play stage, which prevents too many users from clicking through while the ad is playing, but the positive value for γ_2 implies that the variance of the diffusion process diverges after the ad ends (cf. Eq. (5)) so that the diffusion process moves quickly towards the boundary B, causing users to make their choice.

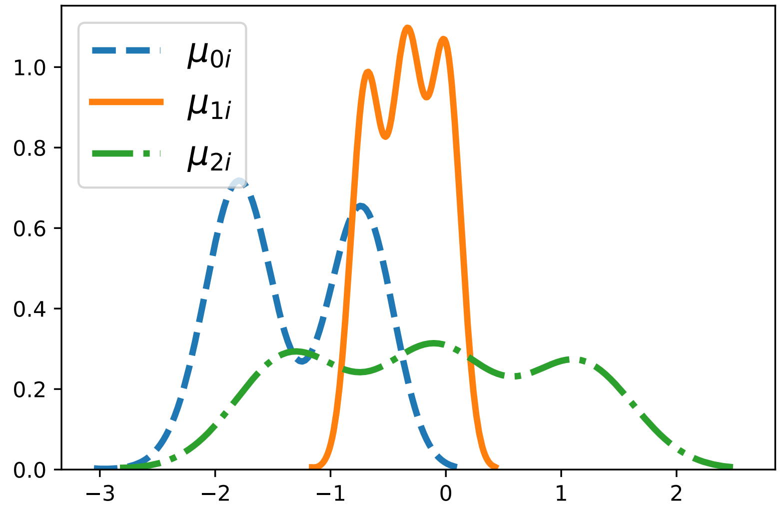
As the parameters μ_0, μ_1, and μ_2 are allowed to vary across users depending on a large number of user characteristics, to ease interpretation we present, in Figure 5, kernel density estimates of the distribution of these parameters across all users. There is considerably less heterogeneity in the drifts during the ad-play stage (μ_1) than during the decision stage (μ_2), which is reasonable as users' attention is directed towards to video ad during the first stage, but free to wander after the ad ends. Indeed, similar findings have arisen in "neuro-cinematics" studies in which brain activity among a set of viewers is more highly correlated during structured movie clips than during non-structured street and landscape scenes, when viewers' focus is

Table 3: Two-stage DDM model estimates

	Other Parameters		Parameters in μ_0		Parameters in μ_1		Parameters in μ_2	
B, Bound	6.8695	(0.0463)						
γ_1	-0.0643	(0.0053)						
γ_2	0.0421	(0.0007)						
σ			0.0312	(0.0036)	1.0000	fixed	1.0000	fixed
Constant			-1.3144	(0.0096)	-0.3357	(0.0082)	-0.1272	(0.0057)
iOS			0.0943	(0.0217)	0.1453	(0.0122)	-0.5284	(0.0137)
Samsung			0.0062	(0.0141)	0.0054	(0.0064)	0.0017	(0.0058)
Android Ver			-0.0141	(0.0083)	-0.0065	(0.0060)	0.0101	(0.0055)
iOS Ver			0.0217	(0.0107)	0.0423	(0.0121)	-0.0886	(0.0176)
Screen Res			-0.0167	(0.0093)	-0.0015	(0.0063)	-0.0027	(0.0058)
WiFi			0.0035	(0.0143)	-0.0022	(0.0062)	-0.0015	(0.0061)
English			0.4768	(0.0088)	0.1553	(0.0073)	-0.6178	(0.0066)
Publisher 1			0.0039	(0.0141)	0.0046	(0.0067)	0.0002	(0.0060)
Publisher 2			-0.0198	(0.0085)	0.0001	(0.0056)	-0.0010	(0.0054)
Publisher 3			0.0056	(0.0159)	0.0067	(0.0080)	-0.0006	(0.0065)
Publisher 4			-0.1146	(0.0085)	-0.0045	(0.0064)	-0.0018	(0.0054)
Publisher 5			-0.0210	(0.0082)	-0.0101	(0.0055)	-0.0107	(0.0055)
Log-Likelihood								

Bootstrapped standard errors in parentheses. All covariates are normalized to have mean zero and variance of 1.

Figure 5: Kernel density plot of the estimated drifts $\hat{\mu}_{0i}$, $\hat{\mu}_{1i}$ and $\hat{\mu}_{2i}$ for all users.



For each user i, we compute the drift parameters as $\hat{\mu}_{0i} = \boldsymbol{X}_i\hat{\boldsymbol{\beta}}_0$, $\hat{\mu}_{1i} = \boldsymbol{X}_i\hat{\boldsymbol{\beta}}_1$ and $\hat{\mu}_{2i} = \boldsymbol{X}_i\hat{\boldsymbol{\beta}}_2$

more likely to be idiosyncratic.[18]

The ranking among μ_0, μ_1, and μ_2 is also noteworthy. For most users, $\mu_0 < \mu_1$, suggesting that the signals conveyed by the ad are persuasive in the sense that they outstrip most users' prior inclinations regarding the advertised app. The ranking between μ_1 and μ_2 is more varied, there being users who experience post-ad signals which have lower, equal, or larger means compared to the ad. Since users in our sample are drawn from a variety of publishers' originating apps, this suggests that the influence of advertised app may be better or worse depending on which app the users were engaged with when they were shown the ad.

4.2.1 The Value of Response Time Data: Out-of-Sample Validation

A key contribution of this paper is to explicitly incorporate response times into a choice model to estimate agents' preferences. To gauge how important the response time data is, we compare the preference estimates from our model to those from a standard random utility model, which does not use response time in estimation. Specifically, we consider how well the preference

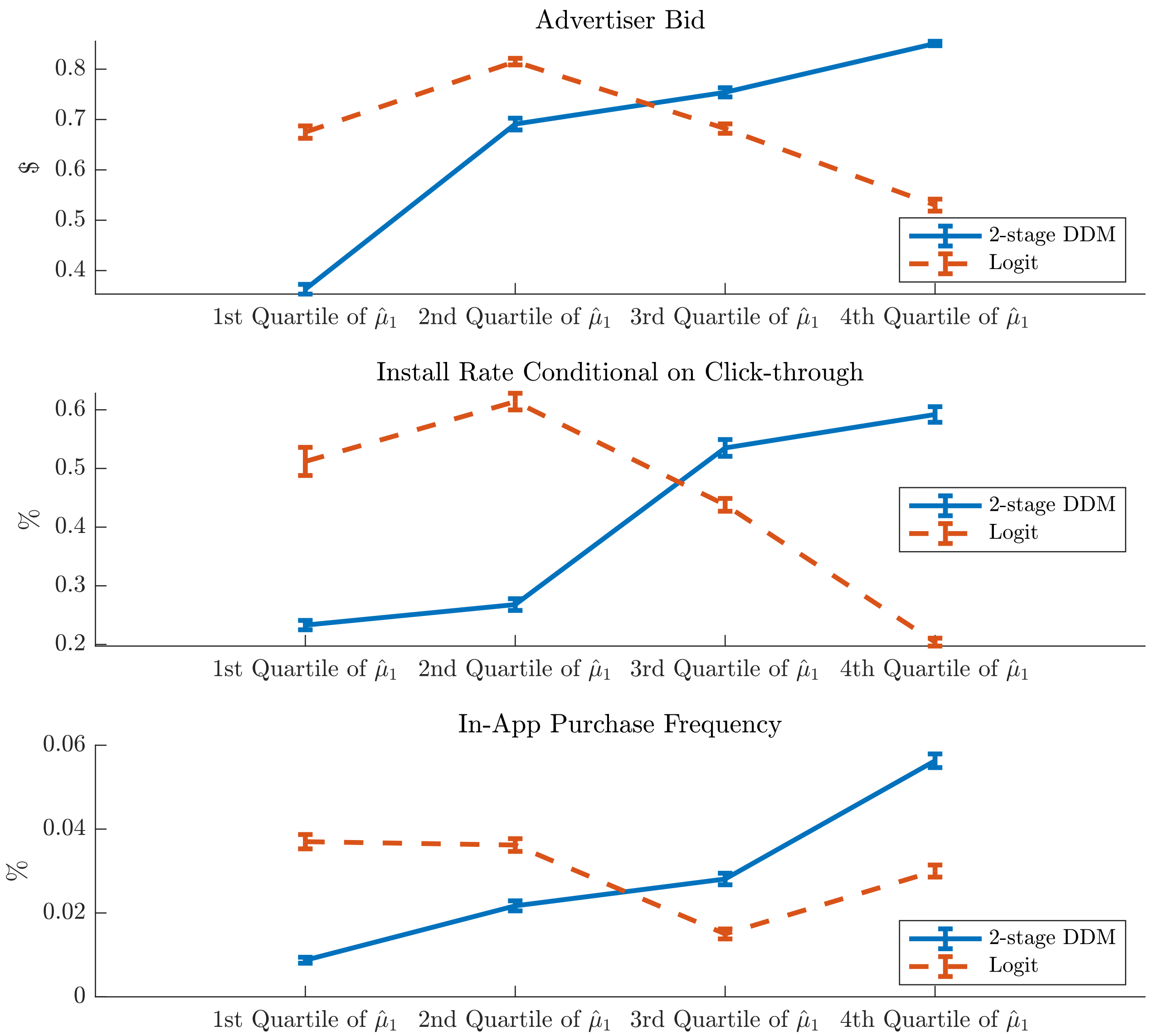
[18]For instance, Hasson, Landesman, Knappmeyer, Vallines, Rubin & Heeger (2008) show that viewers' brain activity is more highly correlated when viewing clips from the classic Western film "The Good, the Bad, and the Ugly" than street scenes from a park in New York City.

estimates relate to three additional "out-of-sample" covariates related to users' valuation and engagement with the advertised app. These variables are: (i) *Advertiser bid*, how much advertisers bid for each user; (ii) *Install*, an indicator for whether a user who clicks through also ends up installing the app; and (iii) *In-app purchase*, an indicator for whether a user who installs the app makes a cash purchase within the app (these purchases include upgrading to a premium version of the app, or buying tokens which allow the user to gain certain gameplay privileges). Perhaps most importantly, accurate predictions about each of these variables are relevant to both the advertiser and the platform.

For the two-stage DDM model, we focus on the estimates of μ_1, users' drift parameters during the ad-play stage, as this parameter reflects most closely the users' valuation for the advertised app. In Figure 6, the upward-trending blue lines show that μ_1 is *positively* related to all three of these variables: users with values of μ_1 in the upper two quartiles are more valuable to advertisers, as they elicit a higher bid (around \$0.80) from advertisers compared to below-median users in the lowest two quartiles (around \$0.50). Moreover, above-median users appear to be more engaged with the game, as they have a higher propensity to install the advertised app conditional on clicking-through (around 56% vs. 25%) and are also more likely to make cash purchases within the game.

For comparison, in Table 6, we also graph the analogous results for a binary logit model which only uses choice data for estimation (click-through vs. click-back). While this model is routinely used for choice prediction and to recover valuations from choice experiments, it performs rather poorly here: the orange lines in the figure generally trend downwards, implying that the utilities recovered from the logit model relate *negatively* to these three out-of-sample measures. That is, the users with above-median utilities from the logit model are actually those who fetch *lower* bids from advertisers, who are *less* likely to install the app conditional on click-through, and are *less likely* to upgrade the game after installation. Compared to the two-stage DDM, one problematic issue with the Logit model in this setting is that, by construction, it is agnostic about the timing of the decision: it cannot separate estimates during the ad-play stage from estimates during the post-ad stage, and instead must collapse this temporally-rich decision process into an instantaneous estimate of utility.

Figure 6: Relation between estimated preferences and out-of-sample engagement metrics.



Bars denote ± 2 standard deviations. The numerical values corresponding to graphs are reported in Table 6 in Appendix.

5 Counter-factuals:Does manipulating attention to ads increase clicks?

We use our estimates to simulate counterfactuals which address a long-running question as to whether drawing attention to an alternative results in higher demand. While most of the papers in the literature address the question using lab-experimental methods, we consider redesigns of the video ads in which viewers are forced to watch portions of the ad – that is, they are unable to "skip" the ad until a specified time is passed.[19] Indeed, such attempts at "requiring" users to pay attention to ads have proliferated in mobile advertising, via "non-skippable" ads in which users are required to play some portion of the ad before they are permitted to click back.[20] Non-skippable ads are encountered in a number of mobile and online platforms; for instance, YouTube interrupts videos with ads, and viewers are forced to watch the initial five seconds of the ad before they can skip back to the video that they were watching.

Based on our estimation results, we simulate a range of counterfactual scenarios where we vary the number of seconds that viewers are forced to watch the ad before they can click-back. In these scenarios, users are prevented from clicking-back during the first x seconds, with x ranging from 0 (corresponding to the benchmark skippable ad) to 15 seconds. In each case, we implement the counterfactual by removing the lower barrier during the initial x seconds of the ad, so that the diffusion process continues even when it goes below the barrier during those seconds. We refrain from going beyond 15 seconds, as forcing people to watch the ad for such a long time may lead to negative side-effects – ad annoyance, viewer inattention, and so on – which are not explicitly incorporated in our model.[21]

Table 4 shows the counterfactual click-through rates from the simulations. On average across all users, forcing them to watch the initial seconds of the ad indeed increases the click-through rate (Column A). However, the difference is small – compared to 3.07% at the zero-second benchmark, we see that forcing users to watch the first 6 seconds increases the CTR only to 3.08%, and watching 15 seconds raises the CTR up to 3.22%, an increase of only

[19]For instance, Krajbich et al. (2010) and Krajbich (2019) have performed eye-tracking experiments which show that increased visual attention/fixation to an object indeed increases the propensity of subjects to choose it by 1-2%. The effects were concentrated in a sub-set of subjects for which the manipulation was most effective (measured by the amount of time they attended to the option.

[20]Dukes, Liu & Shuai (2018) present a theoretical analysis of non-skippable ads.

[21]Moreover, ads in which users are forced to watch for such a long amount of time are typically “incentivized”, so that viewers are given rewards in the game (an extra life, time, or in-game currency) in return for viewing such an ad.

Table 4: Counterfactual click-through rates, from making ad partially non-skippable

Treatments:	(A)	(B)	(C)	(D)	(E)
Force users to watch: (secs.)	***Counterfactual click-through probabilities (average) across:*** All users	1st qtile $\hat{\mu}_1$	2nd qtile $\hat{\mu}_1$	3rd qtile $\hat{\mu}_1$	4th qtile $\hat{\mu}_1$
0 (benchmark)	3.09%	2.63%	2.59%	3.04%	4.00%
2	3.09%	2.64%	2.59%	3.04%	4.00%
4	3.09%	2.65%	2.59%	3.04%	4.00%
6	3.11%	2.69%	2.61%	3.04%	4.00%
8	3.14%	2.77%	2.63%	3.06%	4.00%
10	3.19%	2.92%	2.69%	3.09%	4.00%
12	3.26%	3.08%	2.76%	3.13%	4.00%
15	3.42%	3.56%	2.90%	3.19%	4.00%

0.15% relative to the benchmark.

But these aggregate results mask a lot of heterogeneity across users. In columns (B)-(E) of Table 4, we report the counterfactual click-through rates separately for groups of users classified by their estimated values of μ_1 (as in Table 6 above). Clearly, the "marginal" clicks gained by forcing users to watch the initial seconds of the ad are primarily those in the lowest two quartiles of the distribution of estimated μ_1: CTRs in the two lowest quartiles increased the most, but are practically unchanged in the upper two quartiles. These results are consistent with laboratory studies which document choice effects of roughly 1-2% in the subset of subjects for which the manipulation was most effective, and null effects in the remaining sample (Krajbich, 2019).

Moreover, as we saw in Table 6 above, the users with lower values for μ_1 were less valuable for advertisers, as these users generated lower bids from advertisers. Apparently, the additional users who click-through when they are forced to watch some of the ad are not those which the advertisers are keenest to contact. This is confirmed in Table 5, which reports the average advertising revenue per thousand impressions (corresponding to the CPM measure employed in industry), across all users as well as broken down by quartiles according to the estimated value of μ_1. The CPM's in the table lie squarely within the ballpark for video ads; for instance,

Table 5: Counterfactual click-through revenues, from making ad partially non-skippable

Treatments:	(A)	(B)	(C)	(D)	(E)
Force users to watch: (secs.)	***Counterfactual revenue per thousand impressions (average) across:*** All users	1st qtile $\hat{\mu}_1$	2nd qtile $\hat{\mu}_1$	3rd qtile $\hat{\mu}_1$	4th qtile $\hat{\mu}_1$
0 (benchmark)	\$9.43 (0.00%)	\$1.71 (0.00%)	\$4.80 (0.00%)	\$9.83 (0.00%)	\$20.20 (0.00%)
2	\$9.43 (0.00%)	\$1.71 (0.00%)	\$4.80 (0.00%)	\$9.83 (0.00%)	\$20.20 (0.00%)
4	\$9.43 (0.00%)	\$1.72 (0.58%)	\$4.80 (0.00%)	\$9.83 (0.00%)	\$20.20 (0.00%)
6	\$9.45 (0.21%)	\$1.74 (1.75%)	\$4.81 (0.21%)	\$9.85 (0.20%)	\$20.20 (0.00%)
8	\$9.48 (0.53%)	\$1.80 (5.26%)	\$4.85 (1.04%)	\$9.90 (0.71%)	\$20.20 (0.00%)
10	\$9.55 (1.27%)	\$1.89 (10.53%)	\$4.92 (2.50%)	\$9.99 (1.63%)	\$20.20 (0.00%)
12	\$9.63 (2.12%)	\$1.99 (16.37%)	\$5.03 (4.79%)	\$10.11 (2.85%)	\$20.20 (0.00%)
15	\$9.79 (3.82%)	\$2.31 (35.09%)	\$5.20 (8.33%)	\$10.31 (4.88%)	\$20.20 (0.00%)

the CPM for YouTube was $7.50 in 2014.[22] The results show that re-designing the ad to be non-skippable has a much larger effect on users with below-median μ_1 ($+11.4\%$ moving from 0 to 15 seconds) than above-median μ_1 ($+0.6\%$). Overall, the aggregate revenue increase from making the ad non-skippable is rather small, equal to a $0.13 increase in CPM (the revenue per thousand impressions).

6 Conclusions

We study how choice and response time data from the field can be combined to estimate a structural model of smartphone users' responses to mobile advertisements. We consider a two-stage drift-diffusion model in which the combination of response time with choice data allows separate identification of the diffusion processes characterizing users' preferences when the ad is playing, as well as when users face a subsequent decision to click-through on the ad. Preferences estimated from our two-stage model corroborate external measures of users' preferences toward and valuations of the advertised app, in contrast with those from the conventional logit choice model (which only uses users' choice data but not the response times). This provides validation for our two-stage DDM and, more broadly, supports the usefulness of

[22] See Kaufman (2014).

incorporating endogenous response time data into choice analyses.

Using our estimates, we assess the effectiveness of attention manipulations on demand by means of a counterfactual requirement to watch the initial portion of an ad before exiting, corresponding to the "non-skippable" ads used on some online platforms. We find that while the click-through rates would be higher if the ad were made non-skippable for the initial 10-15 seconds, the overall benefits are modest. These results are consistent with lab-based experiments which also find only modest benefits from manipulating attention on demand. The reason for this is that the marginal clicks gained by making the ad non-skippable are, overall, those who are less persuaded by the ad. These users also hold the least value to advertiser, in terms of our out-of-sample spending and engagement metrics. This suggests that forcing users to watch part of an ad may not generate much revenue for publishers; on the contrary, such a practice may even annoy users and hence be counterproductive.

We offer several concluding remarks. The two-stage DDM employed in this paper can be enriched in a number of ways. For instance, Fudenberg et al. (2018) show that a DDM with time-varying barriers can be consistent with optimal sequential sampling under uncertainty, and we are exploring that in ongoing work. More broadly, the use of structural estimation and modeling to address the effects of skippable vs. non-skippable ads is novel relative to existing methodologies for determining policy effects in online and mobile platforms.[23] Our structural model allows us to extract information from endogenous variables like response time; otherwise, simply including endogenous response times as an explanatory variable in a regression leads to difficulties in both interpretation of parameters and choice prediction.

[23] See, for instance, Lewis & Rao (2015).

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A Proof of Lemma 1

Proof. Derivation from Shadlen et al. (2006), Equations (10.23) and (10.33), shows that the simple DDM $dZ(t) = \mu dt + \sigma dB(t)$ implies:

$$\bar{y} = \mathbb{E}[y_i] = \frac{1}{e^{-2B\mu/\sigma^2} + 1} \tag{8}$$

$$\bar{t} = \mathbb{E}[t_i^*] = \frac{B\left(e^{B\mu/\sigma^2} - e^{-B\mu/\sigma^2}\right)}{\mu\left(e^{B\mu/\sigma^2} + e^{-B\mu/\sigma^2}\right)} \tag{9}$$

To see that σ^2 is not identified through the moments above, suppose we multiply σ^2 by k^2. Then if we multiply B by k and multiply μ by k, the two equations above remained unchanged. That is, (σ^2, B, μ) and $(k^2\sigma^2, kB, k\mu)$ are both solutions to the equations above.

First, we solve for μ and B by normalizing $\sigma = 1$. If we normalize σ to be $\sigma = \bar{\sigma}$, then $(\bar{\sigma}B, \bar{\sigma}\mu)$ would be the corresponding solution. Given the normalization $\sigma = 1$, when we solve for these equations in terms of μ and B, there are only two real solutions. Among the two solutions, one of the solutions is such that $\hat{\mu}$ is increasing in $\bar{y}$, and the other solution is such that $\hat{\mu}$ is decreasing in $\bar{y}$. Therefore the first solution is the valid one.

Moreover, when we solve for the first solution for μ, we get:

$$\hat{\mu} = \sqrt{\frac{2\hat{y} - 1}{2t}}\sqrt{\log\left(\frac{\hat{y}}{1 - \hat{y}}\right)} \tag{10}$$

Now when $\hat{y} < 0.5$, both $\frac{2\hat{y}-1}{2t}$ and $\log\left(\frac{\hat{y}}{1-\hat{y}}\right)$ are negative. Therefore, we can rewrite Equation 10 as $\hat{\mu} = \sqrt{-1}^2\sqrt{\left|\frac{2\hat{y}-1}{2t}\right|}\sqrt{\left|\log\left(\frac{\hat{y}}{1-\hat{y}}\right)\right|} = -\sqrt{\frac{(2\bar{y}-1)\log\left(\frac{\bar{y}}{1-\bar{y}}\right)}{2\bar{t}}}$. When $\hat{y} > 0.5$, both $\frac{2\hat{y}-1}{2t}$ and $\log\left(\frac{\hat{y}}{1-\hat{y}}\right)$ are positive, and we have $\hat{\mu} = \sqrt{\frac{(2\bar{y}-1)\log\left(\frac{\bar{y}}{1-\bar{y}}\right)}{2\bar{t}}}$. This leads to the following solution (which we additionally verified using numerical solvers):

$$\hat{\mu} = \begin{cases} -\sqrt{\frac{(2\bar{y}-1)\log\left(\frac{\bar{y}}{1-\bar{y}}\right)}{2\bar{t}}} & \text{if } \bar{y} < 0.5 \\ \sqrt{\frac{(2\bar{y}-1)\log\left(\frac{\bar{y}}{1-\bar{y}}\right)}{2\bar{t}}} & \text{if } \bar{y} \geq 0.5 \end{cases}$$

$$\hat{B} = \frac{1}{2}\sqrt{\frac{2\bar{t}\log\left(\frac{\bar{y}}{1-\bar{y}}\right)}{2\bar{y} - 1}}$$

□

B Details of estimation procedure

The structural parameters of the two-stage DDM are estimated by a simulated nonlinear least squares procedure, which we describe in this section. Let $\tau = \{t_0 = 0, t_1, t_2, \ldots\}$ denote a grid of time-points. We discretize time from 0 to 60 seconds into increments of $\Delta t = t_{j+1} - t_j$, equal to a half-second ($\Delta t = 0.5$) for our estimation.[24] For each candidate parameter vector $\theta = (\mu_0, \sigma_0, \mu_1, \mu_2, \gamma_1, \gamma_2, B)$,[25] we draw a starting point Z_0 according for Eq. (4). Let $\tilde{Z}(t)$ denote the overall diffusion process; i.e. it is equal to $Z_1(t)$ for $t \leq 30$ and $Z_2(t)$ for $t > 30$. Then the increments of the accumulation process can be recursively simulated for each $t_j \in \tau$ and user i as:

$$\tilde{Z}_i(t_j) - \tilde{Z}_i(t_{j-1}) = \begin{cases} \mu_1 \Delta t + \gamma_1 \tilde{Z}_i(t_{j-1}) \Delta t + \sigma \Delta W_i(t_j), & \text{for } t \leq 30 \\ \mu_2 \Delta t + \gamma_2 \tilde{Z}_i(t_{j-1}) \Delta t + \sigma \Delta W_i(t_j), & \text{for } t > 30 \end{cases} \tag{11}$$

The increments of the Brownian motion $\Delta W_i(t_j)$ is drawn i.i.d from $\mathcal{N}(0, \Delta t)$ for each $t_j \in \tau$. In consideration of Lemma 1 above, we normalize $\sigma^2 = 1$.

We observe (y_i, t_i^*, X_i), the choice and covariates, for each user i. Correspondingly, for each user, we generate S (=20000) independent sample paths for the diffusion process. For each user i and each sample path $s = (1, \ldots, S)$, we record $(y_{i,s}, t_{i,s}^*)$, where $t_{i,s}^*$ is the time the sample process s first hits either the upper bound or the lower bound (whichever comes first) for user i. Similarly, $y_{i,s} = 1$ if the sample process hits the upper bound at $t_{i,s}^*$ and $y_{i,s} = 0$ otherwise. For each user, we compute the predicted choice and response time as respectively $\tilde{y}_i(\theta) \equiv \frac{1}{S}\sum_{s=1}^{S} y_{i,s}$ and $\tilde{t}_i^* \equiv \frac{1}{S}\sum_{s=1}^{S} t_{i,s}^*$. Thus we estimate θ by minimizing the sum of squared differences across all users:

$$\min_{\theta} \sum_{i=1}^{n} (y_i - \tilde{y}_i(\theta))^2 + (t_i^* - \tilde{t}_i^*(\theta))^2.$$

Despite the large number of simulations, this problem is highly parallelizable and we utilize GPU computing to simulate the diffusion process. We use an approach similar to Norets (2012) to avoid repeated simulations at each parameter value, further reducing the computational burden. This procedure exploits the assumed index form on the user-specific drifts, i.e. $\mu_{i0} = \boldsymbol{X}_i \boldsymbol{\beta}_0$ and $\mu_{i1} = \boldsymbol{X}_i \boldsymbol{\beta}_1$. We pre-solve and store the mapping between the parameter vector $\theta = (\mu_0, \sigma_0, \mu_1, \mu_2, \gamma_1, \gamma_2, B)$ and the outcome variables of interest: (i) the average response time and (ii) the probability of click-through. Subsequently, we approximate the expected choice and response time at a candidate parameter vector $(\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \gamma_0, \gamma_1, B)$ as a weighted average of pre-solved choices and response times, computed at the k-nearest neighbors in the space of $(\mu_0, \mu_1, \gamma_0, \gamma_1, B)$.

C Engagement Metrics Reported in Table 6

[24] We have performed simulations to ensure that this discretization is accurate enough for our purposes.

[25] As explained below, in the preferred specification we allow the drift terms μ_1 and μ_2 to be linear functions of user-specific covariates.

Table 6: Users' estimated preferences and out-of-sample variables

DDM parameter estimators (first-stage drift μ_1) and external engagement metrics				
	1st quartile μ_1	2nd quartile μ_1	3rd quartile μ_1	4th quartile μ_1
Advertiser bid	\$0.363	\$0.691	\$0.754	\$0.851
	(0.00964)	(0.0117)	(0.00914)	(0.00441)
Install rate conditional	0.233	0.268	0.535	0.592
on click-through	(0.00799)	(0.0100)	(0.0143)	(0.0133)
In-app purchase	0.00873	0.0217	0.0281	0.0563
frequency	(0.000712)	(0.00120)	(0.00138)	(0.00163)

Logit estimates and external engagement metrics				
	1st quartile μ_1	2nd quartile μ_1	3rd quartile μ_1	4th quartile μ_1
Advertiser bid	\$0.675	\$0.815	\$0.682	\$0.530
	(0.0123)	(0.00655)	(0.00936)	(0.0119)
Install rate conditional	0.512	0.614	0.438	0.204
on click-through	(0.0240)	(0.0142)	(0.0109)	(0.00671)
In-app purchase	0.0370	0.0362	0.0150	0.0300
frequency	(0.00171)	(0.00152)	(0.00119)	(0.00144)

Standard errors in parentheses.