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Tenth International Olympiad, 1968

1968/1.

Prove that there is one and only one triangle whose side lengths are consecutive integers, and one of whose angles is twice as large as another.

1968/2.

Find all natural numbers x such that the product of their digits (in decimal notation) is equal to $x^2 - 10x - 22$

1968/3.

Consider the system of equations

$ax_1^2 + bx_1 + c = x_2$
$ax_2^2 + bx_2 + c = x_3$
...
$ax_{n-1}^2 + bx_{n-1} + c = x_n$
$ax_n^2 + bx_n + c = x_1$

with unknowns $x_1, x_2, ..., x_n$, where a, b, c are real and $a \neq 0$. Let $\Delta = (b-1)^2 - 4ac$. Prove that for this system

1. if $\Delta < 0$, there is no solution,
2. if $\Delta = 0$, there is exactly one solution,
3. if $\Delta > 0$, there is more than one solution.

1968/4.

Prove that in every tetrahedron there is a vertex such that the three edges meeting there have lengths which are the sides of a triangle.

1968/5.

Let f be a real-valued function defined for all real numbers x such that, for some positive constant a, the equation

$$f(x+a) = \frac{1}{2} + \sqrt{f(x) - [f(x)]^2}$$

(a) Prove that the function f is periodic (i.e., there exists a positive number b such that $f(x+b)=f(x)$ for all x).

(b) For $a=1$, give an example of a non-constant function with the required properties.

1968/6.

For every natural number n, evaluate the sum
$\sum_{k=0}^{\infty}\left\lfloor \frac{n+2^k}{2^{k+1}} \right\rfloor = \left\lfloor \frac{n+1}{2} \right\rfloor + \left\lfloor \frac{n+2}{4} \right\rfloor + ... + \left\lfloor \frac{n+2^k}{2^{k+1}} \right\rfloor + ...$ (The symbol [x] denotes the greatest integer not exceeding x.)