5.1 Introduction

Consider a doctor moving around in a big hospital attending patients. S/he may have to be paged frequently by the monitoring staff attached to critical patients. A service personal carrying a pager often have to be tracked and located in order to attend service requirements of the clients. Ambulances are usually fitted with tracking devices to lead the paramedicos to attend the emergencies and the rescue operations, and ferrying the critical patients to the nearest medical assistance centers or hospitals. Similarly, police petrol vehicles are always equipped with tracking devices, so that the control room can locate the nearest petrol to rush to a place of crime, disturbances or a distress call as fast as possible. In commerce and business, a timely response could provide a competitive edge to a business establishment over its rivals. Therefore, more often then not such organizations equip the executives and the field representatives with mobile cellular phones, or PDAs. All these mobile devices need to be tracked and located to communicate with the users. In the context of computing mobility assumes further dimensions such as a laptop or PDA user continue to access shared file systems or data repositories while on the move in cars, aircrafts or ships. The ramifications of the suggested movements associated with computing can be witnessed in migration of software (called mobile agents), transaction models with mobility support, collaborative computing, etc. Some of the issues are discussed in other chapters of this text. But the key to the various issues arising out mobility support is the tracking of the mobile objects. The object can be an automobile, a cellular phone, a PDA, a laptop or even a piece of software. The location management is concerned with the ability to track and/or locate a moving object with an intention to communicate.

At an abstract level location management involves two operations which are also found to be basic to the usage of a database.

- Look up or search, which must be invoked each time a mobile object is to be located.

- Update, which is invoked each time the object moves.

But there are significant differences in the operations with a database and the location management. The differences are

- A database has precise data. But the location management must have to deal with the imprecision of various forms in location data.

- The update requirements in maintaining location data will depend on the degree of tolerance in of imprecision in the location of a mobile object. Whereas the update requirements in a database is directly linked with the consistency problem and, therefore, have to be done when needed.

- The scale of updates in location management is huge. This basically relates to the fact that there is absolutely no control in proliferation of mobile devices. For example, in a cell size of about half a kilometer radius in an average a thousand to thousand and five hundred mobile devices may be active. Assuming every object makes about five moves in a day, the total number of updates exceeds 10,000 in just one cell calculated at the rate of one insertion and one deletion per move per mobile object.

The three dimension of the approaches to the management of location information and its use can thus be identified as

- *Availability* – refers to availability of location information at all network sites or at some selected places in the network.

- *Imprecision* – refers to the exactness of the location information.

- *Currency* – refers to the regularity of location updates.

An abstract view of the space of location management problem is depicted in figure 5.1. If exact information is maintained each and every network site, the updates have to be tied with every move and disseminated to all the sites, whereas look up becomes immediate. The other extreme case could be no information about location of any object is maintained anywhere. This case is represented by the origin of the frame of reference shown in the above figure. The search for a mobile object in this situation becomes an exhaustive search at each network site. In between the above two extremities several combination of approaches regarding location update and look up can be possible.

It may be mentioned here that the implicitly assumption is location management is supported by a cellular network architecture acting as the backbone. Of course, cellular architecture is not the only possibility. But it is a good candidate to understand the issues that arise in the location management. In absence of a infrastructures backbone architecture, an alternative
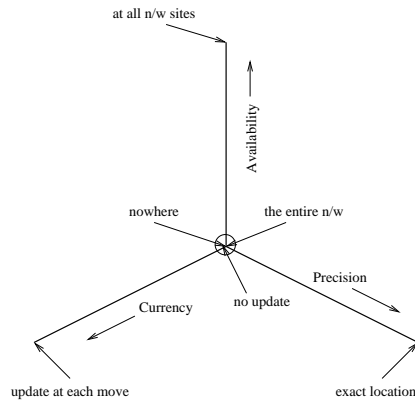
Figure 5.1: The space of location

approach such as global positioning system has to be used to track or locate mobile objects. In the discussion here we assume the mobility support is provided by cellular architecture. The location management is handled in the data link or network layer.

5.1.1 Registration and Paging

There are two distinct types of mobilities involving mobile devices. *Terminal mobility* allows a terminal to be identified by some identity independent of its point of attachment in the network. It allows the terminal to be attached to different points of the fixed network at different time. Therefore, allows mobility with respect to point of attachment with the fixed network.

Personal mobility is associated with a user. A user gets a distinct identity which is independent of terminal s/he uses as well as the point of attachment of that terminal in the network. Personal mobility allows the users to receive or make calls any where through any terminals. To facilitate the tracking, each mobile user has to explicitly register itself to notify the server or the sites on the network fixed about its location. The location registers stores these location data. The granularity of location can be a single cell to a group of cells. When the location is known a user can be tracked by paging. The paging process is a polling by the system by sending signals to likely locations to track the users. By changing the size of registration area, flexibility of combination of paging and registration can be attained.

5.2 Two Tier Structure

In two tier scheme [9] there are two location registers per mobile hosts, (i) *home location register* (HLR) and (ii) *visitors location register* (VLR). Every mobile user is associated with a HLR which is located at a network location prespecified for each user. It is normally a zone where the user was registered first, and may usually be dependent on the normal area of movement of the user. A HLR maintains the current location of an user. Hence, a HLR gets updated every time the user makes a move. Each zone also maintains a set of VLRs each of which is associated with a user currently active in that zone but is not a resident of the zone.

In HLR-VLR scheme the call setup is quite straightforward. A user A in location area LAI_i wanting to communicate with another user B in cell belonging to LAI_j, then the VLR in LAI_i is searched first. If there is no entry for B, the search is then directed to the HLR maintained at LAI_b which is the home location area of B. Thus, the update process coupled with movement and supporting the call setup procedure could be as follows:

- When a user moves from a cell in LAI_i to a cell under LAI_j then the HLR of the user in its home location area gets updated, and

- A VLR entry for the user is made in LAI_j, besides deleting the entry for the user from VLR of LAI_i.

Figure 5.2 illustrates the movement of user A from cell under LAI_i to a cell under LAI_j. The LAI_a denotes the home location of A.

5.2.1 Drawbacks of Fixed Home Addresses

The home location is permanent. Thus long-lived objects can not change home, i.e., resettlement at a different neighbourhood is not allowed. The two-tier approach does not scale up well with highly distributed system. Often the distant home locations may have to be contacted for for look up even in case of moderately mobile objects. Thus, the locality of the moves is not captured well.
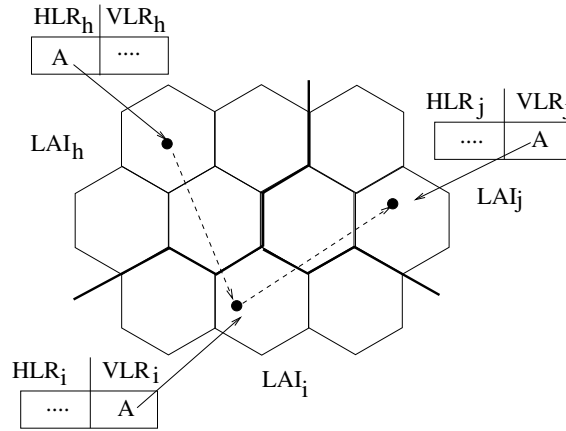
Figure 5.2: Move in two-tier scheme

5.3 Hierarchical Scheme

Hierarchical schemes [4] are designed to take advantage of locality. The location database at an internal node contains location information of users in the set of zones under the subtree rooted at the node. The information stored may simply be a pointer to the lower-level location server or the current location. A leaf node serves a single zone. The organization of location servers in a tree-base hierarchical scheme is illustrate in figure 5.3. A zone can be considered as a location area or a single cell served by a mobile support station.

The performance of any location management scheme is clearly dependent on two factors, namely,

1. The relative frequency of moves of the users.

2. The calls received by each user.

Based on these factors we can define two performance metrics:

(i) *local call to mobility ratio*, and

(ii) *global call to mobility ratio*.

Let C_i be expected number of calls received, and let U_i be location area crossings made by a mobile user over a period of time T. The fraction C_i/U_i
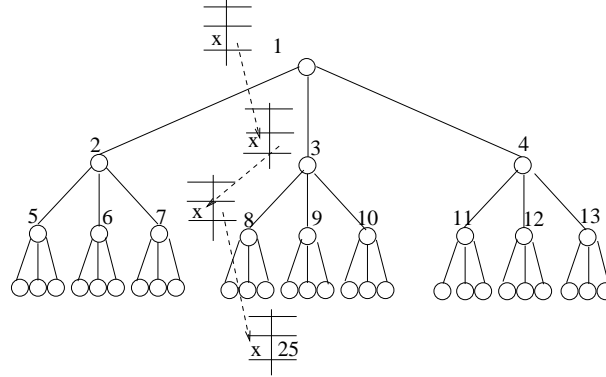
Figure 5.3: Tree-based organization of location servers

is the global call to mobility ratio for time T. If C_{ij} represents expected number of calls from location LAI_j to the user in time T, then the local call to mobility ratio (LCMR) is the ratio C_{ij}/U_i. Local call to mobility ratio $LCMR_{ij}$ at any internal node j of tree hierarchy can be computed as $\sum_k LCMR_{ik}$, where k is a child of j.

5.3.1 Update Requirement

When location information at internal nodes in the hierarchy are stored as pointers, and a MH A moves from a cell in LAI_i under location server LS_i to a new location, say to the cell LAI_j under location server LS_j, the tree paths down:

- from $LCA(LAI_i, LAI_j)$ to LAI_i and

- from $LCA(LAI_i, LAI_j)$ to LAI_j

should be updated.

For example in the figure 5.3 each node represents a location server. Each leaf servers store location information of mobile users in one location area. When a mobile user mu moves from a cell in the location area under server 25 to a cell in location area under server 26, then updates will be needed at the nodes 3, 8, 9, 25, 27.

- At 8, 25 entries for mu should be deleted.

- At 3 entry for *mu* should be updated.

- At 9, 27 new entries for *mu* should be made.

If actual cell id information is stored at each node from the root to leaf, then path from the root to $LCA(LAI_i, LAI_j)$ also has to be updated. For example, at the time *mu* was in the cell under 25, nodes 1, 3, 8, 25 had entries pointing to cell 25 which indicates that *mu* was in cell under 25. So, when *mu* moves to cell under 27, all the nodes 1, 3, 9, 27 must be updated to indicated *mu* is now under cell 27.

5.3.2   Lookup in Hierarchical Scheme

The look up operation in hierarchical scheme can start with search initiated at the neighbourhood, i.e., bottom up in the tree hierarchy. If the mobile object being search for is not found in the current level of the hierarchy then the search process is just pushed up one level up the hierarchy. Since the root maintains information about all the objects, the look up is bound to succeed at a level of the hierarchy. In particular, suppose a search is initiated at a cell under LAI_i for the mobile object which is currently location in a cell in LAI_j. Then, look up starts at LAI_j itself and finally reaches $LCA(LAI_i, LAI_j)$ where an entry for the object will be found. After that following that pointer the search is guided down the hierarchy along the path $LCA(LAI_i, LAI_j)$ to LAI_i, and the object is thus found.

5.3.3   Advantages and Drawbacks

The tree-based hierarchical scheme does not require HLR and VLR type stores to be maintained at every cell. There is support for locality of call setup process. On the other hand, updates have to be done at a number of location servers on different levels of hierarchy. The location information for each mobile object has to replicated at each node on the tree path from the root to the leaf node corresponding to the location area in which the mobile object is located. The load for updates increases monotonically at internal nodes up with the hierarchy. The storage requirement also increases at higher levels.

5.4 Caching

Caching in location management is used primarily to reduce the lookup cost. It also helps to reduce the delay in establishing links. The idea is based on conventional use of caching.

In the two-tier architecture, when a large number of calls originate from a particular MSC to a particular mobile belonging to a different MSC, then the ID of the mobile and its serving VLR's address can be stored at the calling MSC. This helps not only reduce the signaling cost but also to reduce the delay in establishing the connection. Each time a call is attempted from the calling MSC overhead associated with caching is as follows:

- First, the cached information of VLR is checked at calling MSC. If a cache hit occurs, the VLR of the callee is contacted directly.

- When the callee mobile moves out, cache miss will occur. Then the HLR of the callee is contacted to establish the call.

Checking cache at calling MSC adds little to overhead. On the other hand, if cache hit occurs the saving on signal cost and improvement on establishment of calls will be substantial. However, when callee's mobile crosses over to a new location area, old cache entry becomes invalid. So a cache revalidation will be needed by updating the old entry.

From point of view of pure location update (no cache), new VLR address should registered with HLR [5] while old VLR must send cancellation of registration to HLR. So, the total cost is given by:

$$\text{update}_{nocache} = \text{cost}(\text{VLR}_{new} \leftrightarrow \text{HLR}) + \text{cost}(\text{VLR}_{old} \leftrightarrow \text{HLR})$$

In absence of cache, HLR is queried which provides the address of VLR of the new region where the callee mobile is found. So, the cost of lookup is:

$$\text{search}_{nocache} = \text{cost}(\text{VLR}_{caller} \leftrightarrow \text{HLR}) + \text{cost}(\text{HLR} \leftrightarrow \text{VLR}_{callee})$$

If the expected number of calls from a particular MSC to a particular mobile is ρ, then the total cost for establishing calls will be

$$\text{update}_{nocache} + \rho \times \text{search}_{nocache}$$

We need to get an estimate of the expected number of calls ρ received by a mobile from the area under an MSC. This estimation requires analysis of

call traces. A study [7] reveals that a user typically receives 90% of calls from his/her top 5 callers. A location area can be ranked according to the number of calls a user receives from that area. Let f_k denote the proportion of calls the user receives from the location area ranked $k = 0, 1, \ldots, N_{lai}$. Let us assume that call arrival times to a mobile in a service area is exponentially distributed with mean arrival rate λ. Then probability distribution function is: $\lambda e^{-\lambda t}$ Also the residence time of a mobile in a location area be exponentially distributed with mean residence time $1/\mu$, i.e., the probability distribution function for the residence time is $\mu e^{-\mu t}$

Let p_{cache} represent the probability that cached information for a mobile at a location area is correct. In other words, p_{cache} defines the probability that mobile has not moved from its current location since it received the last call from same location area. So,

$$p_{cache} = \text{Prob}[t < t_1] = \int_0^\infty \lambda e^{-\lambda t} \int_t^\infty \mu e^{-\mu t_1} dt_1 dt = \frac{\lambda}{\lambda + \mu}$$

Let C_B represent the lookup cost of connection setup in basic scheme (no caching is used), and C_H be the cost when caching is used. Note that C_H is the cost of cache hit in VLR of calling MSC and retrieving information from callee's VLR. C_B, on the other hand, involves (i) the cost of invalidating VLR entry of calling MSC when mobile crosses to a new location, and (ii) the cost of updating mobile HLR entry.

$$\begin{aligned} C_H &= \text{cost}(VLR_{caller} \leftrightarrow VLR_{callee}) \\ C_B &= \text{cost}(VLR_{caller} \leftrightarrow HLR) + \text{cost}(HLR \leftrightarrow VLR_{callee}) \end{aligned}$$

A cost saving is achieved in caching scheme only if

$$p_{cache} C_H + (1 - p_{cache})(C_B + C_H) \le C_B,$$

where p_{cache} is the probability of a cache hit. From the above inequality, we find that $p_T = \min\{p_{cache}\} = C_H/C_B$. This implies that if caching to be useful in saving cost, the threshold p_T for cache hit should be:

$$p_{cache} > p_T \ge C_H/C_B$$

Calling MSC typically uses Local Call to Mobility Ratio (LCMR) for determining if caching would be cost effective. LCMR is equal to λ/μ. Relating LCMR to threshold for cache hit, we have
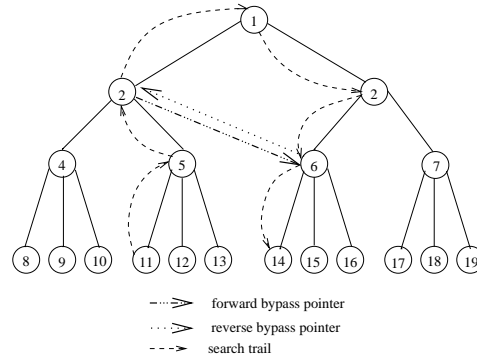
$$\text{LCMR}_T \ge p_T/(1 - p_T)$$

Figure 5.4: Caching in hierarchical location scheme

5.4.1 Caching in Hierarchical Scheme

In a hierarchical architecture, caching can be deployed to reduce the lookup time. Combined with *forward bypass pointers* and *reverse bypass pointers* it is place cache at a higher level of the hierarchy to serving the calls originating from a group of cells rather than a single cell. The tradeoff is the calls have to traverse a longer path if cache is placed at a higher level. The idea is illustrated by figure 5.4. When a call is requested from LAI_i to LAI_j, the call traverses up the tree from LAI_i to $LCA(LAI_i, LAI_j)$ and then downwards to LAI_j. The call setup requires an acknowledgement to be sent from LAI_j back to LAI_i along the same path. All the ancestors of LAI_i in the hierarchy overhear this ack message, and one of them, say LS_i can create a forward bypass pointer to an ancestor, say LS_j, of LAI_j. Likewise, LS_j can create a reverse bypass pointer to LS_i. The cache can be deployed at LS_i. Then subsequent calls from LAI_i reach the location database LS_j for a user in LAI_j via the shorter route using the forward bypass pointer at LS_i. Similarly, the acknowledgement message from LAI_j can traverse a shorter path using the reverse bypass pointer from LS_j to LS_i. If the level of LS_i is high then all the calls originating from its subtree to the cells in the subtree of LS_j can use the bypass pointers. But each call may have to traverse a longer path up to LS_i and spend longer lookup time since time to locate the callee in the subtree of LS_j would depend on its size.

5.5   Forwarding Pointers

For a mobile user receiving relatively less calls compared to the moves, it is expensive to update all the location servers holding the location of the user on every move. It may be cheaper to leave a forwarding pointer to the new location at the previous location of the user. Then any call arriving at old location can be re-routed to the current location by the forward pointer. While the update of database entries holding user location can be updated selectively.

In two-tier architecture, there may be excessive update traffic between the location of current VLR of the user to the HLR location of a user when it is far away from home frequently, and moves in the neighbourhood of the current locations. In order to avoid that, forward pointers can be left at VLR of the serving location. When a call to the user reaches its home location sever, the call is diverted to the present location from its home location following a chain of forward pointers. The chain of forward pointers are allowed to grow up to a fixed predetermined length of K. When the user revisits a location, the potential loop condition in forward pointers chain is avoided by an implicit compression. The approach of forwarding is applied on a per user basis.

Forwarding pointer strategy can also be applied to the hierarchical architecture. In a *simple forwarding* strategy, the pointers are placed in the leaf level. No update is done up the tree path as a user moves from one location to another. But a forwarding pointer is left at the previous location to the current location. In a *level forwarding* strategy, the forwarding pointer is placed at a higher level location server in the hierarchy. In this case, more updates are required to delete the database entries of lower level location servers of the previous location, and also to insert the current location into lower level ancestor location servers of the current location. Figure 5.5 illustrate the two schemes for placing the forward pointers. The solid pointers indicate the simple forwarding and dashed pointers illustrate the level forwarding. Assuming mobile user mu to be initially located in cell 11 decides to move to a the cell 14, then simple forwarding places a forward pointer at the old leaf level location server 11 as indicated by the solid pointers. Whereas in the case of level forwarding, a forward pointer is placed at the ancestor location server 2 at level 3 which points to the location server 3 which is an ancestor of the new location. In this case, updates are more than that required in simple forwarding. This is because, entries concerning new location of mu
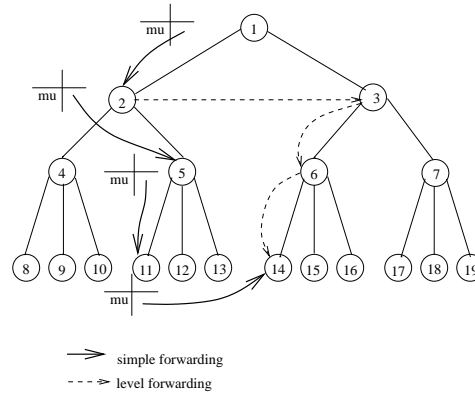
Figure 5.5: Forwarding pointers in hierarchical scheme

should be made at nodes 3, 6, and 14 and database entries for *mu* at nodes 2, 5, and 11 are to be deleted.

5.6 Replication

Replication is another mechanism to reduce the cost for lookup. The basic criterion for replication of location of a user at selected nodes in the hierarchy is the large call to mobility ratio.

In a two-tier architecture replication may be employed if the cost of replication is not more than the cost for non-replication. If many calls originate from LAI_j to a mobile user then it may be cost effective to replicate the location of user at LAI_j. However, if the user moves frequently then the maintenance of replicas incurs heavy cost. Note that a replica does not merely have location information. A replica also includes service information parameters such as call blocking, call forwarding, QoS requirements like the minimum channel quality and the acceptable bandwidth. Replicas are kept both at the receiver and the sender. Thus replicas should not be considered merely as extension of cache entries. If C_{ij} is the expected number of calls made from cell LAI_j to user over a period of time T, α is saving on replication at LAI_j, β is the cost per update, and U_i is the number of updates then

$$\alpha.C_{ij} \geq \beta.U_i$$

should hold in order that the replication of the user at LAI_j to be cost effective. The replicas of the user are kept at all the frequent callers which satisfy the above inequality. The set of the caller locations is called a *working set* for that user. Every time a call is made to the user:

- from a member of the user's working set no update is needed,

- from a non-member then if the inequality is found to be true then that cell is added to the working set of the user.

On the other hand if the user makes a move:

- inequality is evaluated for every member of the working set

- if it fails to hold for any member LAI_k then it is removed from the working set.

The storage requirement for a single user's profile of size F in basic multi-tier location database having L levels is $F + ptr \times (L-1)$, where ptr is the size of a pointer (user ID + database ID). If the profiles are replicated then the cumulative storage requirements should not exceed the storage space available in the database [3].

Apart from storage constraint, the decision to place a replica should also be based on minimization of network communication cost. In two-tier model, local call to mobility ratio is used for this purpose. A user i's profile is replicated a database j only if LCMR$_{ij}$ exceeds a minimum threshold, say R_{min}. In hierarchical database it is impossible to arrive at a single parameter for databases at different levels of hierarchy. However, using an additional parameter R_{max} it is possible to arrive at a decision. The computation of $LCMR_{ij}$ for hierarchical database is done by a simple bottom up summing the $LCMR$ values of its children. Clearly, if high $LCMR$ value is the criterion for the selection of replication then when a node is selected for placing a replica, all its ancestor nodes also should be selected for replica placement. Therefore, this selection process results in excessive updates at higher levels of databases. This calls for setting a number of constraints including high (R_{max}) and low (R_{min}) marks for $LCMR$ values to determine the nodes in the hierarchy which may be selected for placing replicas. The rules for selecting replication site are as follows [3]:

1. If LCMR$_{ij} < R_{min}$, replica of i's profile is not placed site j.

2. If $\text{LCMR}_{ij} \geq R_{max}$ then always place replica of i's profile at site j if the constraints on L and N are satisfied, where L represents hierarchy level, N is the bound on the number of replicas.

3. If $R_{min} \leq \text{LCMR}_{ij} < R_{max}$, then the decision to place i's profile at site j will depend on database topology.

The other constraints are the level of location server in the hierarchy, and the maximum number of replicas to be placed. Unconstrained number of replications can put severe pressure in maintenance of the replication even for a moderately mobile user. The reader is referred to [3] for details of analysis and the algorithm for placing replication.

5.7 Personal Mobility

In the context of location management, there is trade off between search and update. Most location management schemes are based on the approach to achieve balance between the two. To what extent the the trade off can swing between update and search will depend on the bound signaling requirements for location management. But the question is how to define a bound on signaling requirement? Furthermore, even if a bound can be defined, is it possible to reach the bound? In order to seek answers to these questions, it is important to understand that terminal mobility and personal mobility are tightly coupled. In reality a mobile equipment at best is a portable device, it cannot move on its own. The movement of a mobile is caused by the movements of the user. The movement of a person or the user of a mobile can be considered as a path in some random process.

5.7.1 Random Process, Information and Entropy

Let us explore a bit about randomness of a process. If the volume of information content of a random process is high then the unpredictability is low. The probability of occurrence of an event contains the amount of information about the event. For example, if the probability of occurrence of an event A is known to be more than the probability of occurrence of another event B, then the amount of information available about A is more than that available for B. In other words, A's occurrence is more predictable than B's occurrence. This qualitative measure of information can be interpreted in terms

of *surprisal* [8]. Because it provides a measure of an element of surprise in occurrence of one event compared to another. If the information content is high then the surprisal is low. Similarly, if the information content is low then surprisal is high. For example, if an event has a probability 1, then it occurs always or its surprisal is zero. On the other hand, an surprisal an event with zero probability is infinity. This implies that surprisal of an event A defined by the inverse of the probability of occurrence of A.

5.7.2   Shannon's Entropy Formulation

Shannon [6] formulated the surprisal in terms of entropy. Shannon's formulation capture the following two important aspects:

1. The extent of randomness is determined by the size of entropy.

2. The more random a process is, the greater is its unpredictability

From a practical prospective, it implies that a large information base is needed for predicting any future events in a system with large entropy. Since a terminal movement is tied to the personal mobility of its user, for predicting future locations of a mobile terminal we need more information about the movements of its user. So, Shannon's entropy formulation can be used the calculating the optimal bounds.

The amount of information is measured in number of bits. For example, 3000 bits are needed in order to transmit the results of 1000 rollings of an unbiased hypothetical eight sided die. If the die is known to be biased, and the probability distribution is known, then a variable length encoding can be used. Since, the information bits are transmitted together, the encoding should be such that it possible to disambiguate the block of bits representing the results of different rollings of the die. This implies that the encoding must have prefix property which ensures that no code is a prefix of any code.

For example, let the probability distribution for a biased die be:

$$p(i) = \begin{cases} 1/2^i, \text{ for } i \leq 7 \\ 1/2^{i-1}, \text{ for } i = 8, \end{cases}$$

Since, half the number of rollings result in 1 showing up, the shortest code should be used for 1. Likewise, the code for the rarest event, a rolling that results in 8, could be the longest. A possible encoding scheme with the above mentioned properties would be:

Results of rolling	1	2	3	4	5	6	7	8
Code	0	10	110	1110	11110	111110	1111110	1111111

The above encoding satisfies the prefix property. The average number of bits needed for encoding the result of 1000 rollings is

$$\frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + \ldots + 7 \times \frac{1}{128} + 8 \times \frac{1}{128} = 1.984$$

So, with more information, the average number of bits required for transmitting the result 1000 rolling of the biased 8 sides hypothetical die is reduced from 3000 to 1984 bits.

As another example, consider flipping of a biased coin. Let head show up just once in 1000 flips. Suppose the coin is tossed one million times. Let a 0 represent the fact that the result of a toss is a head, and similarly let 1 represent a tail. Without using any clever encoding, 1 bit will be required for the result of each toss. So, transmitting the result of one million tosses requires a million bits. Since a head shows up only once in 1000 flips of the biased coin, we may just record the sequence number of tosses that resulted in a head. The missing sequence numbers will then represent tails. Any number between 1 to 10^6 can be represented by at most $\log 10^6 = 20$ bits. Therefore, the information transfer for the result of 1000 flippings of biased coin will need just 20 bits. A total of 20000 bits will be needed for transmitting the results of one million tosses.

From the above examples, let us try to abstract out the answer to the general case of information coding for a random event. Consider a conventional die with six faces to find an answer to the above question. If the die is unbiased, the probability of occurrence of any value is $p = 1/6$, the number of bits required $= \log 6 = -\log(1/6) = -\log p$. The result of one throw requires $\log 6 = 2.58$ bits. It is not immediately apparent how the result any particular throw of a die can be encoded by less than 3 bits. However, if we group g successive throws, the results can coded by less than 6^g bits. For example, the number of possible outcomes for a group of three throws $= 6^3 = 216 < 255$, and 0 to 255 can be coded using 8 bits. Thus, for a biased probability distribution, the number of bits required optimal code is determined by

$$-\sum_x p(x) \times \log p(x).$$

The examples we have discussed so far, point to the fact that a rare event has a very low probability. Hence, the information content related to occurrence of a rare event is high. In the coin toss example, the information content in occurrence of a head is $-\log(1/1000) = \log 1000 = 9.9658$. So 10 bits will be needed to represent occurrence of a head. As against this information contents in the probability of appearance of a tail is $-\log(999/1000) = 0.0044$ bit. So the average information content is

$$(1/1000) \times 10 + (999/1000) \times 0.0044 = 0.0114 \text{bit.}$$

Let us try to generalize the method of determining information content as outlined in the above example. Suppose, the probability of an outcome A_i of a random event A is $p(A_i)$. Then the expected value of self information in A

$$H(A) = \sum_{i=1}^{n} p(A_i) \times \log\left(\frac{1}{p(A_i)}\right)$$
$$= -\sum_{i=1}^{n} p(A_i) \times \log p(A_i)$$

According to Shannon [6], $H(A)$ represents entropy of A.

Now consider X and Y, a pair of discrete random variables with joint probability distribution $p(x, y)$, where $x \in \mathcal{X}$, $y \in \mathcal{Y}$ then joint entropy of X and Y is given by the expression:

$$H(X, Y) = -\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \log p(x, y)$$

The conditional entropy $H(Y|X)$ represents information content of a random event Y given that some event X has occurred. $H(Y|X)$ can be expressed by the formula:

$$H(Y|X) = \sum_{x \in \mathcal{X}} p(x) H(Y|X = x) = -\sum_{x \in \mathcal{X}} p(x) \sum_{y \in \mathcal{Y}} p(y|x) \log p(y|x)$$
$$= -\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \log p(y|x)$$

Joint entropy and conditional entropy are closely related. Joint entropy can be expressed as the sum of entropies of the first random variable and the

conditional entropy of second random variable given the first.

$$H(X,Y) = -\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x,y) \log p(x,y)$$

$$= -\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x,y) \log(p(x)p(y|x))$$

$$= -\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x,y) \log p(x) - \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x,y) \log p(y|x)$$

$$= -\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x,y) \log p(x) + H(Y|X)$$

$$= -\sum_{x \in \mathcal{X}} p(x) \log p(x) + H(Y|X)$$

$$= H(X) + H(Y|X).$$

The generalization of the above result, known as chain rule, says:

$$H(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} H(X_i|X_{i-1}, \ldots, X_1).$$

If the random variables are known to be independent then according to the chain rule:

$$H(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} H(X_i).$$

5.7.3 Mobility Pattern as a Stochastic Process

In order to capture the personal mobility, we may view a user's movement as a random process. With a cellular infrastructure, a user's movement can be captured as a sequence of cells. In a GSM type network, each symbol will represent a location area identity (LAI) consisting of several cells. There is a possibility that certain substrings of LAIs are repeated in the sequence of LAIs representing a user's movement. The repetitions represent locality of movement. By characterizing mobility as probabilistic sequence, mobility can interpreted as a stochastic process.

At first we need to be clear on two issues:

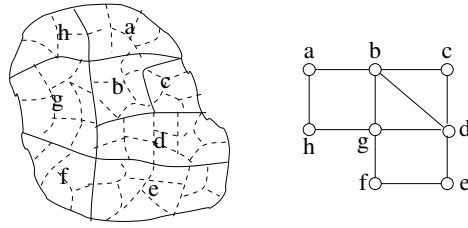1. How a user would move in a service area?

Figure 5.6: GSM type location area map (source [1]).

2. How movement can be recorded?

Figure 5.6 illustrates an example for a service area consisting of eight LAIs a, b, c, d, e, f, g, h. The shapes of actual cells are not quite hexagonal, but have irregular geometric contour. The contour of a cell is determined by actual field measurement of signal strengths. Multiple cells are grouped into an LAI. The topology of LAIs in coverage area can be abstracted in form of a graph shown along side. Each node in the graph represents an LAI. Two nodes are connected with edge if and only if the corresponding LAIs are adjacent to each other. With the graph abstraction as explained above, the mobility pattern of a user is represented as a walk in the graph. A walk is formed by update of location as a user moves and enters new LAIs.

How location update can be recorded? The mobiles send their updates to network subsystem in periodic intervals or on crossing the boundary of an LAI. Thus, location updates could be distance based, time based, movement based or combinations thereof.

- Distance based update: in the distance based updates, mobiles keep track of the Euclidean distance from the time last update was sent. Then if distance travelled from the last update crosses a threshold D, the mobile sends an update.

- Movement based: a mobile sends an update if it has performed n cell crossings since the last update.

- Time based: a mobile sends periodic update.

Let us consider an example to understand how different updates schemes would generate the updates. Suppose the service is started at 9.00AM. An example of LAI crossings is provided in table below.

Crossings in morning			
Time	11:04	11:32	11:57
Crossing	$a \to b$	$b \to a$	$a \to b$

Crossings in afternoon			
Time	3:18	4:12	4:52
Crossing	$b \to a$	$a \to b$	$b \to c$

Crossings in evening				
Time	5:13	6:11	6:33	6:54
Crossing	$c \to d$	$d \to c$	$c \to b$	$b \to a$

With the movement history shown in previous slide, the LAI sequences reported by various update schemes will be as shown below.

Update scheme	: Movement history
$T = 1$hr	: $aaabbbbacdaaa\ldots$
$T = 1/2$hr	: $aaaaabbbbbbbaabcddcaaaa\ldots$
$M = 1$	: $abababcdcba\ldots$
$M = 2$	: $aaacca\ldots$
$T = 1$hr, $M = 1$	: $aaababbbbbaabccddcbaaaa\ldots$

The movement history of a user can be represented by a string $v_1, v_2, v_3, \ldots$, where each symbol v_i, $i = 1, 2, \ldots$, denotes the LAI reported by the mobile in ith update. The symbols are drawn an alphabet set $\mathcal{V}$ representing the set of LAIs covering the entire service area. The mobility of a user is characterized as a stationary stochastic process $\{V_i\}$, where V_i's form a sequence random variables, and each V_i is take a value v_i from set $\mathcal{V}$. A stochastic process is stationary if the joint probability distribution does not change when shifted in time or space. If observed over time, a normal user of a mobile device is most likely to exhibit the preference for visiting any known sequence of LAIs in a time invariant manner. The correlation between visiting adjacent LAIs of the sequence remain unchanged across all periods of time. Consequently, personal mobility pattern can be treated as a stationary stochastic process. Thus,

$$\Pr[V_1 = v_1, V_2 = v_2, \ldots, V_n = v_n] = \Pr[V_{l+1} = v_1, V_{l+2} = v_2, \ldots, V_{l+n} = v_n].$$

The above general model could aid in learning, if a universal predictor can be constructed. Let us examine the possible models for evolving a universal predictor. The common models used for interpreting the movement history are:

- Ignorant Model (IM)

- Identically Independent Distribution (IID)

- Markov Model (MM)

IM disbelieves and disregards past history. So the probability of any LAI residence is the same, i.e., 1/8 for each of the 8 LAIs for the example.

IID model assumes that the value random variables defining a stochastic process are Identically and Independently Distributed. It uses relative frequencies of symbols as estimates of the residence probabilities of the LAIs. Assuming time and movement based scheme the probabilities for the string *aaababbbbbaabccddcbaaaa*, are:

$$p(a) = 10/23, p(b) = 8/23, p(c) = 3/23, p(d) = 2/23,$$
$$p(e) = p(f) = p(g) = p(h) = 0.$$

The string consists of 23 symbols, the probability of a symbol is determined by its relative frequency, since it is independent of that for other symbols.

The simplest Markov model is a Markov chain where distribution of the random variable depends only on distribution of the previous state. So, this model assumes the stochastic process to be a time-invariant Markov chain defined by:

$$Pr[V_k = v_k | V_1 = v_1, \ldots, V_{k-1} = v_{k-1}]$$
$$= Pr[V_k = v_k | V_{k-1} = v_{k-1}]$$
$$= Pr[V_i = v_i | V_{i-1} = v_{i-1}]$$

for arbitrary choices for k and i. Since, the LAIs e, f, g, h are not visited at all, each acquire zero probability. The effective state space is $\{a, b, c, d\}$. One step transition probabilities are:

$$P_{i,j} = Pr[V_k = v_j | V_{k-1} = v_i],$$

where $v_i, v_j \in \{a, b, c, d\}$, are estimated by relative counts. So, movement profile can be represented by the corresponding transition probability matrix:

$$P = \begin{bmatrix} 2/3 & 1/3 & 0 & 0 \\ 3/8 & 1/2 & 1/8 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$
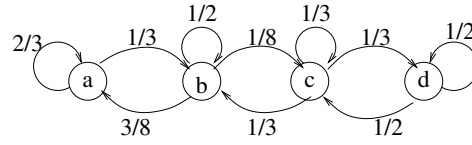
Figure 5.7: One step state transition diagram for personal mobility.

Note that a occurs 2 out of 3 times in context of a and 3 times out of 8 times in context b. The values of other transition probabilities can be found likewise. Thus, the state transitions with the respective probabilities can be viewed as shown in Figure 5.7. Let $\Pi = [p(a)\ p(b)\ p(c)\ p(d)]^T$ be steady state probability vector. Solving $\Pi = \Pi \times P$ with $p(a) + p(b) + p(c) + p(d) = 1$, we obtain $p(a) = 9/22$, $p(b) = 4/11$, $p(c) = 3/22$ and $p(d) = 1/11$.

To summarize the above discussion, we notice that IM can never be adaptive. IID is the first step toward adaptive modeling. The optimal paging strategy dependents on the independent probabilities of symbols $\{a, b, c, d, e, f, g, h\}$. Order-1 Markov model carries information to the extent of one symbol context. For uniformity, we may refer to IM as order-(-1) Markov model, and IID as order-0 Markov model respectively. Order-2 Markov model can be constructed by counting the frequencies of symbols appearing in order-2 contexts for the sequence $aaababbbbbaabccddcbaaaa$. The table below provides frequencies of different symbols for all three contexts.

Order-0	Order-1		Order-2		
$a(10)$	$a\|a(6)$	$b\|c(1)$	$a\|aa(3)$	$a\|ba(2)$	$a\|cb(1)$
$b(8)$	$b\|a(3)$	$c\|c(1)$	$b\|aa(2)$	$b\|ba(1)$	$d\|cc(1)$
$c(3)$	$a\|b(3)$	$d\|c(1)$	$a\|ab(1)$	$a\|bb(1)$	$d\|cd(1)$
$d(2)$	$b\|b(4)$	$c\|d(1)$	$b\|ab(1)$	$b\|bb(3)$	$b\|dc(1)$
	$c\|b(1)$	$d\|d(1)$	$c\|ab(1)$	$c\|bc(1)$	$c\|dd(1)$

According to order-1 model the probability of taking the route $a \to b \to c \to b \to c \to d$ is:

$$= (9/22) \times (1/3) \times (1/8) \times (1/3) \times (1/8) \times (1/3)$$
$$= 1/4224 = 2.37 \times 10^{-4}$$

It is unlikely that any user will ever take such a zig-zag route. Though the probability is very low; still it is not zero. However, if order-2 model is

---

applied then the proposed route will be impossible. So, the richness of the information helps.

The question is how much of the past history should be stored so that it could lead to a good prediction? Storing the movement history for every movement of a user is not practical. The conditional entropy is known to be a decreasing function of the number of symbols in a stationary process [2], implying that the advantage of higher order contexts dies out after a finite value. To appreciate this fact, let us compute the entropy for the three different models for the example for personal mobility we have used in the text.

- Order-(-1) model:

  V_is are independently and uniformly distributed, so $p(v_i) = 1/8$ for all $v_i \in \{a, b, c, d, e, f, g, h\}$ in the chosen running example. Due to independence of events, $p(v_n|v_1, \ldots v_{n-1}) = p(v_n)$. Therefore, the joint probability $p(v_1, v_2, \ldots, v_n) = \{p(v_1)\}^n$, and both per symbol entropy $H(\mathcal{V})$, and the conditional per symbol entropy $H'(\mathcal{V})$ are same. Using the definition

  $$H(\mathcal{V}) = \lim_{n \to \infty} \frac{1}{n} H(V_1, V_2, \ldots, V_n) = \log 8 = 3.$$

  So, we get the per symbol entropy rate as 3bits for order-(-1) model.

- Order-0 model:

  In this case, V_is are Independently and Identically Distributed. Due to independence $p(v_n|v_1, \ldots v_{n-1}) = p(v_n)$. Therefore,

  $$\begin{aligned}
  H(\mathcal{V}) = H'(\mathcal{V}) &= -\sum_{v_i} p(v_i) \log p(v_i) \\
  &= (10/23) \times \log(23/10) + (8/23) \times \log(23/8) \\
  &\quad + (3/23) \times \log(23/3) + (2/23) \times \log(23/2) \\
  &\approx 1.742.
  \end{aligned}$$

  Therefore, the per symbol entropy rate for order-0 model is 1.742 bits which is much better than order-(-1) model.

- Order-1 model:

In this case, V_is form Markov chains. So $p(v_n|v_1 \ldots v_{n-1}) = p(v_n|v_{n-1}) = P_{v_{n-1},v_n}$. Substituting steady state probabilities $p(a) = 9/22$, $p(b) = 4/11$, $p(c) = 3/22$ and $p(d) = 1/11$, we find

$$
\begin{aligned}
H'(\mathcal{V}) = -\sum_{v_i} p(v_i) \left(\sum_j P_{i,j} \log P_{i,j} \right) &= \frac{9}{22} \left(\frac{2}{3} \log \frac{3}{2} + \frac{1}{3} \log \frac{3}{1} \right) \\
&+ \frac{4}{11} \left(\frac{3}{8} \log \frac{8}{3} + \frac{1}{2} \log \frac{2}{1} + \frac{1}{8} \log \frac{8}{1} \right) \\
&+ \frac{3}{22} \left(3 \times \frac{1}{3} \log \frac{3}{1} \right) + \frac{1}{11} \left(2 \times \frac{1}{2} \log \frac{2}{1} \right) \approx 1.194.
\end{aligned}
$$

Thus using order-1 MM, we get a per symbol entropy rate of 1.194 bits. It improves the entropy rate substantially over order-(-1) and order-0 models.

Note that 3 bits are sufficient to represent any symbol from a space of eight symbols $\{a, b, c, d, e, f, g\}$. So order-(-1) model can not resolve uncertainities in any of the three bits. But both order-0 and order-1 models exhibit richness in information and gradual decrease in entropy rates. The entropy rates $H(\mathcal{V})$ and $H'(\mathcal{V})$ are same in both order-(-1) and order-0 MM due to independence of the events related to symbols.

A mobile's location is unknown for the duration between two successive update. The approach in LeZi update is to delay update if the path traversed by mobile is familiar. The information lag should not impact paging, because system can use prefix matching to predict location with high probability.

5.7.4   Lempel-Ziv algorithm

LeZi update is based on Lempel-Ziv's text compression algorithm [10]. The algorithm provides a universal model for variable-to-fixed length coding scheme. The algorithm consists of an encoder and decoder. The encoder incrementally parses the text into distinct phrases or words (which have not been observed so far). A dictionary is gradually builds as phrases keep entering. LeZi update's encoder is identical to the encoder of Lempel-Ziv's algorithm. It runs at mobile terminal.

```
//  Encoder at mobile or compressing algorithm.
```

```
dictionary = null;
phrase w = null;

while (true) {
    wait for next symbol v;
    if (w.v in dictionary) w = w.v;
    else {
        encode < index(w), v>;
        add w.v to dictionary; w = null;
    }
}
```

The encoding process can be best explained by executing it on an string of symbols. Let the example string be:

$$aaababbbbbaabccddcbaaaa$$

The table below illustrates how encoding each phrase is realized.

Index	Prefix	Last symbol	Input phrase	Output
1	Λ	a	a	(0, a)
2	a	a	aa	(1, a)
3	Λ	b	b	(0, b)
4	a	b	ab	(1, b)
5	b	b	bb	(3, b)
6	bb	a	bba	(5, a)
7	ab	c	abc	(4, c)
8	Λ	c	c	(0, c)
9	Λ	d	d	(0, d)
10	d	c	dc	(9, c)
11	b	a	ba	(3, a)
12	aa	a	aaa	(2, a)

The first incoming phrase is a, Λ or the null phrase is its prefix. The null phrase is assumed to be a prefix of any phrase having one symbol, and the index of Λ is set to 0. So, a is coded as $0a$. The index of an incoming phrase is determined by the position of the phrase in the dictionary which is largest proper prefix of the current phrase. A proper prefix excludes the last symbol in a phrase. For example, let us see how the next incoming phrase aa is

encoded. The largest proper prefix of *aa* is *a*. Since the position of *a* is 1 in the dictionary, the index of the incoming phrase is 1. Therefore, appending *a* to 1, we get the encoding of *aa* as 1*a*. Therefore using encoding algorithm the string *aaababbbbbaabccddaaaa* is encoded as: 0*a*, 1*a*, 0*b*, 1*b*, 3*b*, 5*a*, 4*c*, 0*c*, 0*d*, 9*c*, 3*a*, 2*a*.

A code word consists of two parts (i) an index, and (ii) a symbol. The index represents the dictionary entry of the phrase which is the prefix of the code word. The prefix completely matches with the code word symbol-wise except for the last symbol. Therefore, the decoding consists of finding the prefix and appending the last symbol to it. The major distinction between LeZi update and Lempel-Ziv's algorithm is in the decoder part. In the case of LeZi update the decoder executes at the network side. The decoder appears below.

```
// Decoder at the system side.
// It basically decompresses the string

while (true) {
   wait for the next code word <i, s>;
   decode phrase = dictionary[i].s;
   add phrase to dictinary;
   increment frequency of every prefix of the phrase;
}
```

To see how the decoding process works, we consider code word specified in terms of tuples <index, last_symbol> and illustrate the decoding with help of the table below.

Input tuple	Prefix phrase	Last symbol	Output phrase
(0, a)	Λ	a	a
(1, a)	a	a	aa
(0, b)	Λ	b	b
(1, b)	a	b	ab
(3, b)	b	b	bb
(5, a)	bb	a	bba
(4, c)	ab	c	abc
(0, c)	Λ	c	c
(0, d)	Λ	d	d
(9, c)	d	c	dc
(3, a)	b	a	ba
(2, a)	aa	a	aaa

When the code word tuple is received, the index part is extracted first. For example index part in the input tuple $(0, a)$ is 0. The index is then used to retrieve the phrase from the dictionary. Since, index of Λ is 0, the phrase retrieved is Λ. Then symbol of input codeword is contatenated with retrieved phrase, i.e. a is appended to λ. Since $\Lambda + a = a$, the decoding of $(0, a)$ outputs the phrase a. Similarly, when the code word $(5, a)$ is received, the phrase having index 5 is extracted from the dictionary, and the symbol a is appended to it producing the output *bba*.

The decoding process helps to build conditional probabilities of larger contexts as larger and larger phrases are inserted into the dictionary. So, dictionaries are maintained at both system as well as each mobile terminal. The mobile terminal sends the updates only in coded form. It delays sending of update until a pre-determined interval of time has elapsed. The updates are processed in chunks and sent to the network as a sequence code words of the form $C(w_1)C(w_2)C(w_3) \ldots$, where each phrase w_i, for $i = 1, 2, \ldots$, is a non-overlapping segment of symbols from the string $v_1 v_2 v_3 \ldots$ that represents the LAIs visited by the mobile since the time of sending the last update to the network. So, LeZi update can be seen as a path based update scheme instead of LAI based update.

5.7.5   Incremental parsing

The coding process is closely inter-twined with the learning process. The learning process works efficiently by creating the dictionary and searching it
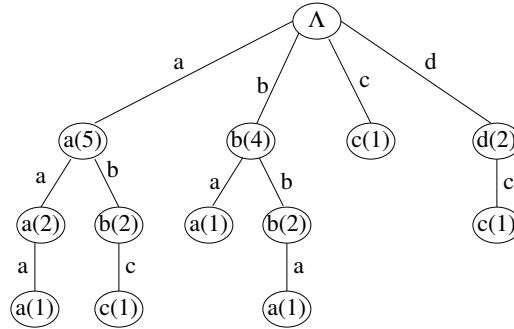
Figure 5.8: Trie built by classical Lempel-Ziv algorithm.

for the existence of incoming phrases. An input string $v_1 v_2 \ldots v_n$ is parsed into k distinct phrases $w_1, w_2, \ldots, w_k$ such that the prefix (all symbos except the last one) of the current incoming phrase w_j is one of the previously occurring phrases w_i, $1 \le i < j$. So, the context statistics related to all prefixes can be updated during the parsing of the current phrase itself. In addition, the prefix property also allows to store the history efficiently in a trie.

Figure 5.8 depicts the trie produced by classical Lempel-Ziv algorithm for the string in the example. The numbers alongside the symbols represent the frequency computed by Lempel-Ziv algorithm. To see how frequencies are obtained, consider the parsing of aaa. It leads to frequency counts of a, aa being incremented by 1 each, as both are prefixes of the phrase aaa. Of course, the frequency count of aaa is set to 1. Total count of a's occurrences is obtained from phrases a, aa, ab, abc and aaa. Each contributes 1 to frequency count of a. Therefore, a total frequency count is 5 as shown in the figure. To provide a variation, consider how the frequency count of b is determined. The symbols b appears as a prefix in phrases b, ba, bb and bba, each of which contributes 1 to b's frequency. Thus, b frequency is 4.

A new dictionary entry can be created by appending one symbol to an already existing phrase in the dictionary. An existing phrase terminates at a node of the trie. An appended symbol to an existing phrase appears as a label of an edge leading from terminating node of the phrase to another. As the incremental parsing progresses larger and larger phrases are stored in the dictionary. Consequently, conditional probabilities among the phrases starts to build up. Since there is limit to the richness of higher order Markov model,

Lempel-Ziv symbol wise model eventually converge to a universal model.

As far as personal mobility is concerned, the location updates can be viewed as that of generating a new symbol for the sequence representing the movement history of the form $v_1 v_2 \ldots v_n$. So, the movement history can be parsed into distinct substrings and new update can be inserted into a trie which gradually builds the personal mobility pattern.

However, we can not use Lempel-Ziv compression based model in a straightforward way. One serious problem with the above compression model is that it fails to capture conditional entropy early on due to following reasons:

- It works on one phrase at a time.

- Decoding algorithm counts only the frequencies of prefixes of a decoded phrase.

- It remains unaware about the contexts that straddle phrase boundaries.

All of the above slow down the rate of convergence. LeZi update uses an enhanced trie, where frequencies of of all prefixes of all suffixes are updated. So instead of using the decoder of the previous section LeZi update uses a slightly altered decoder.

```
// Decoder at the system side.
// It basically decompresses the string

while (true) {
   wait for the next code word <i, s>;
   decode phrase = dictionary[i].s;
   add phrase to dictionary;
   increment frequency of every prefix of every suffix the phrase;
}
```

By doing so, it captures the straddling effect.

The revised frequency counting method for phrases in LeZi update is as follow:

- Find all the prefix of all the suffixes of each incoming phrase.

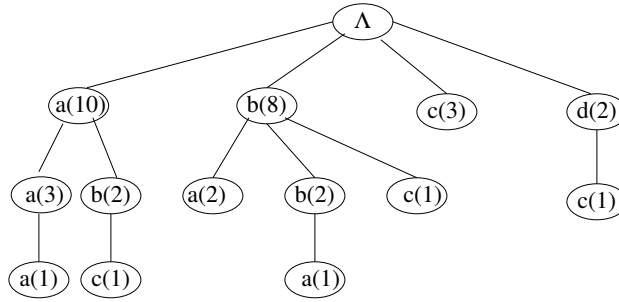- Increment the frequency each time a particular prefix is encountered starting from zero.

Figure 5.9: Enhanced trie.

Let us examine the effect of considering all prefixes of all the suffixes in generating frequency counts for symbols in the example string. For the phrase *aaa*:

- Suffixes are *aaa*, *aa* and *a*, so the frequency counts of all prefixes of *aaa*, *aa*, and *a* are incremented.

- The frequency counts of *aa* incremented by 2.

- The frequency counts of *a* incremented by 3.

The total count for *a* can be found by considering phrases *a*, *aa*, *ab*, *ba*, *abc*, *bba*, and *aaa*. The enhanced trie obtained by LeZi update method is provided by Figure 5.9.

In order to estimate the effectiveness of the new model for personal mobility against the model that is based on the classical Lempel-Ziv compression scheme, let us compute the entropies for each case. The conditional entropy without considering suffixes (see figure 5.8):

$$H(V_1) = \frac{5}{12} \log \frac{12}{5} + \frac{1}{3} \log 3 + \frac{1}{12} \log 12 + \frac{1}{6} \log 6$$
$$\approx 1.784. \text{ bits, and}$$

$$H(V_2|V_1) = \frac{5}{12} \left(2 \times \frac{1}{2} \log 2 \right) + \frac{4}{12} \left(\frac{1}{3} \log 3 + \frac{2}{3} \log \frac{3}{2} \right)$$
$$\approx 0.723 \text{ bits.}$$

Other two terms of $H(V_2|V_1)$ being 0 are not included in the expression. The estimate for $H(\mathcal{V}) = (1.784 + 0.723)/2 = 1.254$ bits. The conditional probabilities of all order-2 contexts are 0.

When all prefixes of all suffuxes are considered, the frequency count will be as shown in trie of Figure 5.9. With enhanced tries, we still have

$$H(V_1) = \frac{10}{23} \log \frac{23}{10} + \frac{8}{23} \log \frac{23}{8} + \frac{3}{23} \log \frac{23}{3} + \frac{2}{23} \log \frac{23}{2}$$
$$\approx 1.742. \text{ bits, and}$$

$$H(V_2|V_1) = \frac{10}{23} \left(\frac{3}{5} \log \frac{5}{3} + \frac{2}{5} \log \frac{5}{2} \right) + \frac{8}{23} \left(2 \times \frac{2}{5} \log \frac{5}{2} + \frac{1}{5} \log 5 \right)$$
$$\approx 0.952 \text{ bits.}$$

Therefore, the estimate for $H(\mathcal{V}) = (1.742 + 0.952)/2 = 1.347$ bits when all the prefixes of all the suffixes are considered, implying that the suggested enhancements carry more information. Hence, location update based on enhancements is expected to perform better than just using simple Lempel-Ziv compression method.

5.7.6   Probability assignment

The prediction of a location for mobile terminal is guided by probability estimates of its possible locations. The underlying principle behind probability computation is PPM (prediction by partial matching). PPM uses the longest match between the previously seen strings and the current context. Our interest is in estimating the probability of occurrence of the next symbol (LAI) on the path segment to be reported by the next update. The segments are LAI-sequences generated when traversing from the root to the leaves of the sub-tries representing the current context. The estimated conditional probabilities for all LAIs at the current context gives the conditional probability distribution.

Suppose no LeZi type update received after receiving *aaa* and we want to find the probability of predicting the next symbol as *a*. The contexts that can be used are all suffixes of *aaa* except for itself, namely, *aa* and *a* and Λ. PPM tells that to determine the probabilities only the previously seen phrases need be considered. Therefore, the possible paths that can be predicted with the contexts *aa*, *a* and Λ are:

aa (Order-2)	a (Order-1)	Λ (Order-0)		
	a|a(2)	a(5)	ba(2)	d(1)
a|aa(1)	aa|a(1)	aa(2)	bb(1)	dc(1)
Λ|aa(2)	b|a(1)	ab(1)	bba(1)	Λ(1)
	bc|a(1)	abc(1)	bc(1)	
	Λ|a(5)	b(3)	c(3)	

Start form the highest order, i.e. context *aa*, the probability *a* is 1/3. Then fall back with probability 2/3 to order 1 (with null prediction), where the probability of *a*'s occurrence is $2/10 = 1/5$. Now fall back to order-0 (with null prediction), which has probability $5/10 = 1/2$. The probability of *a*'s occurrence in order-0 is 5/23. So blended probability of next symbol being *a* is

$$\frac{1}{3} + \frac{2}{3}\left(\frac{1}{5} + \frac{1}{2}\left(\frac{5}{23}\right)\right) = 0.5319$$

To examine a variation, consider the occurrence of phrase *bba* after receiving *aaa*. The phrase *bba* does not occur in any of the contexts 1 or 2. The probabilities of escape from the order-1 and the order-2 contexts with null prediction are:

- *Order 2 context*: (with phrase *aa*), the probability of null prediction is 2/3 (out of 3 occurrence of *aa*).

- *Order 1 context*: (with phrase *a*), the probability of null prediction is 1/2, because Λ occurs 5 times out of 10.

The idea is to predict starting with a pre-determined highest order of context, and escape to the lower orders until order 0 is reached.

Therefore, the blended probability of phrase *bba* is

$$0 + \frac{2}{3}\left(0 + \frac{1}{2}\left(\frac{1}{23}\right)\right) = 0.0145$$

Since *a* occurs once and *b* occurs twice in the phrase *bba*, the individual probabilities of symbols *a* and *b* respectively are: $(1/3)\times 0.0145 = .0048$, and $0.0145\times (2/3) = .0097$.

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