Agenda for discussion

The lecture contains:
Measurement of **wireless signal** and **traffic intensity**.

- Units for signal strength measurement.
- Principles of wireless signal propagation
- SIR and co-channel interferences in cellular system.
- Modeling and measurement of traffic intensity.

Signal measurements

Unit of measurement

- DeciBel (dB): measures relative strengths of radio signals.
- 10 DeciBel $=$ 1 Bel represents power ratio 1:10.
- Power ratio 1:100 equals 2 Bels or 20 deciBels.
- Power gain due to amplification is measured by relative power strengths of input power P_{input} and amplified power P_{amp}.
- In log scale, $\log_{10}(P_{amp}/P_{input})$ measures relative power strength due to amplification in Bels.
- Eg., if an amplifier outputs 100W with an input of 100 mW, then power gain is $\log_{10}(100/0.1) = \log_{10} 1000 = 3$ Bels or 30 deciBels.

Signal measurements

Illustrative example

Problem

Suppose, a micro-wave system uses a 10w transmitter. The transmitter is connected by a cable with 0.7dB loss to a 13dB antenna. Let atmospheric loss be 137dB on transmission. The receiver antenna with 11dB gain connected to cable with 1.3dB loss to the receiver. Then the power at the received can be calculated as follows:

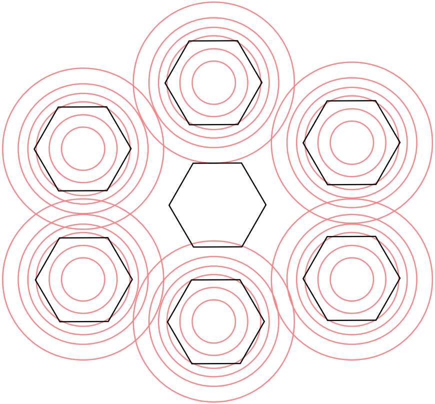
Signal measurements

Illustrative example

Solution

- 10 watts = 10000 mW.
- $10\log_{10}(10000/1) = 40$dB.
- Then the relative strength of power at the transmitter side = (40 - 0.7 + 13 - 137)dB = -84.7dB.
- On the receiver side (11 - 1.4)dB loss.
- So the net power received by the receiver = (-84.7 + 9.6)dB = -75.1dB.

Co-channel interference



Co-channel interference

Measuring signal to interference

- Quality of received signals from the current BS affected by interference from signals of its nearby BS which uses the same frequency.
- Co-channel interference is measured by Signal to Interference Ratio (SIR) at mobile terminals. This ratio is

$$S/I = S/\left(\sum_{i=1}^{i_0} I_i\right),$$

Co-channel interference

Signal attenuation

- In free space, signal strength decays according to a power law involving distance between transmitter and the receiver.

  d: is the mutual distance of the transmitter and the receiver.
  P_0: is the power received at the reference point which is at distance d_0 from the transmitter.
  - Then the average received power P_r at the receiver from the transmitting antenna is given by:

$$P_r \propto P_0 \left(\frac{d}{d_0}\right)^{-n},$$

where n is the path loss exponent.

Co-channel interference

Signal propagation

- In log scale:

$$\log_{10} P_r = \log_{10} P_0 - n \log_{10} \left(\frac{d}{d_0}\right).$$

- So, in deciBel units:

$$P_r(dB) = P_0(dB) - 10n \log_{10} \left(\frac{d}{d_0}\right).$$

Note: n is in Bel so $10n$ is deciBel equivalent.

Co-channel interference

SIR for co-channel interference

- D_i: the distance of an MT from ith co-channel cell.
- R: is the radio range of current BS.
- Signal attenuation of co-channel cell is proportional to D_i^{-n}.
- Signal received from the current BS is proportional to R^{-n}.

Co-channel interference

SIR for co-channel interference

- Assuming all interfering co-channel BSs at equal distance from MT, i.e. all D_is are same, the SIR (in dB) is:

$$\begin{aligned} S/I &= 10\log_{10}\left(R^{-n}/\left(\sum_{i=1}^{i_0} D_i^{-n}\right)\right) \\ &= 10\log_{10}(D/R)^n/i_0 \\ &= 10\log_{10}\left(\left(\sqrt{3N}\right)^n/i_0\right). \end{aligned}$$

Note: MT located at the center of the current cell.

Co-channel interference

SIR for co-channel interference

- 18dB: minimum SIR for good voice quality.
- 4: path loss exponent.

$$
\begin{aligned}
S/I &= 10\log_{10}\left(\left(\sqrt{3N}\right)^4 / i_0\right) \\
&= 10\log_{10}\left(\left(\sqrt{3N}\right)^4 / (N-1)\right) \\
&= 10\log_{10} 9 + 20\log_{10} N - 10\log_{10}(N-1)
\end{aligned}
$$

- For $N = 7$, the above expression evaluates to 18.66.

Co-channel interference

SIR with cell sectorization

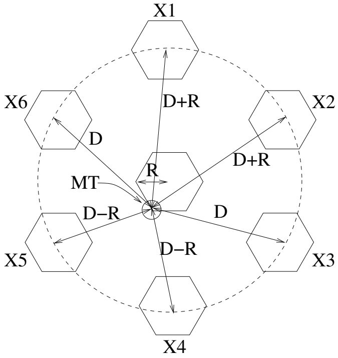
- With 120^o sectors, the number of co-channel cells is reduced from 6 to 2 for $N = 7$.
- Therefore, the SIR is:

$$S/I = \frac{1}{2}\left(\sqrt{3N}\right)^n$$

- Implying that SIR increases with sectorization by 3 times.

Co-channel interference

Topological consideration for SIR



The previous expression is derived assuming MT to be located at the center of its current cell. But in worst case scenario may be:

Co-channel interference

Topological consideration for SIR

- The distances between MT and the BSs of different co-channel cells will be in the range $\{D - R, D, D + R\}$.
  - Two co-channel cells at a distance $D - R$
  - Two at a distance D and
  - Two others at a distance $D + R$.

Co-channel interference

Topological consideration for SIR

- Thus, the ratio of power strengths of current BS and the other interfering BSs, is

$$
\begin{aligned}
S/I &= \frac{R^{-4}}{2(D-R)^{-4} + 2D^{-4} + 2(D+R)^{-4}} \\
&= \frac{1}{2(\sqrt{21}-1)^{-4} + 2(\sqrt{21})^{-4} + 2(\sqrt{21}+1)^{-4}} \\
&= 49.56
\end{aligned}
$$

- So, the value of SIR $= 10\log_{10} 49.56 = 17$ dB.
- Implying the voice quality will not be good.

Traffic intensity

- Traffic intensity varies over the day.
- **Grade of Service** (GoS) is directly related to traffic intensity.
- TI is measured in a unit called **Erlang**.
- One Erlang: traffic volume for one hour.

Example

Eg. consider 40 calls/hour with each of average duration of 5 minutes, then the traffic in Erlang:

$$\text{Traffic in hour } = (40 \times 5)/60 = 3.33 \text{ Erlangs}$$

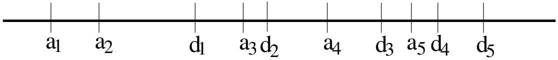
Erlang B model

Notations

- In a lossy system, GoS is computed by *Erlang B* traffic model.
- λ: arrival rate, and μ: service rate.
- $1/\lambda$: average time between arrival of two consecutive requests
- $1/\mu$: average service time.
- Eg., if average duration of connection is 3 minutes, then $\mu = 20$ and.$1/\mu = 3/60 = 0.05$ hour.

Erlang B model

Modeling



- Depicts connection requests and servicing of requests for 5 users.
- Interval $I_i = a_{i+1} - a_i$ represent the inter-arrival time.
- Duration of service represented intervals $S_1 = d_1 - a_1$, $S_2 = d_2 - d_1$, $S_3 = d_3 - d_2$, $S_4 = d_4 - d_3$, $S_5 = d_5 - d_4$.
- The arrival rate and service rate are given by expressions $1/E(I_i)$ and $1/E(S_i)$.

Erlang B model

Modeling

- The inter-arrival times for conneservicing are modeled by **Poisson distribution**.
- The rate λ of a Poisson process is the average number of number events per unit time over a long period.
- The probability of n call requests arriving during an interval of time $[0, t)$ under Poisson process is,

$$Pr_n[t] = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \text{ for } n = 0, 1, \ldots.$$

Erlang B model

Modeling

- Under Poisson arrivals, the call requests arriving during two non-overlapping intervals are independent.
- I.e., $Pr_n[t2 - t1]$ and $Pr_n[t4 - t3]$ are independent,
- For every $t \geq 0$, and $\delta \geq 0$:

$$\begin{aligned} Pr[n_{t+\delta} - n_t = 0] &= 1 - \lambda\delta + O(\delta^2) \\ Pr[n_{t+\delta} - n_t = 1] &= \lambda\delta + O(\delta^2) \\ Pr[n_{t+\delta} - n_t \geq 2] &= O(\delta^2) \end{aligned}$$

- O(δ^2): probability of more than one call request arriving, which is 0, since $\lim_{\delta\to 0}\left\{O\left(\frac{O(\delta^2)}{\delta}\right)\right\} = 0$

Erlang B model

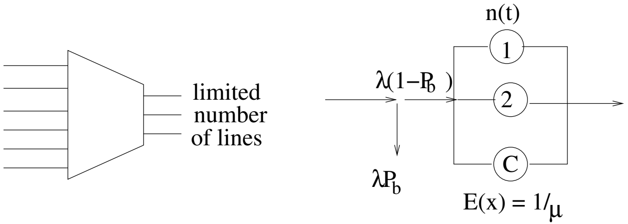
Markov chain

We can use Markov chain to represent channel occupancy.

- The number of channels is C can service C requests concurrently.
- Therefore, it is $M/M/C/C$ **queuing** system with following parameters:
  - Arrival process is Poisson with arrival rate λ.
  - The service time is exponential with servicing rate μ.
  - The number of servers or the channels for serving the connection requests is C.
  - The capacity (number clients which may be in the queue) is C.

Erlang B model

Markov chain



Erlang B model

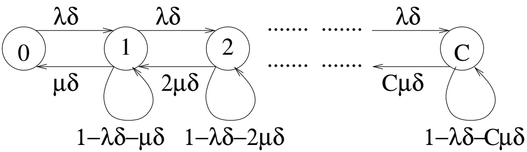
Markov chain

- Suppose 0 channels being used by the system.
- Over a small interval, system may continue in 0 state is 1-$\lambda\delta$.
- The probability there will be change to 1 state (1 channel in use) is $\lambda\delta$.

Erlang B model

Markov chain

- If one channel is in use (state 1), then the probability for transition to state 0 will be $\mu\delta$.
- The system will continue in state 1 with $1 - \lambda\delta - \mu\delta$.



Erlang B model

Erlang B formula

- After a sufficiently long time, system reaches **steady state**.
- What is the steady state probability for i channels being occupied?
  - System switches from i to $i-1$ with servicing of a call.
  - System switches back to i from $i-1$ with a call arrival.
- So, under the steady state condition:

$$\lambda\delta P_{i-1} = i\mu\delta P_i, i \leq C$$

which also known as **global balance equation**

Erlang B model

Erlang B formula

- $\sum_0^C P_i = 1$, so, $P_0 = 1 - \sum_{i=1}^C P_i$
- Solving balance equation for $i = 1, 2, \ldots, C$

$$P_i = P_0 \left(\frac{\lambda}{\mu}\right)^i \frac{1}{i!}$$

- Thus,

$$P_0 = \left(\frac{\mu}{\lambda}\right)^C C! P_C = 1 - \sum_{i=1}^{C} P_i$$

Erlang B model

Erlang B formula

- Substituting P_is in terms of P_0 and rearranging terms in the equation in the preceding slide, we get

$$P_0 = \frac{1}{\sum_{i=0}^{C} \left(\frac{\lambda}{\mu}\right)^i \frac{1}{i!}}$$

- From balance equation: $P_C = P_0 \left(\frac{\lambda}{\mu}\right)^C \frac{1}{C!}$.
- So,

$$P_C = \frac{\left(\frac{\lambda}{\mu}\right)^C \frac{1}{C!}}{\sum_{i=0}^{C} \left(\frac{\lambda}{\mu}\right)^i \frac{1}{i!}}$$

Erlang B model

Erlang B formula

- Traffic intensity is $A = \lambda(1/\mu)$ is measure of congestion.
- Replacing λ/μ in preceding expression for P_C:

$$P_C = \frac{A^C \frac{1}{C!}}{\sum_{i=0}^{C} A^i \frac{1}{i!}}$$

- Above equation is called Erlang B formula.
- This formula is also known as **blocked call cleared formula** as it determines the GoS for traffic system with no queueing for blocked calls.

Erlang B model

Call blocking probability

- Suppose there are 200 connection requests per hour in peak time, i.e., the arrival rate is $\lambda = 200$.
- Average call duration be 3 minutes, or 0.05 hour, i.e., the service rate $\mu = 20$.
- It gives $A = \frac{\lambda}{\mu} = 200/20 = 10$.

Erlang B model

Call blocking probability

- Note that average number of requests per hour λ, and the average call duration is $1/\mu$.
- Hence, the ratio $A = \lambda/\mu$ being equal to the product of the average number of requests and the average duration of the calls, is called the **busy hour traffic**.

Erlang B model

Illustrative examples

Problem

Suppose 200 requests per hour are received during peak period, i.e. $\lambda = 200$. Let average call duration be 3mts. What will be probability for call blocking when 25 channels are used?

Solution

$A = \lambda/\mu = 10$. So using Erlang B formula we have probability of call blocking given by:

$$\frac{10^{25} \times \frac{1}{25!}}{\sum_0^{25} 10^i \frac{1}{i!}} = 2.927 \times 10^{-5}.$$

Erlang B model

Illustrative examples

Problem

Suppose each user averages 3 call per hour and every call has an average duration of 5 minutes. system with 400 cells is used by a service provider with 20 channels per cell. Find the number of subscriber which service provider can support at 2% blocking.

Erlang B model

Illustrative examples

Solution

- Probability of call blocking = 2%.
- Number of channels used is $C = 20$.
- Traffic intensity per user $= 3\times\ 0.12 = .36$ Erlangs.
- For GoS $= 0.02$, and $C = 20$, total traffic carried traffic is $A = 13.15$ Erlang.
- It implies that the number of concurrent users supported in one cell $= 13.15/.36 = 36$.
- So the service provider can support $= 36\times\ 400 = 14400$ subscribers

Summary

In this module we discussed about the following

- Units for measurement of wireless signal.
- Propagation of wireless signals and its attenuation over distance.
- Derived expression for SIR for co-channel interference in cellular system.
- Measurement traffic intensity using Erlang B model.