Learning Classifiers

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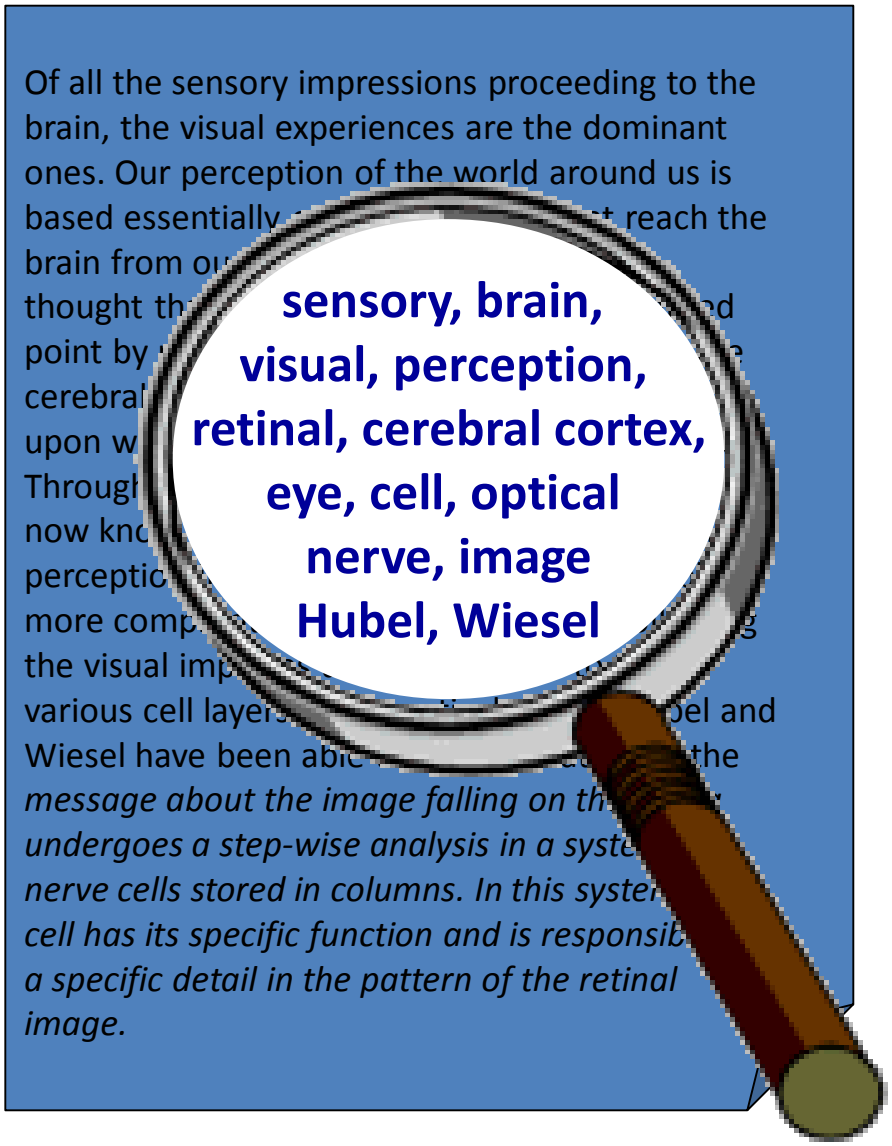
Slide content by Prof. Kristen Grauman, UT Austin

Object → **Bag of 'words'**

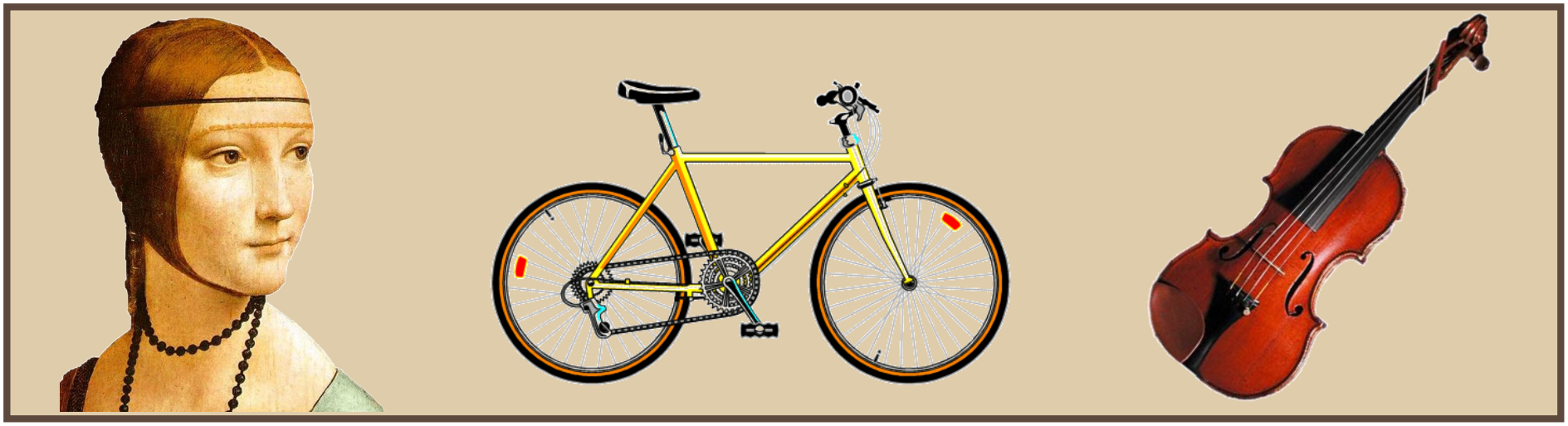
Last Class

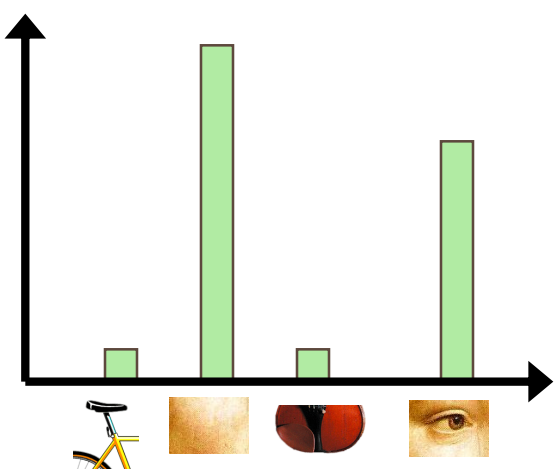
Analogy to documents

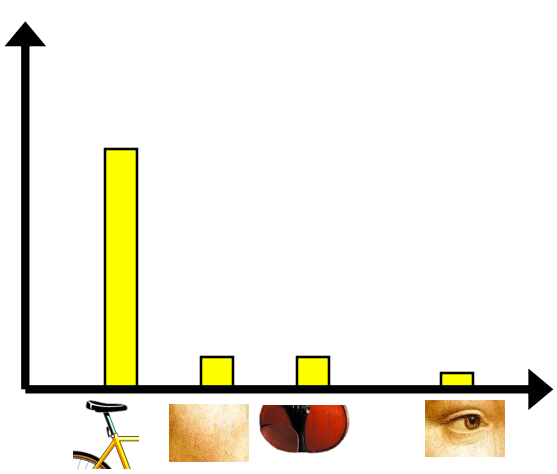


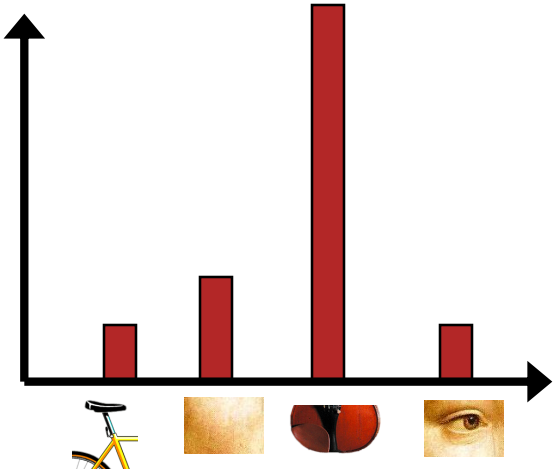
Of all the sensory impressions proceeding to the brain, the visual experiences are the dominant ones. Our perception of the world around us is based essentially … reach the brain from ou… thought th… ed point by … cerebral… upon w… Through… now kno… perceptio… more comp… g the visual imp… various cell layer… bel and Wiesel have been abl… the *message about the image falling on th… undergoes a step-wise analysis in a syste… nerve cells stored in columns. In this syste… cell has its specific function and is responsib… a specific detail in the pattern of the retinal image.*

China is forecasting a trade surplus of $90bn (£51bn) to $100bn this year, a threefold increase on 2004's $32bn. The Commerce Ministry said the surplus would … dicted 30% jump in expo… h a 18% rise in imp… ely to further a… hat China's deliber… the sur… one fac… Xiaochua… more to bo… s stayed within… e value of the yua… % in July and permitted it … w band, but the US wants the yuan to be … d to trade freely. However, Beijing has made … that it will take its time and tread carefull… allowing the yuan to rise further in value.

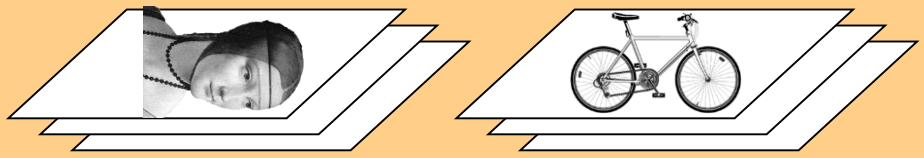






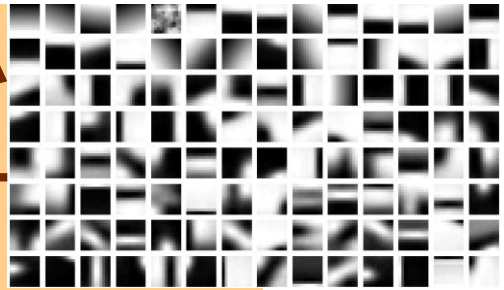


Representation



2.

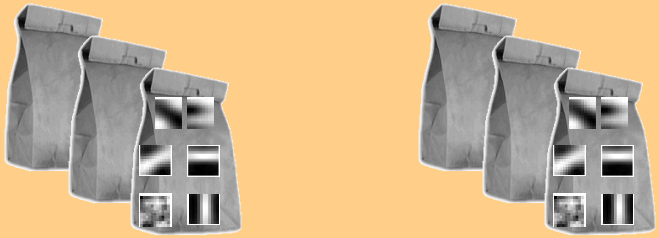
codewords dictionary



1. feature detection & representation

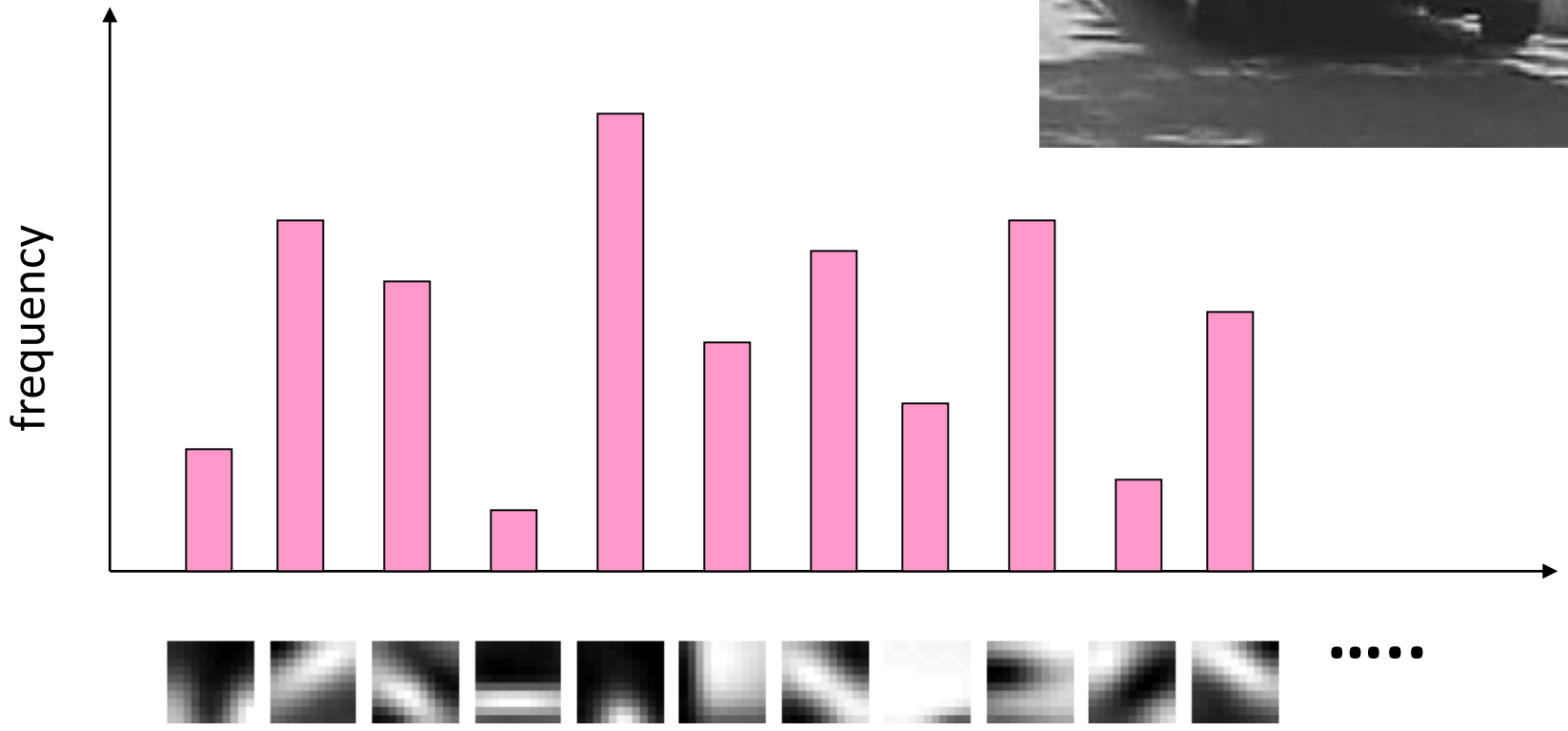
image representation

3.



Last Class

3. Image representation



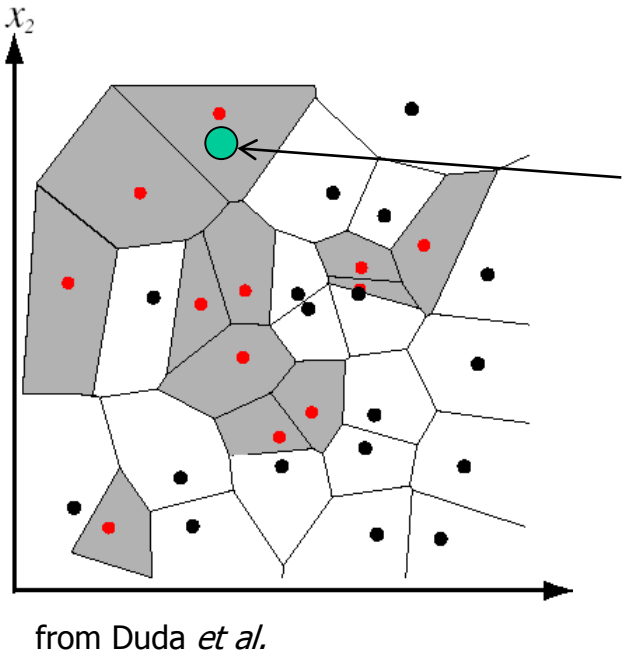
This Class

- Representation
  - How to represent an object category
- Learning
  - How to form the classifier, given training data
- Recognition
  - How the classifier is to be used on novel data

Nearest Neighbor classification

- Assign label of nearest training data point to each test data point

Black = negative
Red = positive



from Duda *et al.*

Novel test example

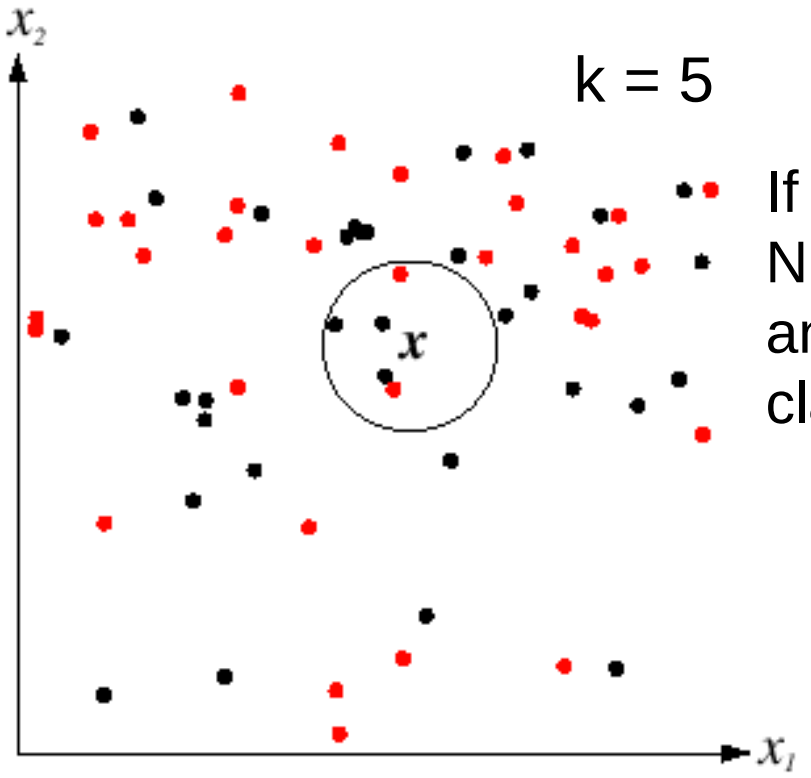
Closest to a positive example from the training set, so classify it as positive.

Voronoi partitioning of feature space
for 2-category 2D data

K-Nearest Neighbors classification

- For a new point, find the k closest points from training data
- Labels of the k points “vote” to classify

Black = negative
Red = positive



If query lands here, the 5 NN consist of 3 negatives and 2 positives, so we classify it as negative.

Source: D. Lowe

A nearest neighbor recognition example

Where in the World?

[Hays and Efros. **im2gps**: Estimating Geographic Information from a Single Image. CVPR 2008.]

Slides: James Hays

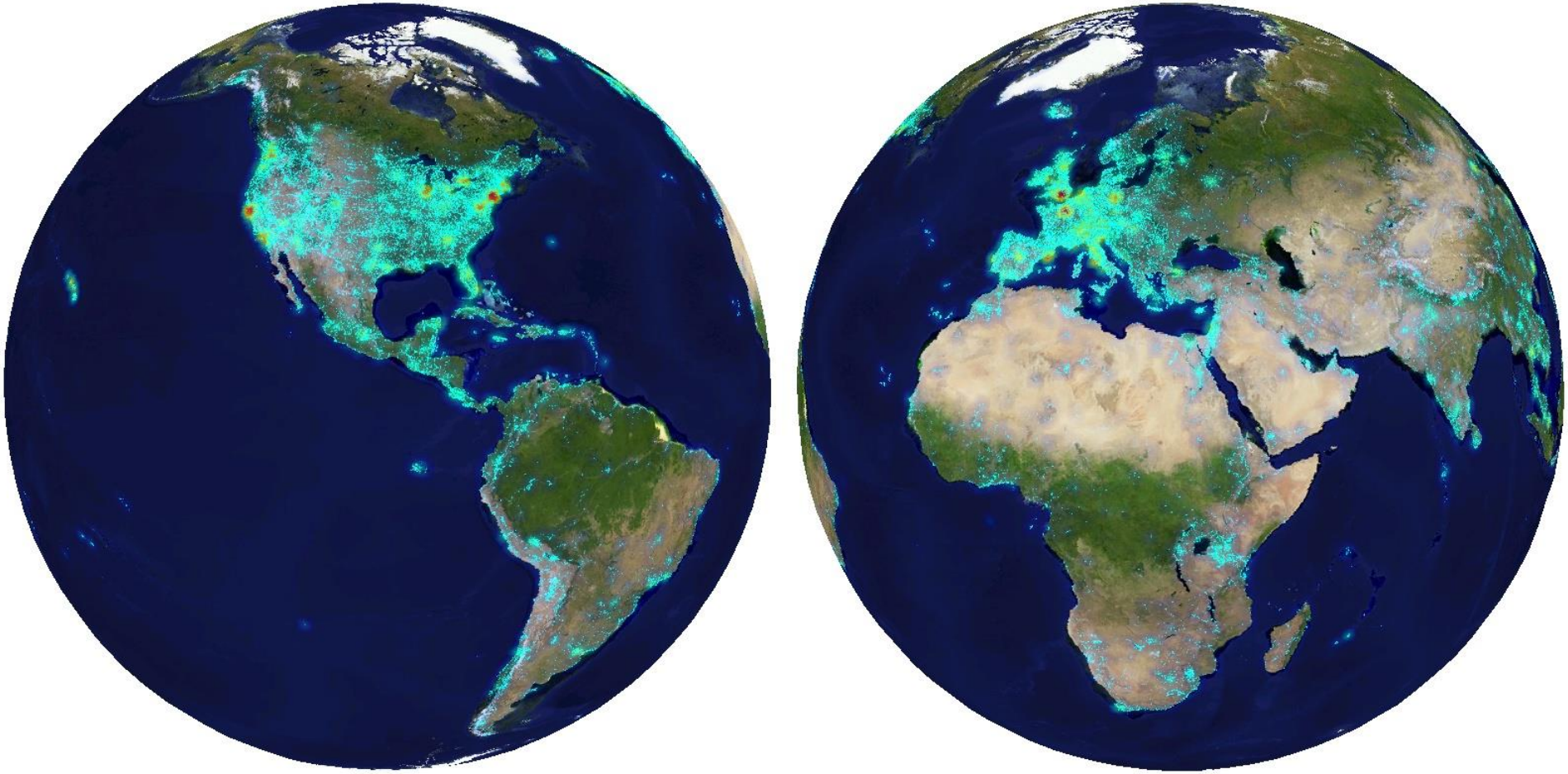
Where in the World?

Slides: James Hays

Where in the World?

Slides: James Hays

6+ million geotagged photos by 109,788 photographers



Annotated by Flickr users

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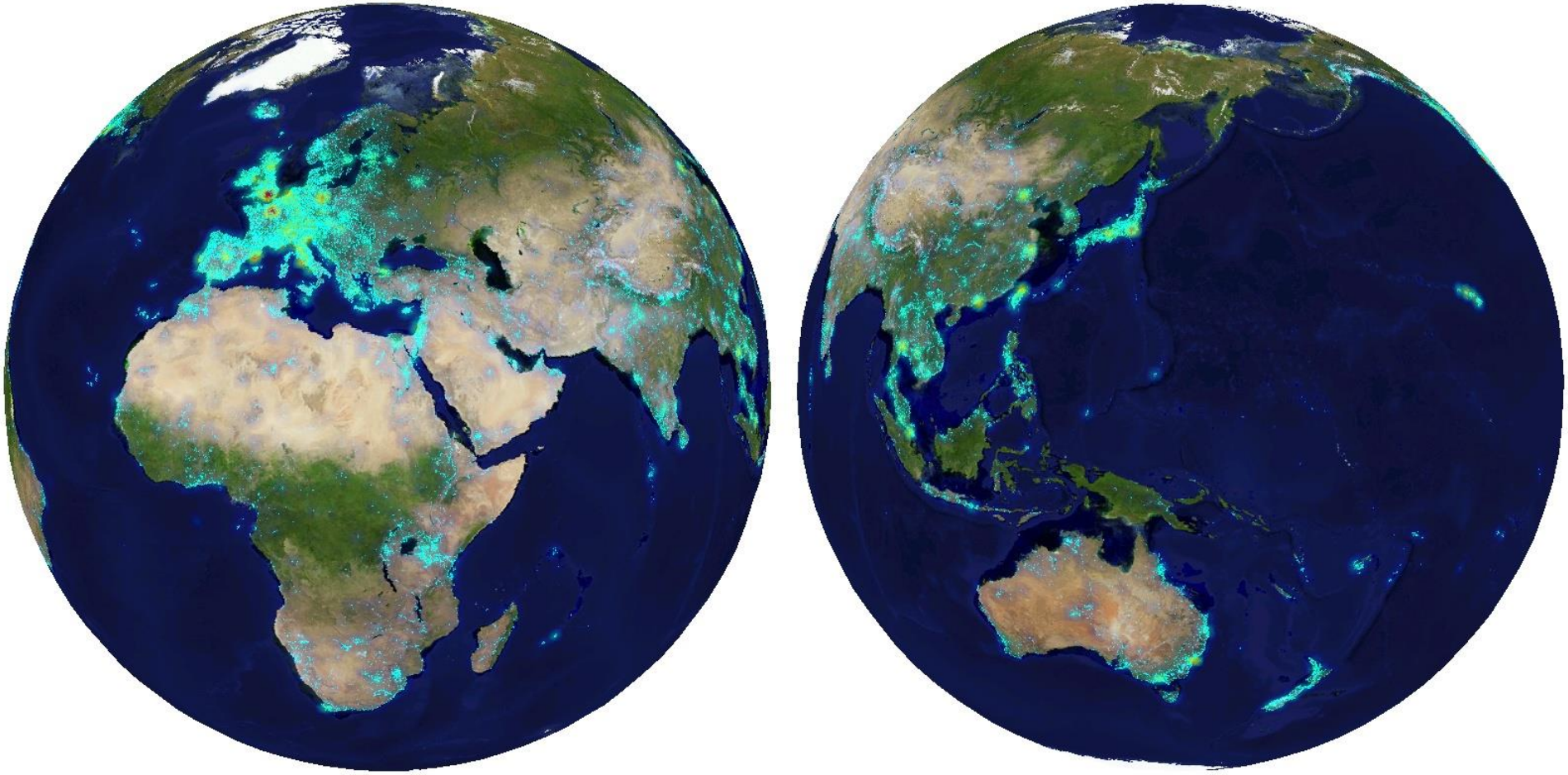
6+ million geotagged photos by 109,788 photographers



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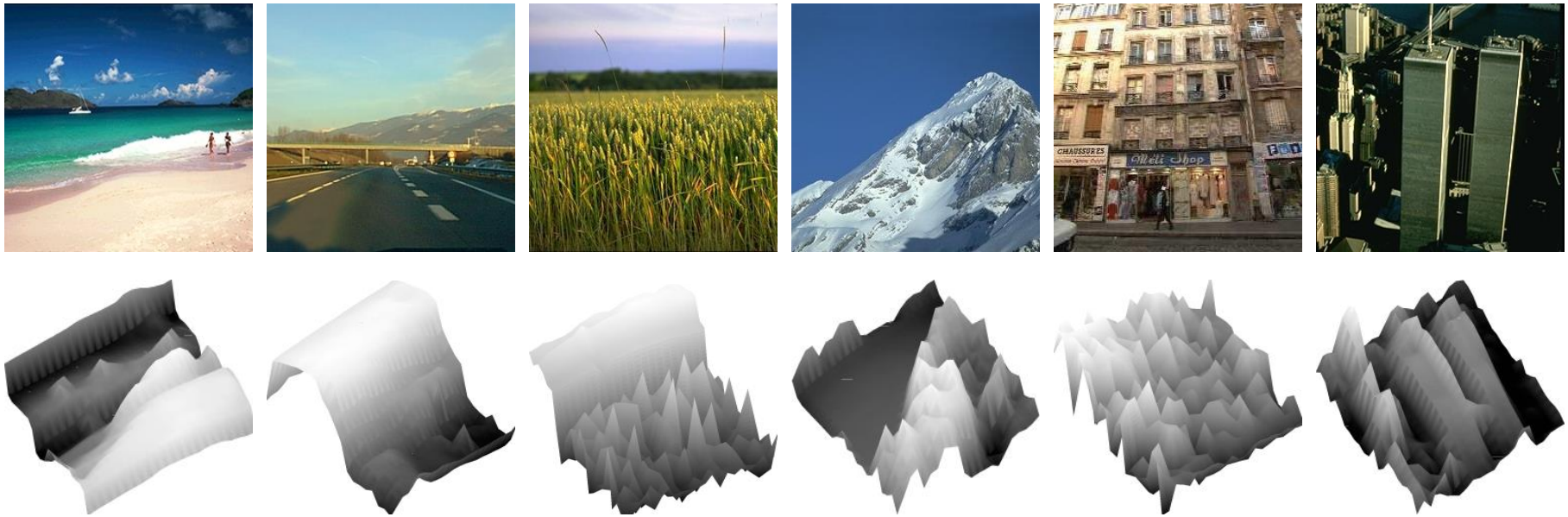
Which scene properties are relevant?

Spatial Envelope Theory of Scene Representation

Oliva & Torralba (2001)

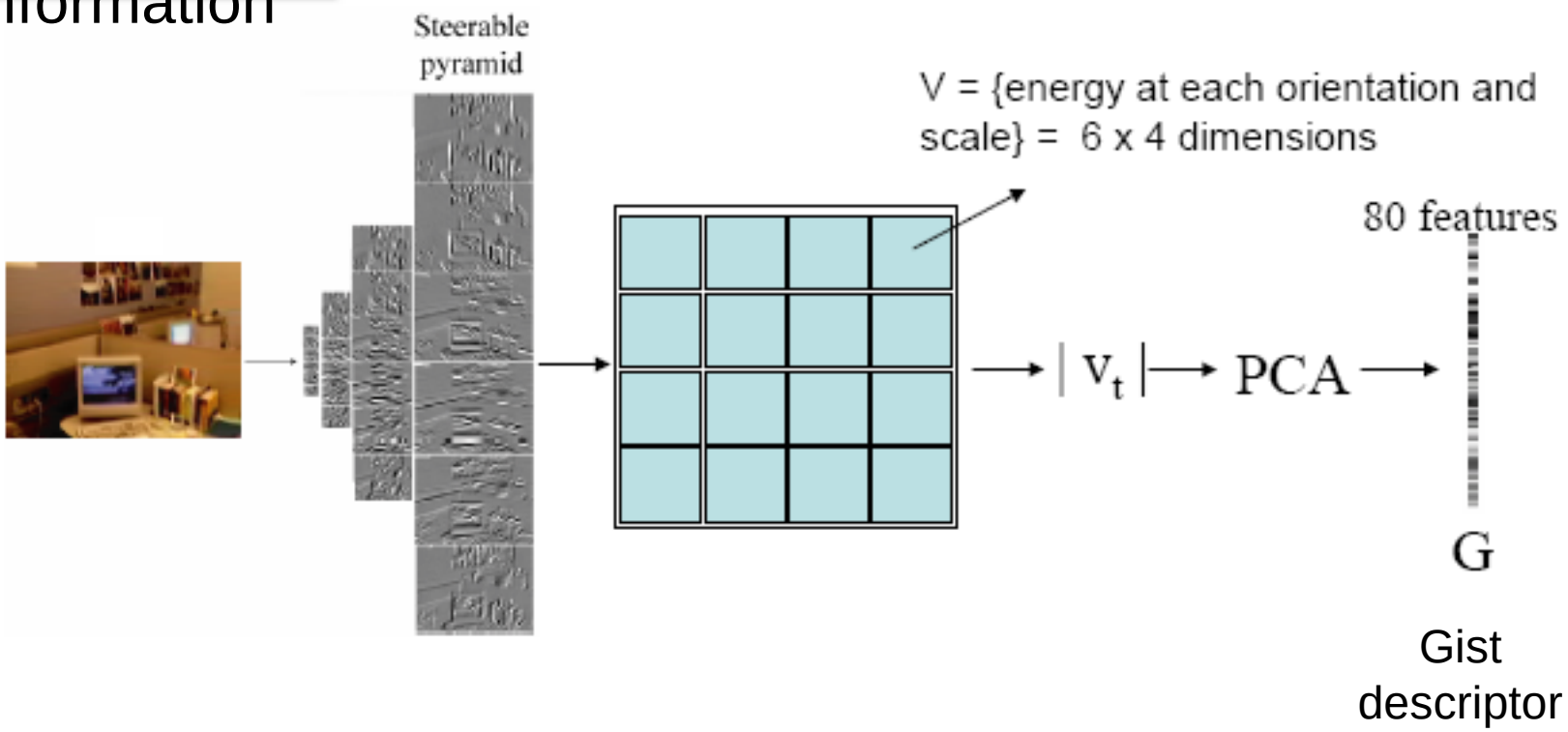


A scene is a single surface that can be represented by global (statistical) descriptors

Slide Credit: Aude Olivia

Global texture: capturing the “Gist” of the scene

Capture global image properties while keeping some spatial information



Oliva & Torralba IJCV 2001, Torralba et al. CVPR 2003

Which scene properties are relevant?

- **Gist scene descriptor**
- **Color Histograms** - L*A*B* 4x14x14 histograms
- **Texton Histograms** – 512 entry, filter bank based
- **Line Features** – Histograms of straight line stats

Scene Matches


Madrid


england


France


Paris


Croatia


heidelberg


Macau


Malta


Cairo


Italy


Italy


Italy


Latvia

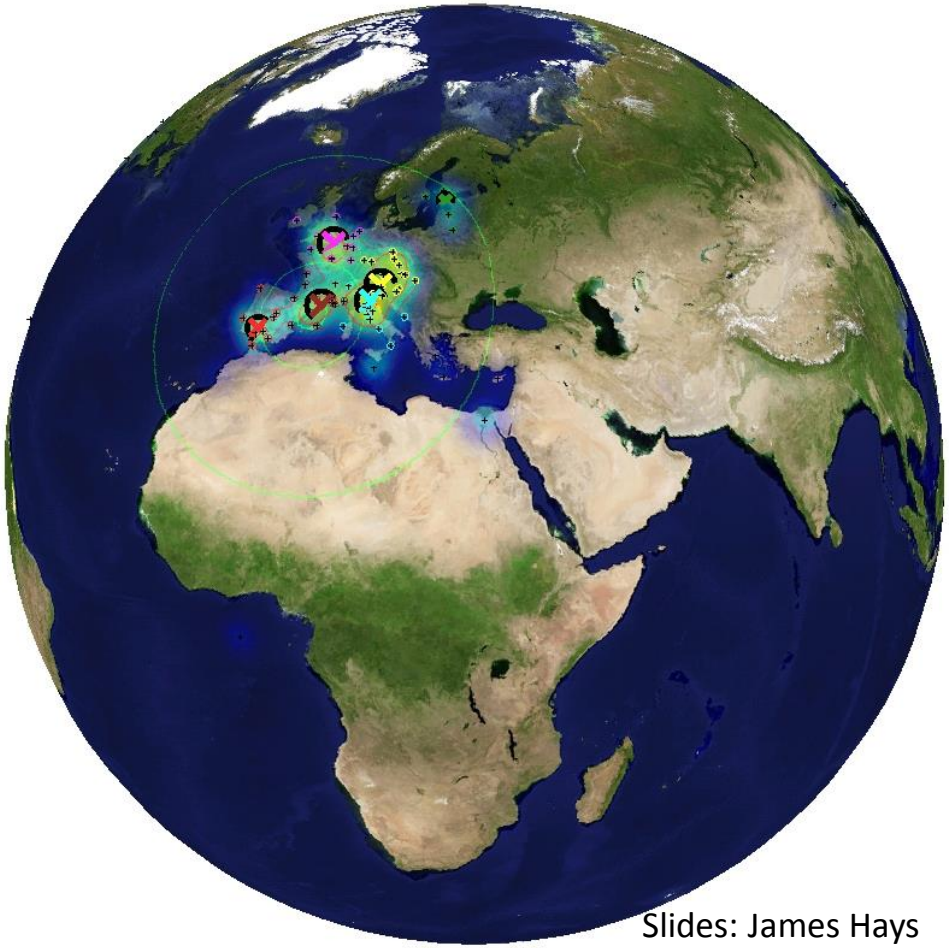

europe


Barcelona


Austria

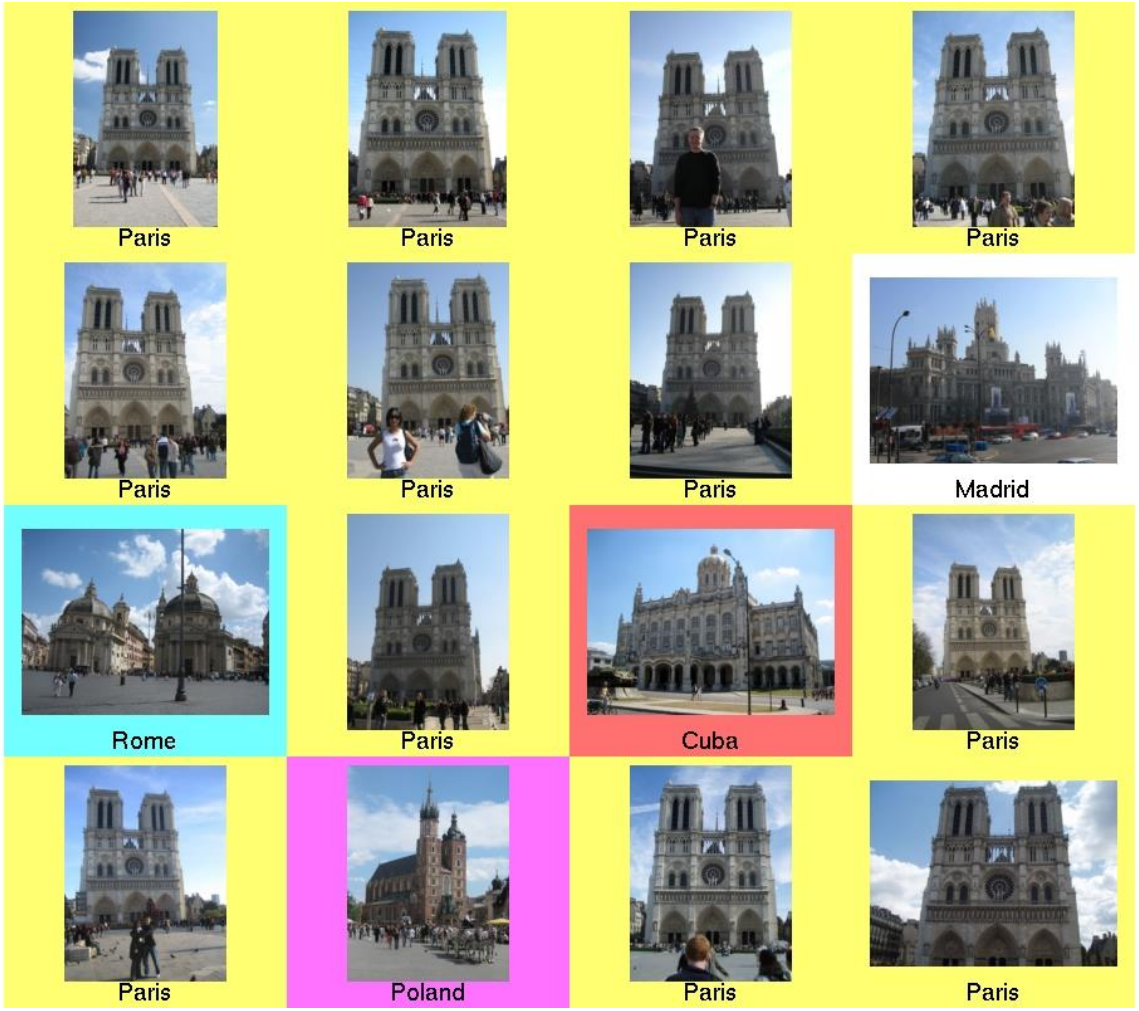
[Hays and Efros. **im2gps**: Estimating Geographic Information from a Single Image. CVPR 2008.]

Slides: James Hays



Slides: James Hays

Scene Matches



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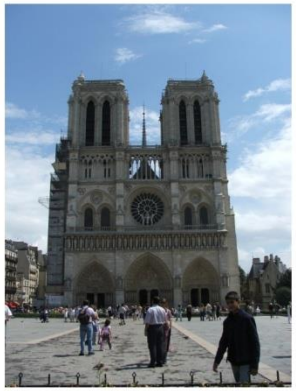

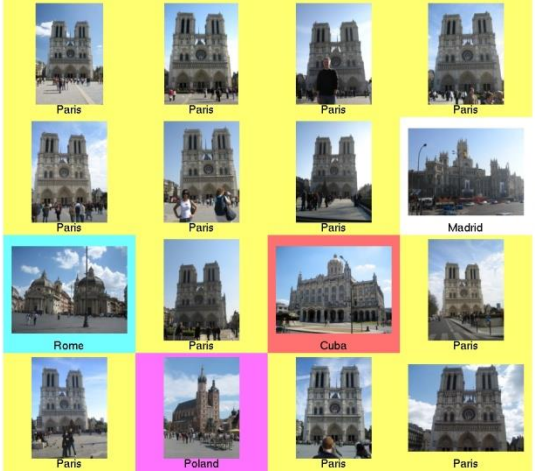

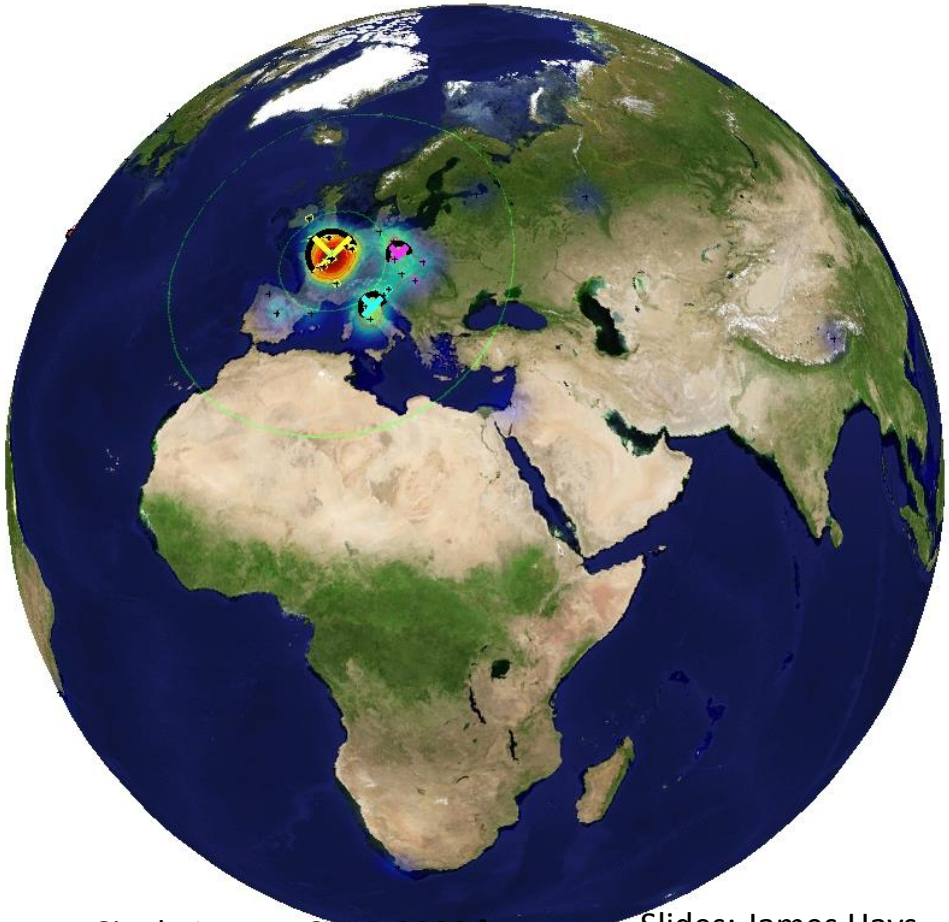

[Hays and Efros. **im2gps**: Estimating Geographic Information from a Single Image. CVPR 2008.]

Slides: James Hays

Scene Matches

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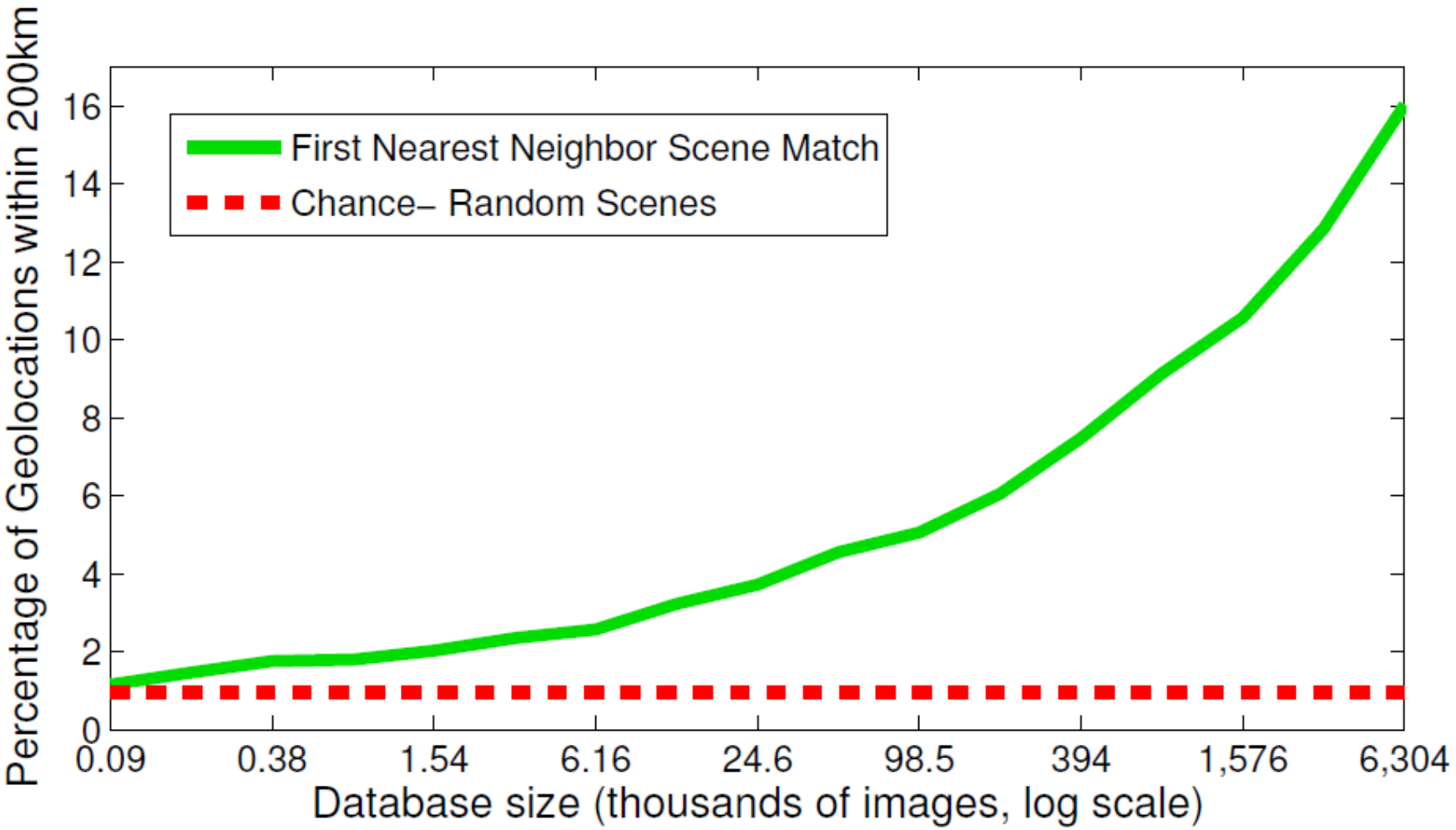
[Hays and Efros. **im2gps**: Estimating Geographic Information from a Single Image. CVPR 2008.]

Slides: James Hays

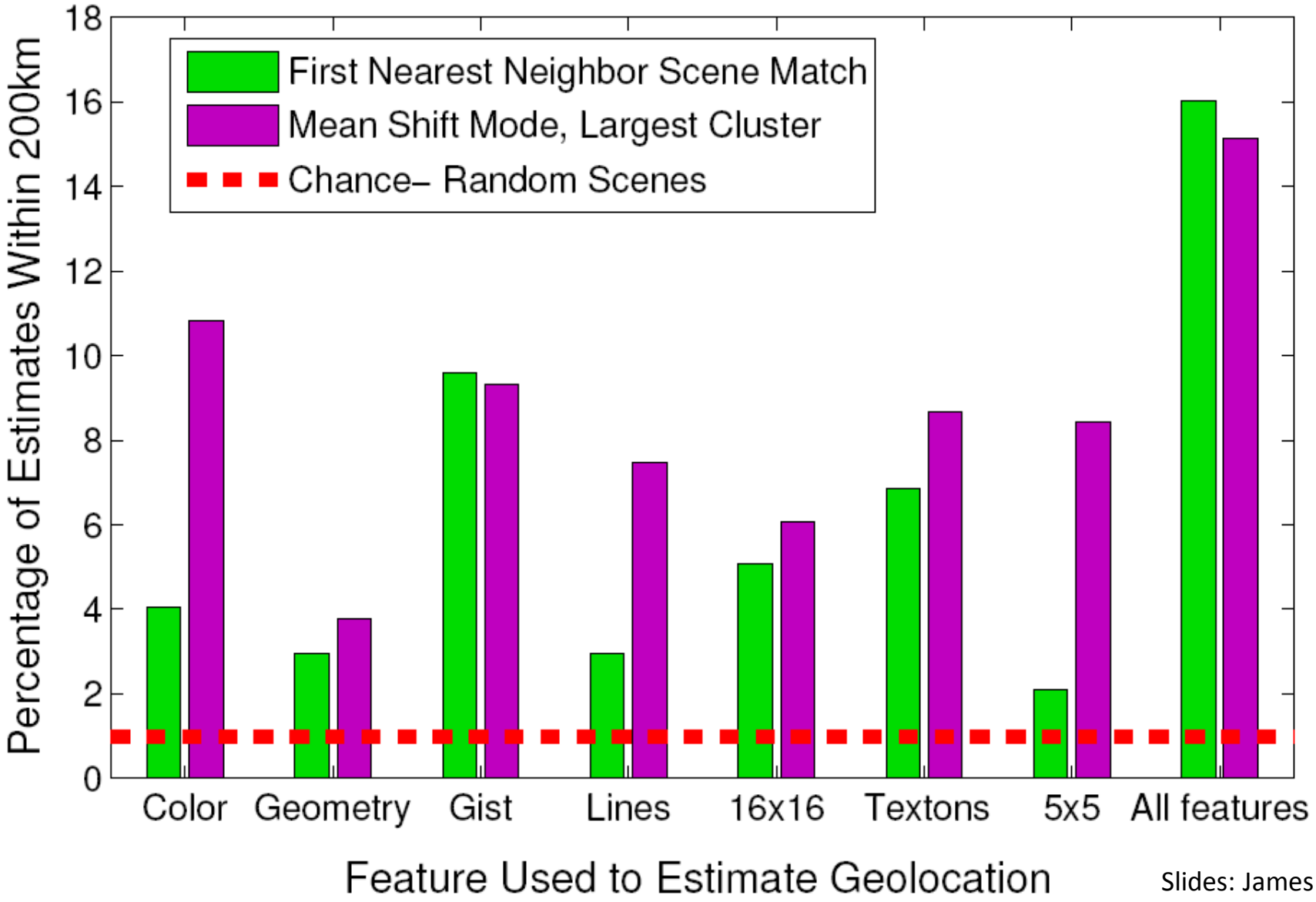
The Importance of Data



[Hays and Efros. **im2gps**: Estimating Geographic Information from a Single Image. CVPR 2008.]

Slides: James Hays

Feature Performance



Slides: James Hays

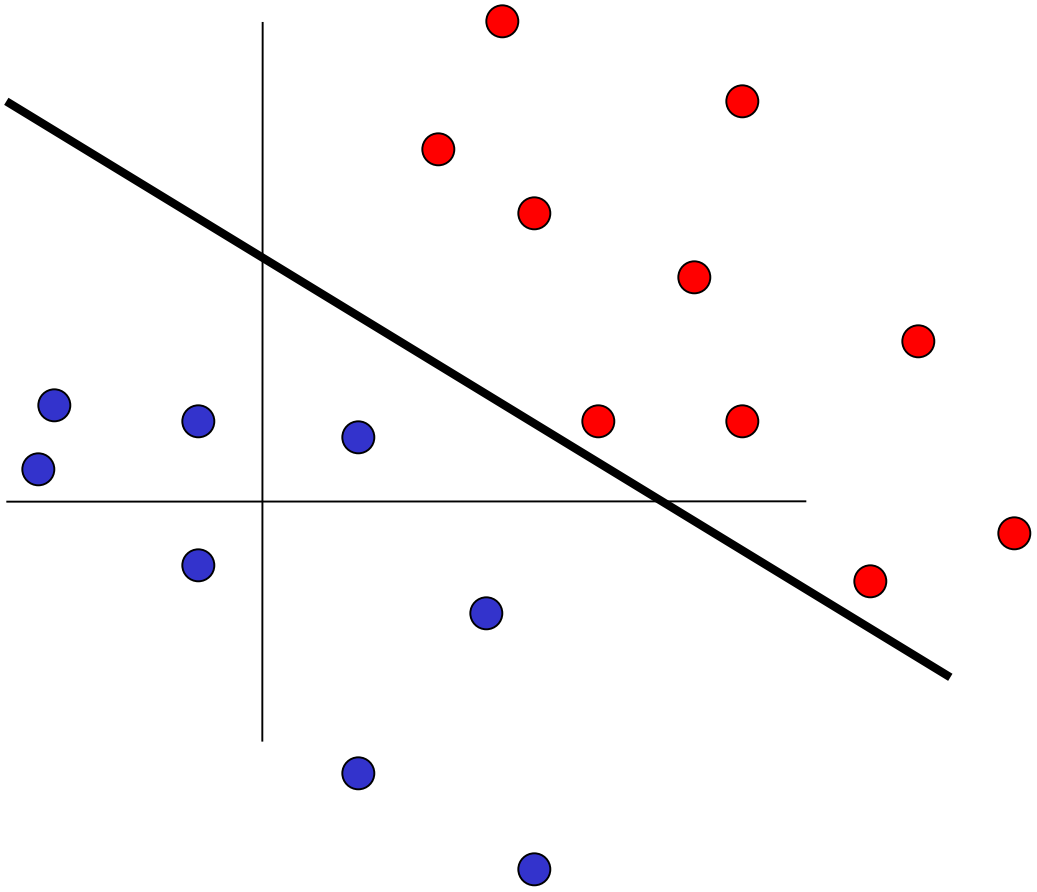
Nearest neighbors: pros and cons

- **Pros**:
  - Simple to implement
  - Flexible to feature / distance choices
  - Naturally handles multi-class cases
  - Can do well in practice with enough representative data
- **Cons:**
  - Large search problem to find nearest neighbors
  - Storage of data
  - Must know we have a meaningful distance function

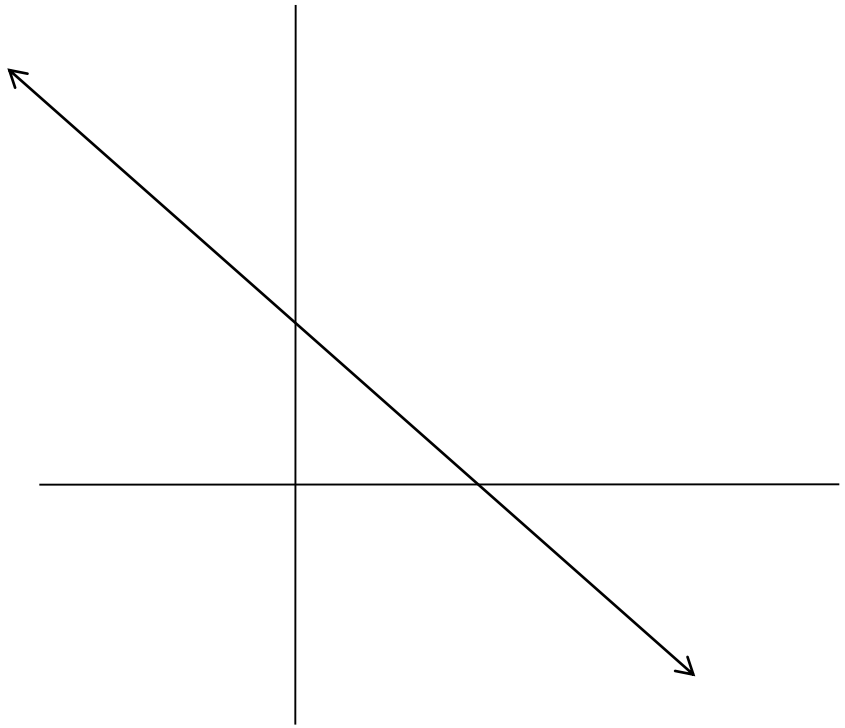
Outline

- Discriminative classifiers
  - Nearest neighbors
  - Support vector machines
  - Boosting

Linear classifiers



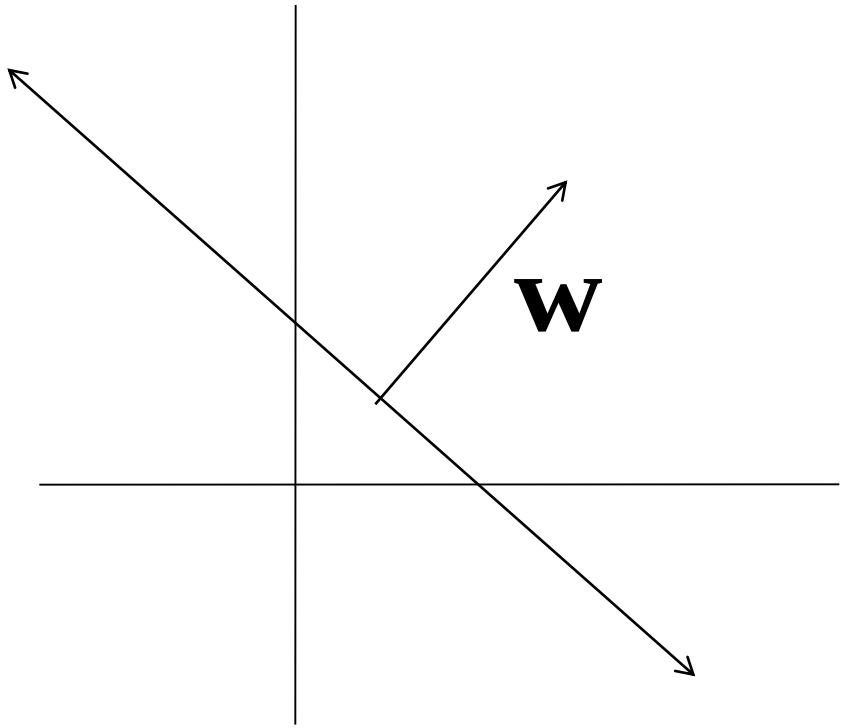
Lines in R^2



Let $\mathbf{w} = \begin{bmatrix} a \\ c \end{bmatrix}$ $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$ax + cy + b = 0$$

Lines in R^2



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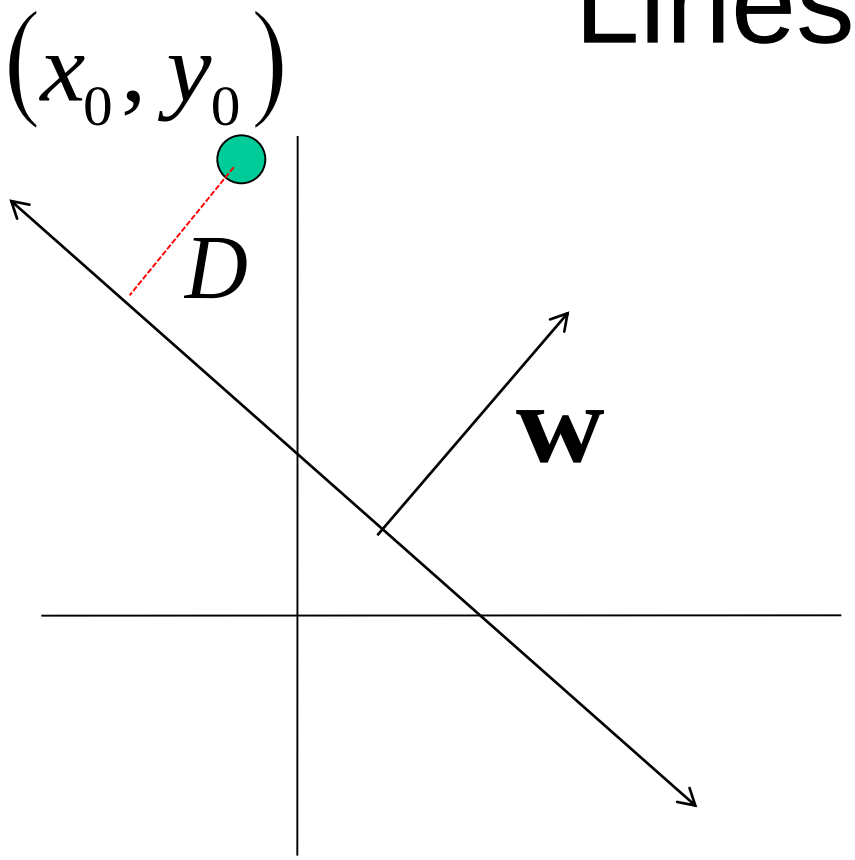
$$\updownarrow$$

$$\mathbf{w} \cdot \mathbf{x} + b = 0$$

Lines in R^2



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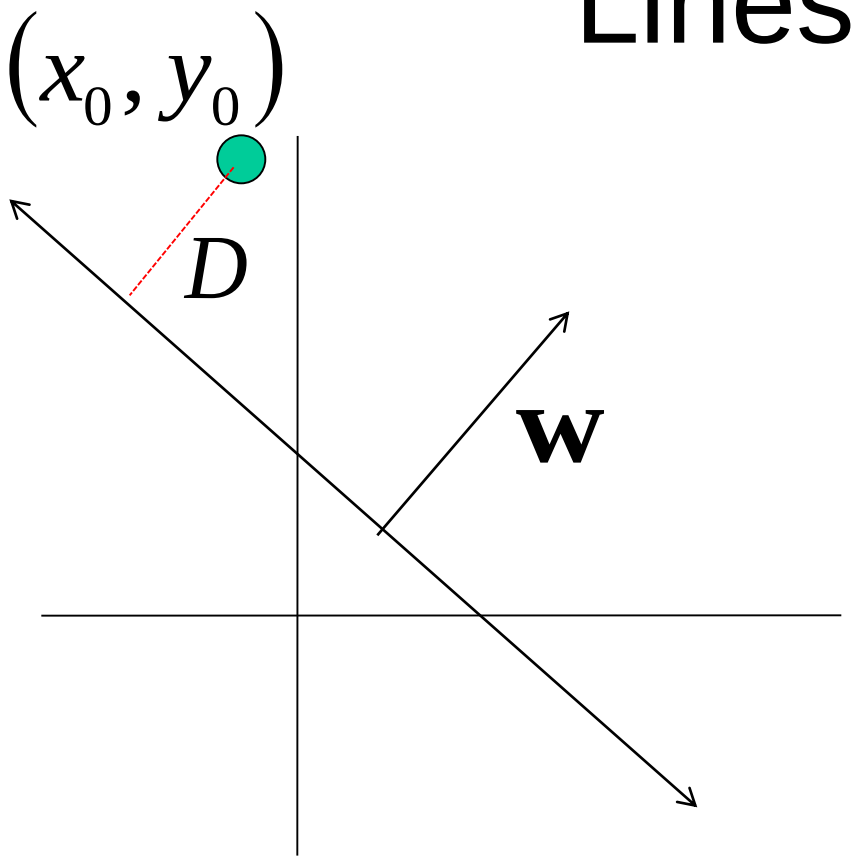

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$$ax + cy + b = 0$$

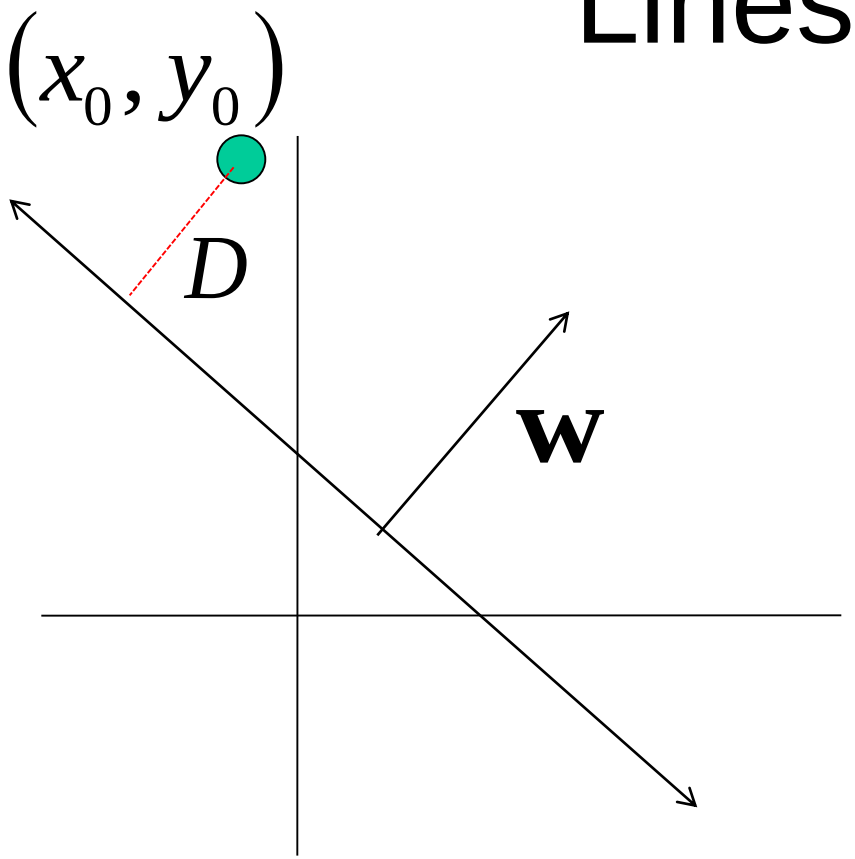
$$\updownarrow$$

$$\mathbf{w} \cdot \mathbf{x} + b = 0$$

$$D = \frac{|ax_0 + cy_0 + b|}{\sqrt{a^2 + c^2}}$$

distance from point to line

Lines in R^2



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$$ax + cy + b = 0$$

$$\updownarrow$$

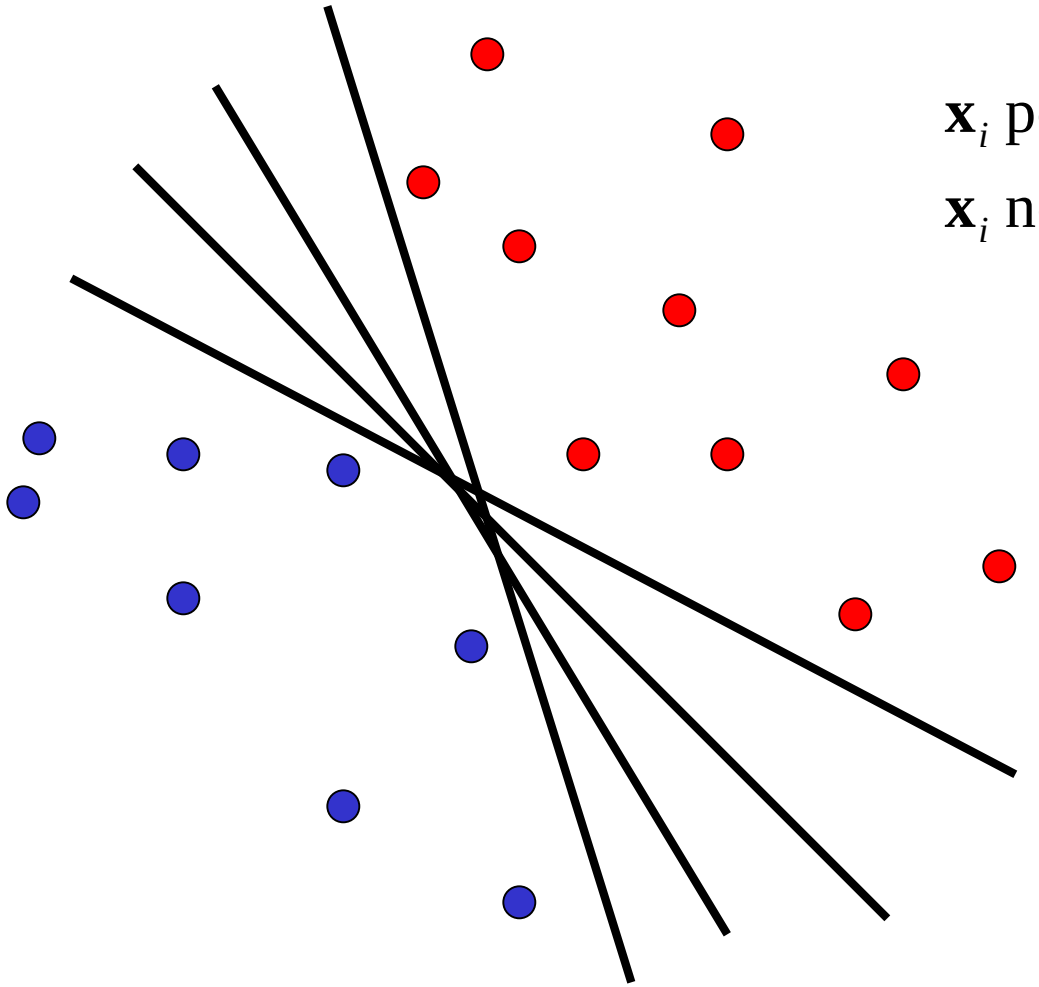
$$\mathbf{w} \cdot \mathbf{x} + b = 0$$

$$D = \frac{|ax_0 + cy_0 + b|}{\sqrt{a^2 + c^2}} = \frac{\mathbf{w}^T\mathbf{x} + b}{\|\mathbf{w}\|}$$

distance from point to line

Linear classifiers

- Find linear function to separate positive and negative examples

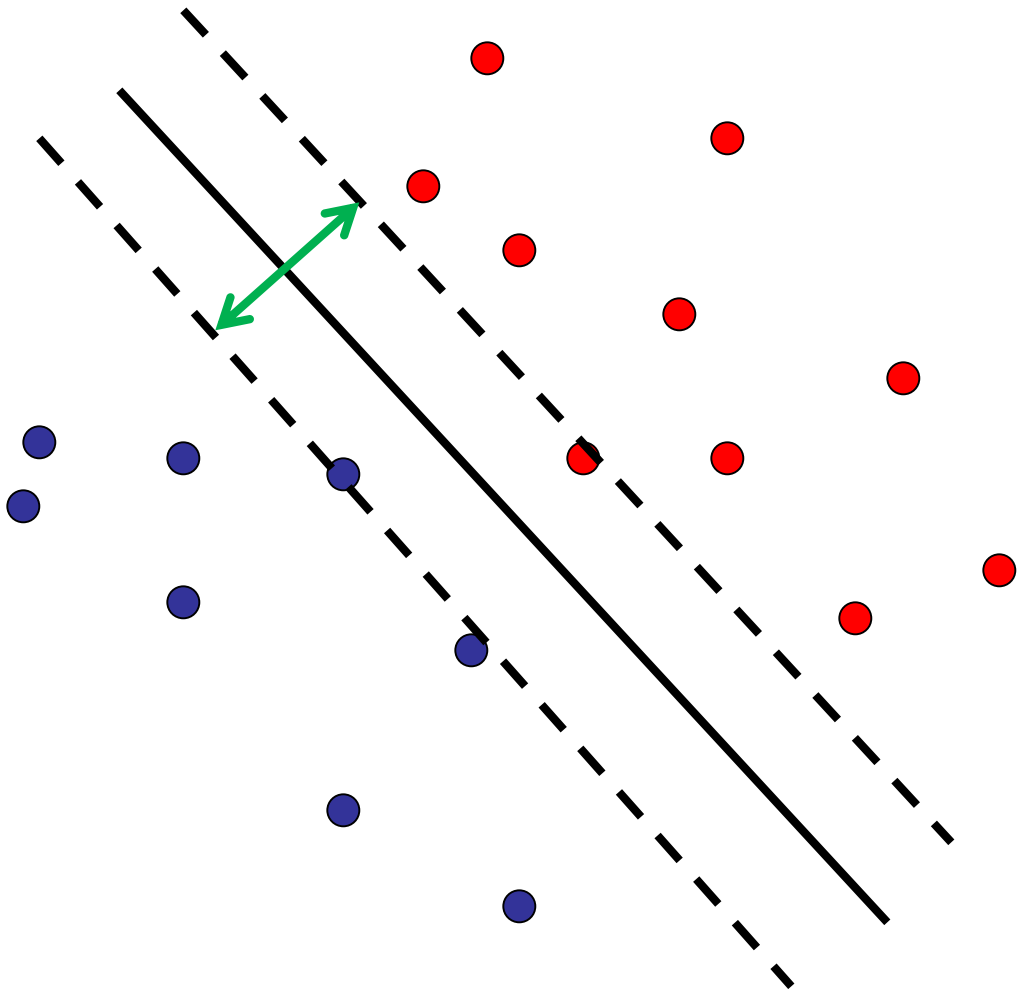


$$\mathbf{x}_i \text{ positive :} \quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 0$$

$$\mathbf{x}_i \text{ negative :} \quad \mathbf{x}_i \cdot \mathbf{w} + b < 0$$

Which line is best?

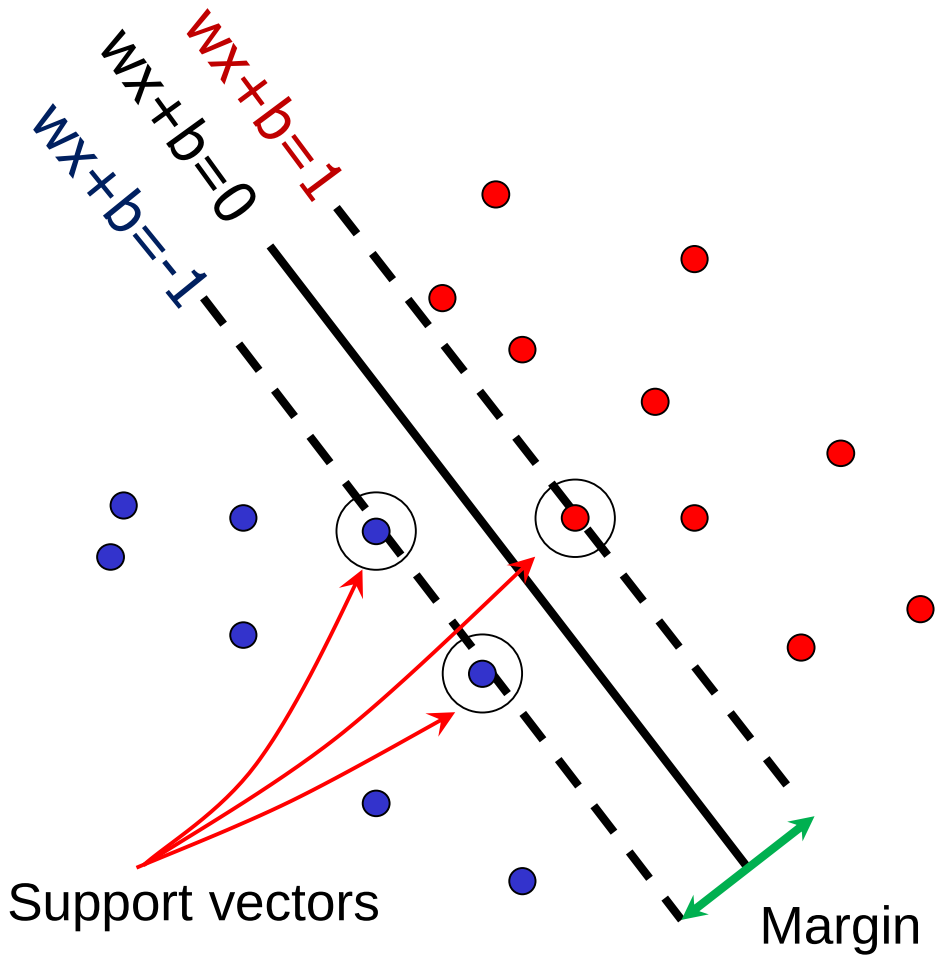
Support Vector Machines (SVMs)



- Discriminative classifier based on *optimal separating line (for 2d case)*
- Maximize the *margin* between the positive and negative training examples

Support vector machines

- Want line that maximizes the margin.



$\mathbf{x}_i$ positive $(y_i = 1)$: $\mathbf{x}_i \cdot \mathbf{w} + b \geq 1$

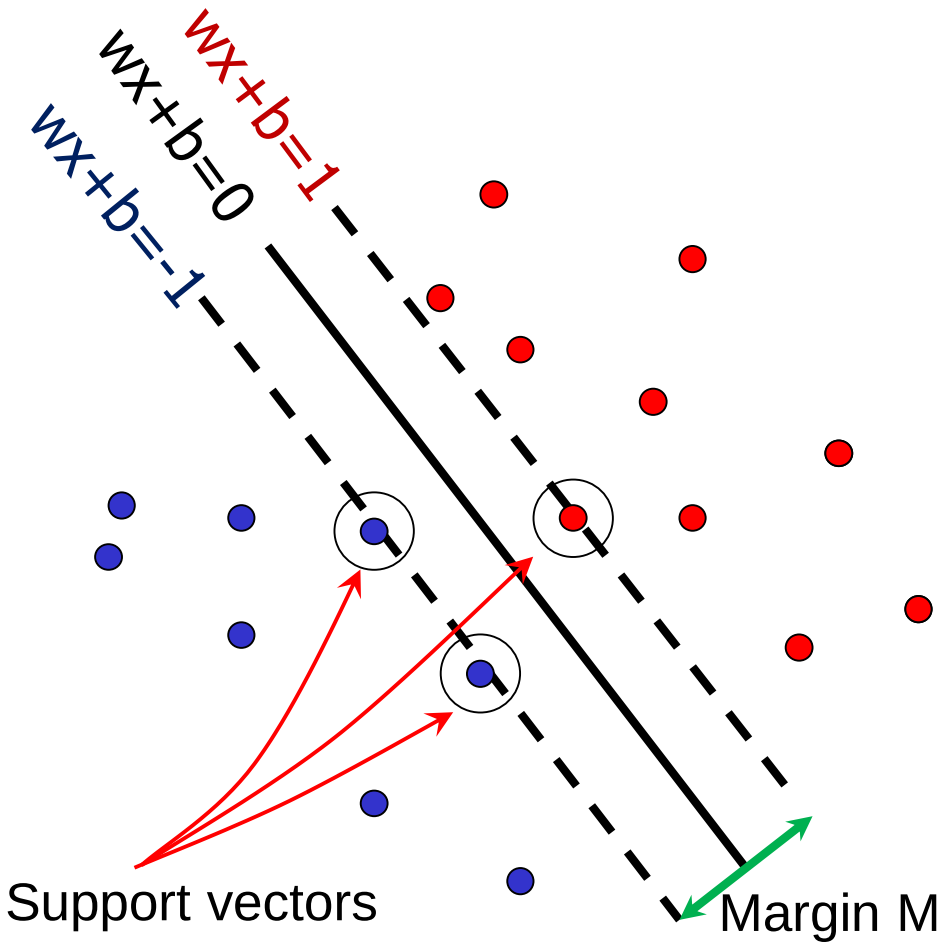
$\mathbf{x}_i$ negative $(y_i = -1)$: $\mathbf{x}_i \cdot \mathbf{w} + b \leq -1$

For support, vectors, $\mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Support vector machines

- Want line that maximizes the margin.



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For support, vectors, $\mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

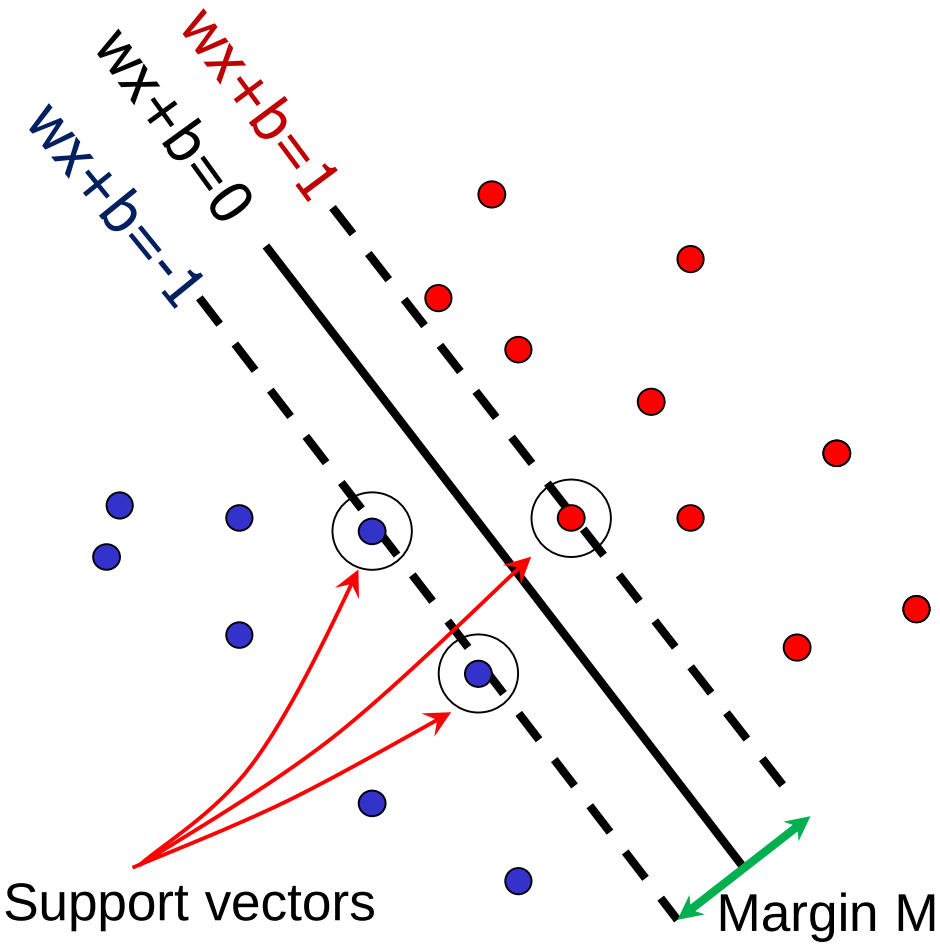
Distance between point and line: $\dfrac{|\mathbf{x}_i \cdot \mathbf{w} + b|}{\|\mathbf{w}\|}$

For support vectors:

$$\frac{\mathbf{w}^T\mathbf{x} + b}{\|\mathbf{w}\|} = \frac{\pm 1}{\|\mathbf{w}\|} \qquad M = \left| \frac{1}{\|\mathbf{w}\|} - \frac{-1}{\|\mathbf{w}\|} \right| = \frac{2}{\|\mathbf{w}\|}$$

Support vector machines

- Want line that maximizes the margin.



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$\mathbf{x}_i$ negative $(y_i = -1)$: $\mathbf{x}_i \cdot \mathbf{w} + b \leq -1$

For support, vectors, $\mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

Distance between point and line: $\dfrac{|\mathbf{x}_i \cdot \mathbf{w} + b|}{\|\mathbf{w}\|}$

Therefore, the margin is $2 / \|\mathbf{w}\|$

Finding the maximum margin line

1. Maximize margin $2/\|\mathbf{w}\|$
2. Correctly classify all training data points:

$$\mathbf{x}_i \text{ positive } (y_i = 1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 1$$

$$\mathbf{x}_i \text{ negative } (y_i = -1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \leq -1$$

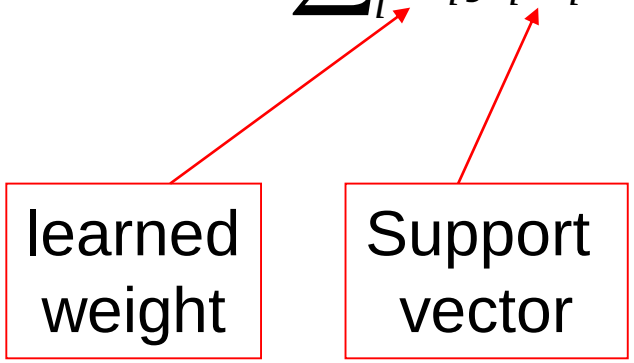
Quadratic optimization problem:

Minimize $\frac{1}{2}\mathbf{w}^T\mathbf{w}$

Subject to $y_i(\mathbf{w}\cdot\mathbf{x}_i+b) \geq 1$

Finding the maximum margin line

- Solution: $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$



Finding the maximum margin line

- Solution: $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$

$b = y_i - \mathbf{w} \cdot \mathbf{x}_i$ (for any support vector)

$$\mathbf{w} \cdot \mathbf{x} + b = \sum_i \alpha_i y_i \mathbf{x}_i \cdot \mathbf{x} + b$$

- Classification function:

$$f(x) = \text{sign}\ (\mathbf{w} \cdot \mathbf{x} + \text{b})$$

$$= \text{sign}\left(\sum_i \alpha_i \mathbf{x}_i \cdot \mathbf{x} + b\right)$$

If f(x) < 0, classify as negative, if f(x) > 0, classify as positive

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1

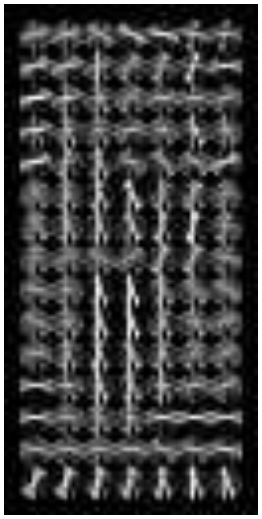
Questions

- **What if the features are not 2d?**
- What if the data is not linearly separable?
- What if we have more than just two categories?

Questions

- What if the features are not 2d?
  - Generalizes to d-dimensions – replace line with "hyperplane"
- What if the data is not linearly separable?
- What if we have more than just two categories?

Person detection with HoG's & linear SVM's



- Map each grid cell in the input window to a histogram counting the gradients per orientation.
- Train a linear SVM using training set of pedestrian vs. non-pedestrian windows.

Code available:
http://pascal.inrialpes.fr/soft/olt/

Dalal & Triggs, CVPR 2005

Person detection with HoG's & linear SVM's

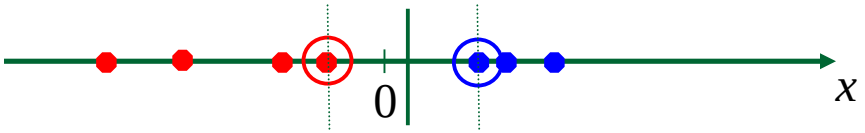
- Histograms of Oriented Gradients for Human Detection, Navneet Dalal, Bill Triggs, International Conference on Computer Vision & Pattern Recognition - June 2005
- http://lear.inrialpes.fr/pubs/2005/DT05/

Questions

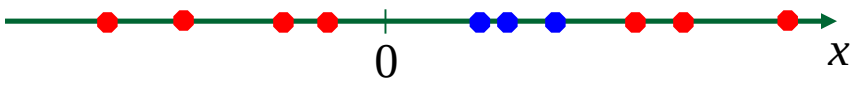
- What if the features are not 2d?
- **What if the data is not linearly separable?**
- What if we have more than just two categories?

Non-linear SVMs

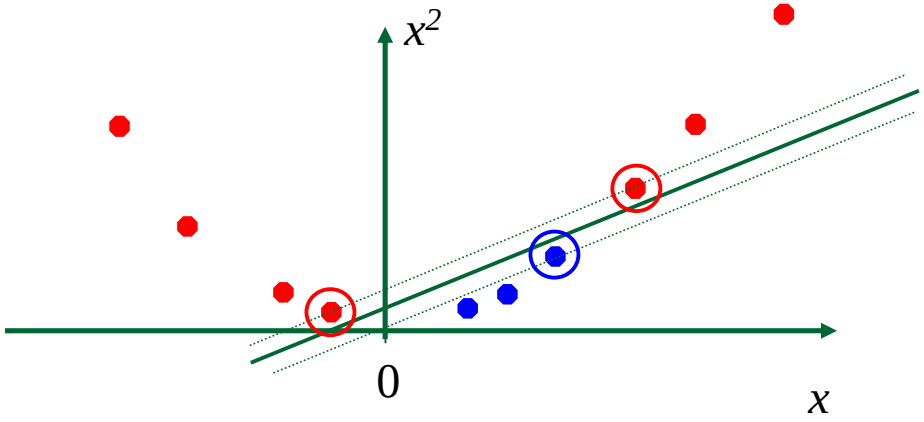
- Datasets that are linearly separable with some noise work out great:



- But what are we going to do if the dataset is just too hard?

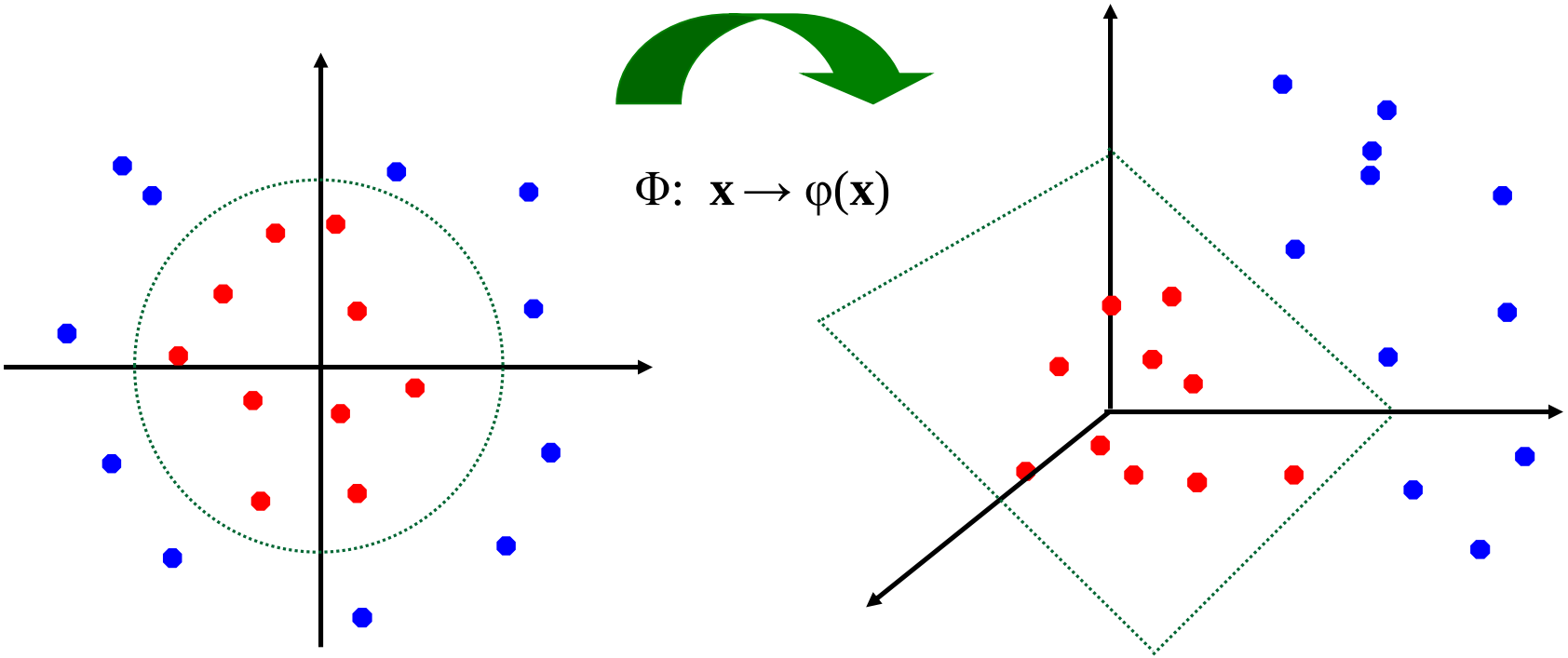


- How about… mapping data to a higher-dimensional space:



Non-linear SVMs: feature spaces

- General idea: the original input space can be mapped to some higher-dimensional feature space where the training set is separable:



Slide from Andrew Moore's tutorial: http://www.autonlab.org/tutorials/svm.html

The "Kernel Trick"

- The linear classifier relies on dot product between vectors $K(x_i,x_j)=x_i^T x_j$

- If every data point is mapped into high-dimensional space via some transformation Φ: $x \rightarrow \varphi(x)$, the dot product becomes:

$$K(x_i,x_j)= \varphi(x_i)^T \varphi(x_j)$$

- A *kernel function* is similarity function that corresponds to an inner product in some expanded feature space.

Slide from Andrew Moore's tutorial: http://www.autonlab.org/tutorials/svm.html

Example

2-dimensional vectors $x=[x_1 \;\; x_2]$;

let $K(x_i,x_j)=(1 + x_i^T x_j)^2$

Need to show that $K(x_i,x_j)= \varphi(x_i)^T \varphi(x_j)$:

$K(x_i,x_j)=(1 + x_i^T x_j)^2$,

$= 1+ x_{i1}^2 x_{j1}^2 + 2\, x_{i1}x_{j1}\, x_{i2}x_{j2} + x_{i2}^2 x_{j2}^2 + 2x_{i1}x_{j1} + 2x_{i2}x_{j2}$

$= [1 \;\; x_{i1}^2 \;\sqrt{2}\, x_{i1}x_{i2} \;\; x_{i2}^2 \;\sqrt{2}x_{i1} \;\sqrt{2}x_{i2}]^T$

$[1 \;\; x_{j1}^2 \;\sqrt{2}\, x_{j1}x_{j2} \;\; x_{j2}^2 \;\sqrt{2}x_{j1} \;\sqrt{2}x_{j2}]$

$= \varphi(x_i)^T \varphi(x_j)$,

where $\varphi(x) = [1 \;\; x_1^2 \;\sqrt{2}\, x_1x_2 \;\; x_2^2 \;\; \sqrt{2}x_1 \;\sqrt{2}x_2]$

from Andrew Moore's tutorial: http://www.autonlab.org/tutorials/svm.html

Nonlinear SVMs

- *The kernel trick*: instead of explicitly computing the lifting transformation $\boldsymbol{\varphi}(\mathbf{x})$, define a kernel function K such that

$$K(\mathbf{x}_i, \mathbf{x}_j) = \boldsymbol{\varphi}(\mathbf{x}_i) \cdot \boldsymbol{\varphi}(\mathbf{x}_j)$$

- This gives a nonlinear decision boundary in the original feature space:

$$\sum_i \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}) + b$$

Examples of kernel functions

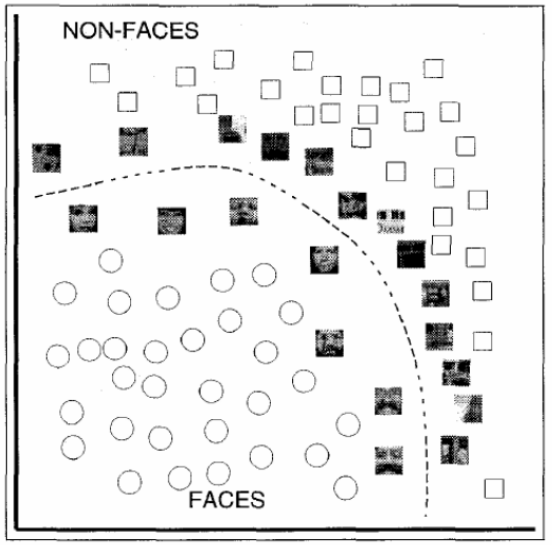
- Linear: $K(x_i, x_j) = x_i^T x_j$

- Gaussian RBF: $K(x_i, x_j) = \exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma^2}\right)$

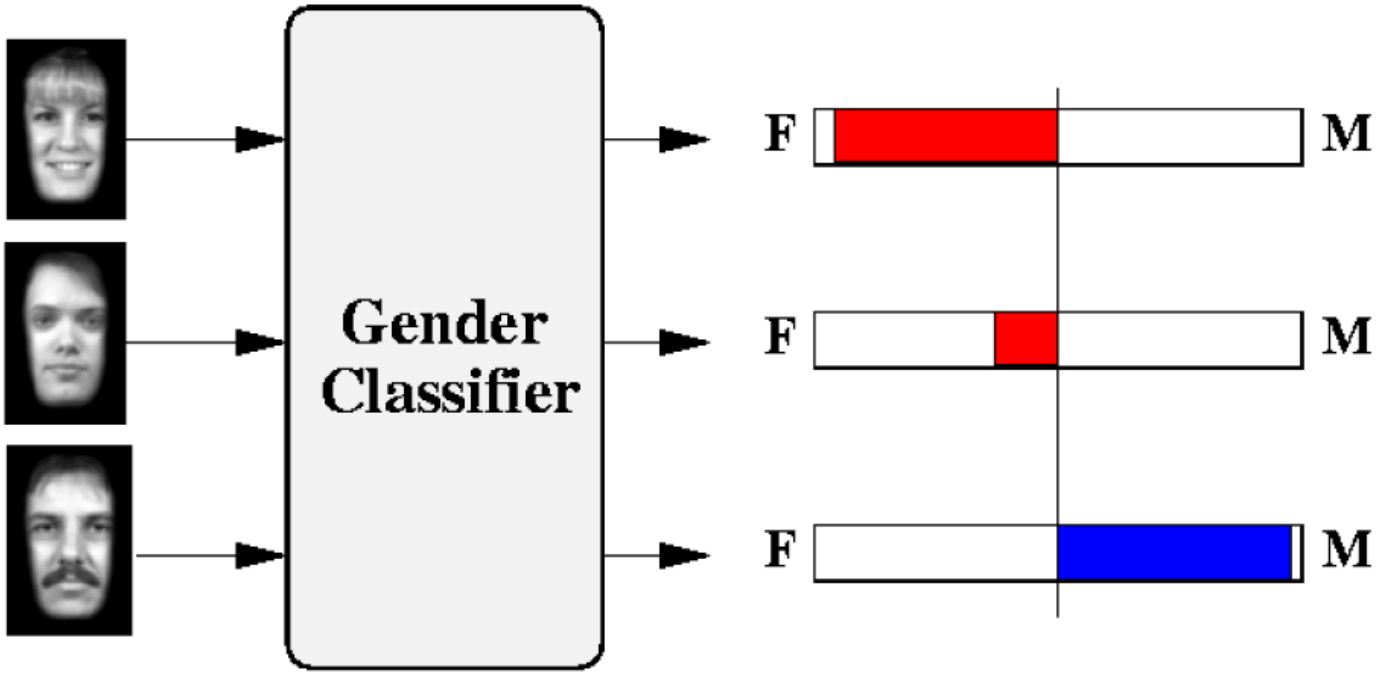
- Histogram intersection: $K(x_i, x_j) = \sum_k \min(x_i(k), x_j(k))$

SVMs for recognition

1. Define your representation for each example.
2. Select a kernel function.
3. Compute pairwise kernel values between labeled examples
4. Use this “kernel matrix” to solve for SVM support vectors & weights.
5. To classify a new example: compute kernel values between new input and support vectors, apply weights, check sign of output.



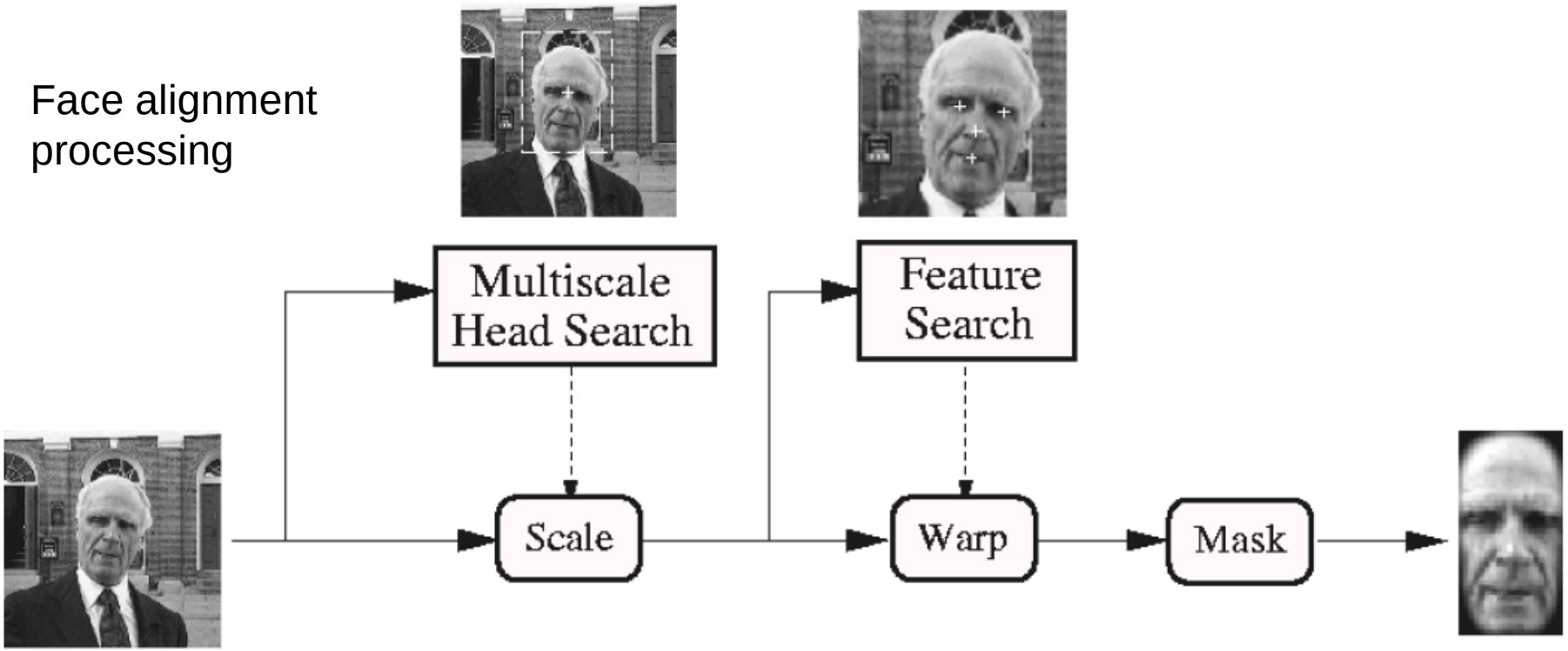
Example: learning gender with SVMs



Moghaddam and Yang, Learning Gender with Support Faces, TPAMI 2002.

Moghaddam and Yang, Face & Gesture 2000.

Face alignment processing



Processed faces

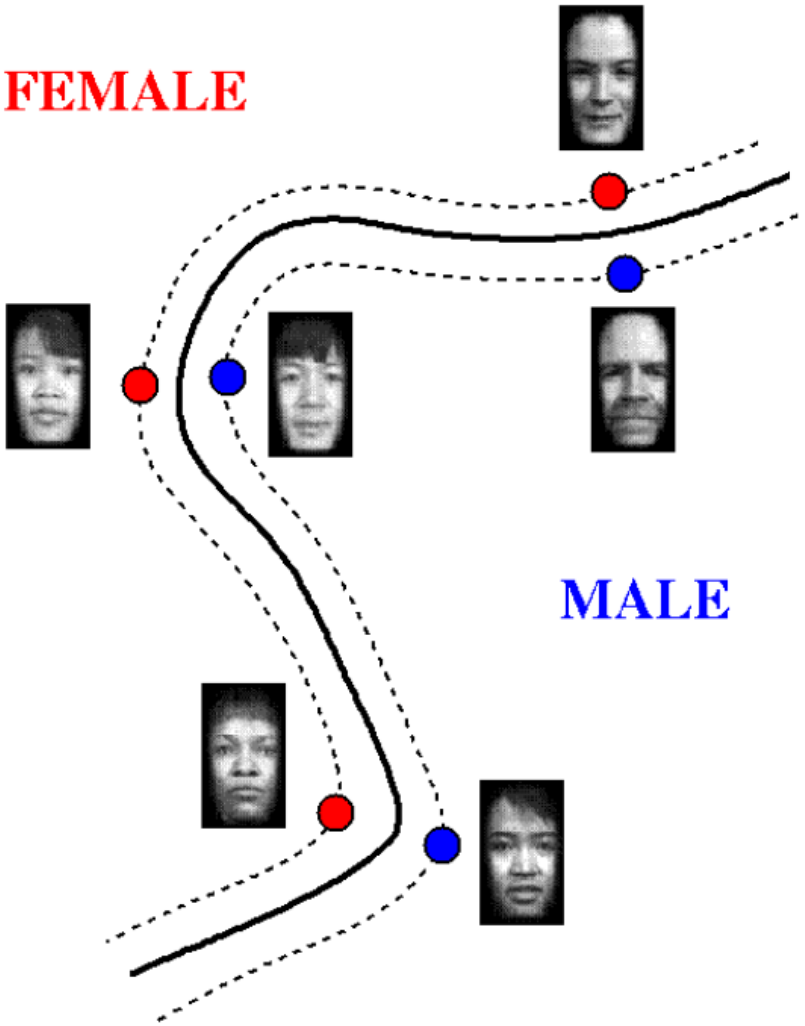
Moghaddam and Yang, Learning Gender with Support Faces, TPAMI 2002.

Learning gender with SVMs

- Training examples:
  - 1044 males
  - 713 females
- Experiment with various kernels, select Gaussian RBF

$$K(\mathbf{x_i}, \mathbf{x_j}) = \exp(-\frac{\left\|\mathbf{x_i} - \mathbf{x_j}\right\|^2}{2\sigma^2})$$

Support Faces



Moghaddam and Yang, Learning Gender with Support Faces, TPAMI 2002.

Classifier Performance

Classifier	Error Rate		
	Overall	Male	Female
SVM with RBF kernel	**3.38%**	**2.05%**	**4.79%**
SVM with cubic polynomial kernel	**4.88%**	**4.21%**	**5.59%**
Large Ensemble of RBF	5.54%	4.59%	6.55%
Classical RBF	7.79%	6.89%	8.75%
Quadratic classifier	10.63%	9.44%	11.88%
Fisher linear discriminant	13.03%	12.31%	13.78%
Nearest neighbor	27.16%	26.53%	28.04%
Linear classifier	58.95%	58.47%	59.45%

Moghaddam and Yang, Learning Gender with Support Faces, TPAMI 2002.

Gender perception experiment: How well can humans do?

- Subjects:
  - 30 people (22 male, 8 female)
  - Ages mid-20's to mid-40's
- Test data:
  - 254 face images (6 males, 4 females)
  - Low res and high res versions
- Task:
  - Classify as male or female, forced choice
  - No time limit

Moghaddam and Yang, Face & Gesture 2000.

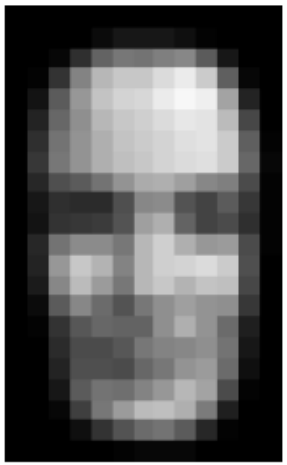
Gender perception experiment: How well can humans do?

Stimuli →

84 x 48

N = 4032

21 x 12



N = 252

Results →

High-Res	Low-Res
6.54%	30.7%
Error	Error

$\sigma = 3.7\%$

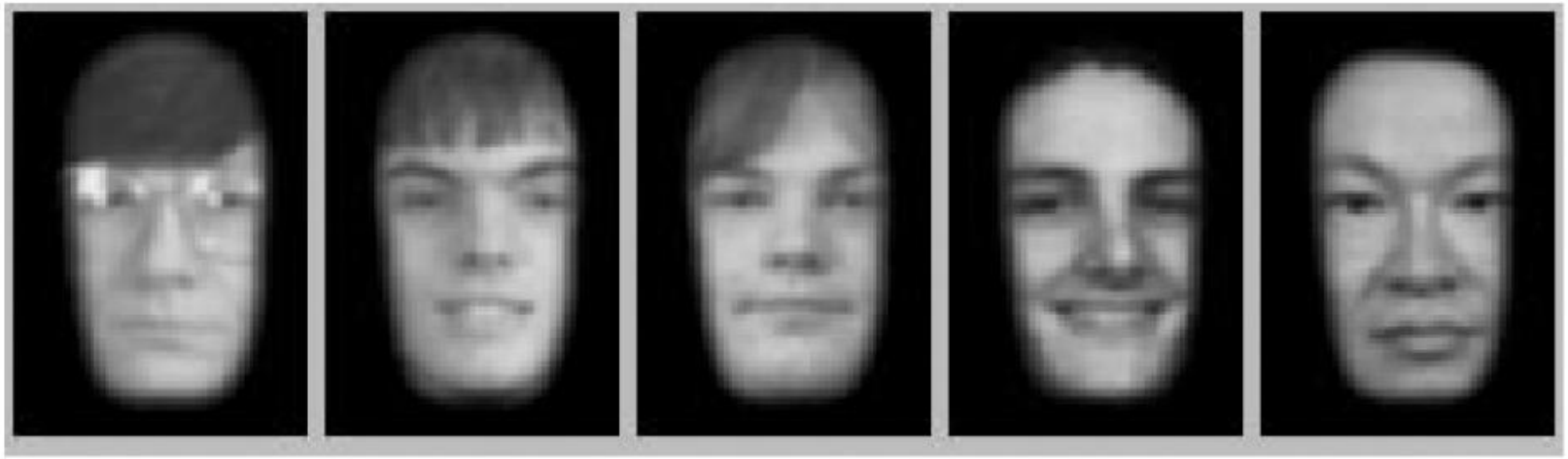
Moghaddam and Yang, Face & Gesture 2000.

Human vs. Machine

Figure 6. SVM vs. Human performance

- SVMs performed better than any single human test subject, at either resolution

Hardest examples for humans



Top five human misclassifications

Moghaddam and Yang, Face & Gesture 2000.

Questions

- What if the features are not 2d?
- What if the data is not linearly separable?
- **What if we have more than just two categories?**

Multi-class SVMs

- Achieve multi-class classifier by combining a number of binary classifiers

- **One vs. all**
  - Training: learn an SVM for each class vs. the rest
  - Testing: apply each SVM to test example and assign to it the class of the SVM that returns the highest decision value

- **One vs. one**
  - Training: learn an SVM for each pair of classes
  - Testing: each learned SVM “votes” for a class to assign to the test example

SVMs: Pros and cons

- Pros
  - Many publicly available SVM packages: http://www.kernel-machines.org/software
  - http://www.csie.ntu.edu.tw/~cjlin/libsvm/
  - Kernel-based framework is very powerful, flexible
  - Often a sparse set of support vectors – compact at test time
  - Work very well in practice, even with very small training sample sizes
- Cons
  - No “direct” multi-class SVM, must combine two-class SVMs
  - Can be tricky to select best kernel function for a problem
  - Computation, memory
    - During training time, must compute matrix of kernel values for every pair of examples
    - Learning can take a very long time for large-scale problems

Adapted from Lana Lazebnik