

Scale

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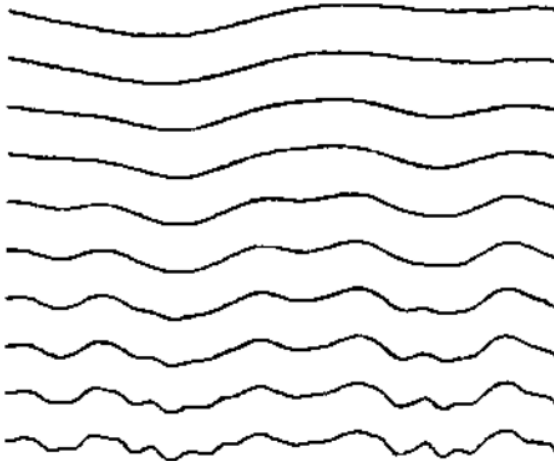
1 Scale Space

2 Burt Adelson pyramid

3 Wavelets

Input for slides includes content by Kyros Kutulakos, Bob Fisher and Witkin

Witkin's one dimensional Gaussian smoothing



Laplacian Mask

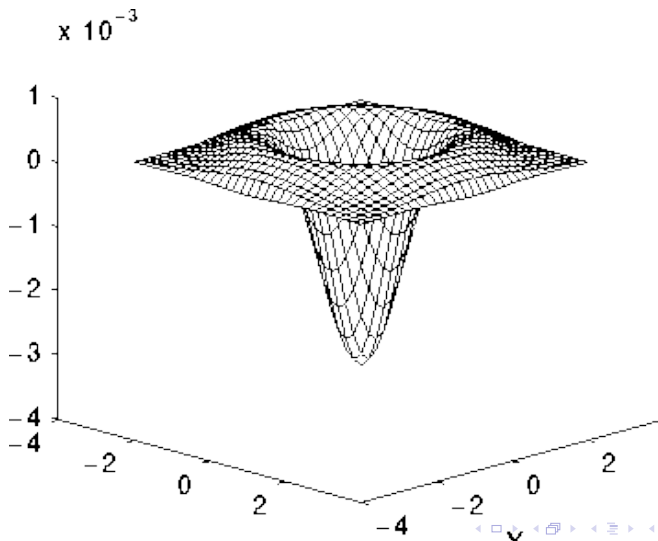
0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

LoG equation

$$LoG(x, y) = -\frac{1}{\pi\sigma^4} \left[1 - \frac{x^2 + y^2}{2\sigma^2} \right] e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

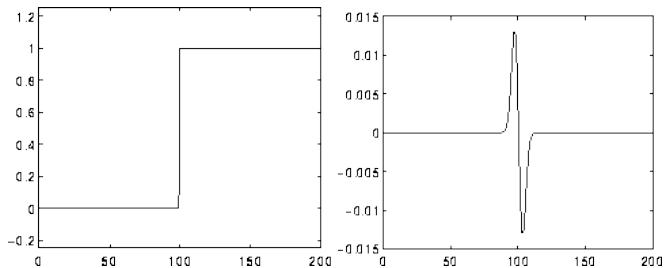
LoG Continuous waveform



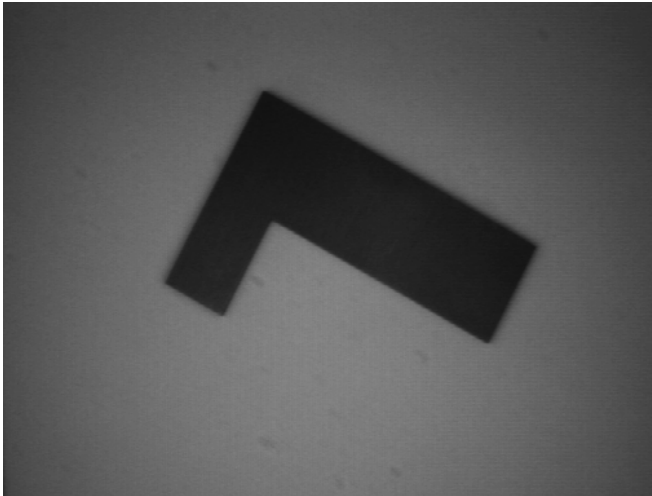
LoG discrete values

0	1	1	2	2	2	1	1	0
1	2	4	5	5	5	4	2	1
1	4	5	3	0	3	5	4	1
2	5	3	-12	-24	-12	3	5	2
2	5	0	-24	-40	-24	0	5	2
2	5	3	-12	-24	-12	3	5	2
1	4	5	3	0	3	5	4	1
1	2	4	5	5	5	4	2	1

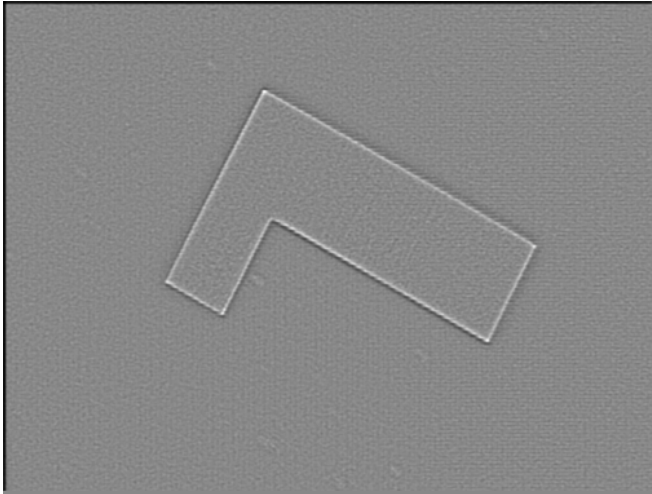
LoG Response



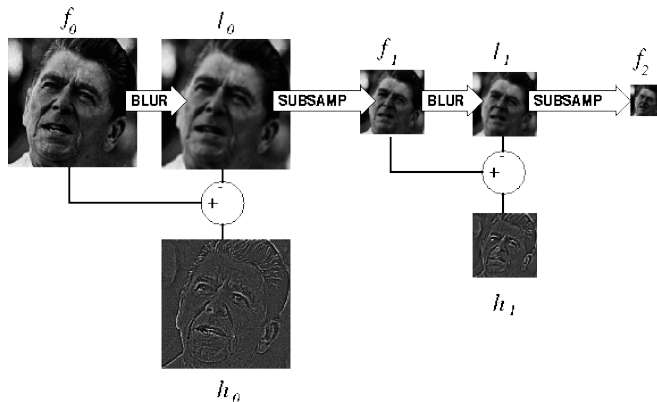
LoG Example Input



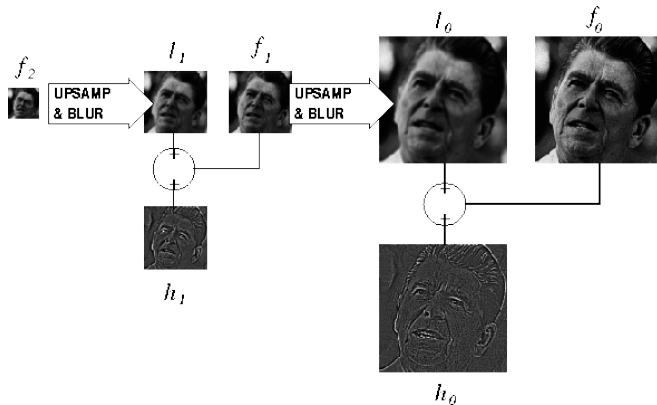
LoG Example Output



Decomposition of Laplacian pyramid

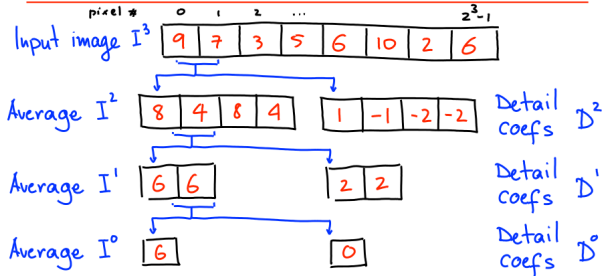


Reconstruction of Laplacian pyramid



Haar wavelet example 1D

1D Haar Wavelet Transform: Recursive Definition



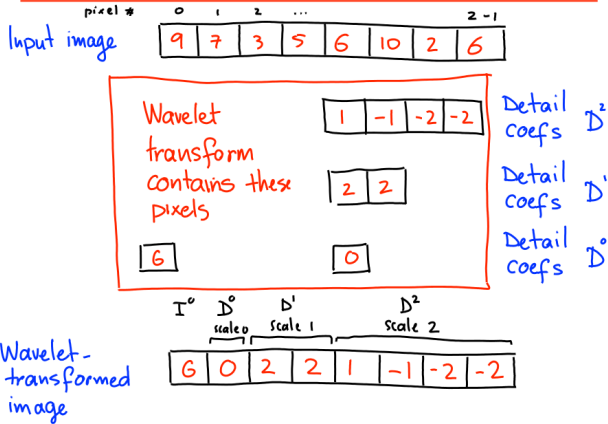
$$I_i^j = \frac{1}{2} (I_{2i}^{j+1} + I_{2i+1}^{j+1})$$

$$D_i^j = \frac{1}{2} (I_{2i}^{j+1} - I_{2i+1}^{j+1})$$

j -th level of "pyramid" contains 2^j pixels

Haar wavelet example 1D

1D Haar Wavelet Transform: Recursive Definition



2D Haar wavelet basis

The 2-D Haar Wavelet Basis

Definition of the first few (coarsest scale) wavelet coefficients of an image of dimensions of $2^N \times 2^N$

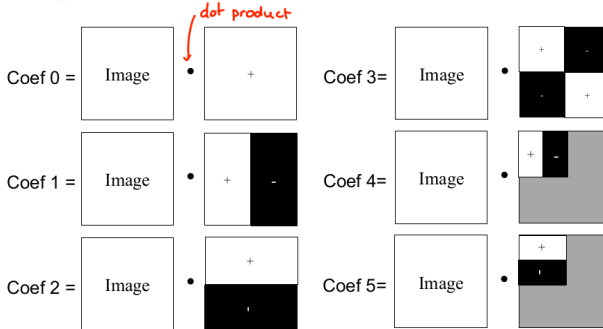
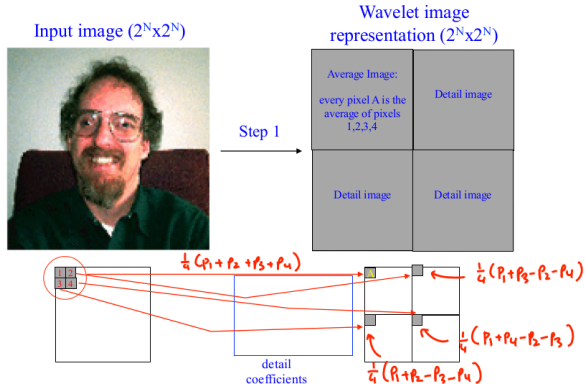


Figure from Kyros Kutulakos

2D Haar wavelet example

The Haar 2-D Wavelet Transform

The 2-D Haar Wavelet Transform corresponds to a modification of this minimal recursive transform



2D Haar inverse wavelet example

Invertibility of the 2D Haar Transform

We can recursively reconstruct the intensities of every 2×2 window from its average and detail coefficients

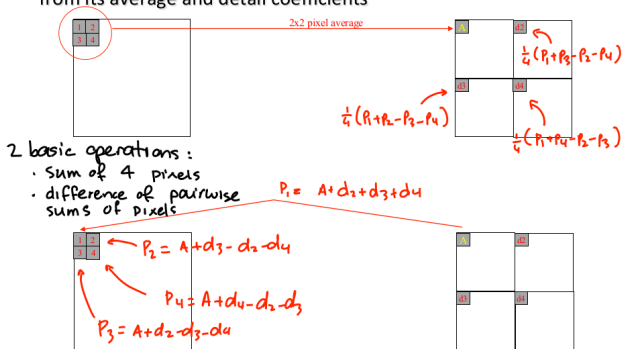
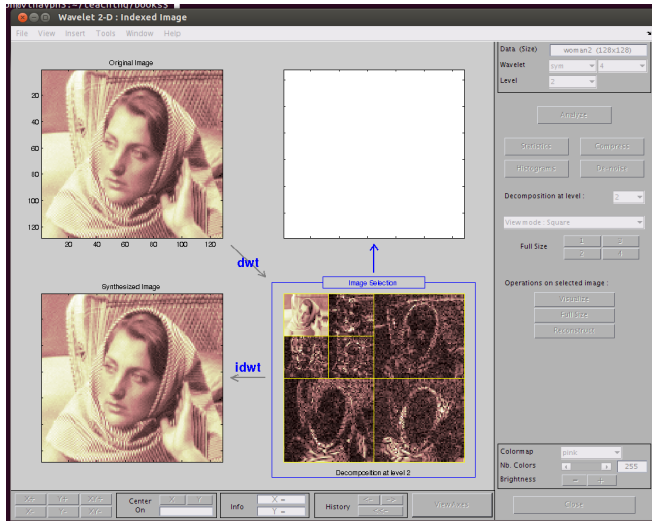
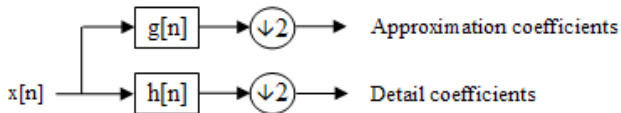


Figure from Kyros Kutulakos

Wavelet decomposition example



Wavelet detail and approximate by filtering



Wavelets Filter bank

