Interest Points

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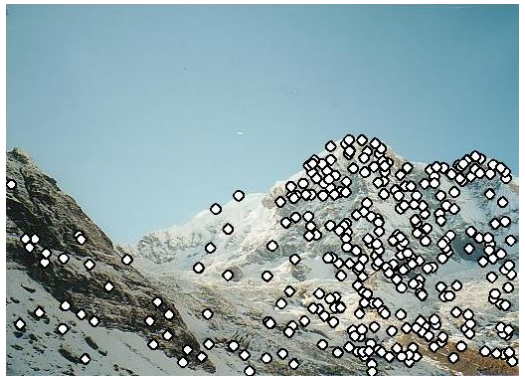
- Slide content based on slides by Steve Seitz, Kristen Grauman, Deva Ramanan

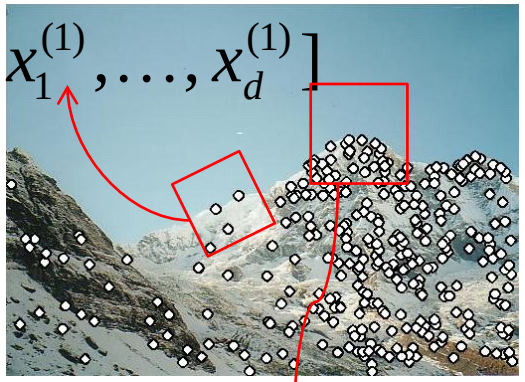
Local features: main components

1) Detection: Identify the interest points



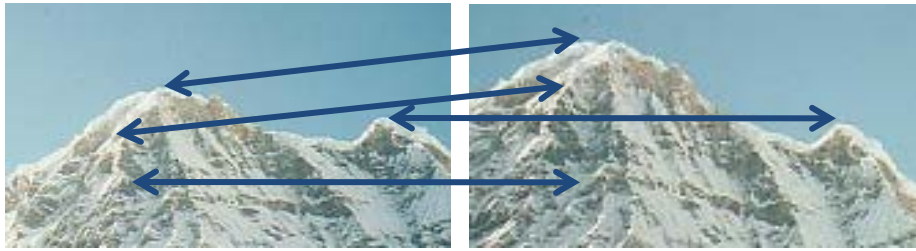
2) Description: Extract vector feature descriptor surrounding each interest point.

$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$



$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}]$$

3) Matching: Determine correspondence between descriptors in two views

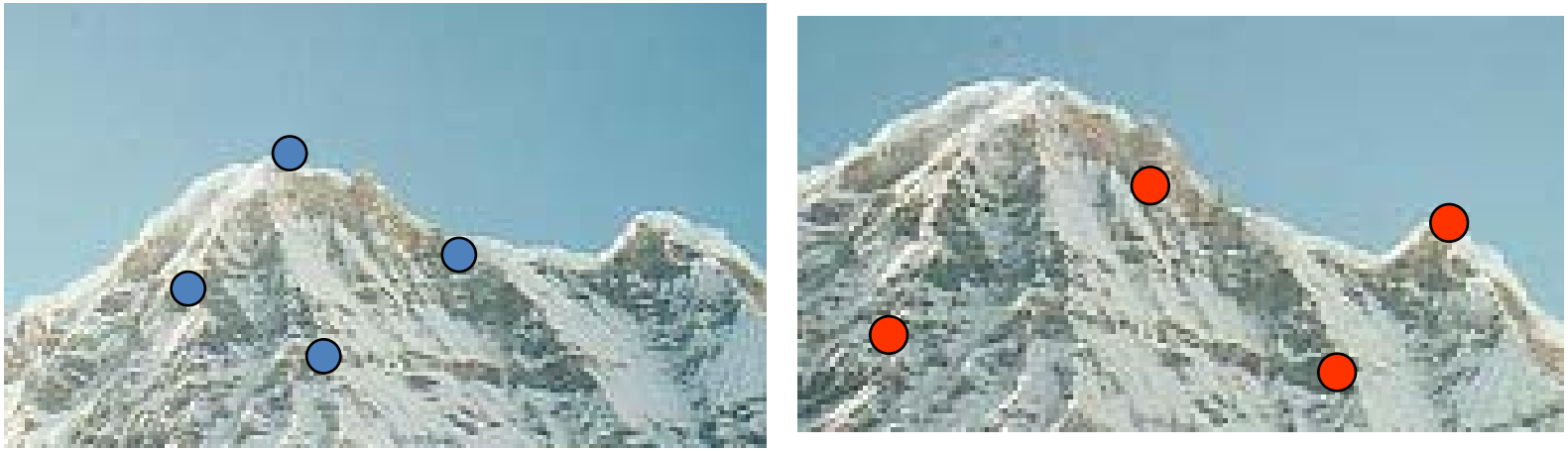


Local features: desired properties

- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature has a distinctive description
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Goal: interest operator repeatability

- We want to detect (at least some of) the same points in both images.

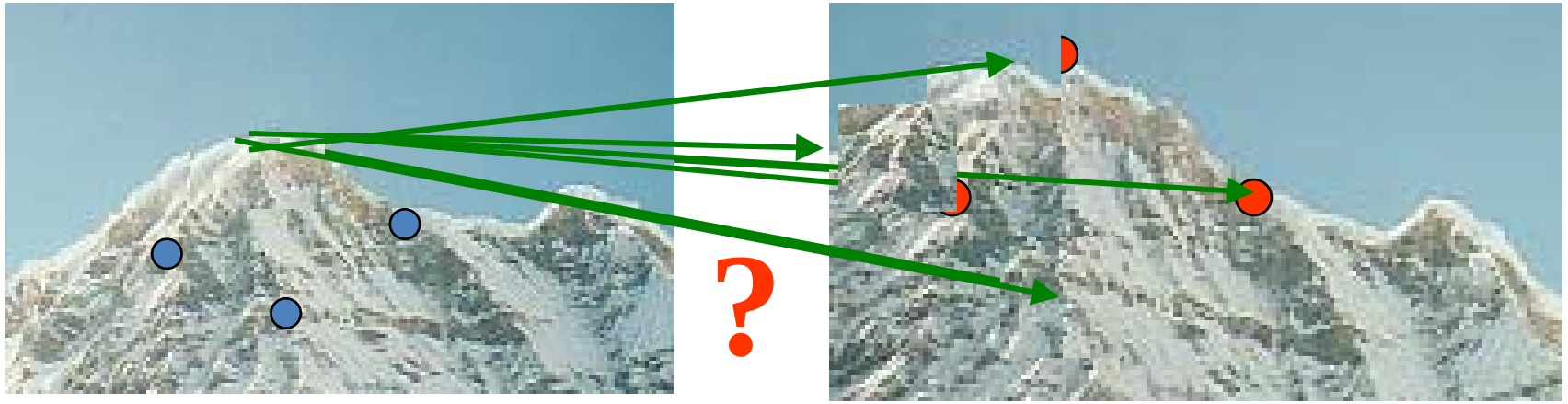


No chance to find true matches!

- Yet we have to be able to run the detection procedure *independently* per image.

Goal: descriptor distinctiveness

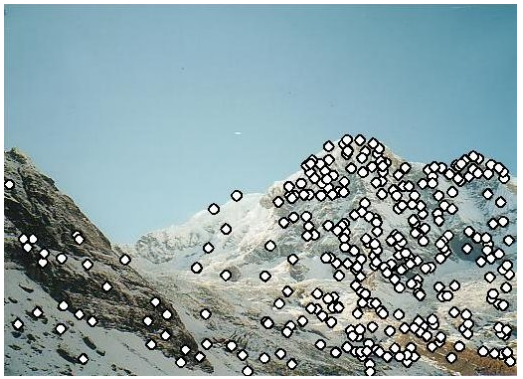
- We want to be able to reliably determine which point goes with which.



- Must provide some invariance to geometric and photometric differences between the two views.

Local features: main components

1) Detection: Identify the interest points



2) Description:Extract vector feature descriptor surrounding each interest point.

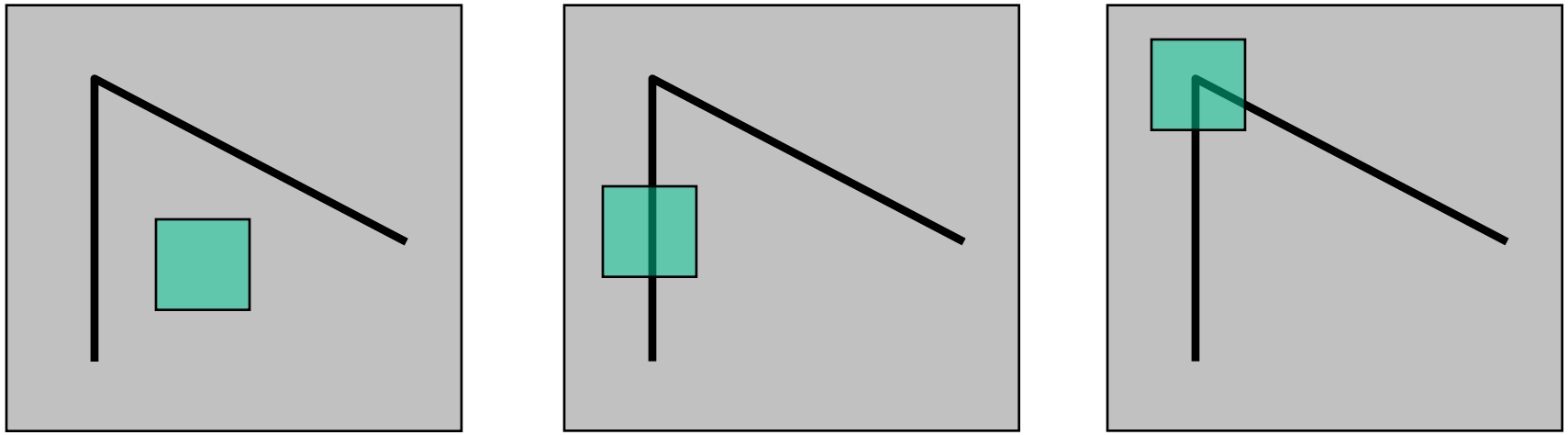
3) Matching: Determine correspondence between descriptors in two views

- What points would you choose?

Local measures of uniqueness

Suppose we only consider a small window of pixels

- What defines whether a feature is a good or bad candidate?

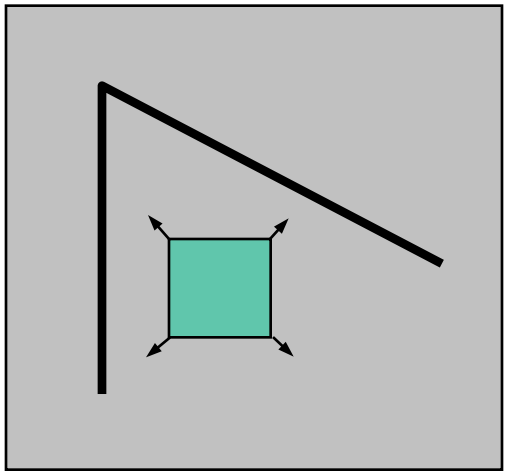


Slide adapted from Darya Frolova, Denis Simakov, Weizmann Institute.

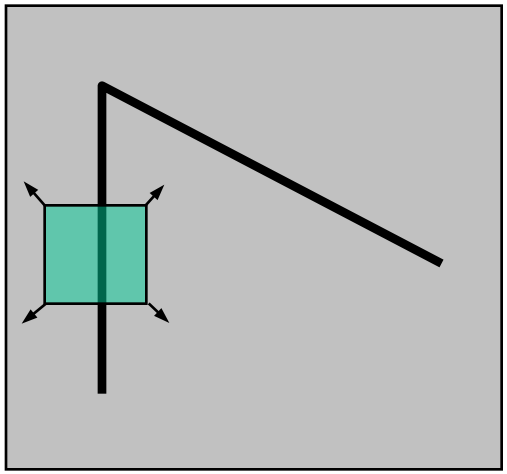
Feature detection

Local measure of feature uniqueness

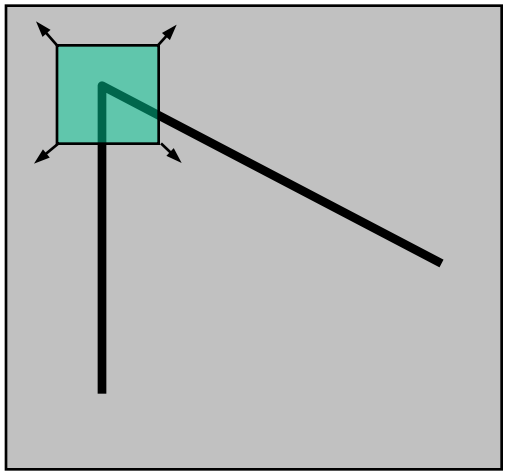
- How does the window change when you shift it?
- Shifting the window in *any direction* causes a *big change*



"flat" region:
no change in all directions



"edge":
no change along the edge direction



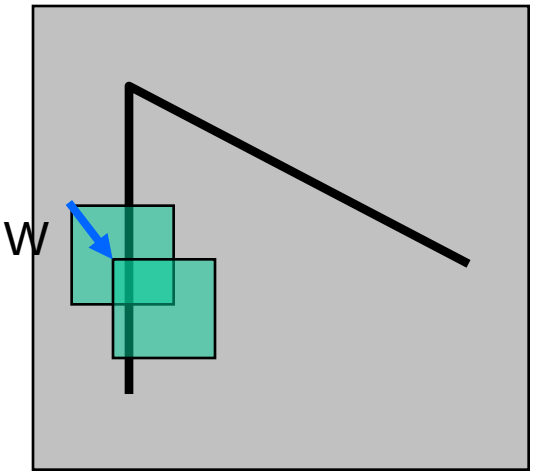
"corner":
significant change in all directions

Slide adapted from Darya Frolova, Denis Simakov, Weizmann Institute.

Feature detection: the math

Consider shifting the window W by (u,v)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD “error” of *E(u,v)*:



$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

Small motion assumption

Taylor Series expansion of I:

$$I(x+u, y+v) = I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u,v) is small, then first order approx is good

$$I(x+u, y+v) \approx I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

$$\approx I(x,y) + \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

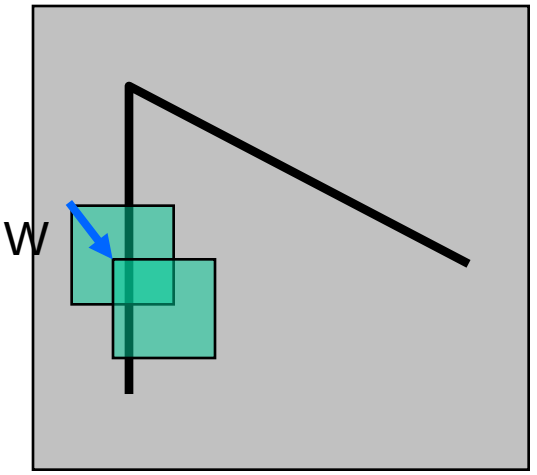
Plugging this into the formula on the previous slide…

$$\text{shorthand: } I_x = \frac{\partial I}{\partial x}$$

Feature detection: the math

Consider shifting the window W by (u,v)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences
- this defines an "error" of E(u,v):



$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^2$$

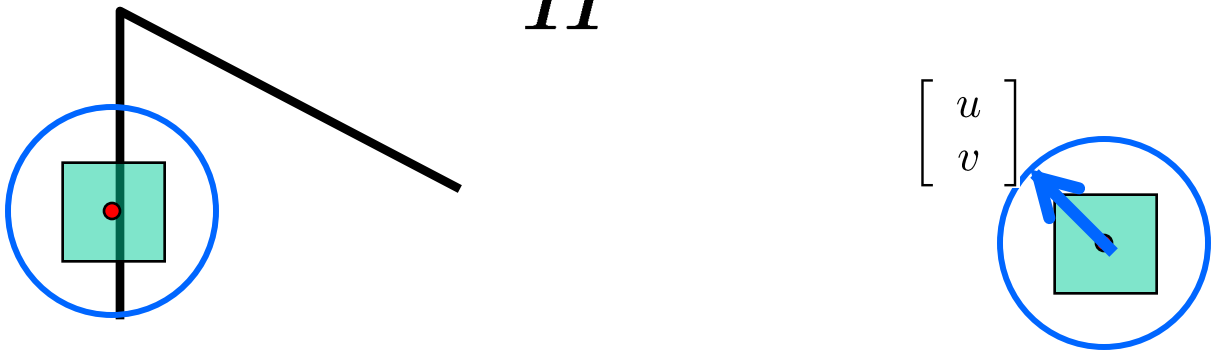
$$\approx \sum_{(x,y)\in W} [I(x,y) + [I_x \; I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x,y)]^2$$

$$\approx \sum_{(x,y)\in W} \left[[I_x \; I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right]^2$$

Feature detection: the math

This can be rewritten:

$$E(u,v) = \sum_{(x,y)\in W} [u\ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$



For the example above

- You can move the center of the green window to anywhere on the blue unit circle
- Which directions will result in the largest and smallest E values?
- We can find these directions by looking at the eigenvectors of *H*

Quick eigenvalue/eigenvector review

The eigenvectors of a matrix A are the vectors x that satisfy:

$$Ax = \lambda x$$

The scalar λ is the eigenvalue corresponding to x

- The eigenvalues are found by solving:

$$det(A - \lambda I) = 0$$

- In our case, *A* = *H* is a 2x2 matrix, so we have

$$det \begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} = 0$$

- The solution:

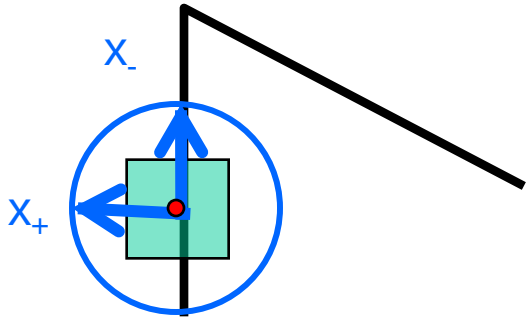
$$\lambda_{\pm} = \frac{1}{2} \left[(h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2} \right]$$

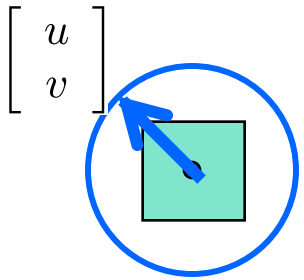
Once you know λ, you find x by solving

$$\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Feature detection: the math

$$E(u,v) = \sum_{(x,y)\in W} [u\ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$





Eigenvalues and eigenvectors of H

- Define shifts with the smallest and largest change (E value)
- x_+ = direction of largest increase in E.
- λ_+ = amount of increase in direction x_+
- x_- = direction of smallest increase in E.
- λ_- = amount of increase in direction x_+

$$Hx_+ = \lambda_+ x_+$$
$$Hx_- = \lambda_- x_-$$

Feature detection: the math

How are λ_+, x_+, λ_-, and x_+ relevant for feature detection?

- What's our feature scoring function?

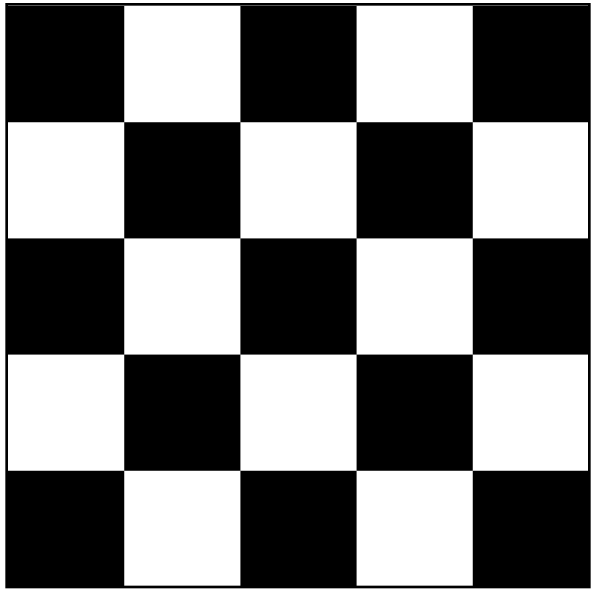
Feature detection: the math

How are λ_+, x_+, λ_-, and x_+ relevant for feature detection?

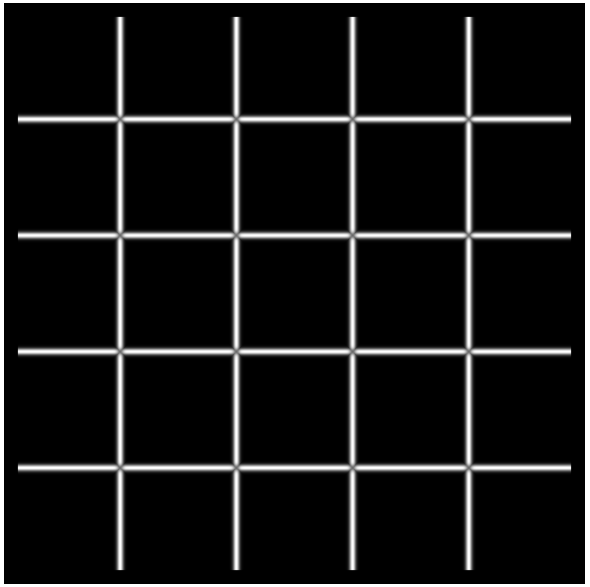
- What's our feature scoring function?

Want *E(u,v)* to be *large* for small shifts in *all* directions

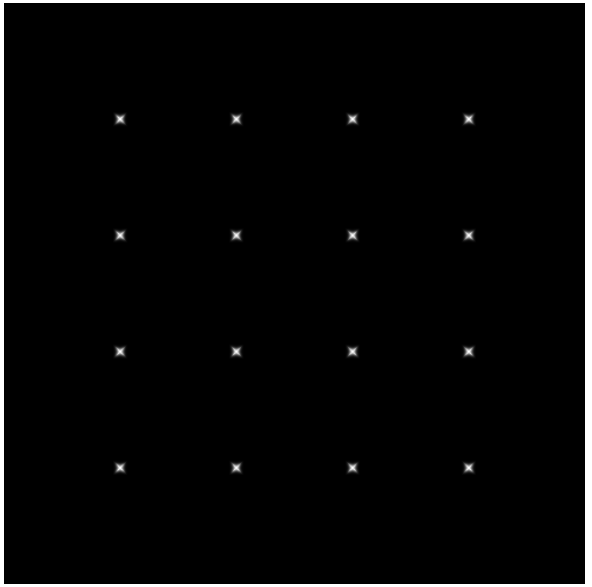
- the *minimum* of *E(u,v)* should be large, over all unit vectors [u v]
- this minimum is given by the smaller eigenvalue (λ_-) of *H*



I



λ_+

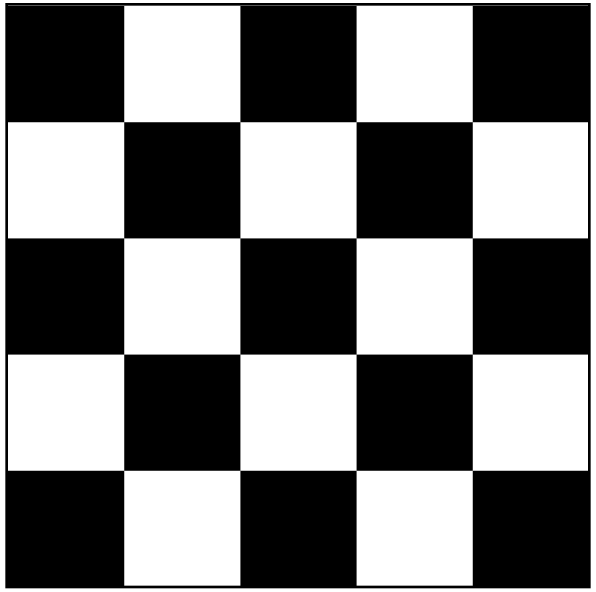


λ_-

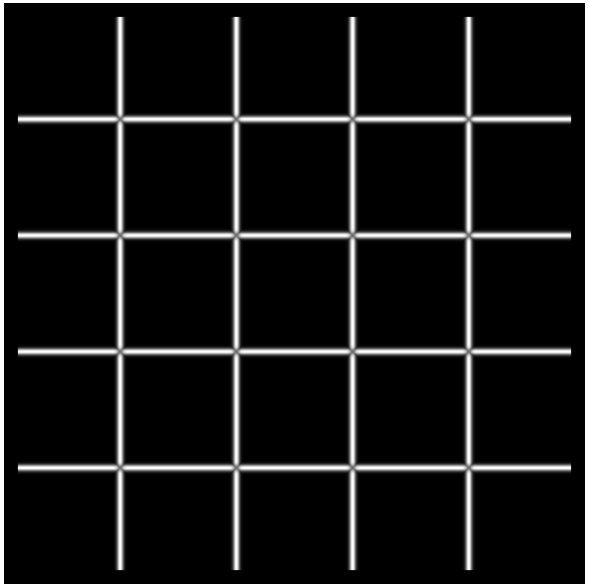
Feature detection summary

Here's what you do

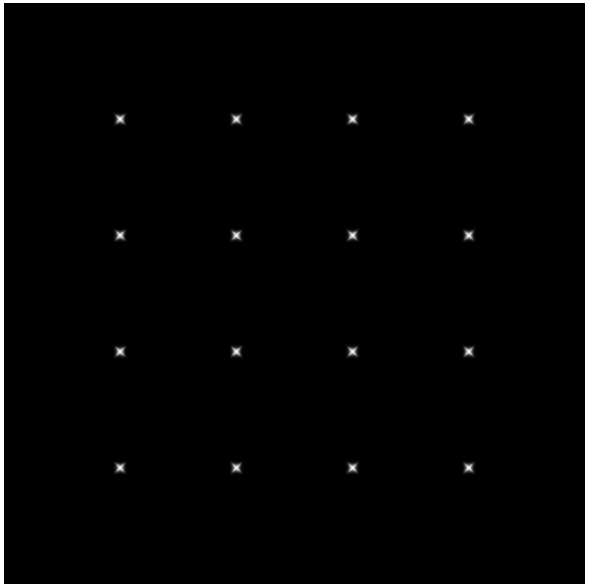
- Compute the gradient at each point in the image
- Create the *H* matrix from the entries in the gradient
- Compute the eigenvalues.
- Find points with large response (λ_- > threshold)
- Choose those points where λ_- is a local maximum as features



I



λ_+



λ_-

Feature detection summary

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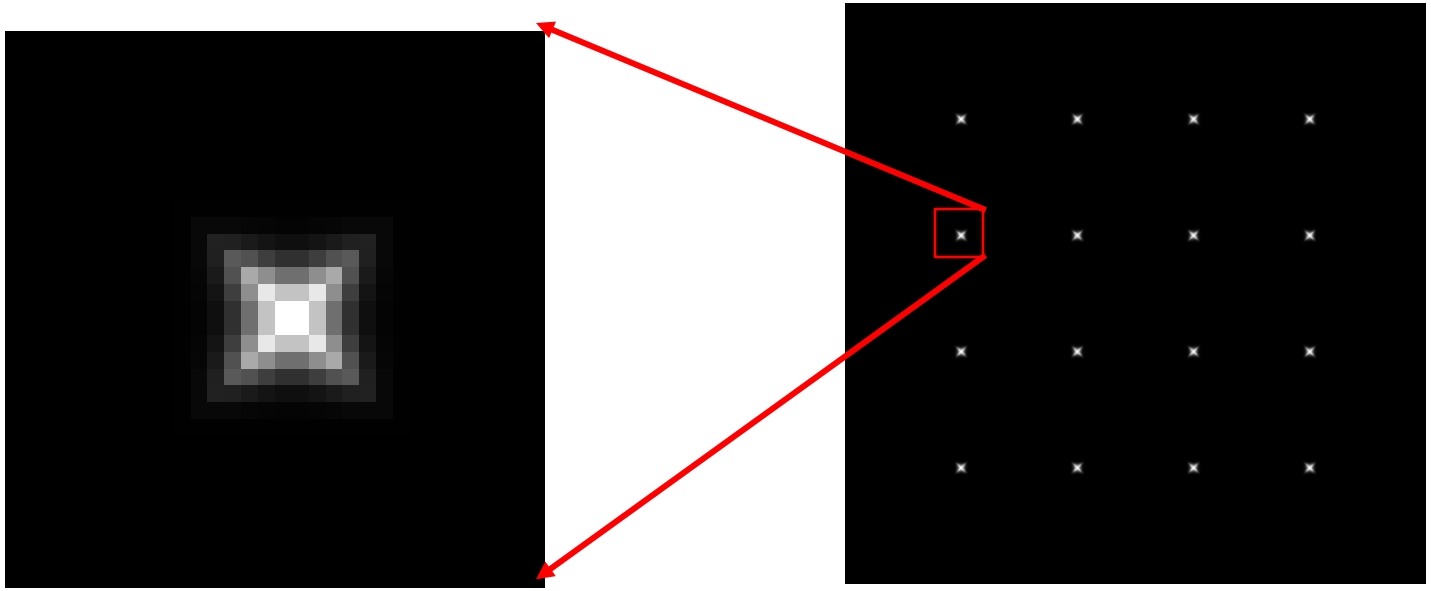


λ_-

The Harris operator

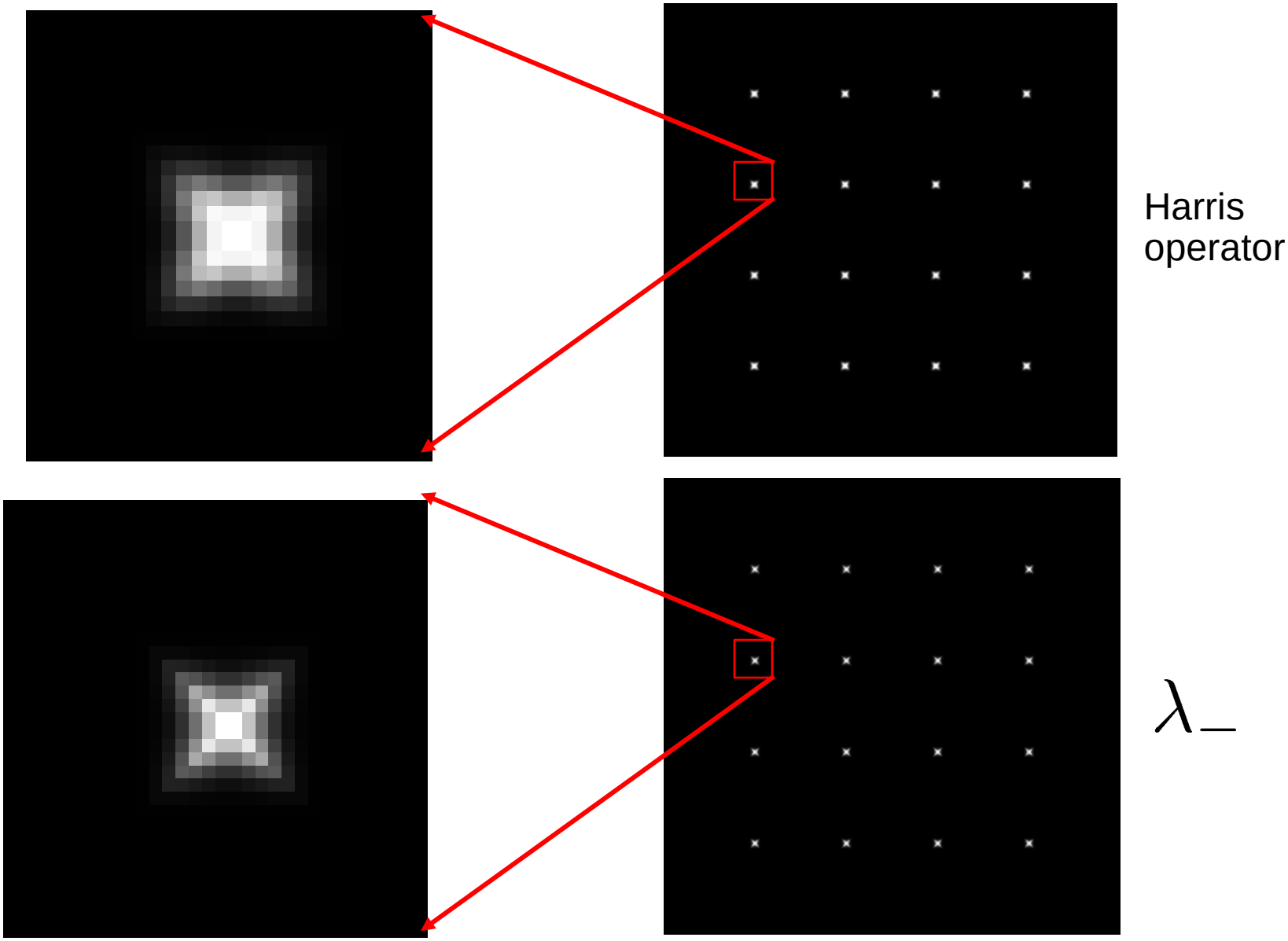
λ_- is a variant of the “Harris operator” for feature detection

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

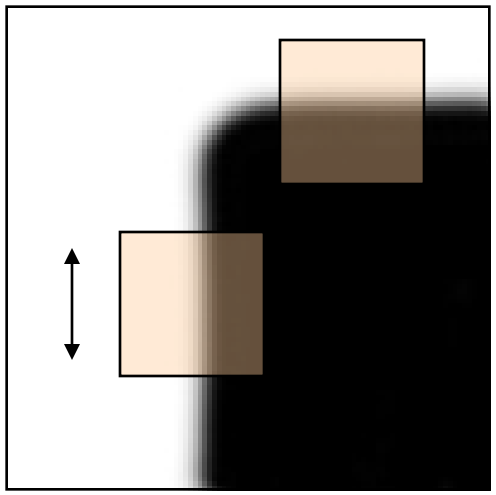
$$= \frac{determinant(H)}{trace(H)}$$

- The *trace* is the sum of the diagonals, i.e., *trace(H)* = $h_{11} + h_{22}$
- Very similar to λ_- but less expensive (no square root)
- Called the “Harris Corner Detector” or “Harris Operator”
- Lots of other detectors, this is one of the most popular

The Harris operator



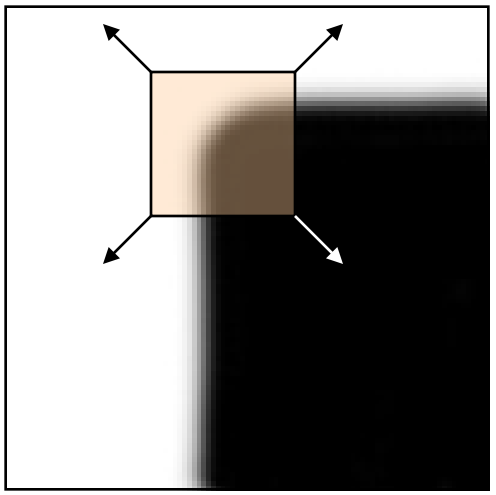
Corner response function



"edge":

$\lambda_1 >> \lambda_2$

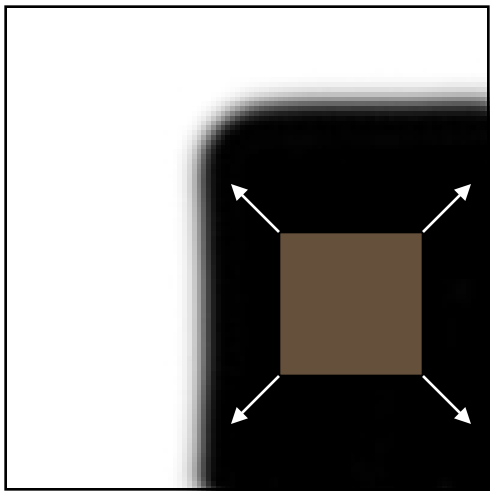
$\lambda_2 >> \lambda_1$



"corner":

λ_1 and λ_2 are large,

$\lambda_1 \sim \lambda_2$;



"flat" region

λ_1 and λ_2 are small

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

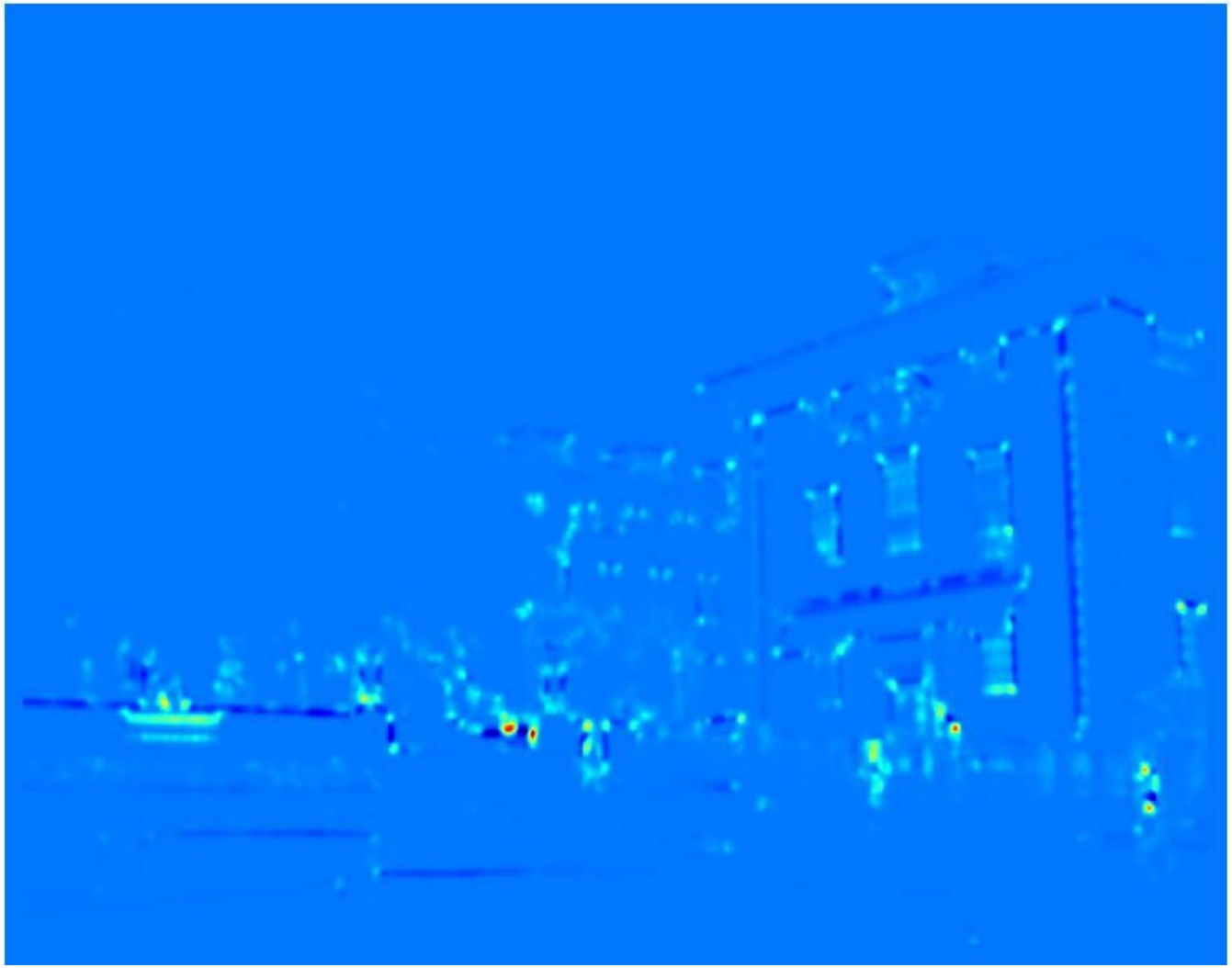
Harris corner detector

1) Compute *E* for each image window to get their *cornerness* scores.
2) Find points whose surrounding window gave large corner response (*f*> threshold)
3) Take the points of local maxima, i.e., perform non-maximum suppression

Example of Harris application

Example of Harris application

- Compute corner response at every pixel.



Example of Harris application

Properties of the Harris corner detector

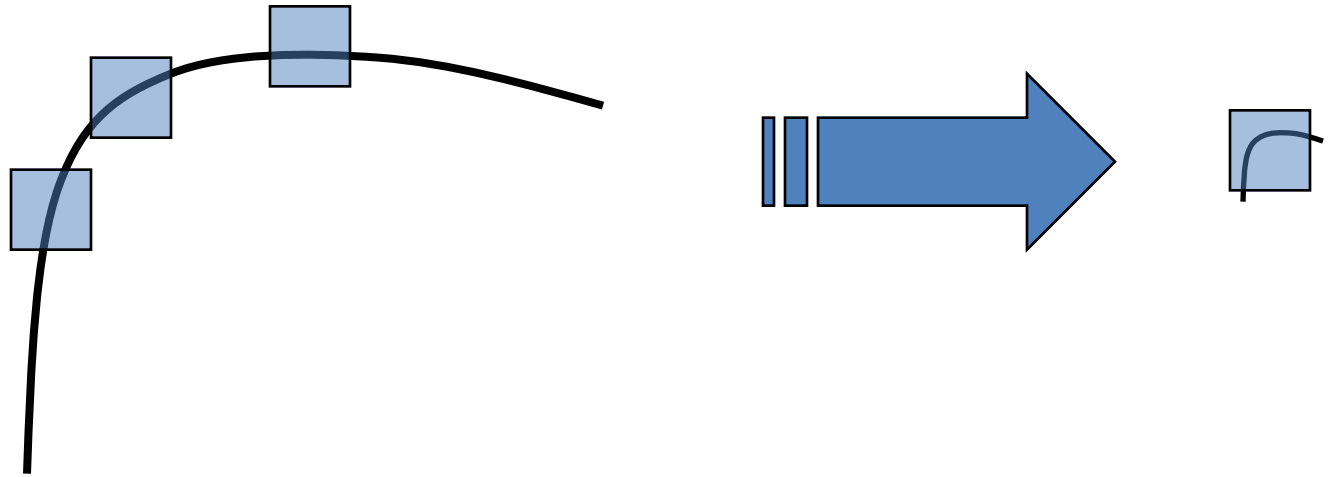
- Rotation invariant? Yes

$$M = X \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} X^T$$

- Scale invariant?

Properties of the Harris corner detector

- Rotation invariant? Yes
- Scale invariant? No



All points will be classified as corner

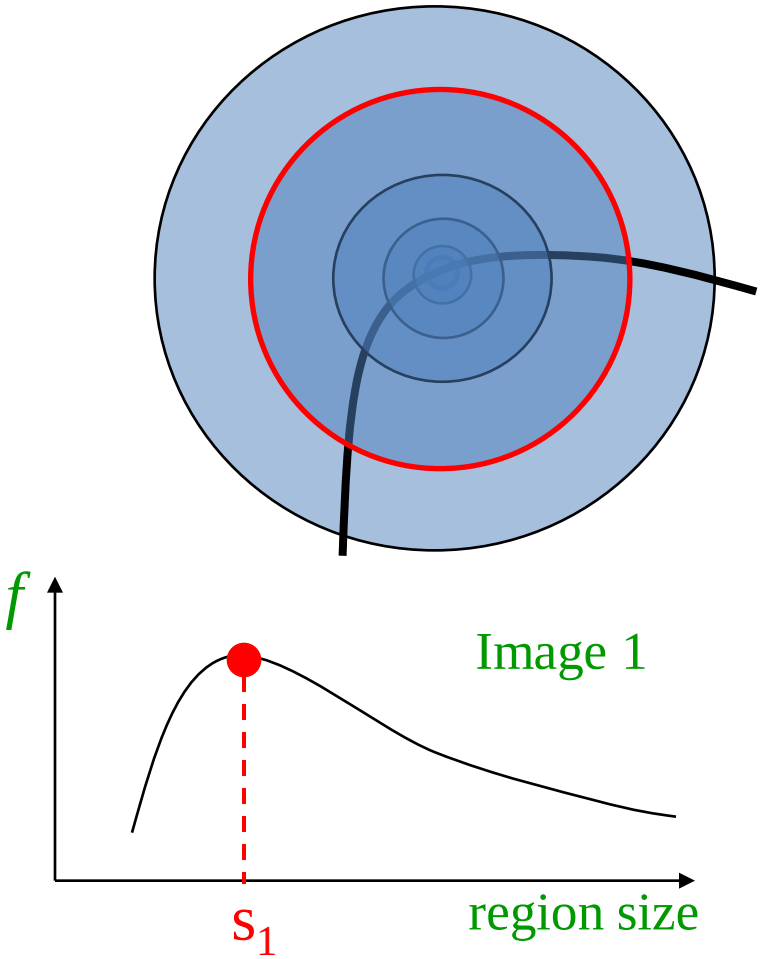
Scale invariant interest points

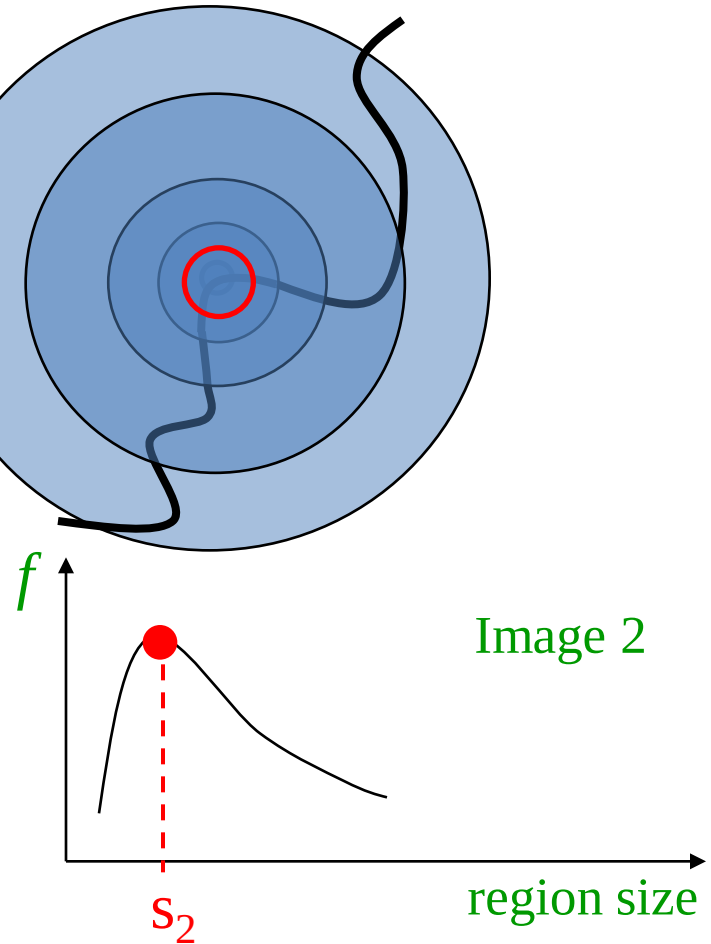
How can we independently select interest points in each image, such that the detections are repeatable across different scales?

Automatic scale selection

Intuition:

- Find scale that gives local maxima of some function *f* in both position and scale.

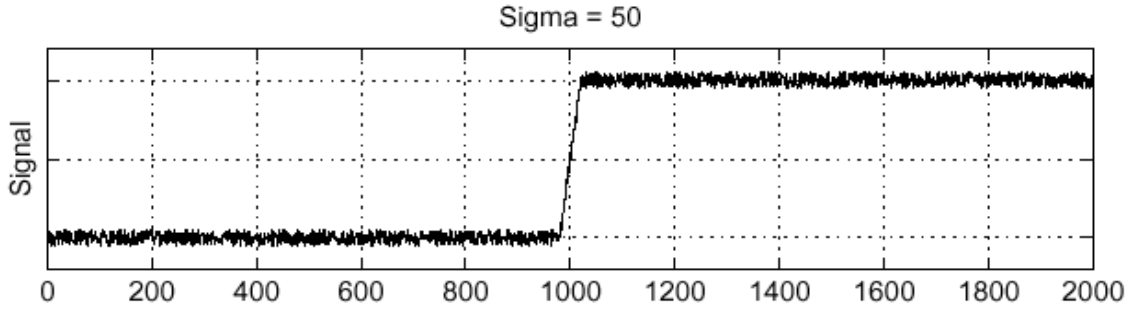




- What can be the “signature” function?

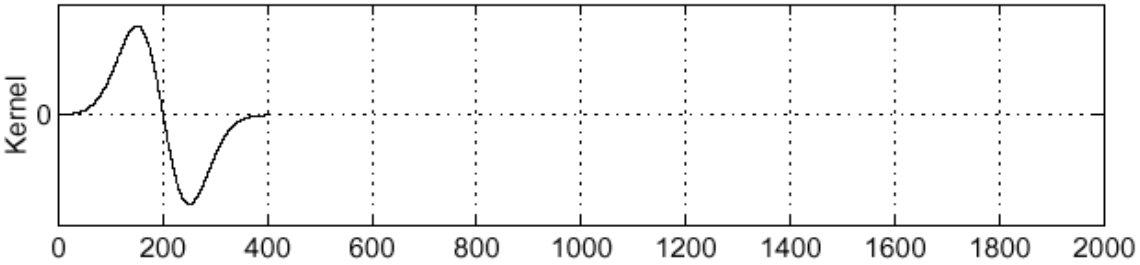
Recall: Edge detection

f



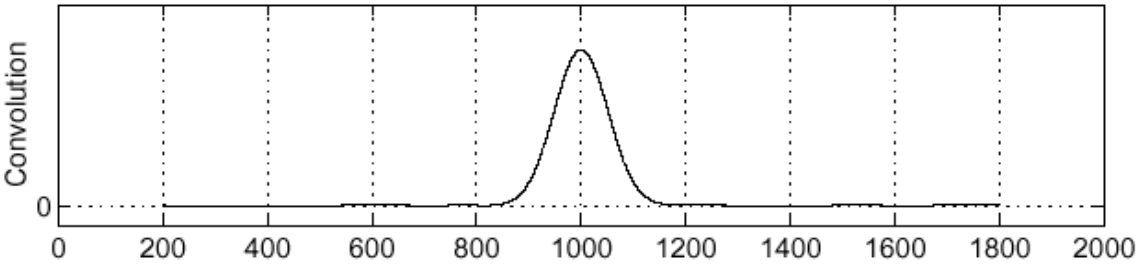
Edge

$\frac{d}{dx} g$



Derivative of Gaussian

$f * \frac{d}{dx} g$

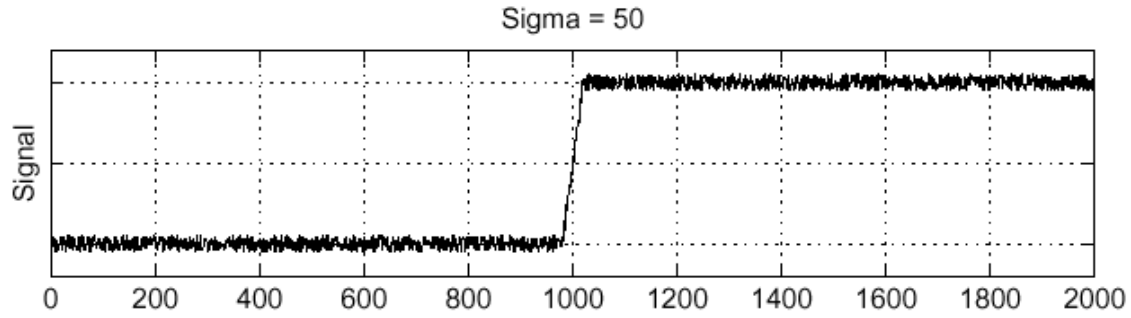


Edge = maximum of derivative

Source: S. Seitz

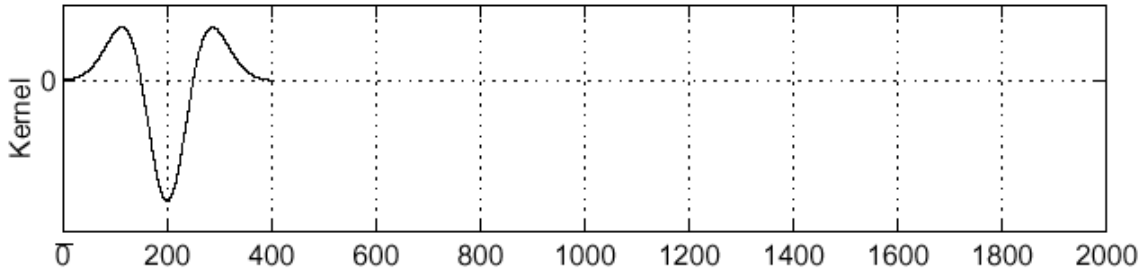
Recall: Edge detection

f



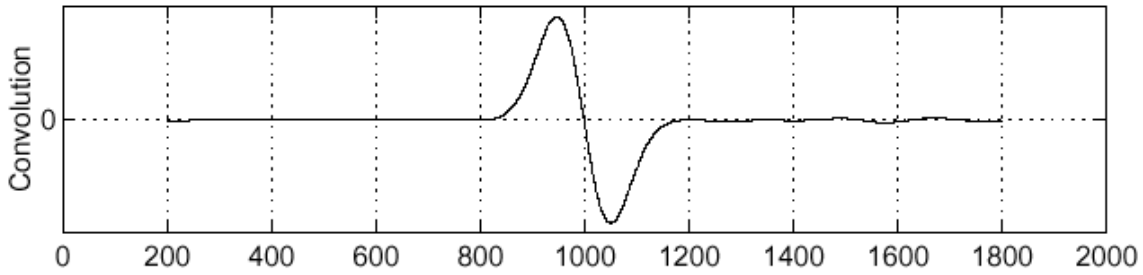
Edge

$$\frac{d^2}{dx^2} g$$



Second derivative of Gaussian (Laplacian)

$$f * \frac{d^2}{dx^2} g$$

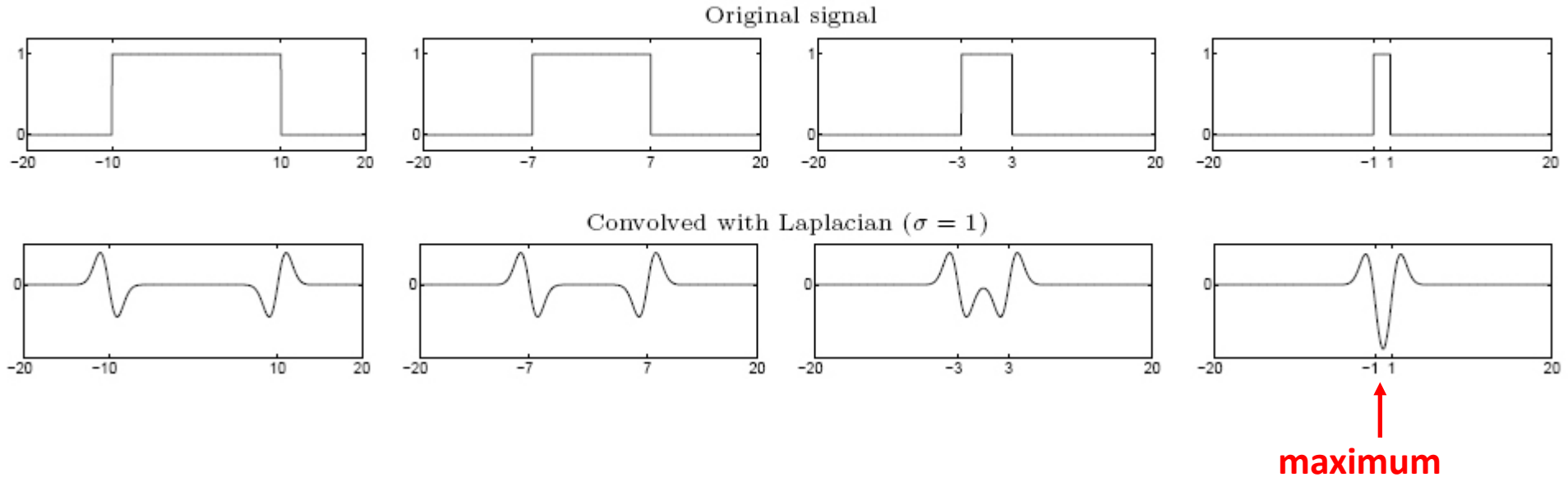


Edge = zero crossing of second derivative

Source: S. Seitz

From edges to blobs

- Edge = ripple
- Blob = superposition of two ripples

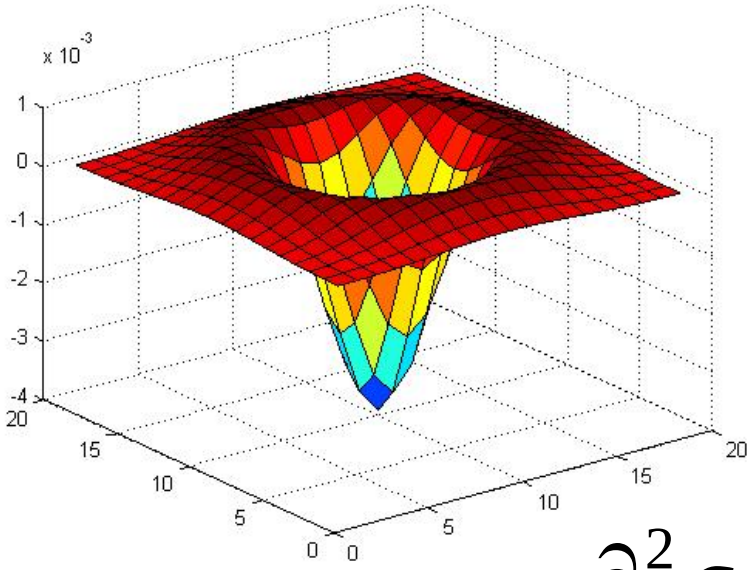


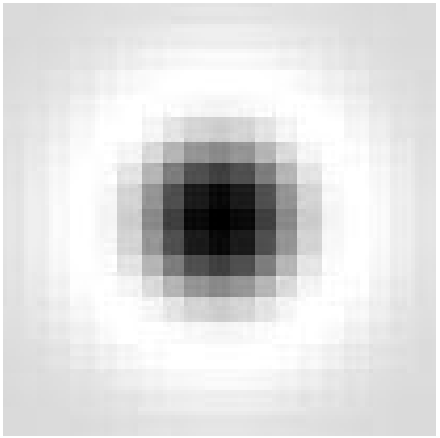
Spatial selection: the **magnitude** of the Laplacian response will achieve a maximum at the center of the blob, provided the scale of the Laplacian is "matched" to the scale of the blob

Slide credit: Lana Lazebnik

Blob detection in 2D

- Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D



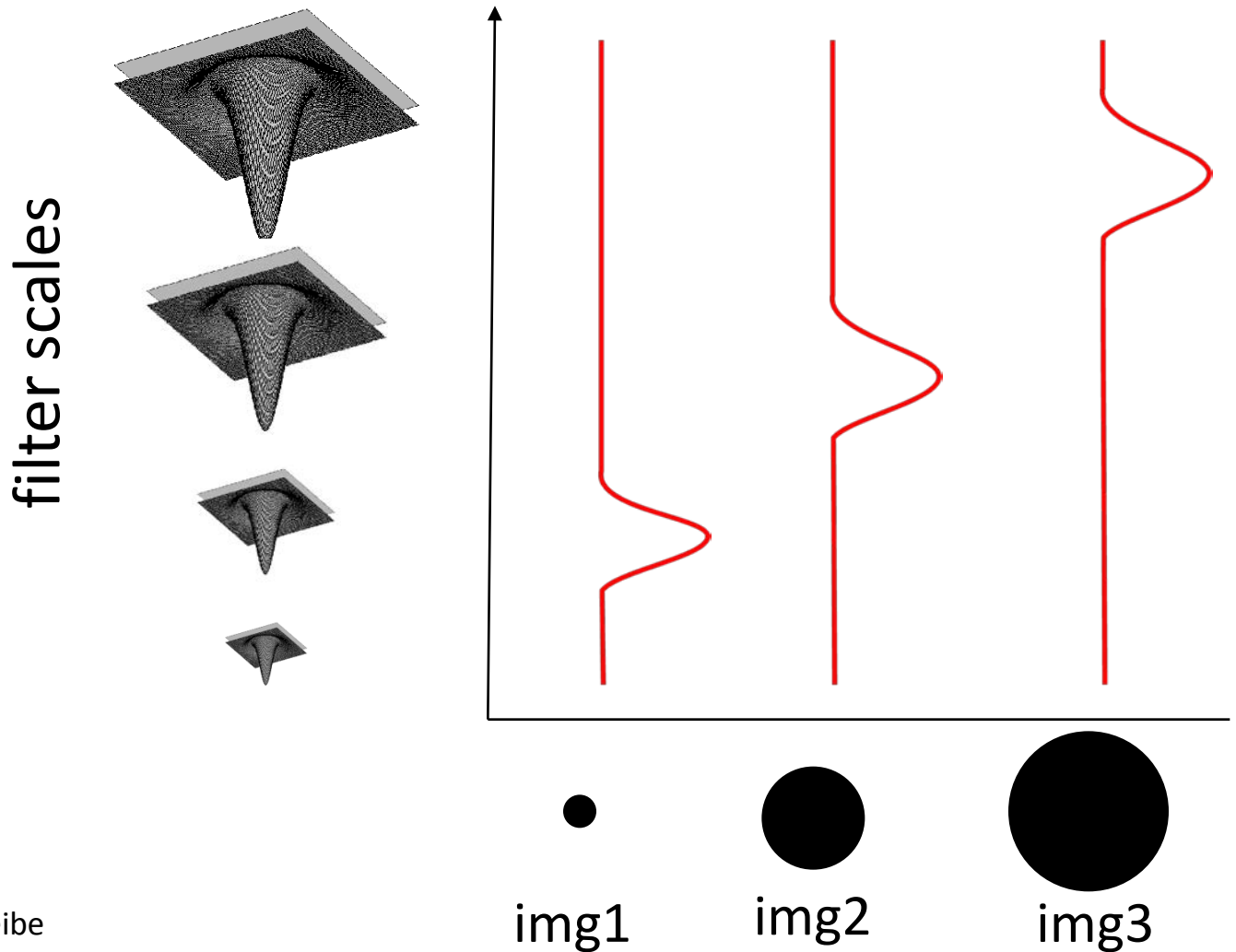


$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

Blob detection in 2D: scale selection

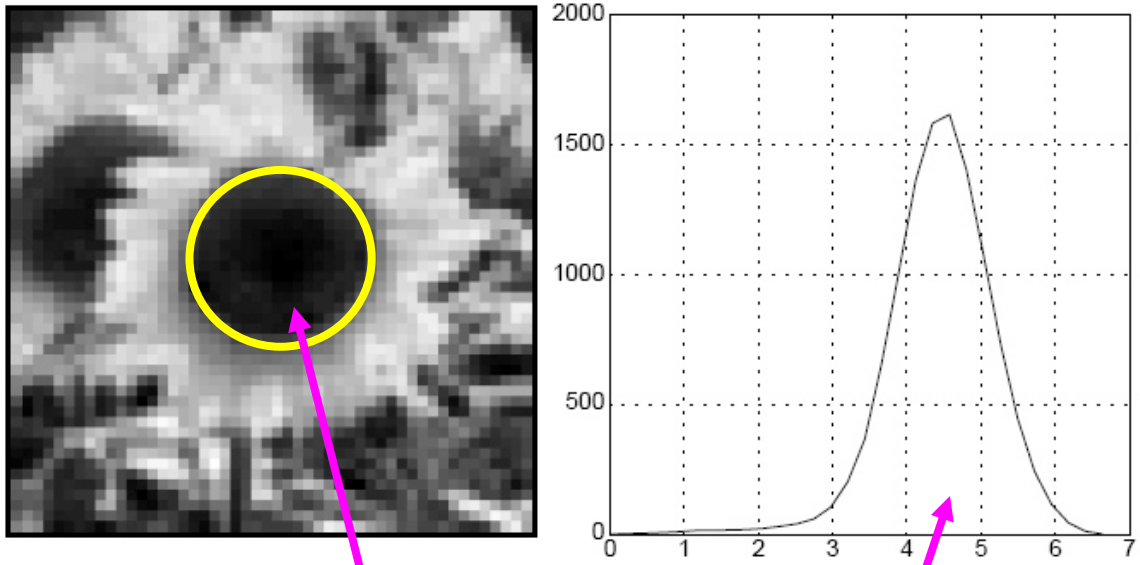
- Laplacian-of-Gaussian = "blob" detector

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$



Blob detection in 2D

- We define the *characteristic scale* as the scale that produces peak of Laplacian response

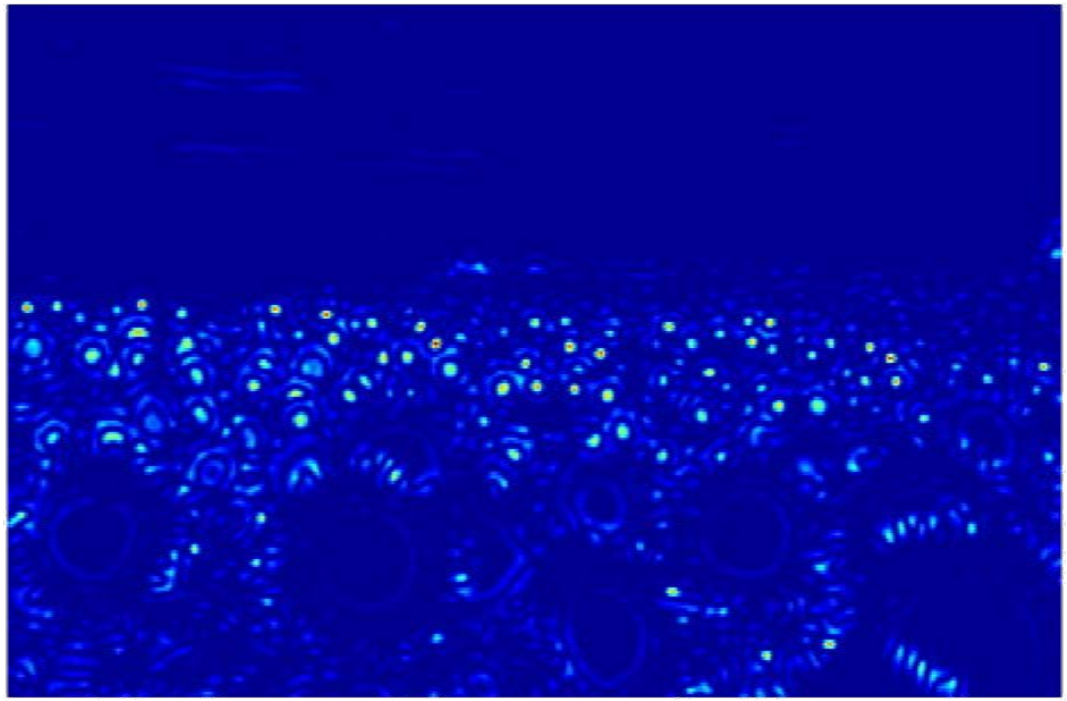


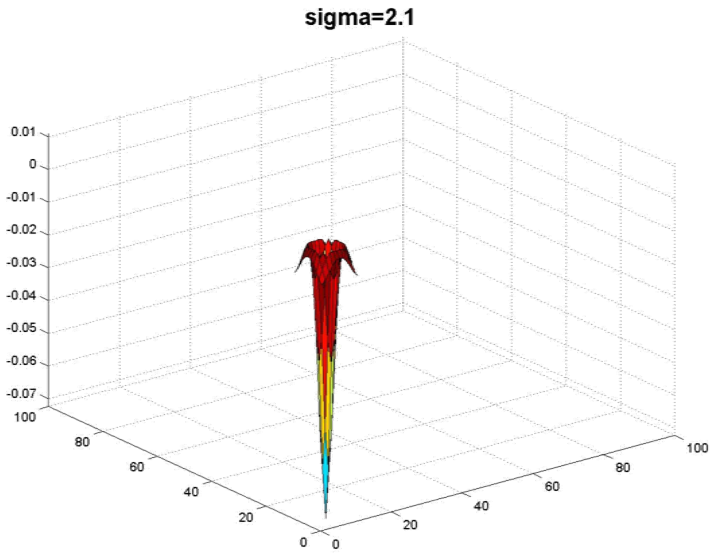
Slide credit: Lana Lazebnik

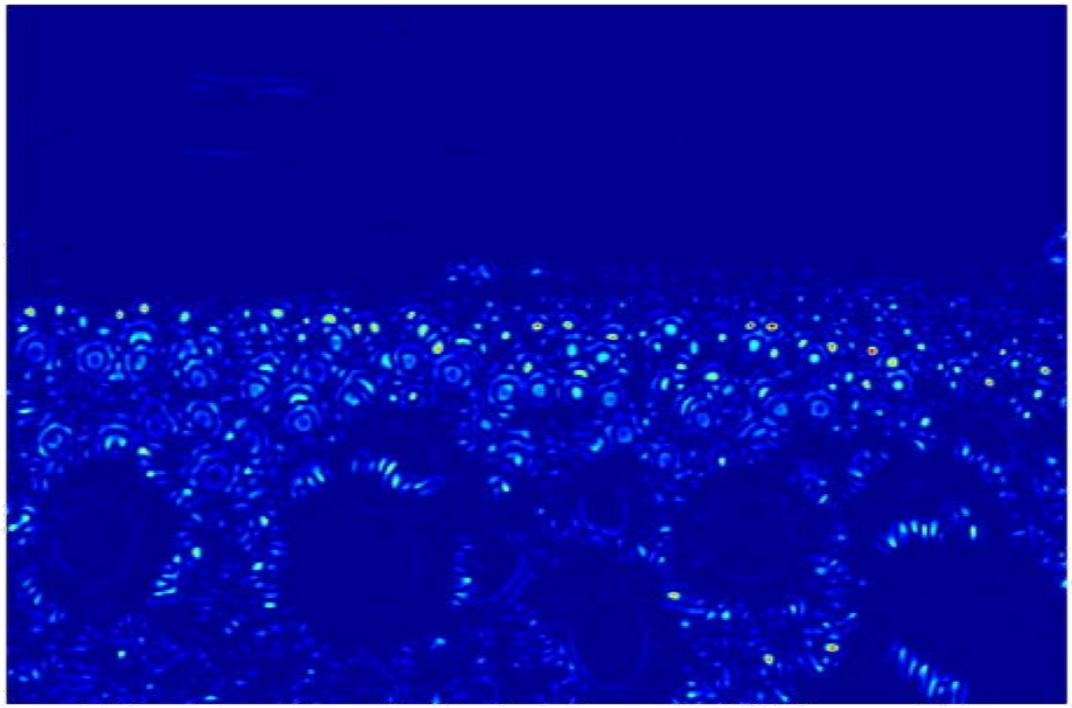
Example

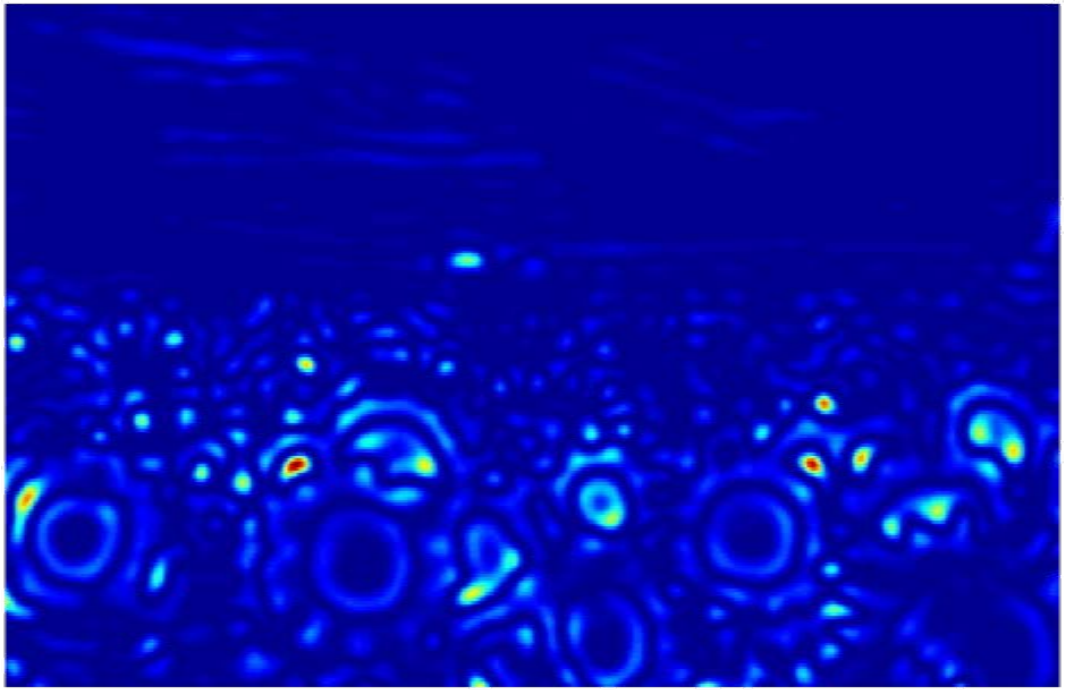
Original image at
¾ the size

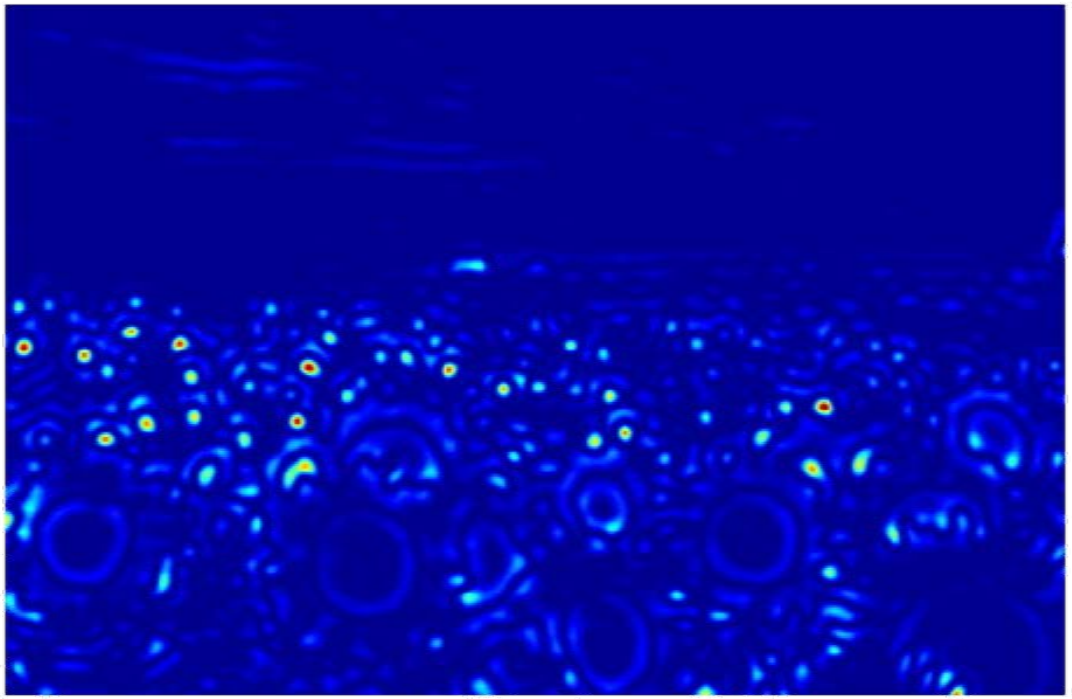
Original image at
¾ the size

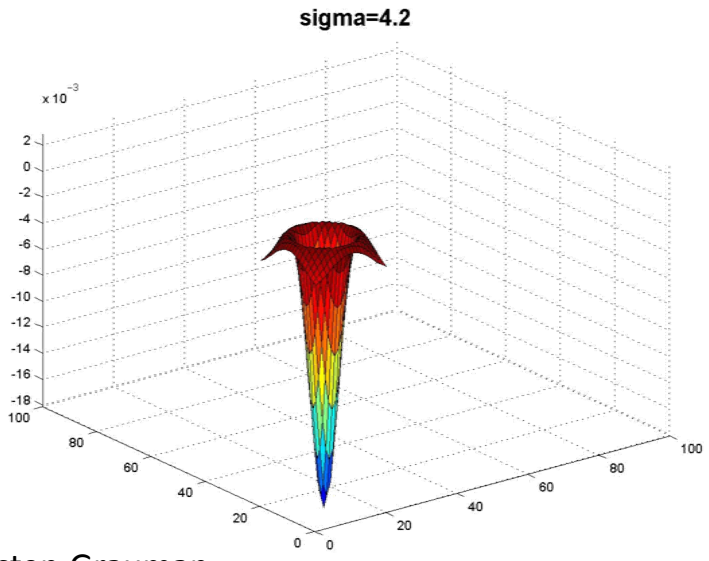








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-6
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20
0
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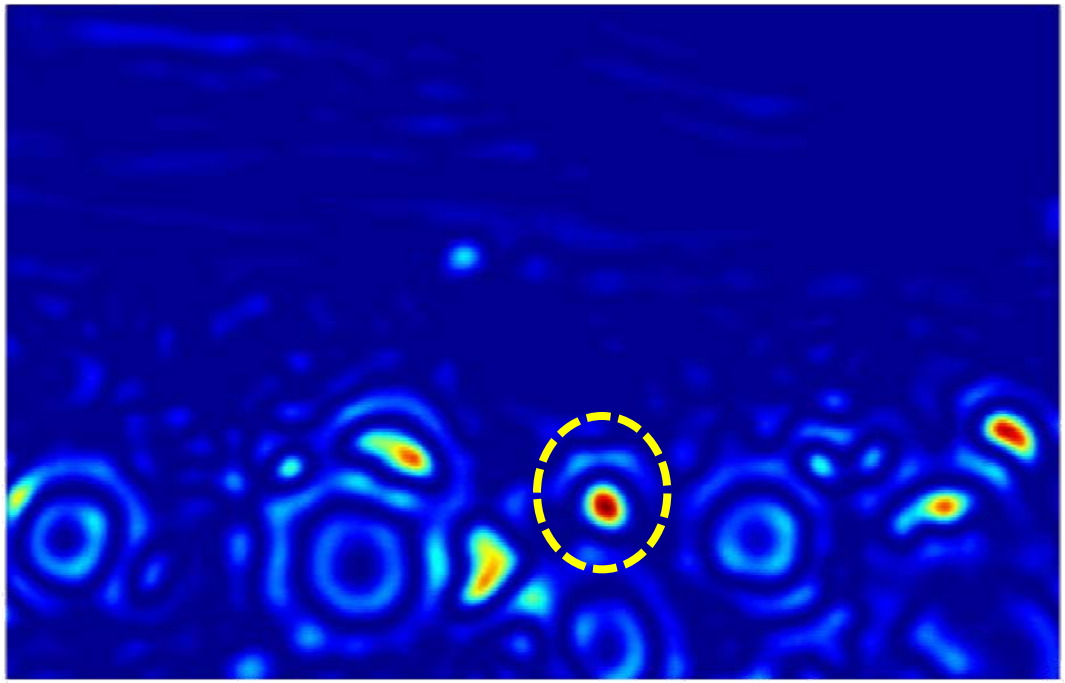
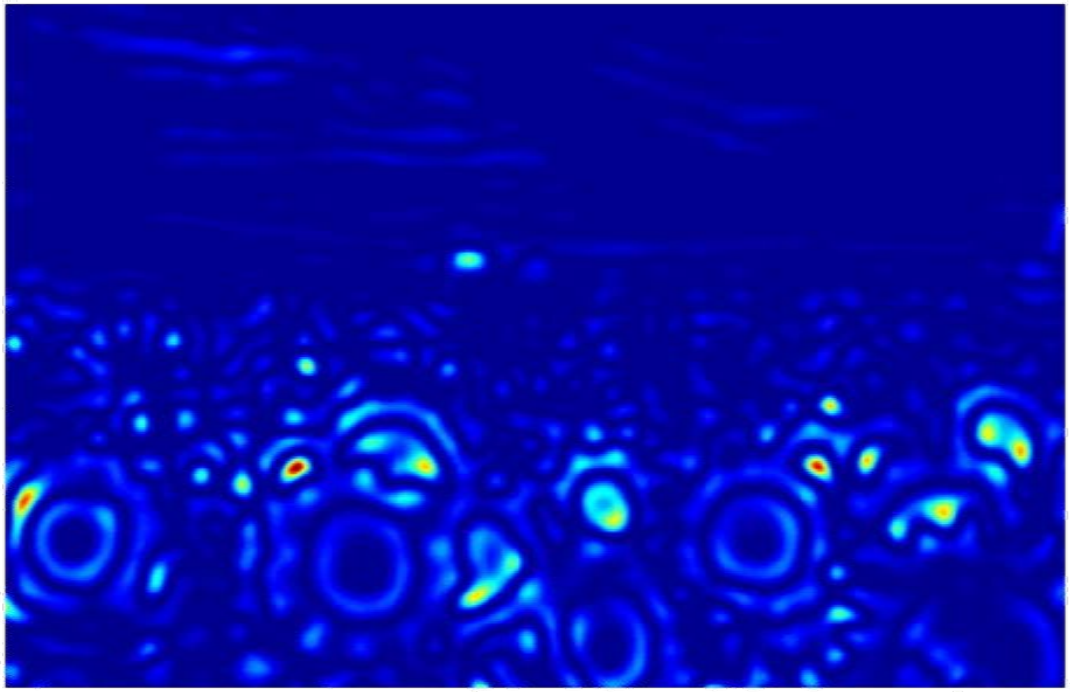
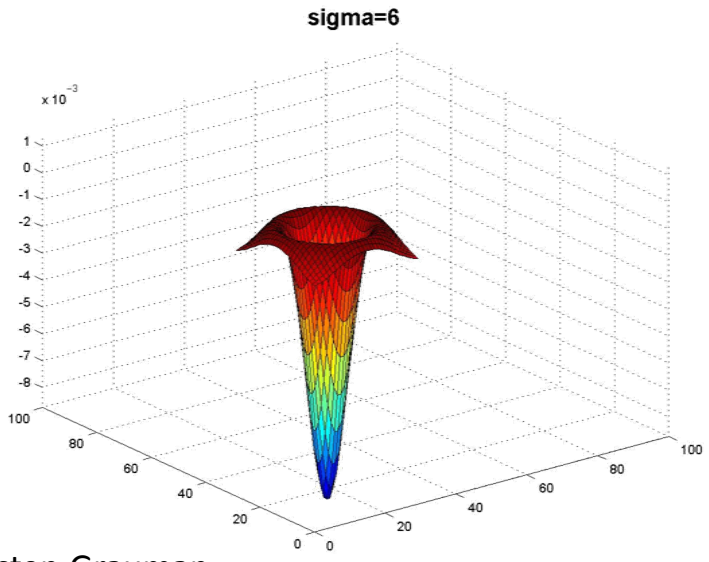
20
40
60
80
100



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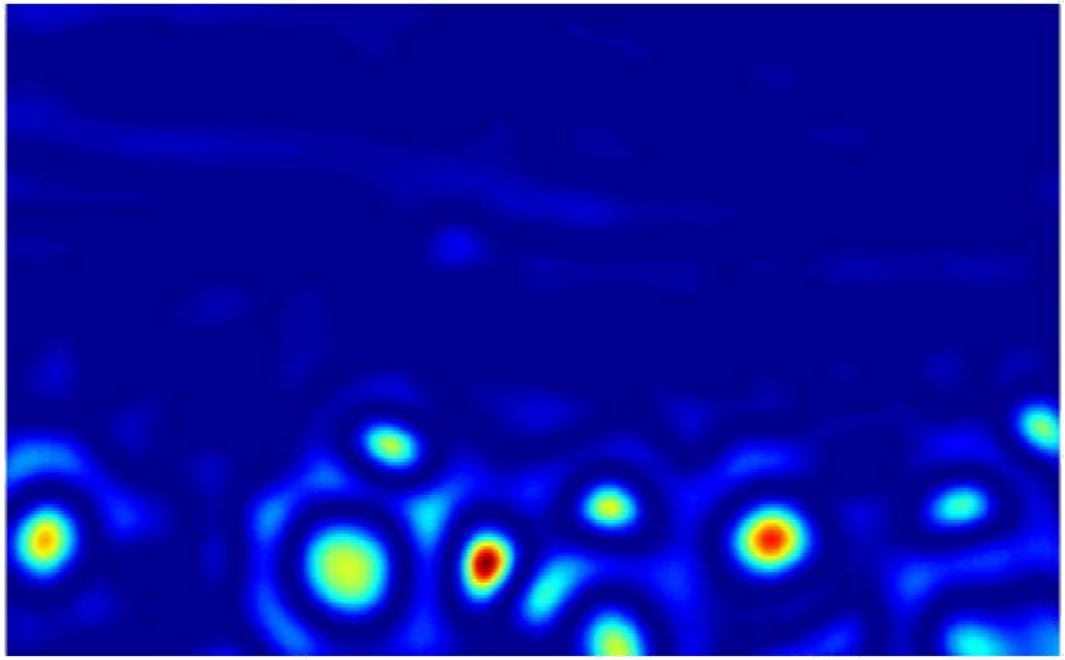
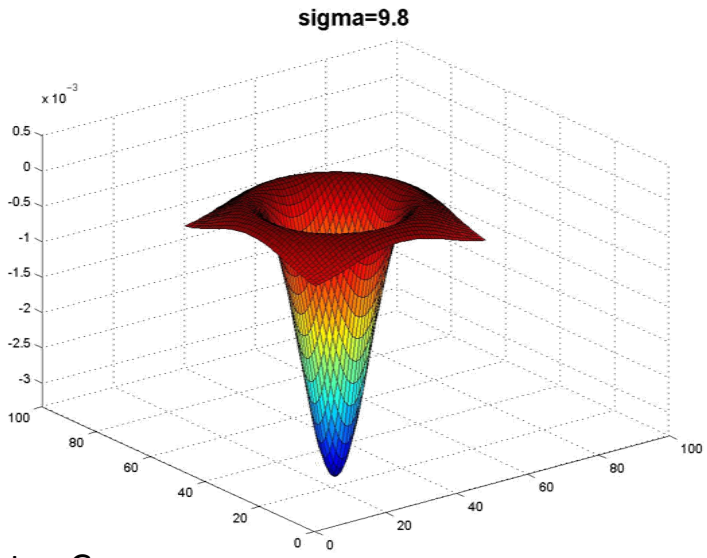
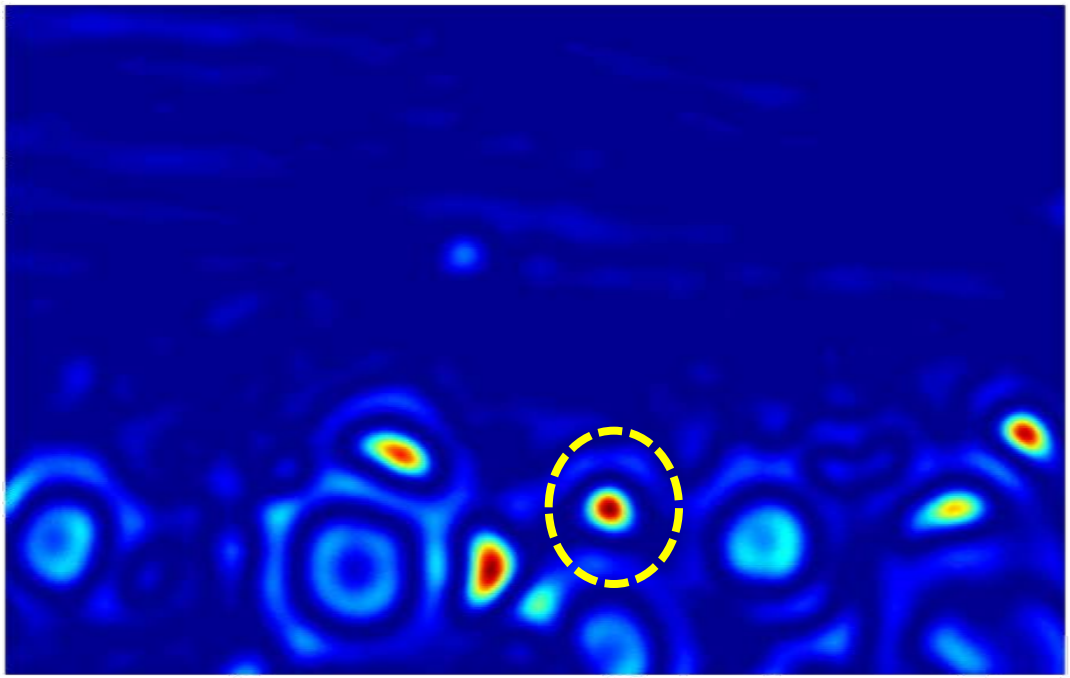
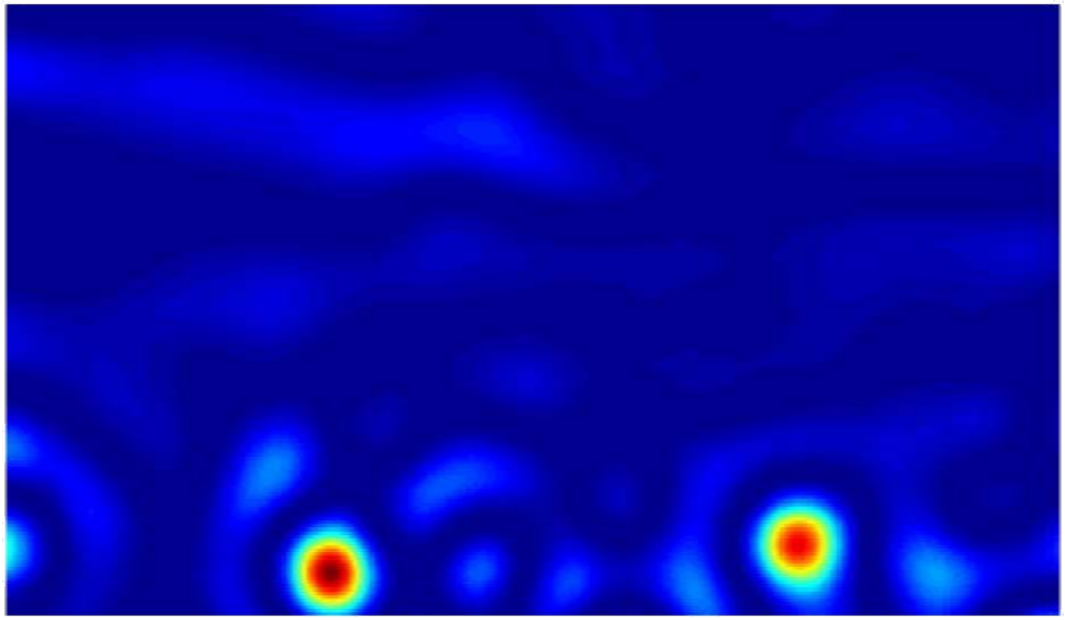
20
40
60
80
100

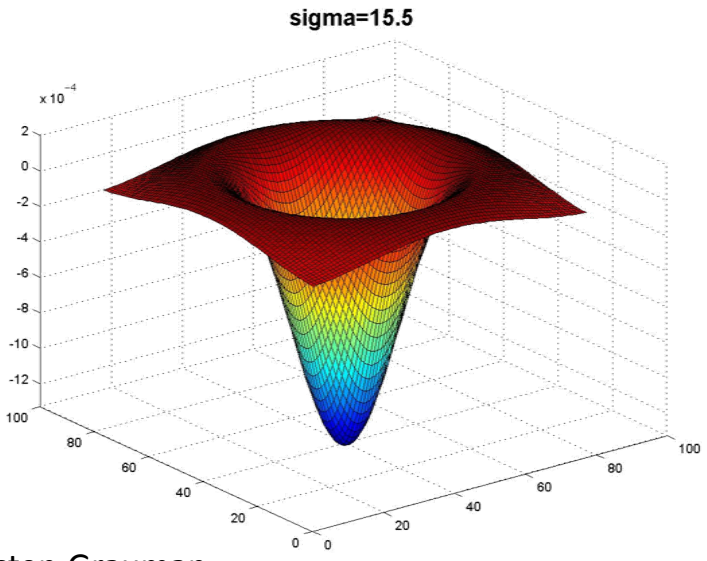






Kristen Grauman




sigma=15.5
x 10^-4
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-4
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-8
-10
-12
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20
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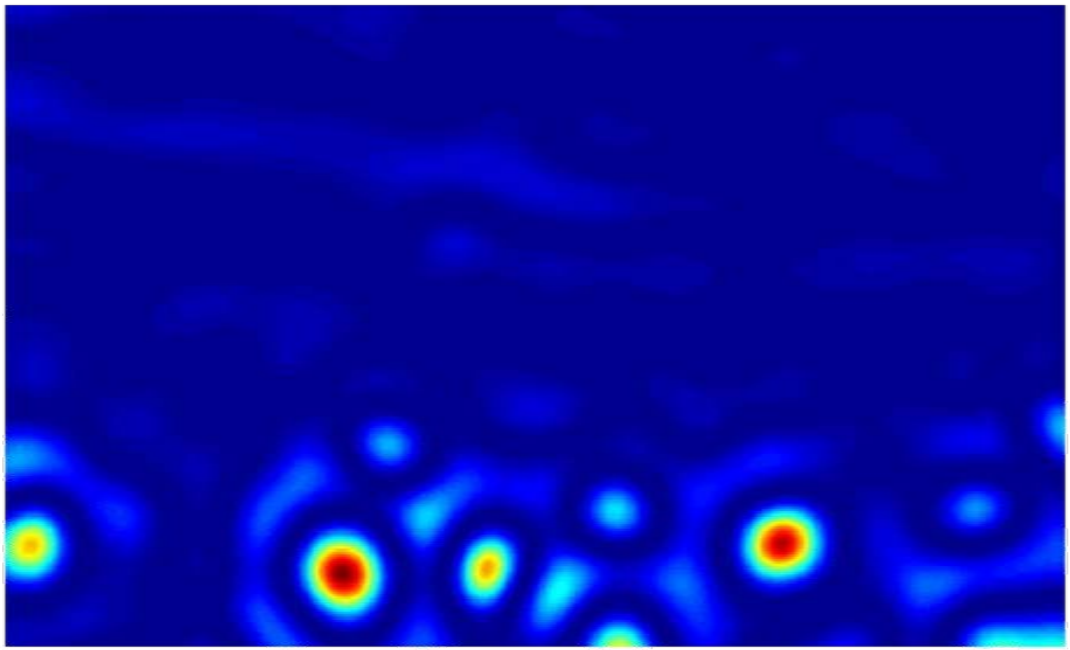
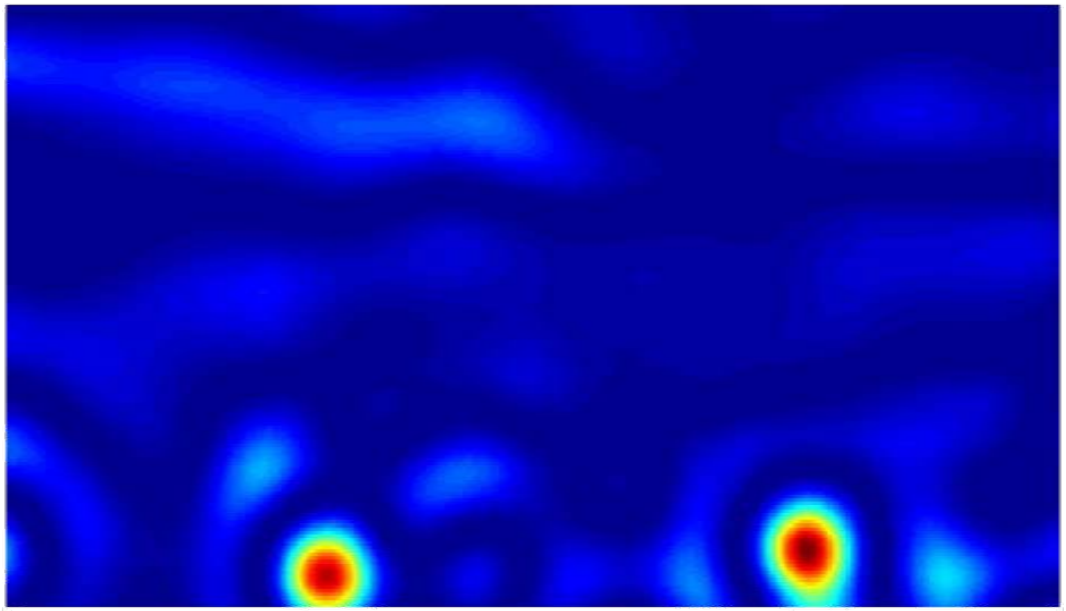
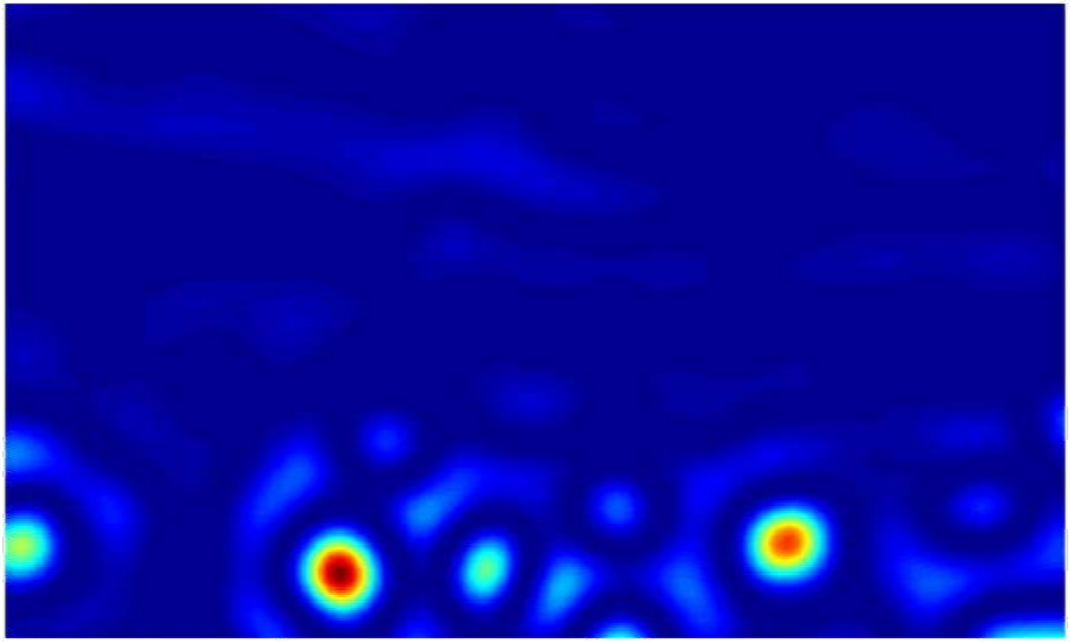
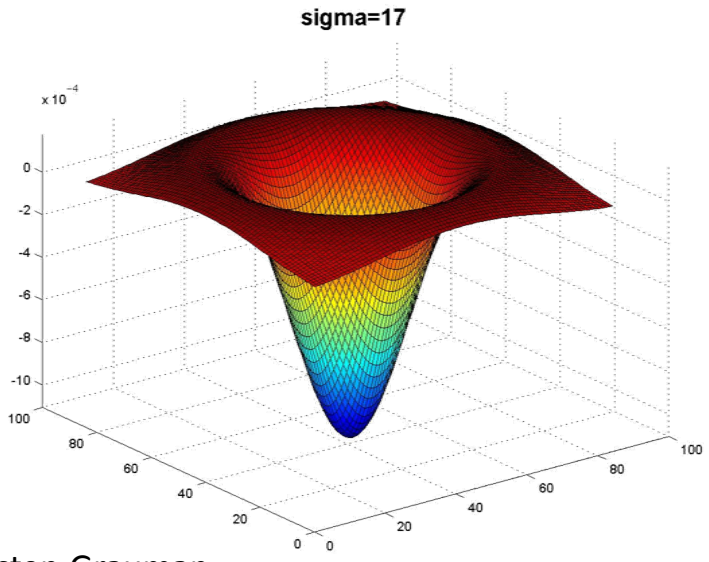
40
60
80
100





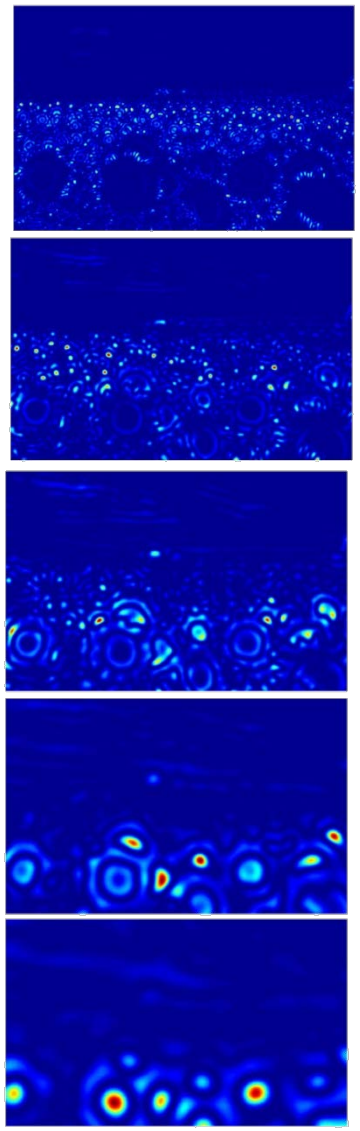


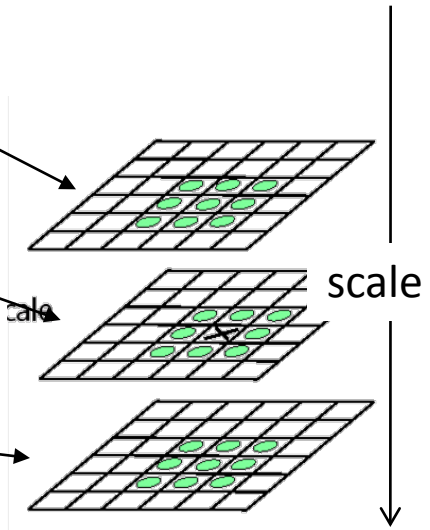

sigma=17
x 10^-4
0
-2
-4
-6
-8
-10
100
80
60
40
20
0
0
20
40
60
80
100

Scale invariant interest points

Interest points are local maxima in both position and scale.

$$L_{xx}(\sigma) + L_{yy}(\sigma)$$





⇒ **List of**
(x, y, σ)

Squared filter response maps

Scale-space blob detector: Example

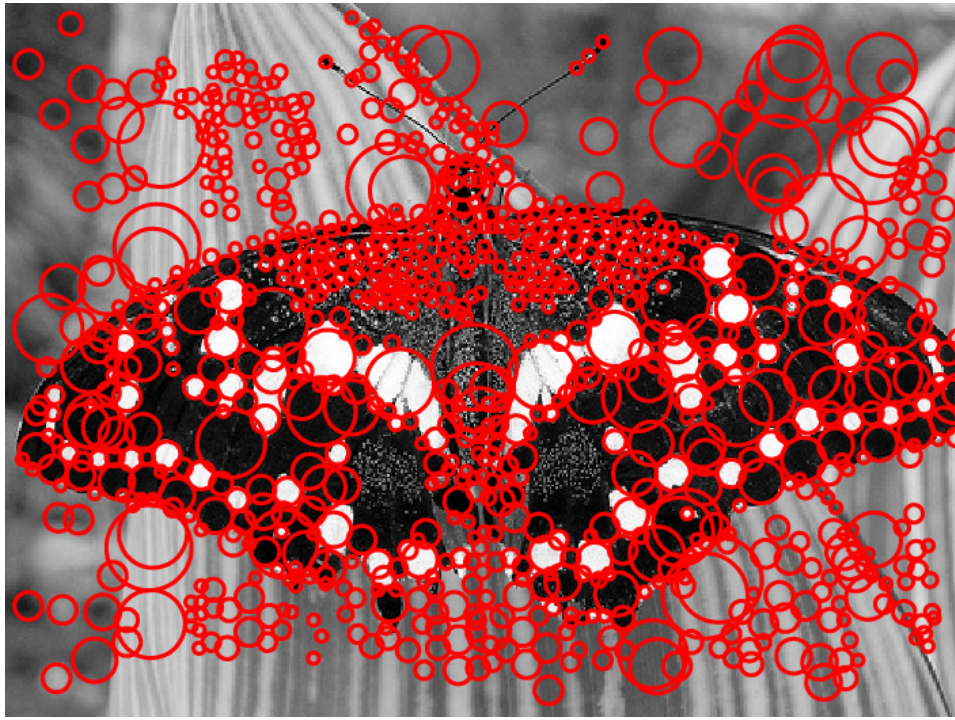


Image credit: Lana Lazebnik

Technical detail

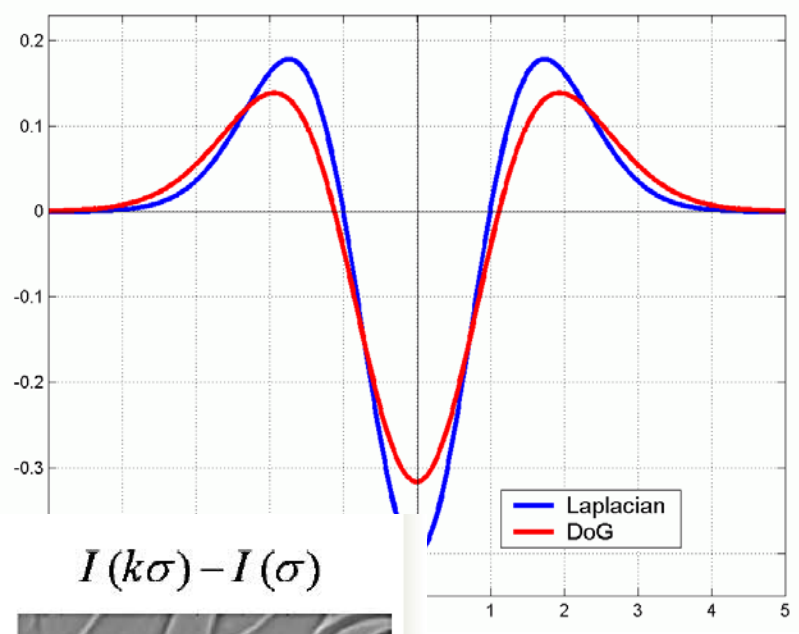
- We can approximate the Laplacian with a difference of Gaussians; more efficient to implement.

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



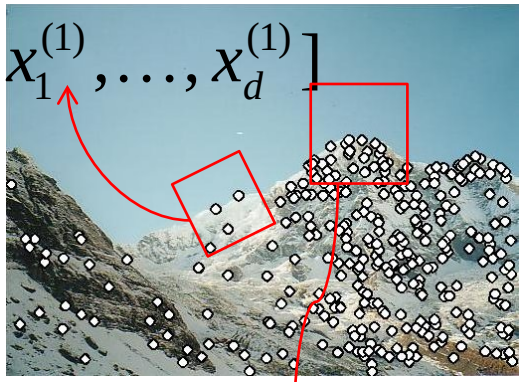
$I(k\sigma)$ $I(\sigma)$ $I(k\sigma) - I(\sigma)$

Local features: main components

1) Detection: Identify the interest points

2) Description: Extract vector feature descriptor surrounding each interest point.

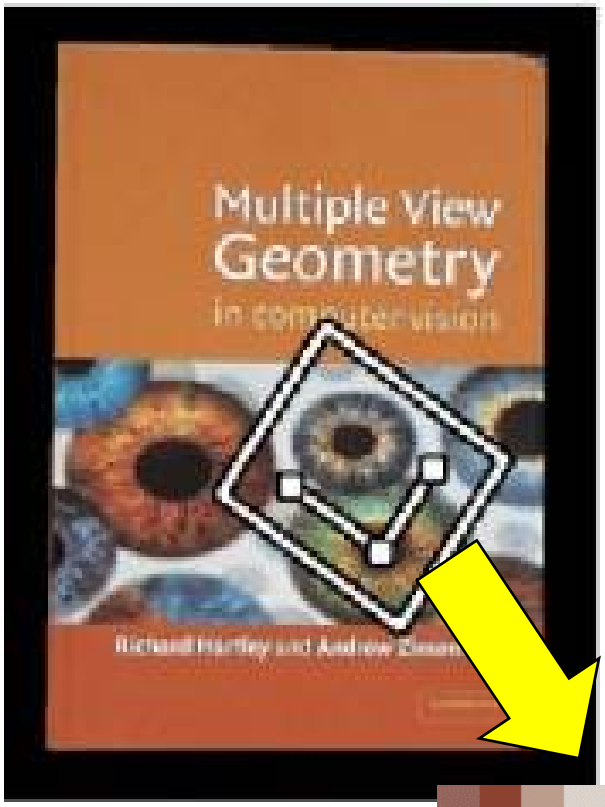
$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$

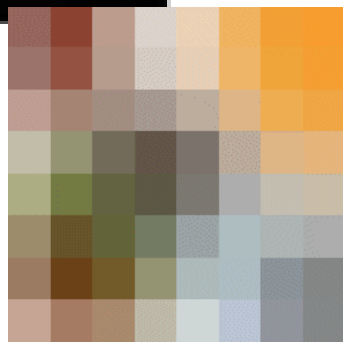


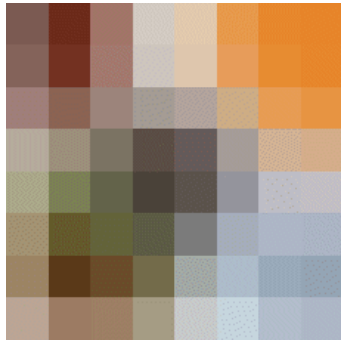
$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}]$$

3) Matching: Determine correspondence between descriptors in two views

Geometric transformations





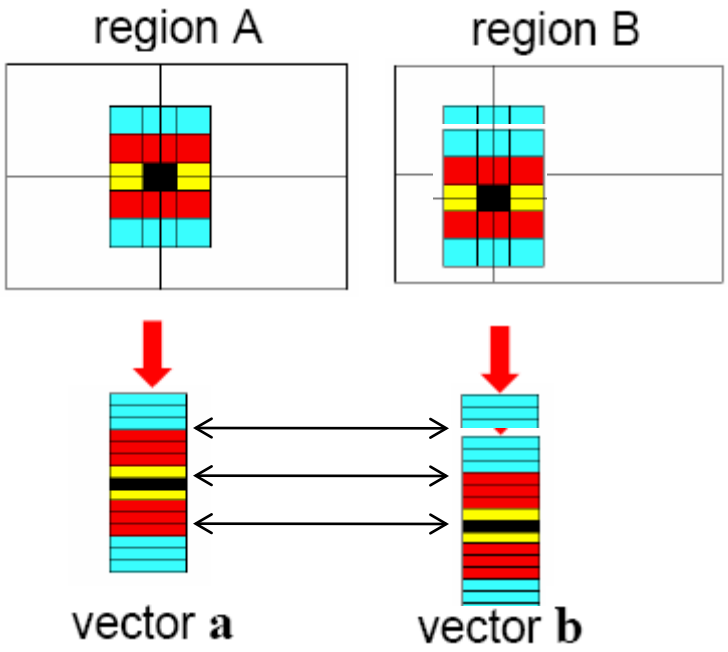


e.g. scale, translation, rotation

Photometric transformations

Figure from T. Tuytelaars ECCV 2006 tutorial

Raw patches as local descriptors

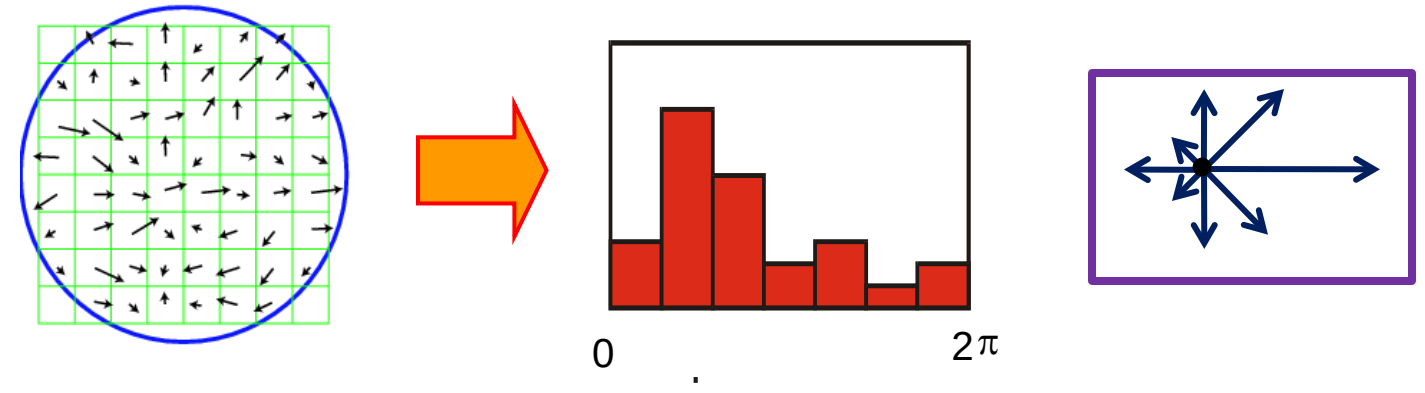


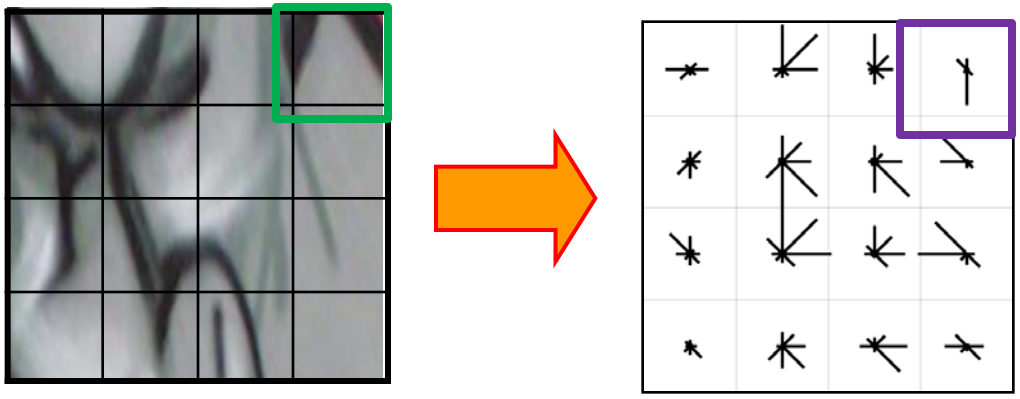
The simplest way to describe the neighborhood around an interest point is to write down the list of intensities to form a feature vector.

But this is very sensitive to even small shifts, rotations.

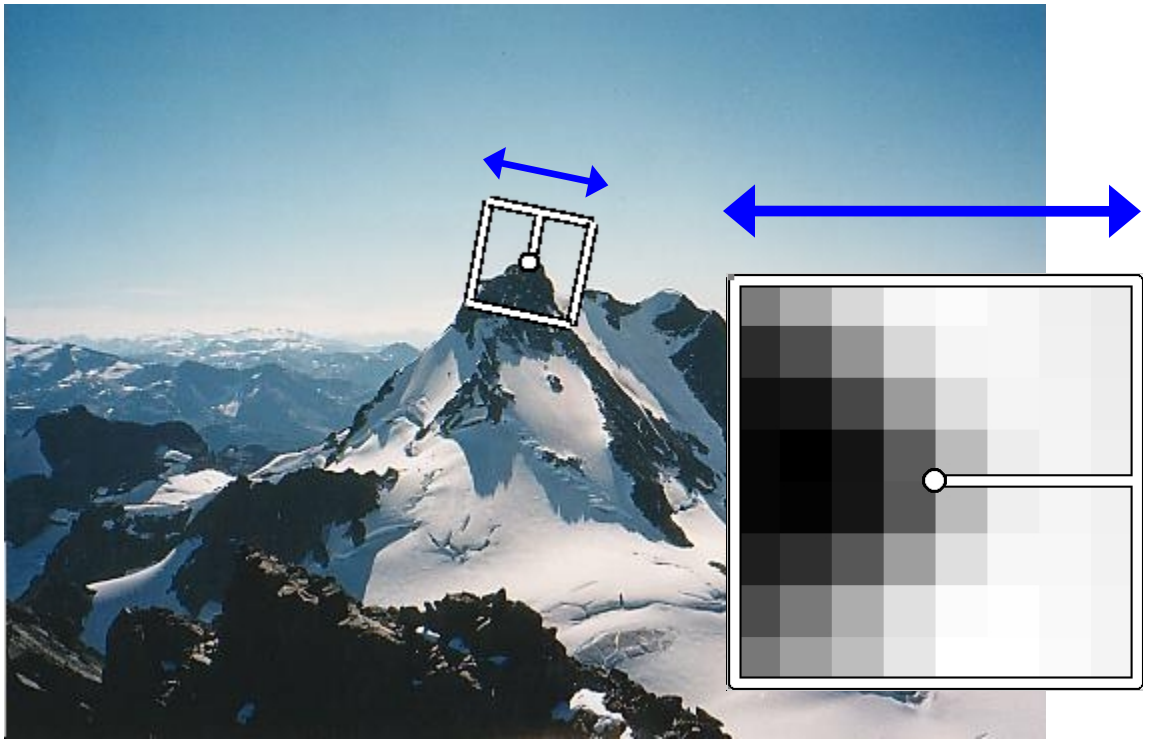
SIFT descriptor [Lowe 2004]

- Use histograms to bin pixels within sub-patches according to their orientation.





Making descriptor rotation invariant



- Rotate patch according to its dominant gradient orientation
- This puts the patches into a canonical orientation.

Image from Matthew Brown

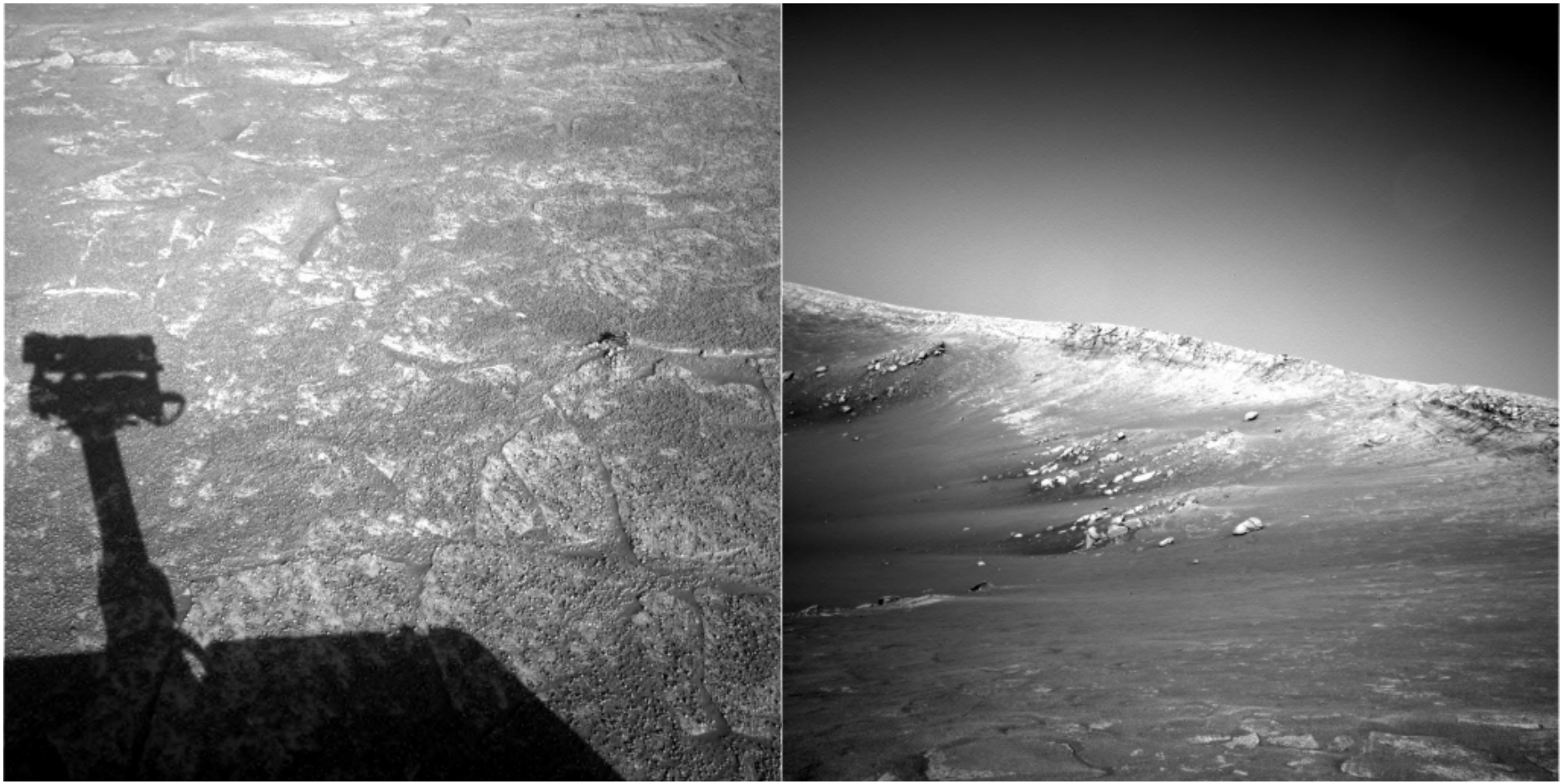
SIFT descriptor [Lowe 2004]

- Extraordinarily robust matching technique
  - Can handle changes in viewpoint
    - Up to about 60 degree out of plane rotation
  - Can handle significant changes in illumination
    - Sometimes even day vs. night (below)
  - Fast and efficient—can run in real time
  - Lots of code available

ck/wiki/index.php/

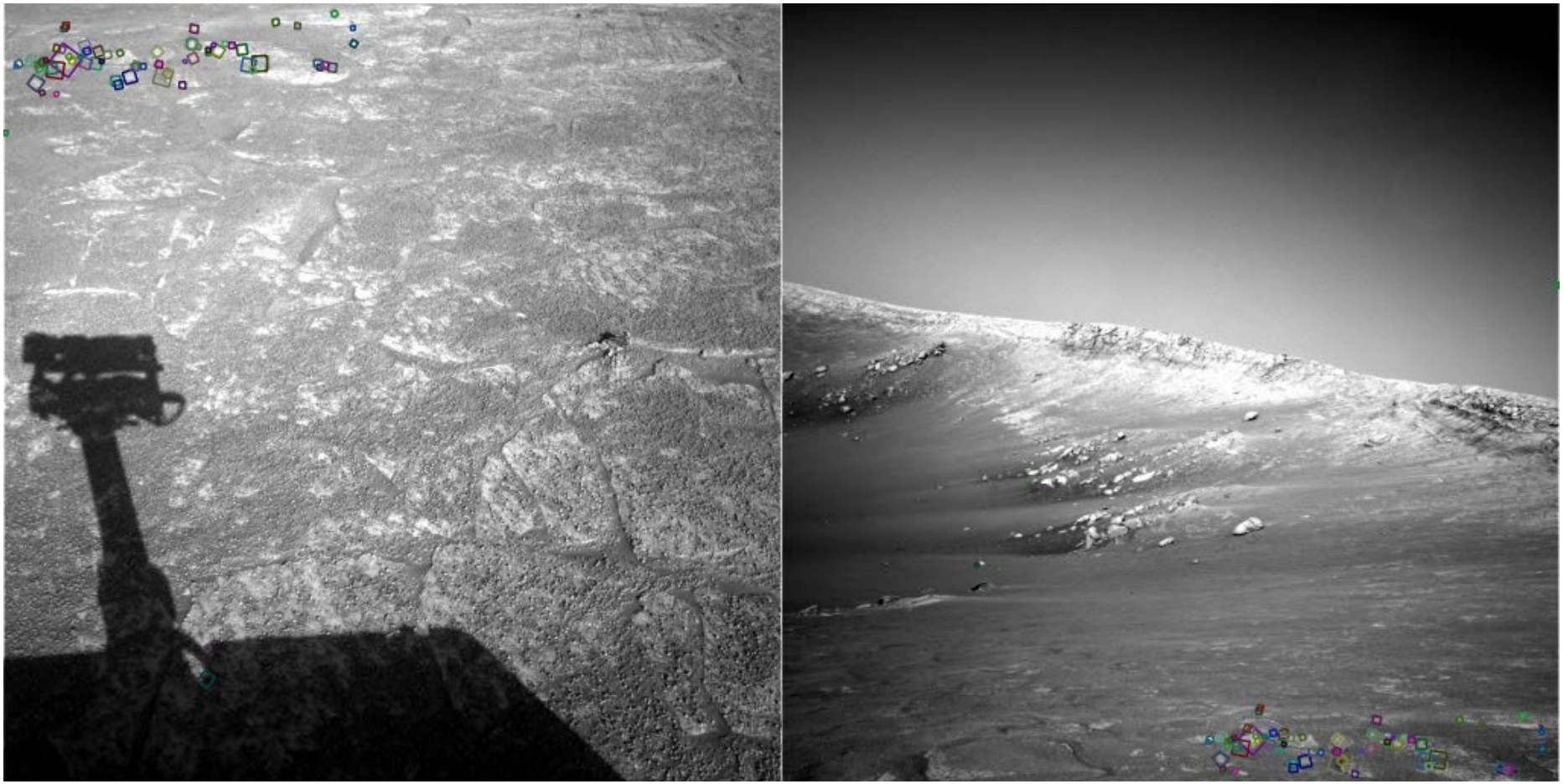
Steve Seitz

Example



NASA Mars Rover images

Example



NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

SIFT properties

- Invariant to
  - Scale
  - Rotation

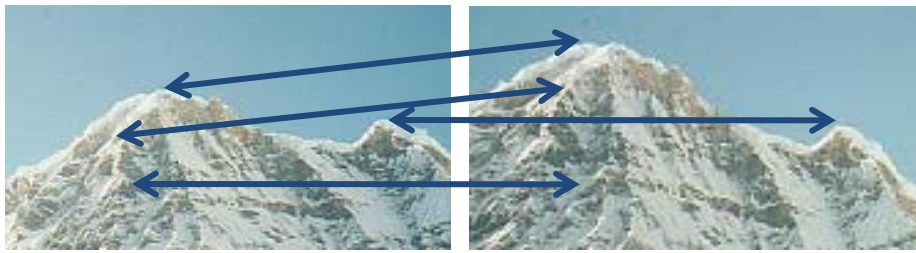
- Partially invariant to
  - Illumination changes
  - Camera viewpoint
  - Occlusion, clutter

Local features: main components

1) Detection: Identify the interest points

2) Description:Extract vector feature descriptor surrounding each interest point.

3) Matching: Determine correspondence between descriptors in two views



Matching local features

Matching local features

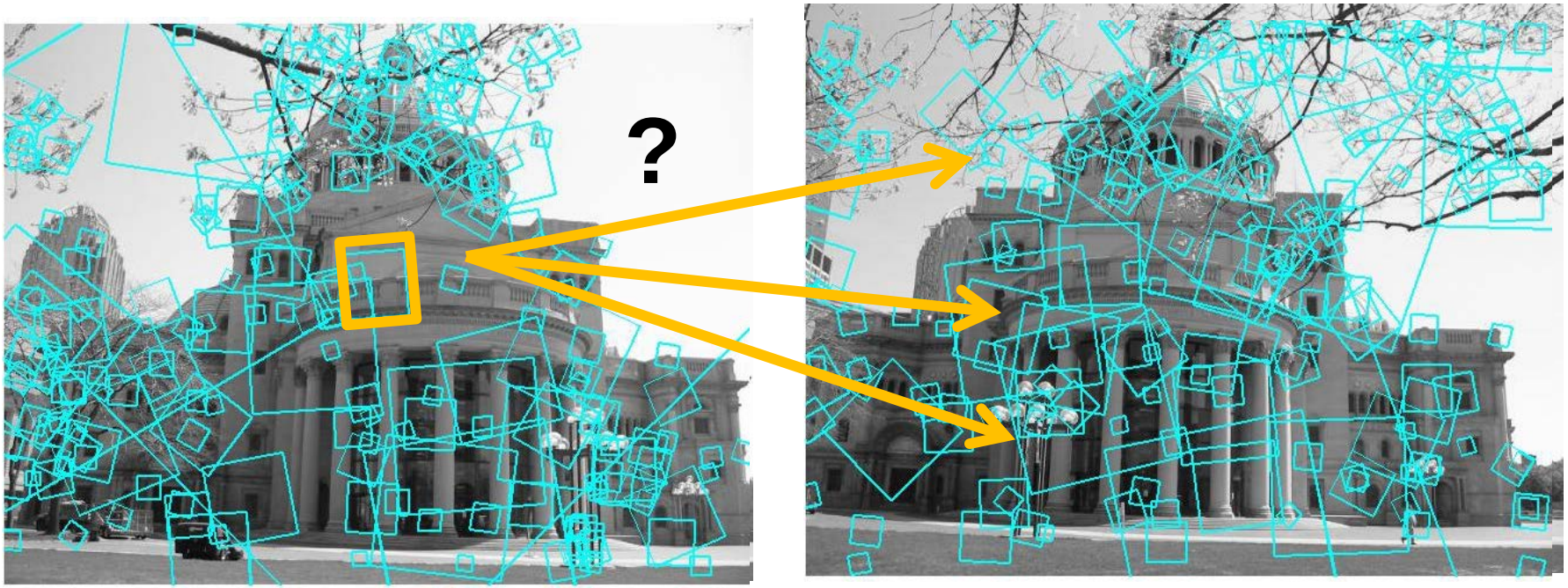


Image 1 Image 2

To generate **candidate matches**, find patches that have the most similar appearance (e.g., lowest SSD)

Simplest approach: compare them all, take the closest (or closest k, or within a thresholded distance)

Ambiguous matches

Image 1

Image 2

At what SSD value do we have a good match?

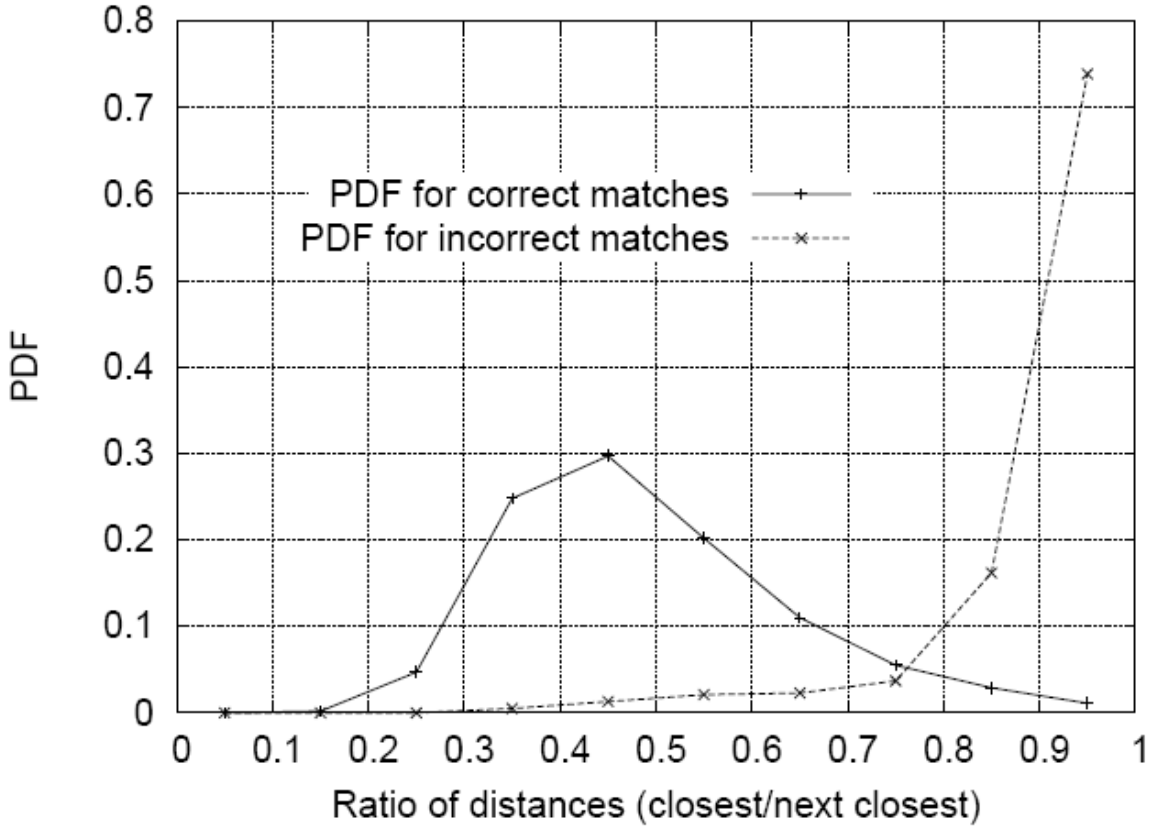
To add robustness to matching, can consider **ratio** : distance to best match / distance to second best match

If low, first match looks good.

If high, could be ambiguous match.

Matching SIFT Descriptors

- Nearest neighbor (Euclidean distance)
- Threshold ratio of nearest to 2nd nearest descriptor

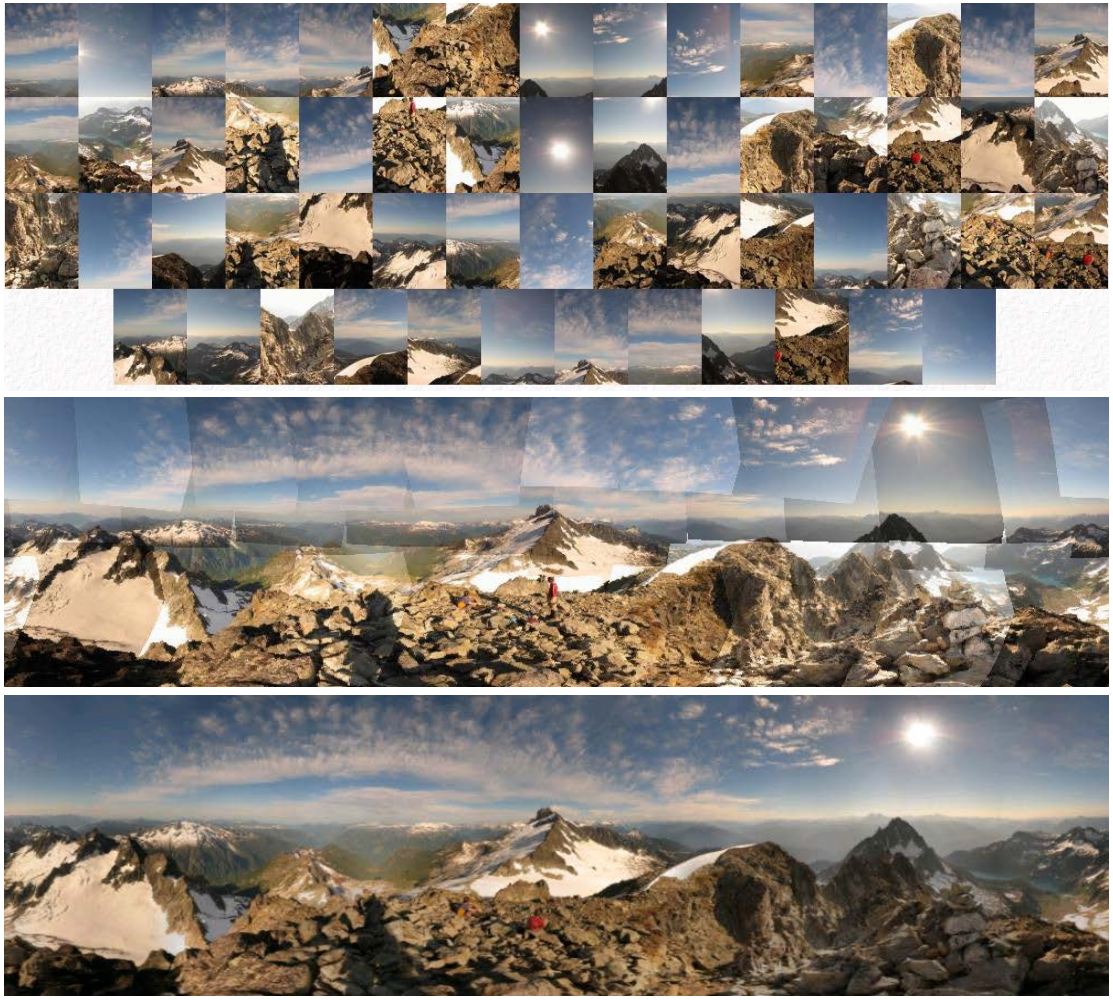


Lowe IJCV 2004

Applications of local invariant features

- Wide baseline stereo
- Motion tracking
- Panoramas
- Mobile robot navigation
- 3D reconstruction
- Recognition
- …

Automatic mosaicing

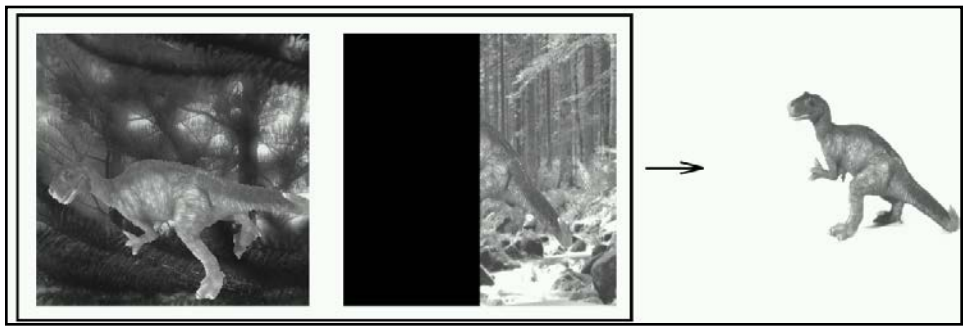


http://www.cs.ubc.ca/~mbrown/autostitch/autostitch.html

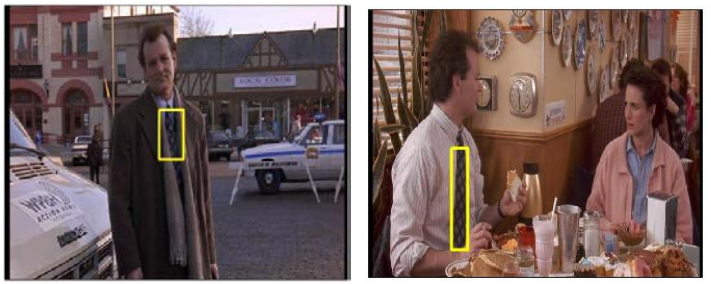
Wide baseline stereo

[Image from T. Tuytelaars ECCV 2006 tutorial]

Recognition of specific objects, scenes



Schmid and Mohr 1997



Sivic and Zisserman, 2003

Rothganger et al. 2003

Lowe 2002

Summary

- Interest point detection
  - Harris corner detector
  - Laplacian of Gaussian, automatic scale selection
- Invariant descriptors
  - Rotation according to dominant gradient direction
  - Histograms for robustness to small shifts and translations (SIFT descriptor)