

Stereo Algorithms

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- Slide credit to Robert Collins

Recall: Simple Stereo System

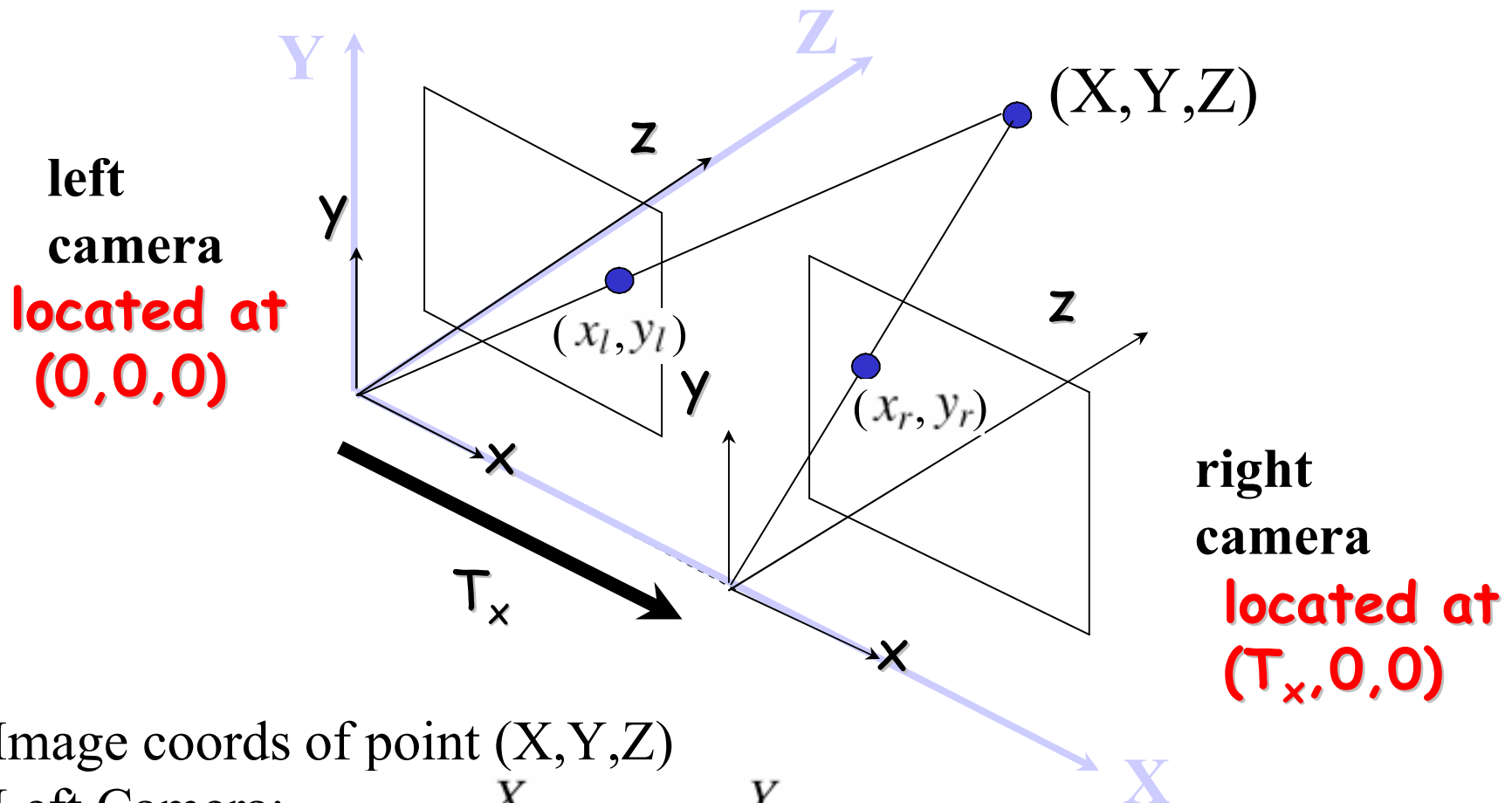


Image coords of point (X, Y, Z)

Left Camera:

$$x_l = f \frac{X}{Z} \quad y_l = f \frac{Y}{Z}$$

Right Camera:

$$x_r = f \frac{X - T_x}{Z} \quad y_r = f \frac{Y}{Z}$$

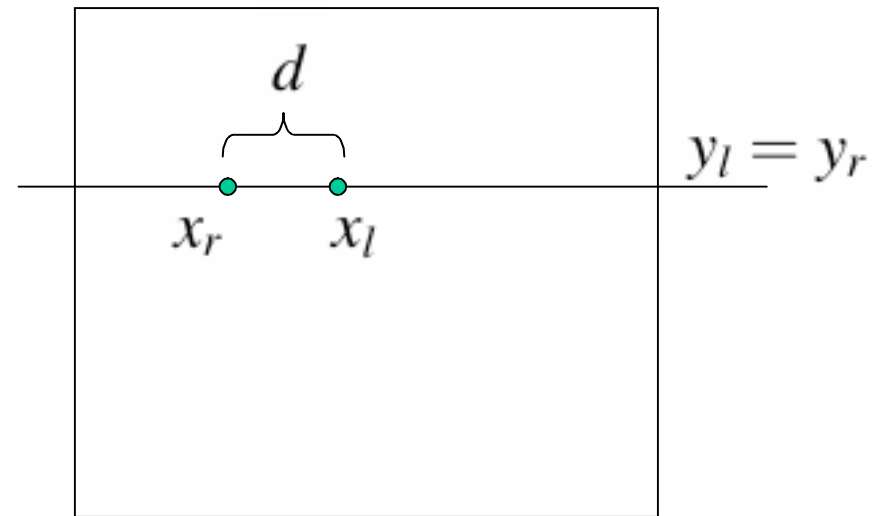
Recall: Stereo Disparity

Left camera

$$x_l = f \frac{X}{Z} \quad y_l = f \frac{Y}{Z}$$

Right camera

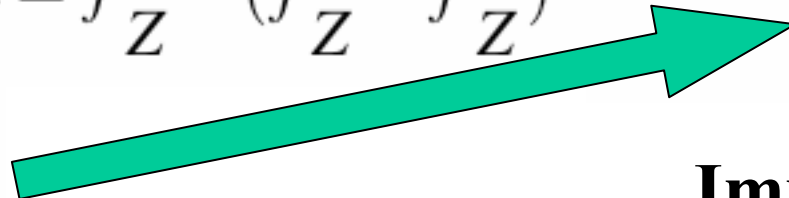
$$x_r = f \frac{X - T_x}{Z} \quad y_r = f \frac{Y}{Z}$$



Stereo Disparity

$$d = x_l - x_r = f \frac{X}{Z} - \left(f \frac{X}{Z} - f \frac{T_x}{Z} \right)$$

$$d = \frac{f T_x}{Z}$$



depth

$$Z = \frac{f T_x}{d}$$

baseline

disparity

Important equation!

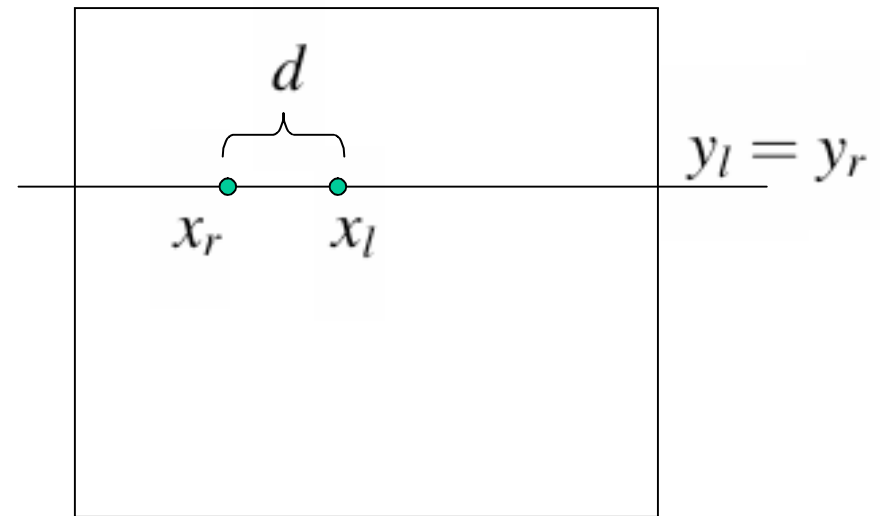
Recall: Stereo Disparity

Left camera

$$x_l = f \frac{X}{Z} \quad y_l = f \frac{Y}{Z}$$

Right camera

$$x_r = f \frac{X - T_x}{Z} \quad y_r = f \frac{Y}{Z}$$



Note: Depth and stereo disparity are inversely proportional

depth

$$Z = \frac{f T_x}{d}$$

disparity

Important equation!

Stereo Example



Left Image



Right Image

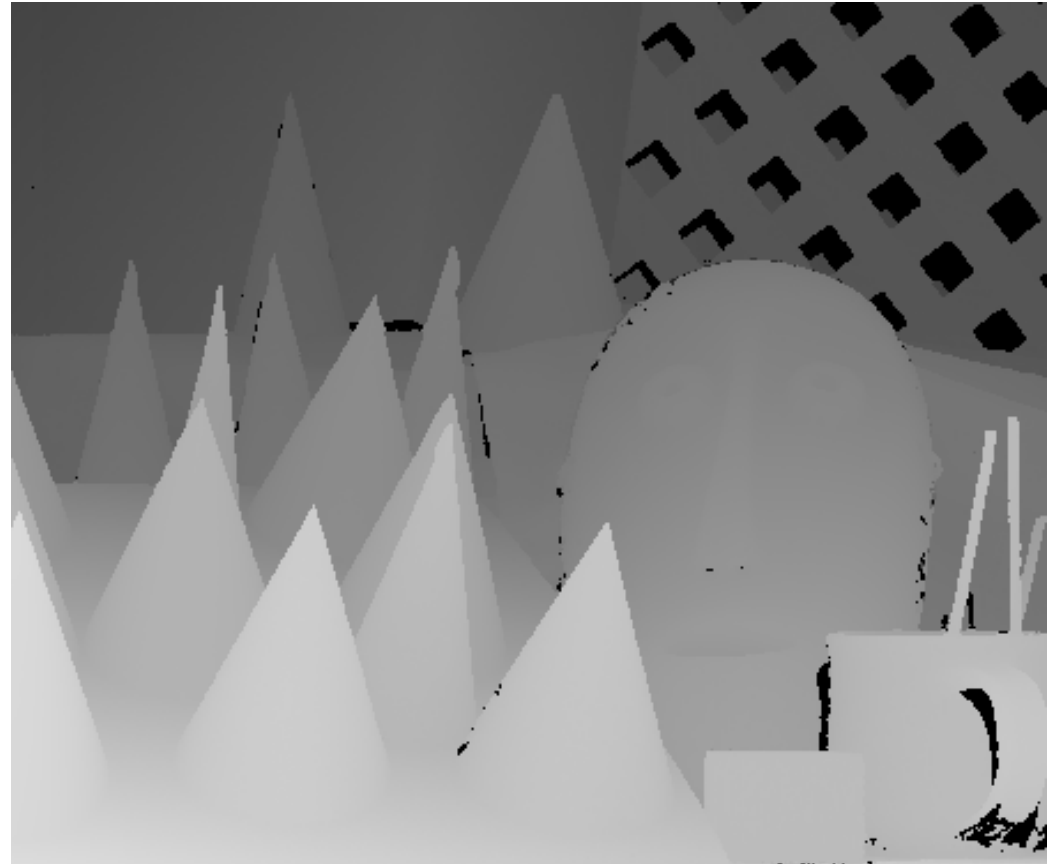
From Middlebury stereo evaluation page

<http://www.middlebury.edu/stereo/>

Stereo Example



Disparity values (0-64)



Note how disparity is larger (brighter) for closer surfaces.

Computing Disparity

- Correspondence Problem:
 - Determining which pixel in the right image corresponds to each pixel in the left image.
 - $\text{Disp} = x_coord(\text{left}) - x_coord(\text{right})$

Recall our discussion of computing correspondences of image patches (Lecture 7).

SSD - sum of squared difference measure

NCC - normalized cross correlation measure

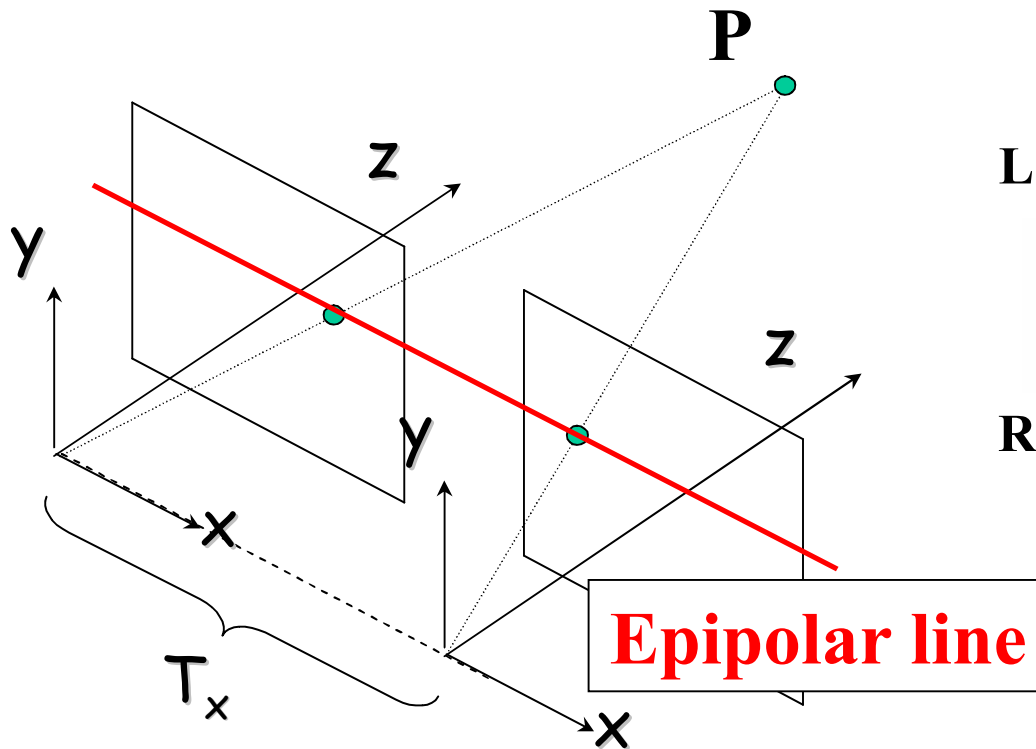
Epipolar Constraint

Important Concept:

For stereo matching, we don't have to search the whole 2D right image for a corresponding point.

The “epipolar constraint” reduces the search space to a one-dimensional line.

Recall : Simple Stereo System



Left camera

$$x_l = f \frac{X}{Z}$$

$$y_l = f \frac{Y}{Z}$$

Right camera

$$x_r = f \frac{X - T_x}{Z}$$

$$y_r = f \frac{Y}{Z}$$

Same Y Coord!

Matching using Epipolar Lines

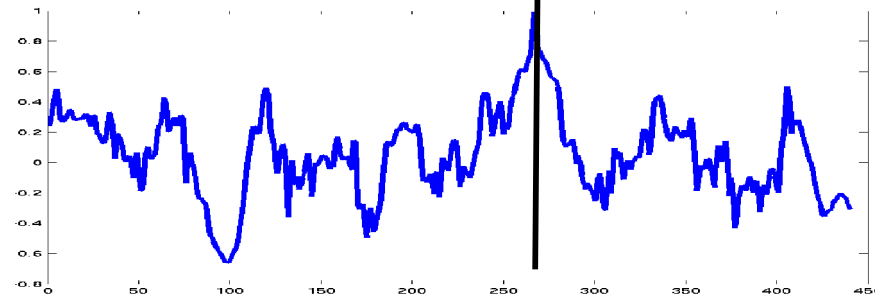
Left Image



Right Image



For a patch in left image
Compare with patches along
same row in right image



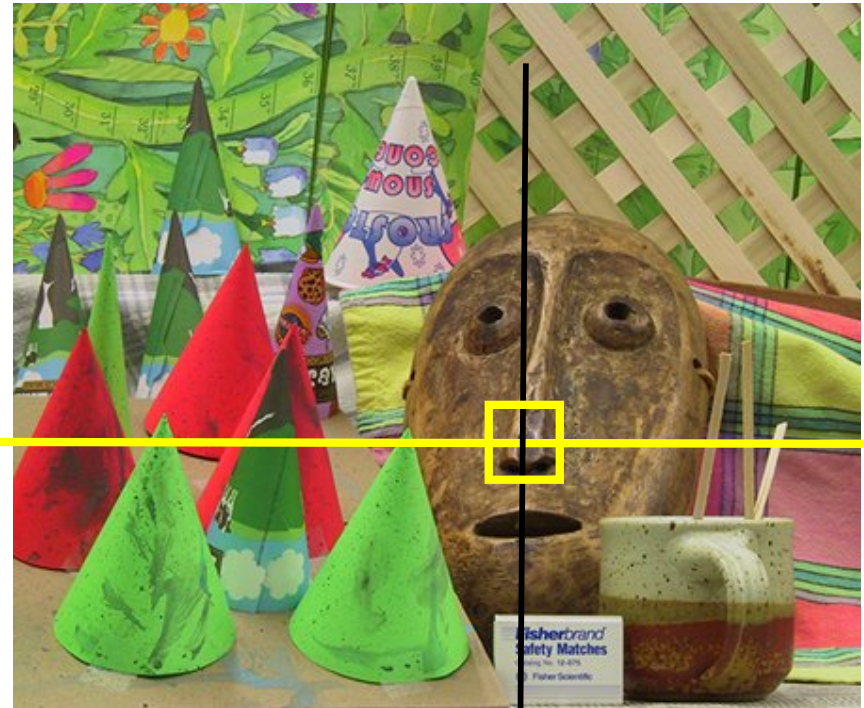
Match Score Values

Matching using Epipolar Lines

Left Image

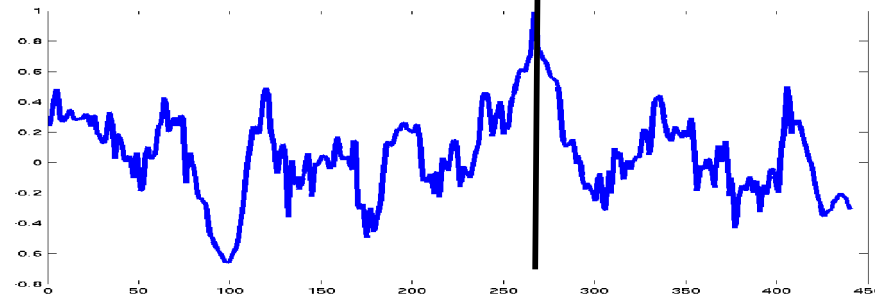


Right Image



Select patch with highest
match score.

Repeat for all pixels in
left image.

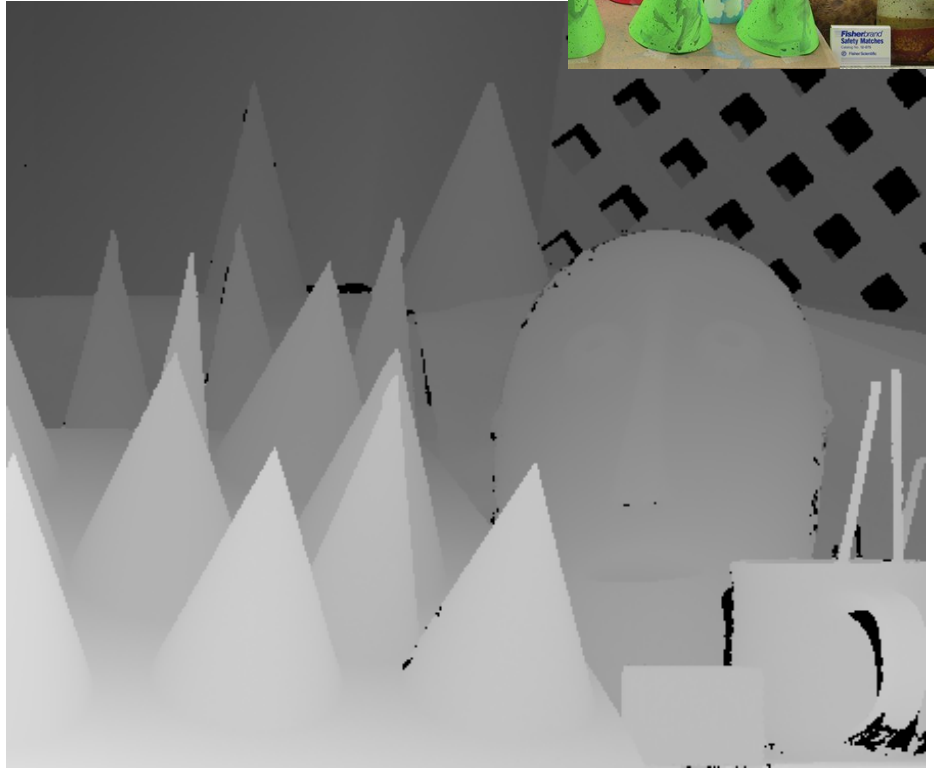


Match Score Values

Example: 5x5 windows NCC match score



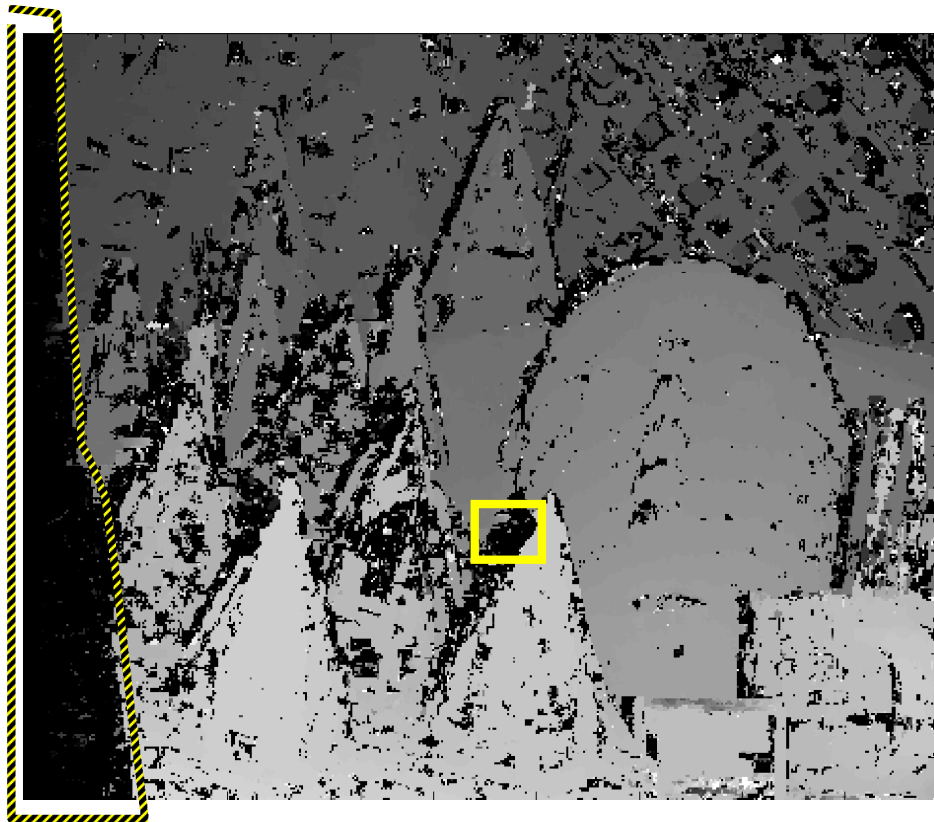
Computed disparities



Ground truth

Black pixels: bad disparity values,
or no matching patch in right image

Occlusions: No matches

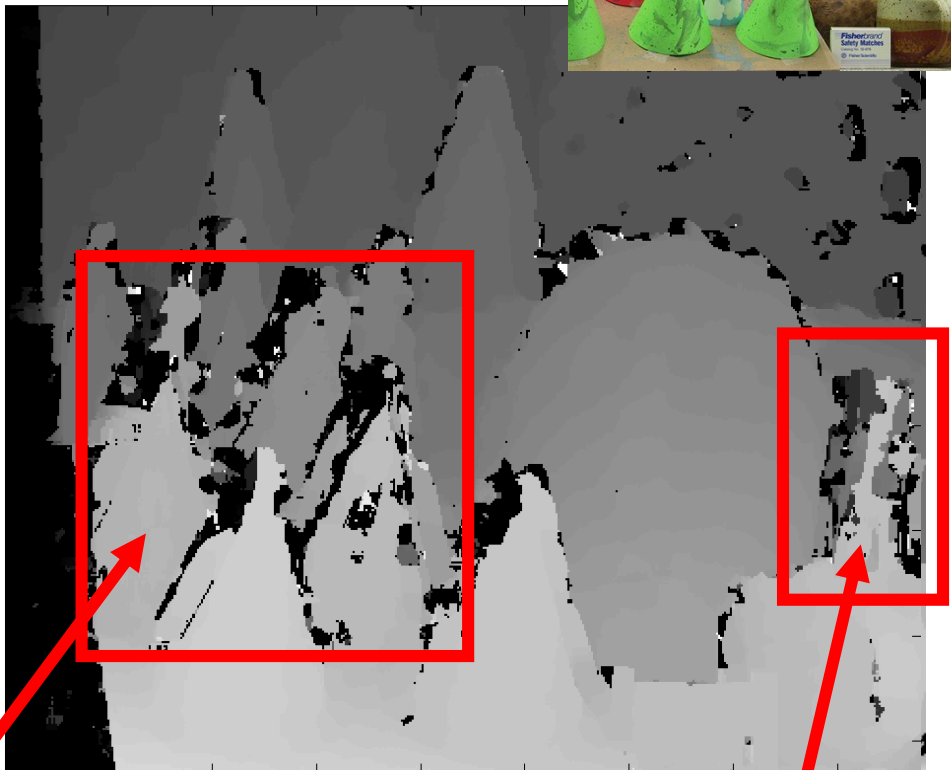


Effects of Patch Size



5x5 patches

Smoother in some areas



11x11 patches

Loss of finer details

Adding Inter-Scanline Consistency

So far, each left image patch has been matched independently along the right epipolar line.

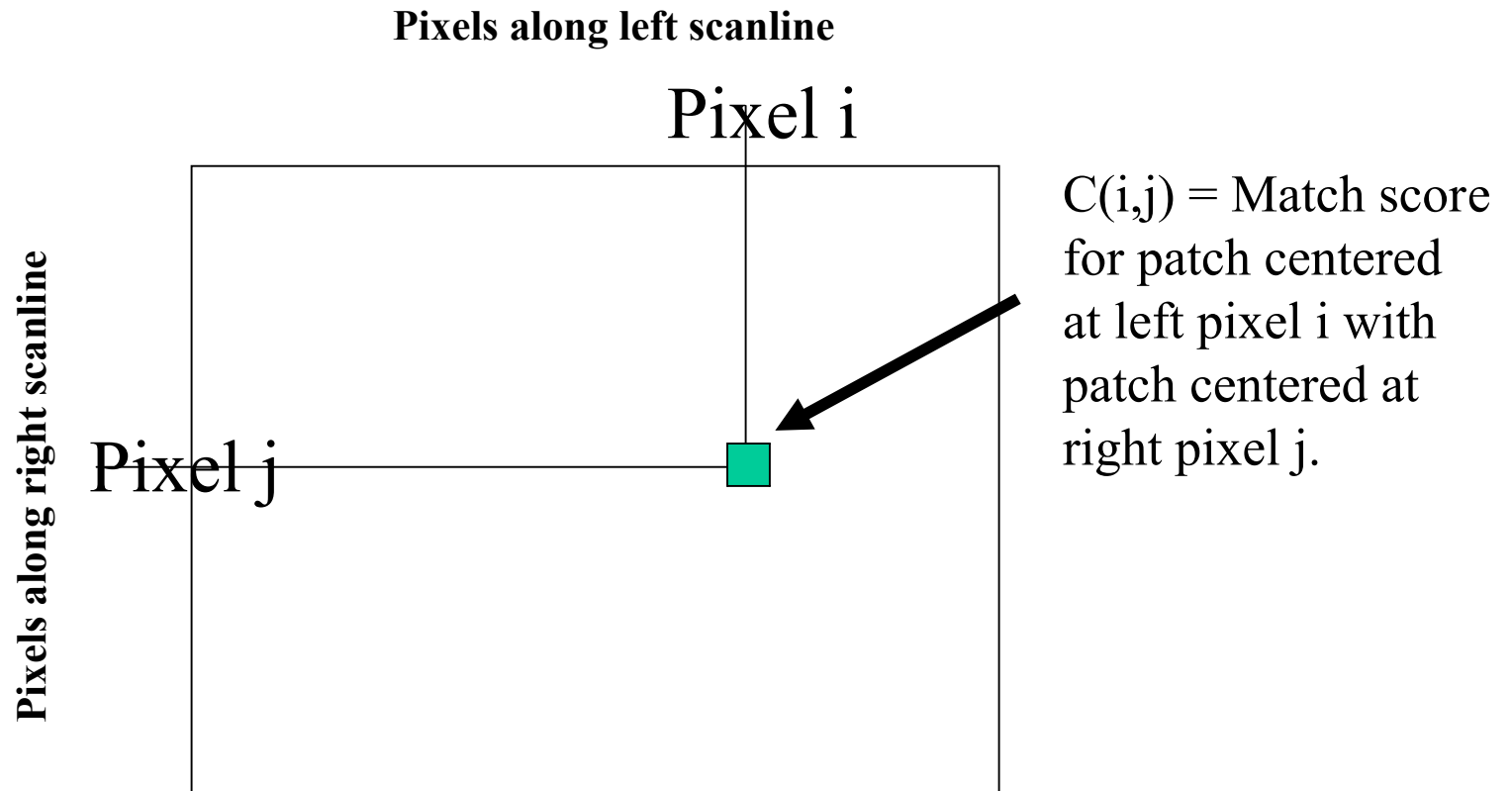
This can lead to errors.

We would like to enforce some consistency among matches in the same row (scanline).

Disparity Space Image

First we introduce the concept of DSI.

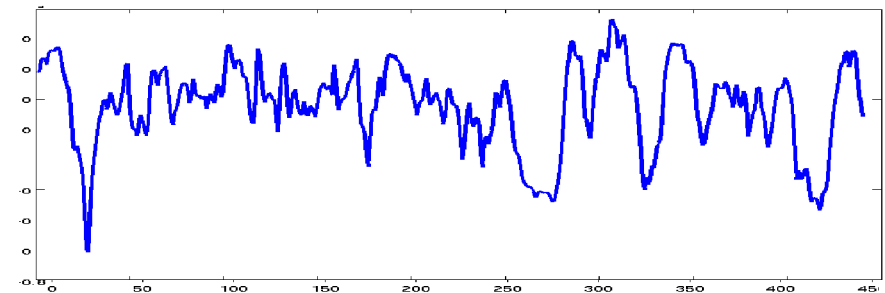
The DSI for one row represents pairwise match scores between patches along that row in the left and right image.



Disparity Space Image (DSI)

Left Image

Right Image

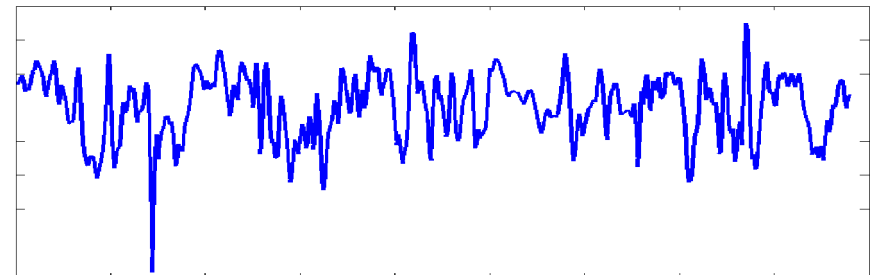


Dissimilarity Values
(1-NCC) or SSD

Disparity Space Image (DSI)

Left Image

Right Image



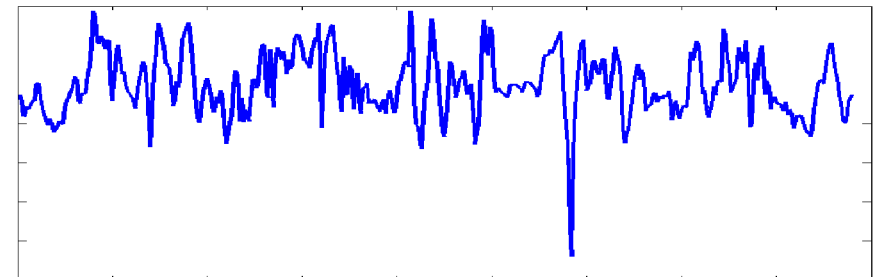
Dissimilarity Values
(1-NCC) or SSD

Disparity Space Image (DSI)

Left Image



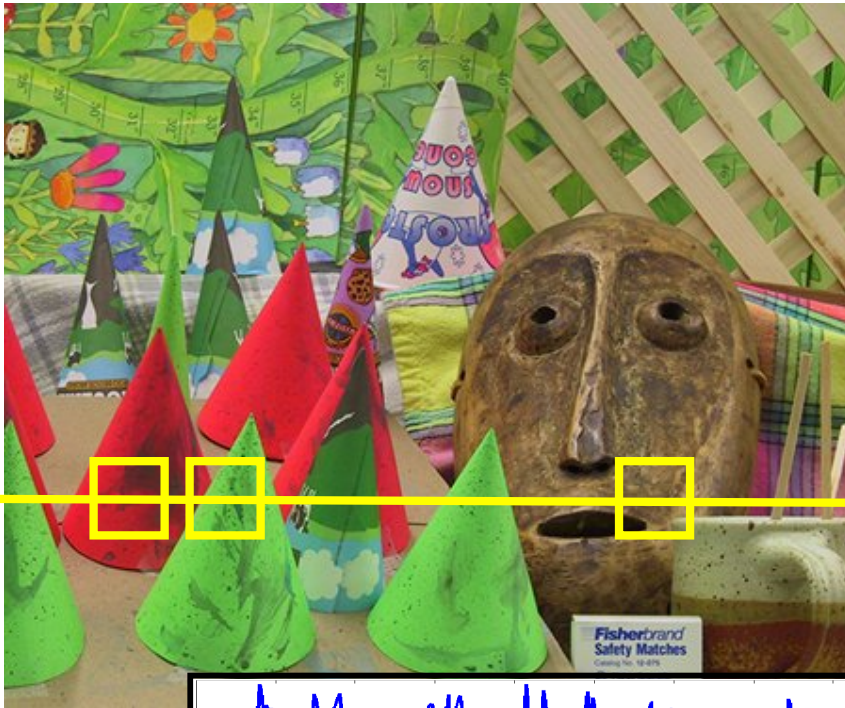
Right Image



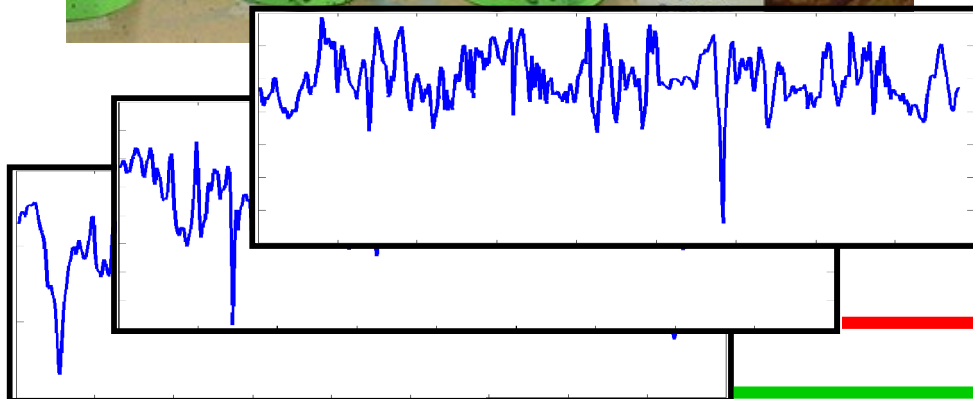
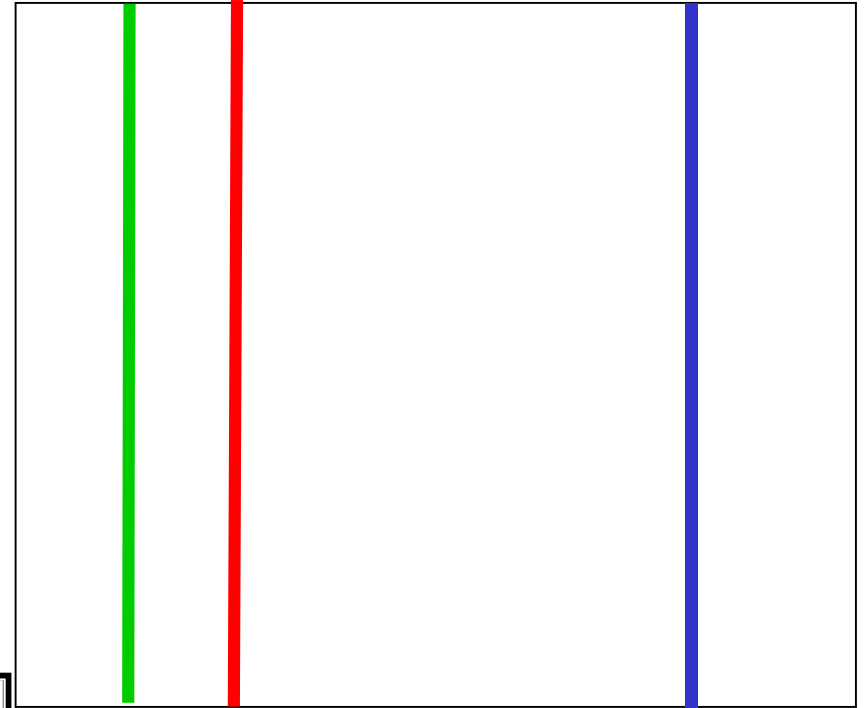
Dissimilarity Values
(1-NCC) or SSD

Disparity Space Image (DSI)

Left Image



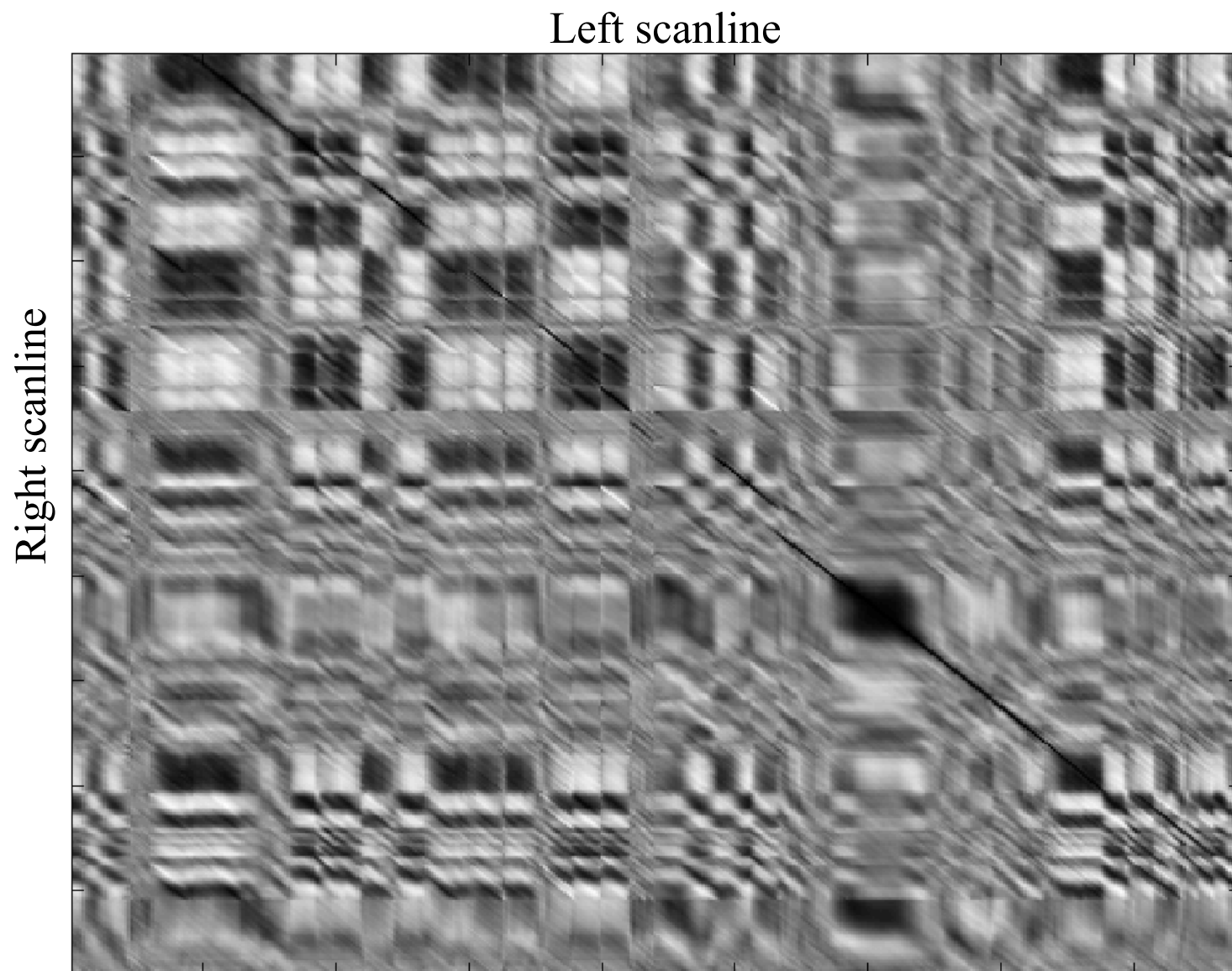
DSI



Dissimilarity Values

Enter each vector of
match scores as a
column in the DSI

Disparity Space Image



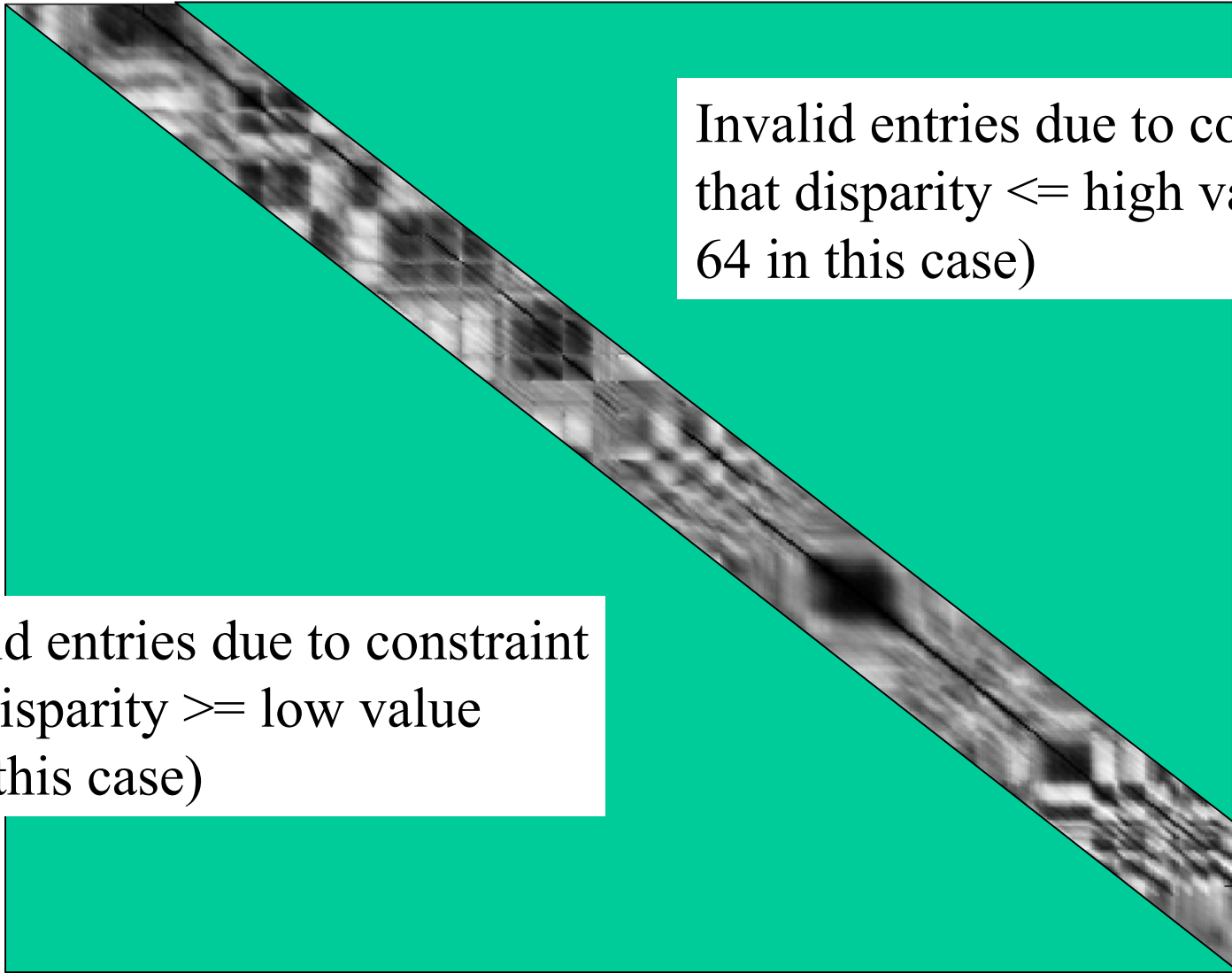
Disparity Space Image

Left scanline

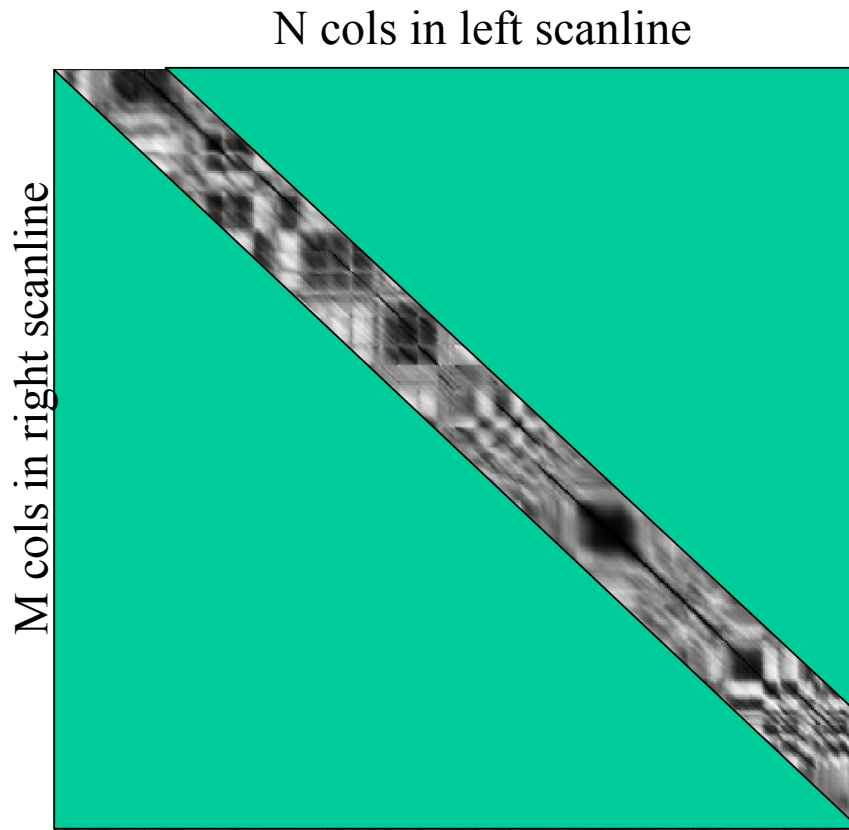
Right scanline

Invalid entries due to constraint
that disparity \leq high value
64 in this case)

Invalid entries due to constraint
that disparity \geq low value
(0 in this case)



Disparity Space Image



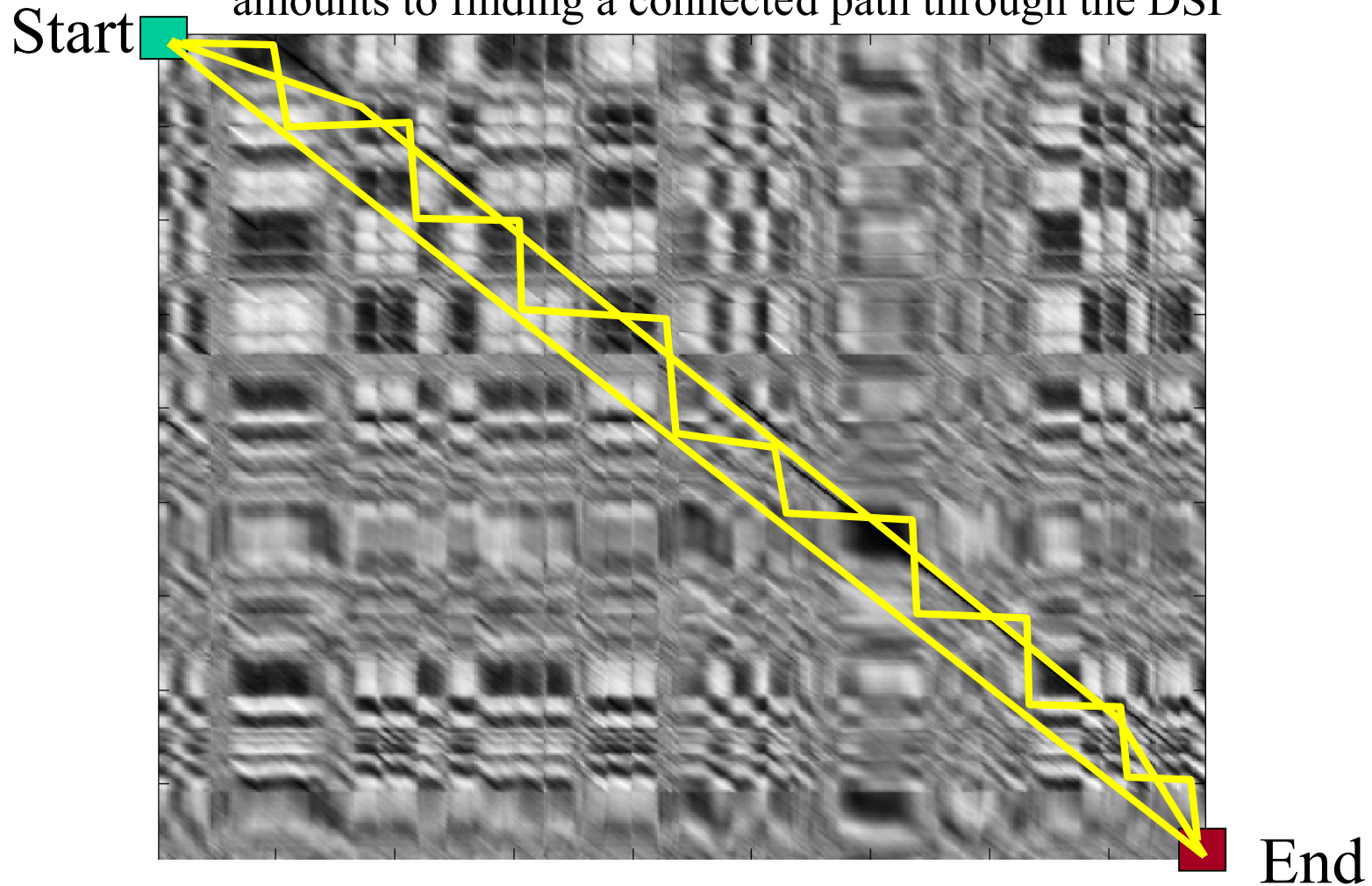
If we rearrange the diagonal band of valid values into a rectangular array (in this case of size $64 \times N$), that is what is traditionally known as the DSI

However, I'm going to keep the full image around, including invalid values (I think it is easier to understand the pixel coordinates involved)



DSI and Scanline Consistency

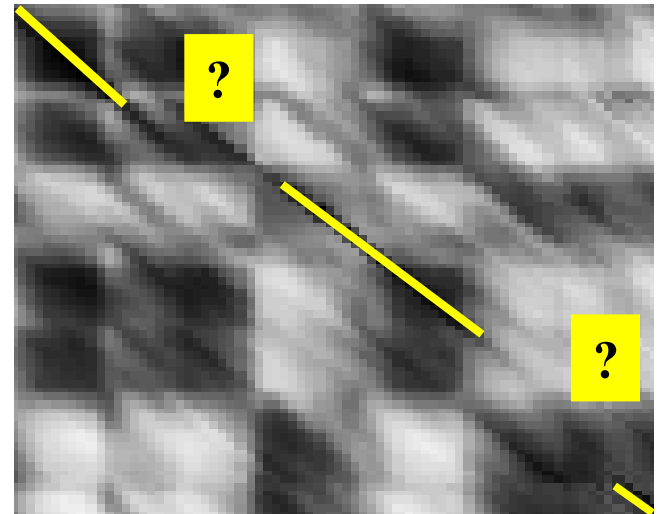
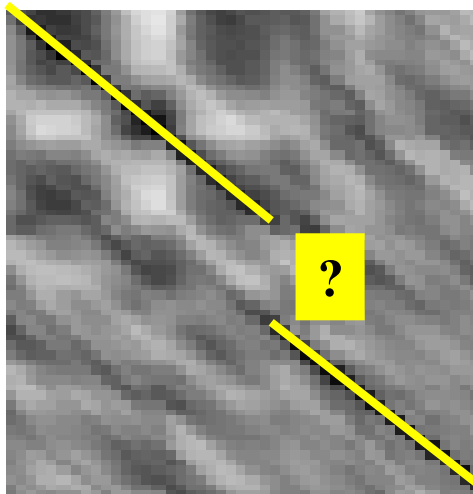
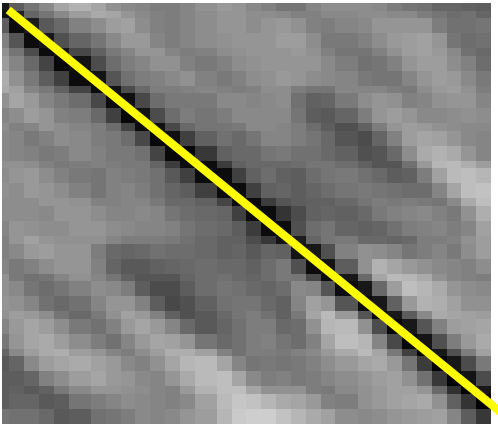
Assigning disparities to all pixels in left scanline now
amounts to finding a connected path through the DSI



Lowest Cost Path

We would like to choose the “best” path.

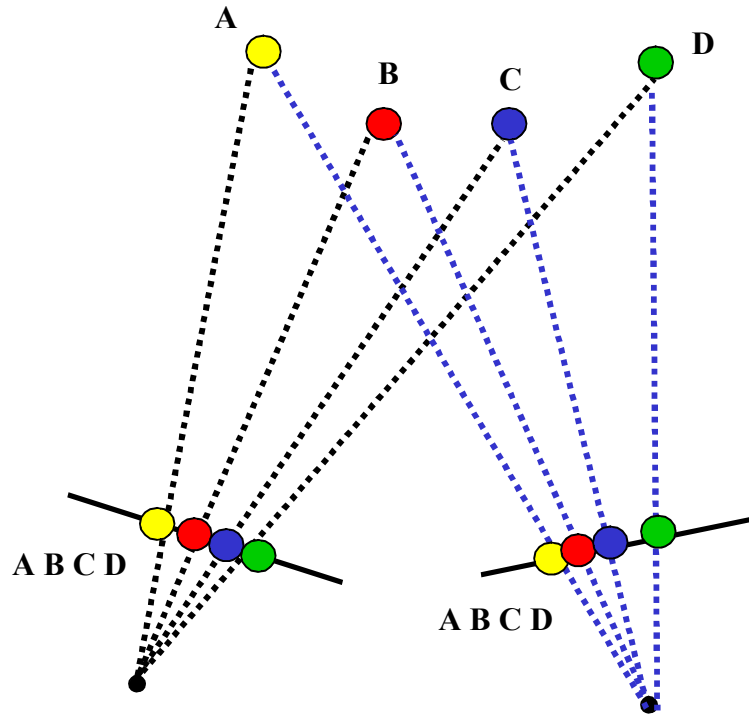
Want one with lowest “cost” (Lowest sum of dissimilarity scores along the path)



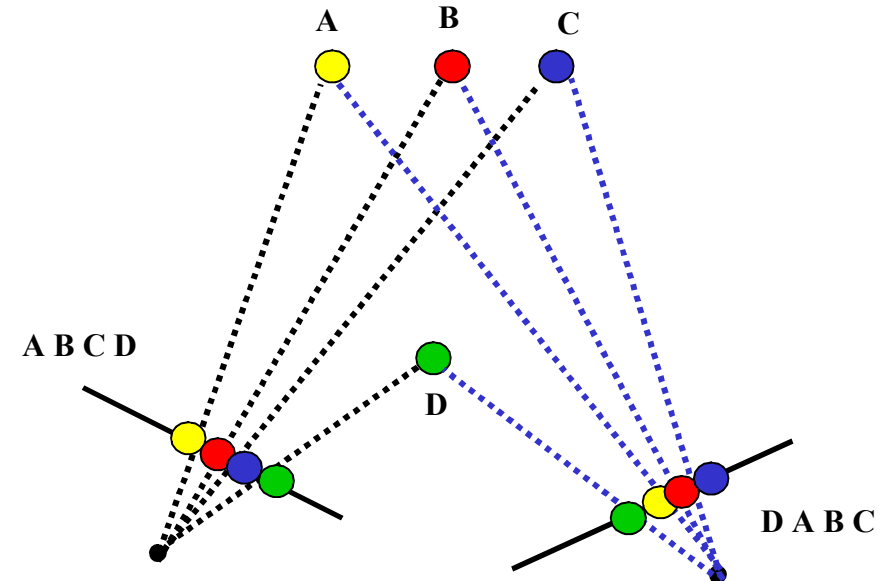
Constraints on Path

It is common to impose an ordering constraint on the path. Intuitively, the path is not allowed to “double back” on itself.

Ordering Constraint

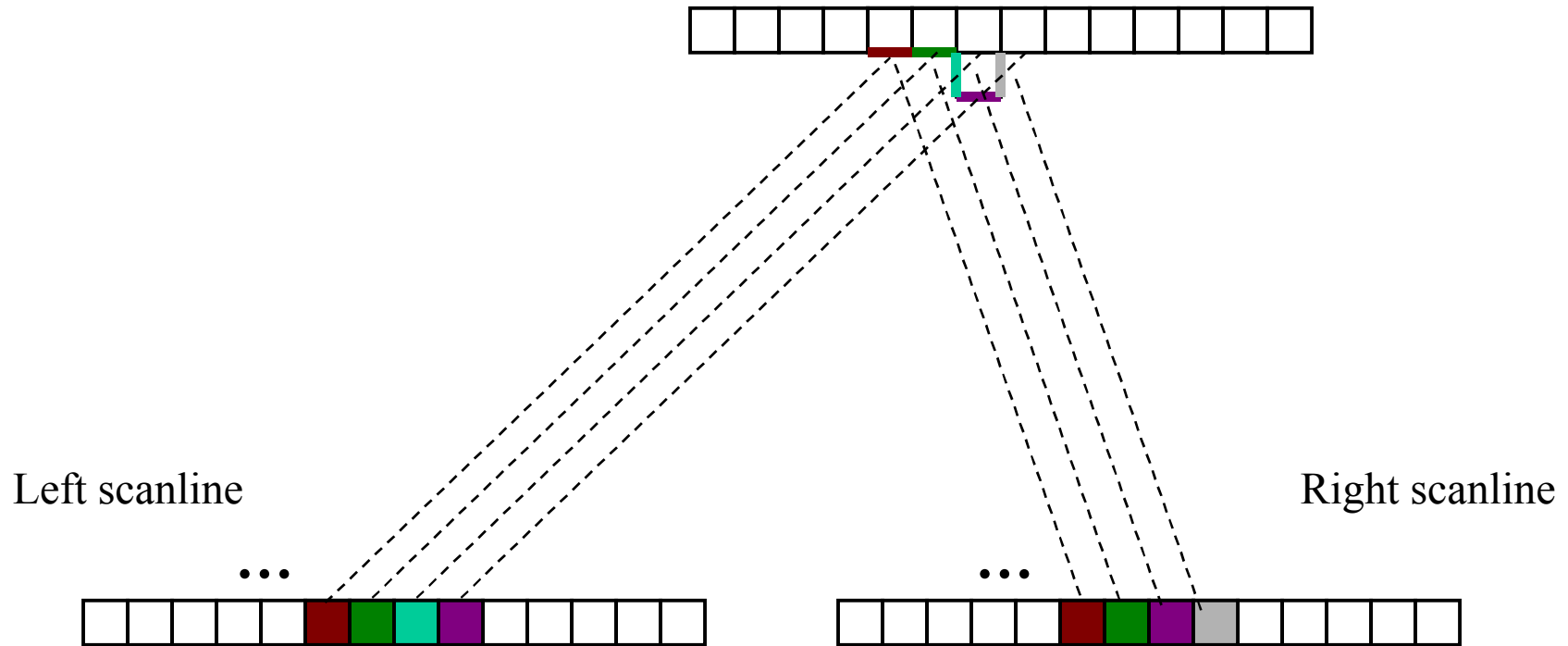


Ordering constraint...

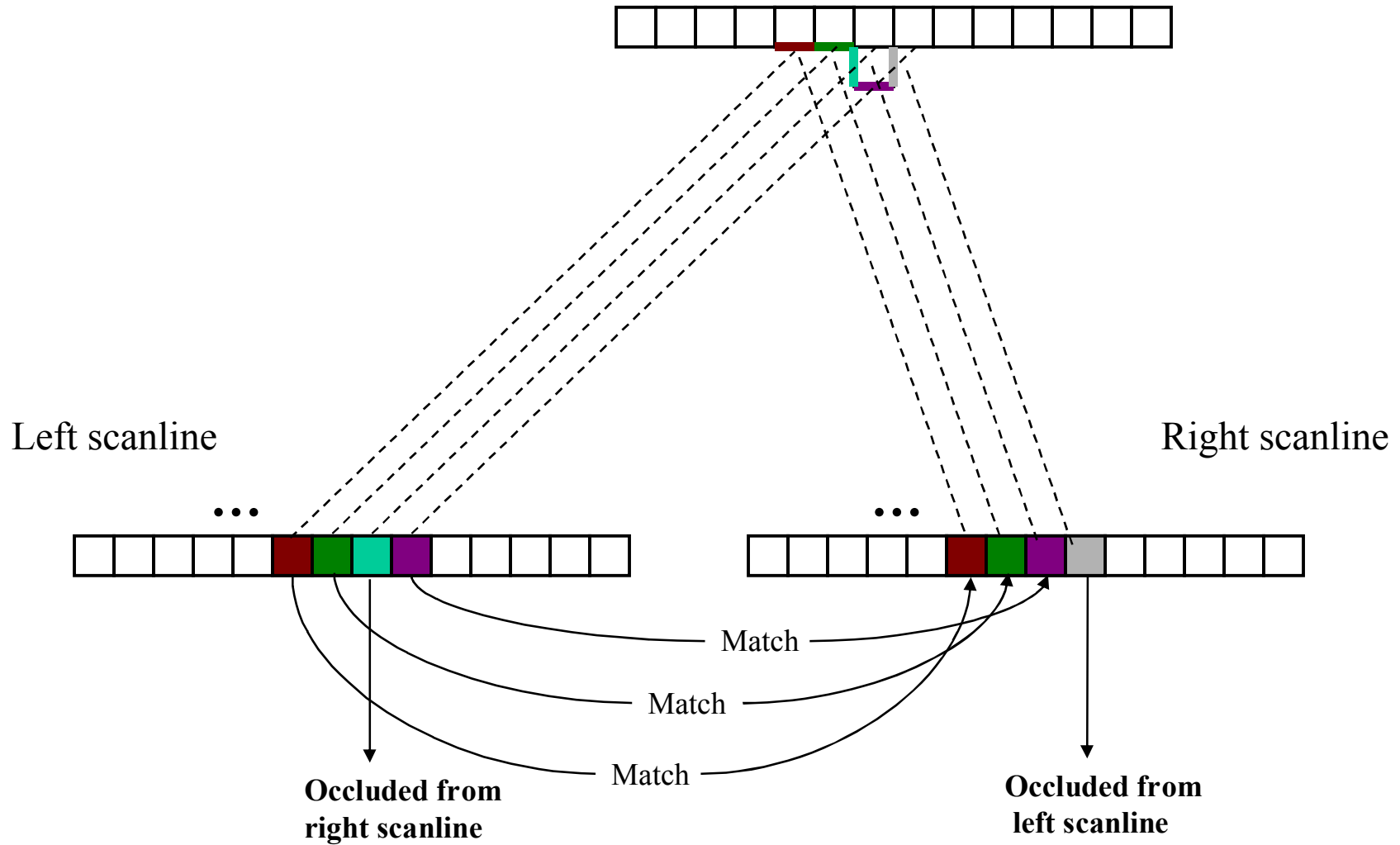


...and its failure

Dealing with Occlusions



Dealing with Occlusions

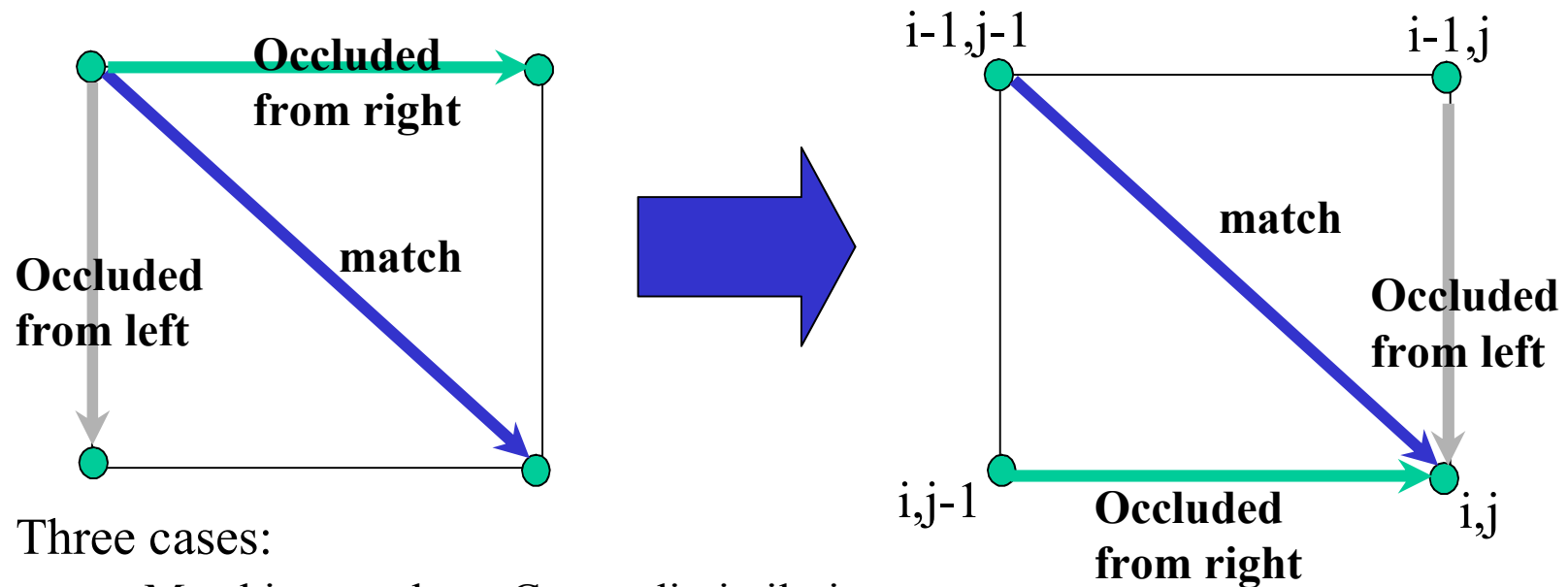


An Optimal Scanline Strategy

- We want to find best path, taking into account ordering constraint and the possibility of occlusions.

**Algorithm we will discuss now is from
Cox, Hingorani, Rao, Maggs, “A Maximum
Likelihood Stereo Algorithm,” Computer
Vision and Image Understanding, Vol 63(3),
May 1996, pp.542-567.**

Cox et.al. Stereo Matching



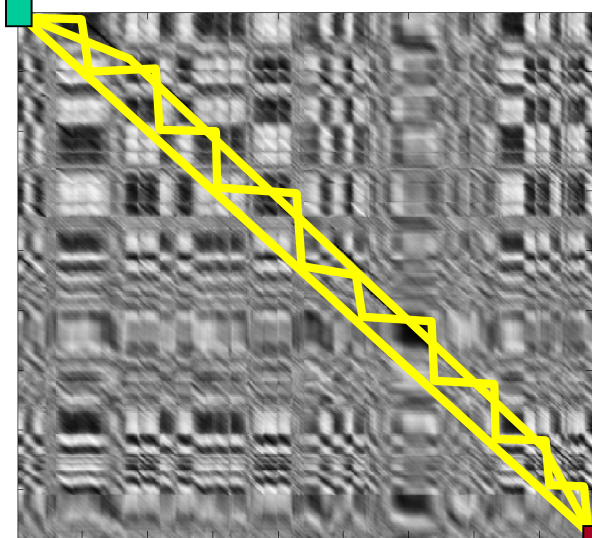
Three cases:

- Matching patches. Cost = dissimilarity score
- Occluded from right. Cost is some constant value.
- Occluded from left. Cost is some constant value.

$$C(i,j) = \min([C(i-1,j-1) + \text{dissimilarity}(i,j), \\ C(i-1,j) + \text{occlusionConstant}, \\ C(i,j-1) + \text{occlusionConstant}]);$$

Cox et.al. Stereo Matching

Start



End

Recap: want to find lowest cost path from upper left to lower right of DSI image.

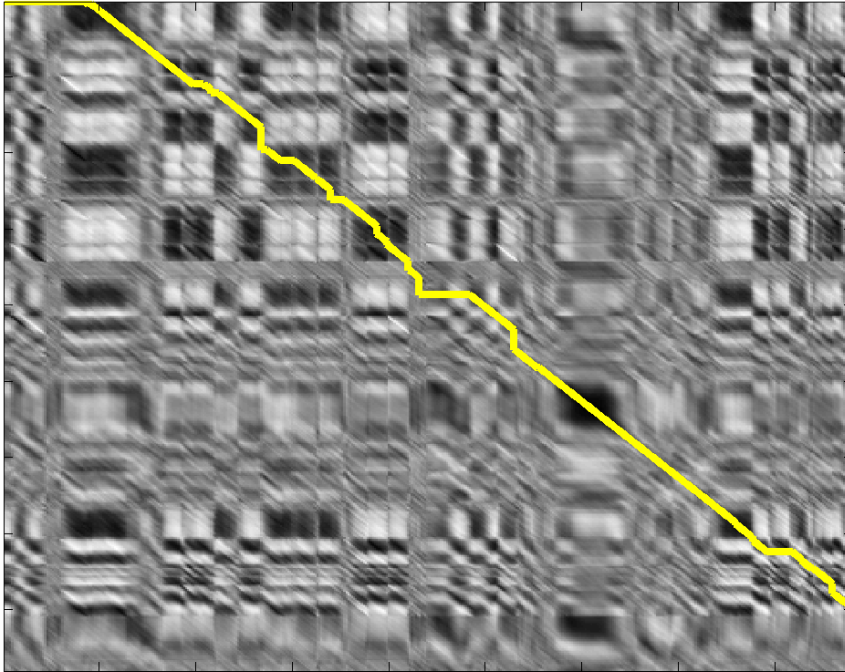
At each point on the path we have three choices: step left, step down, step diagonally.

Each choice has a well-defined cost associated with it.

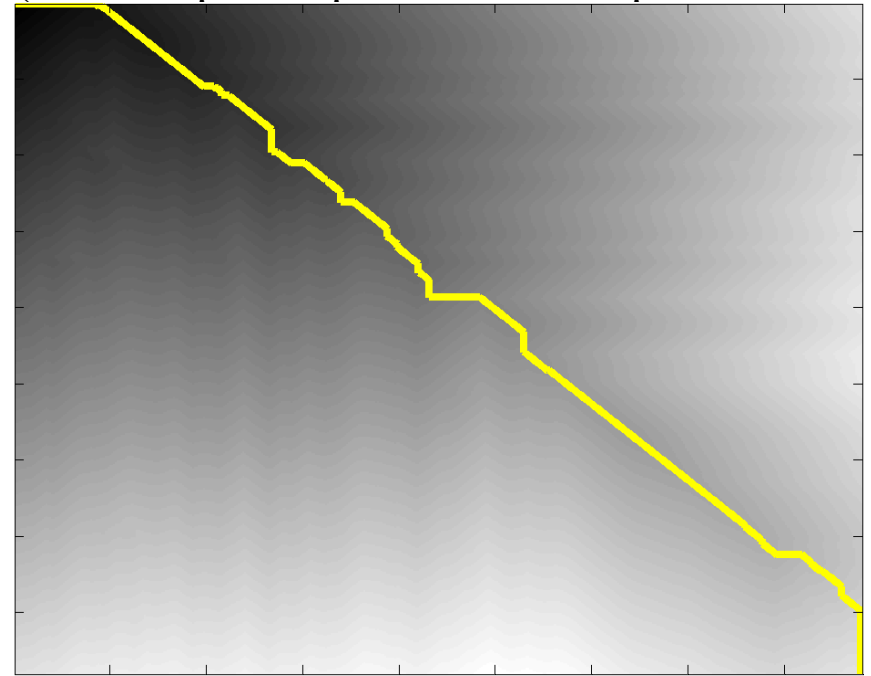
This problem just screams out for Dynamic Programming!
(which, indeed, is how Cox et.al. solve the problem)

Real Scanline Example

DSI



DP cost matrix
(cost of optimal path from each point to END)

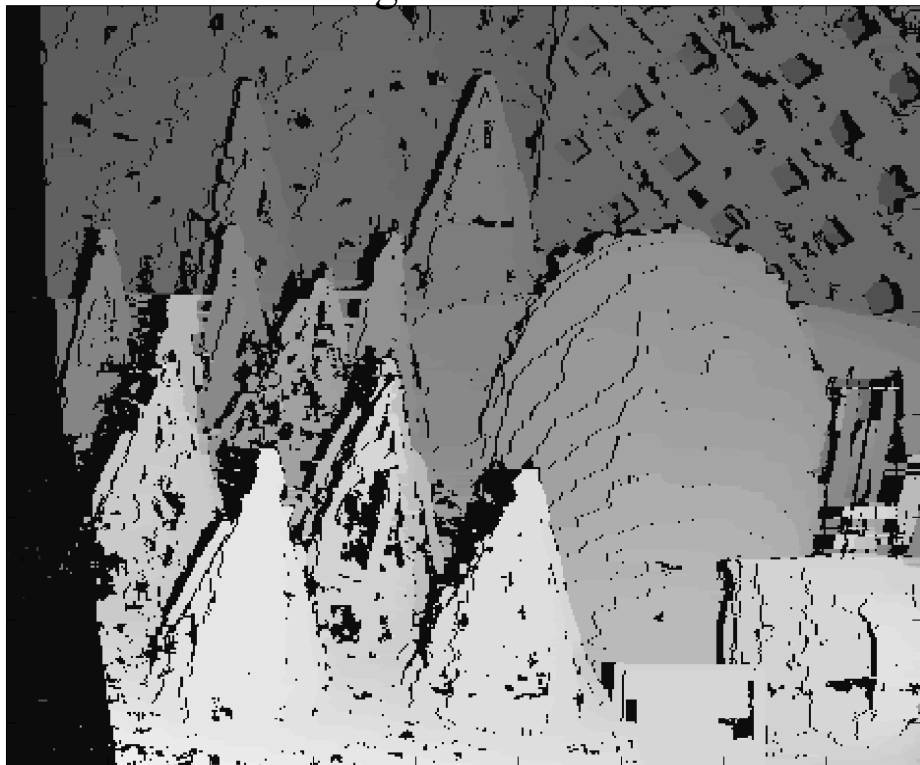


Every pixel in left column now is marked with either a disparity value, or an occlusion label.

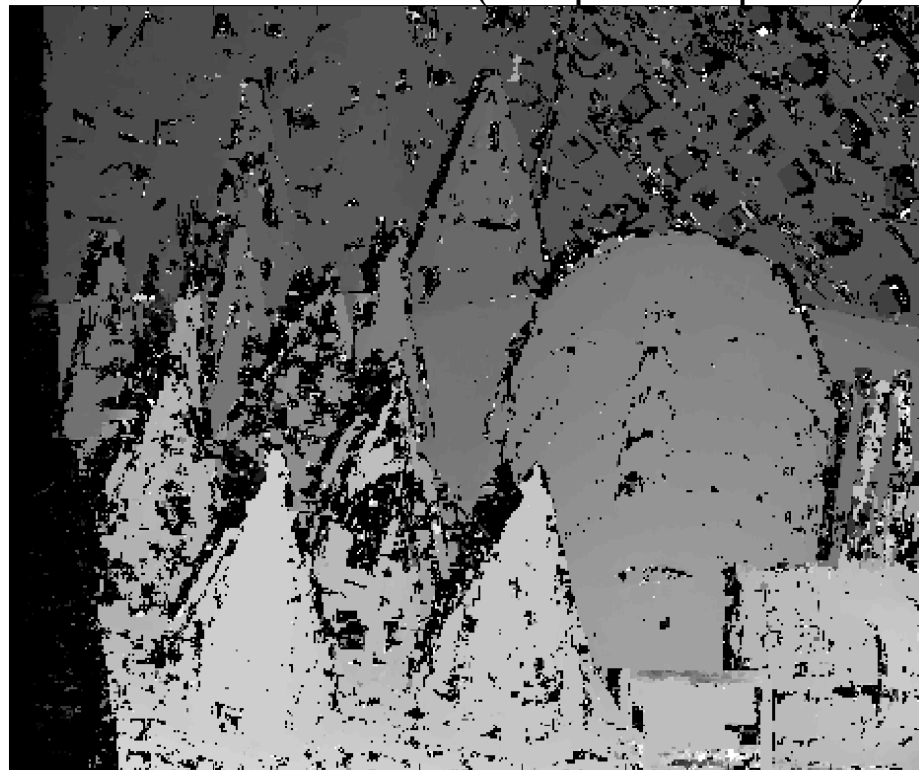
Proceed for every scanline in left image.

Example

Result of DP alg



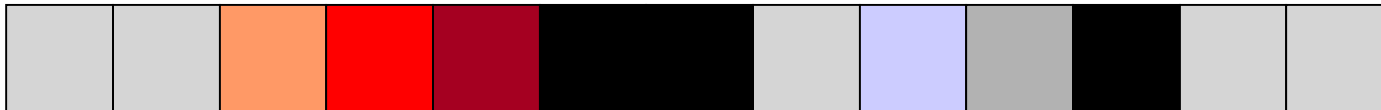
Result without DP (independent pixels)



Result of DP alg. Black pixels = occluded.

Occlusion Filling

Simple trick for filling in gaps caused by occlusion.



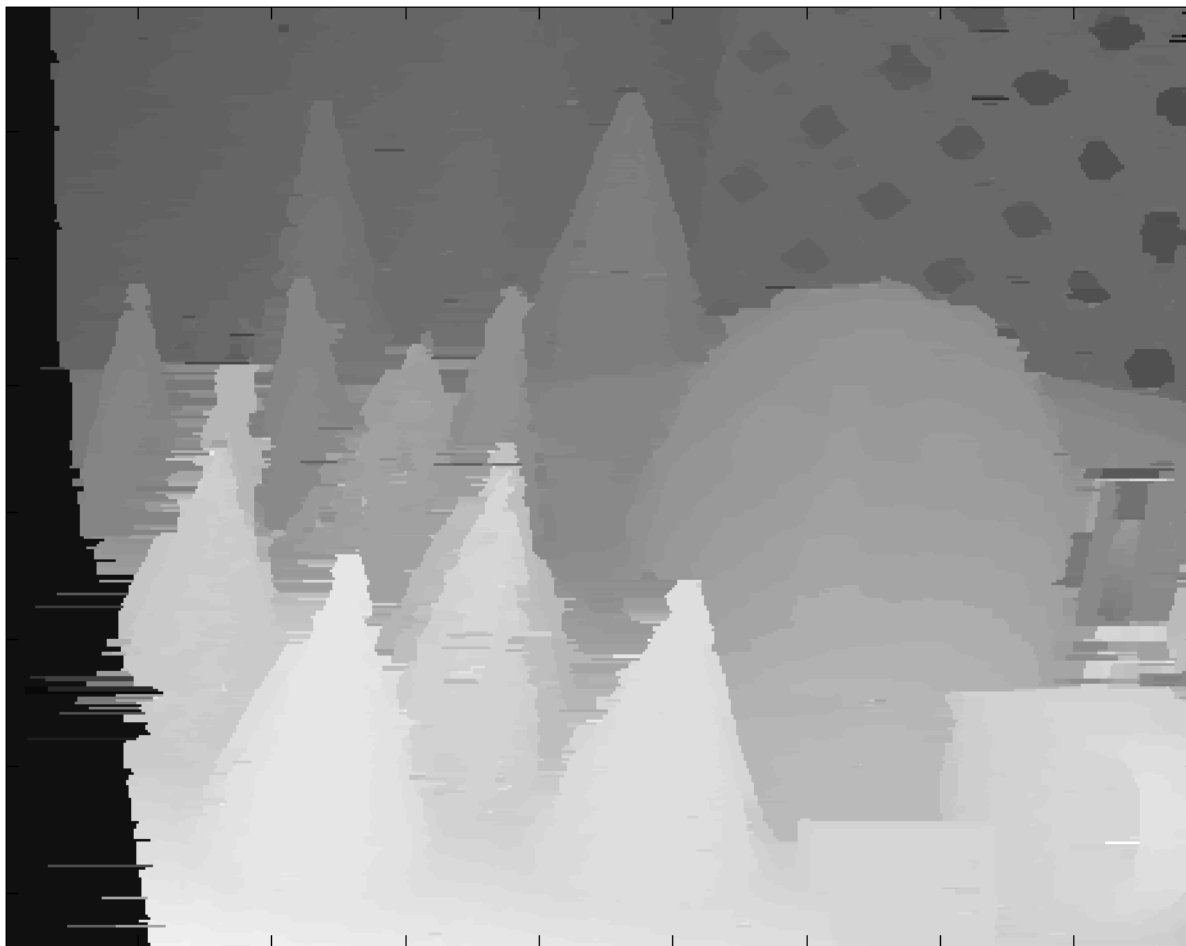
 = left occluded

Fill in left occluded pixels with value from the nearest valid pixel preceding it in the scanline.



Similarly, for right occluded, look for valid pixel to the right.

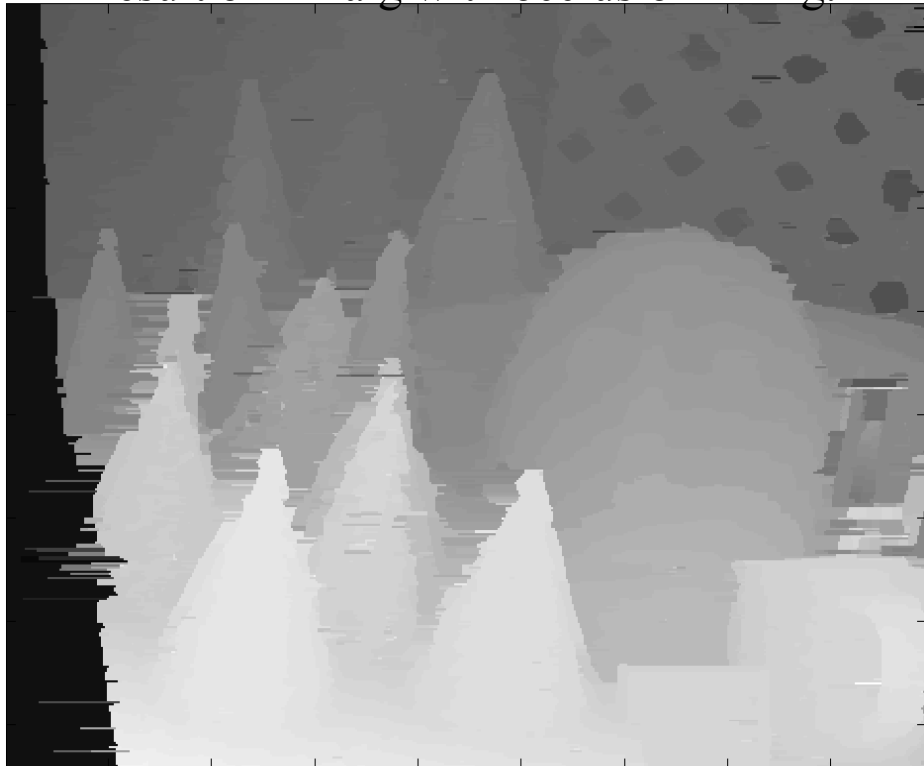
Example



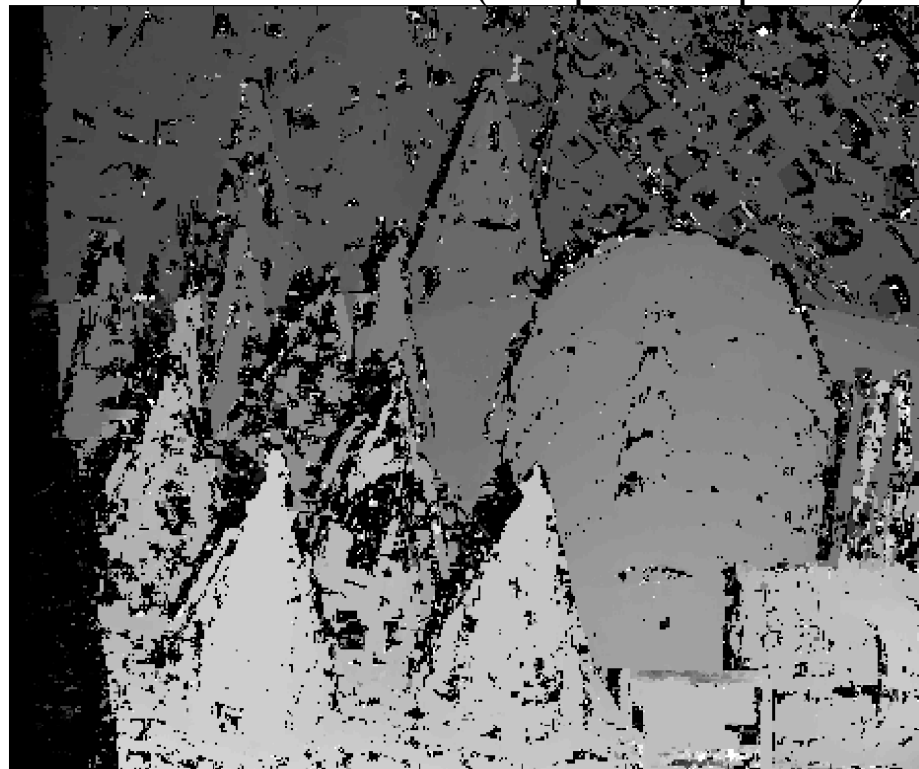
Result of DP alg with occlusion filling.

Example

Result of DP alg with occlusion filling.

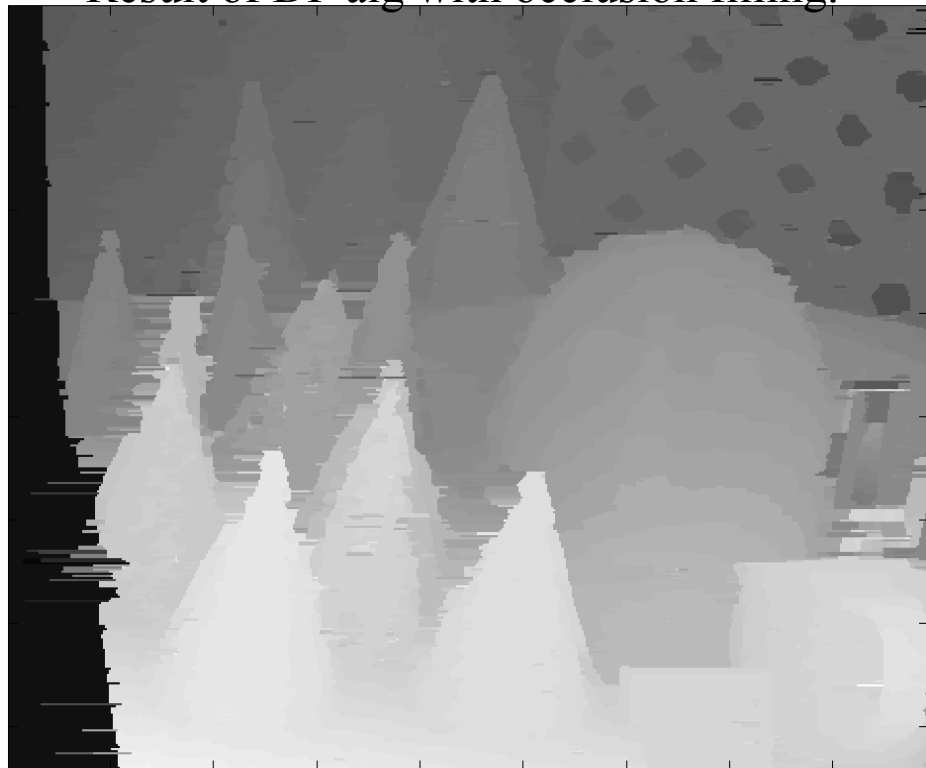


Result without DP (independent pixels)

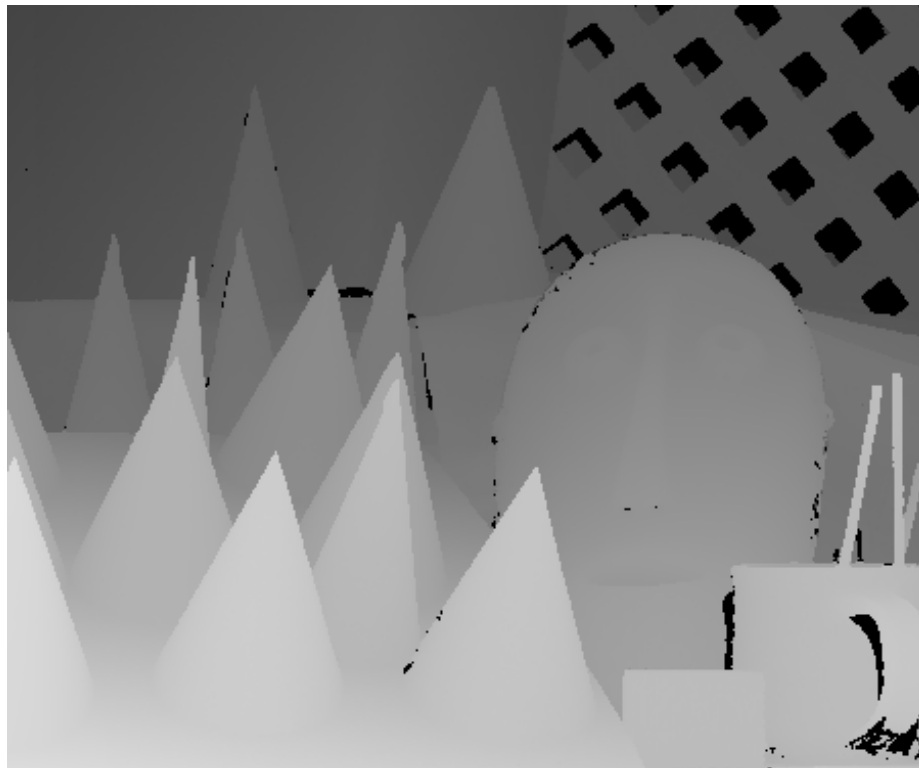


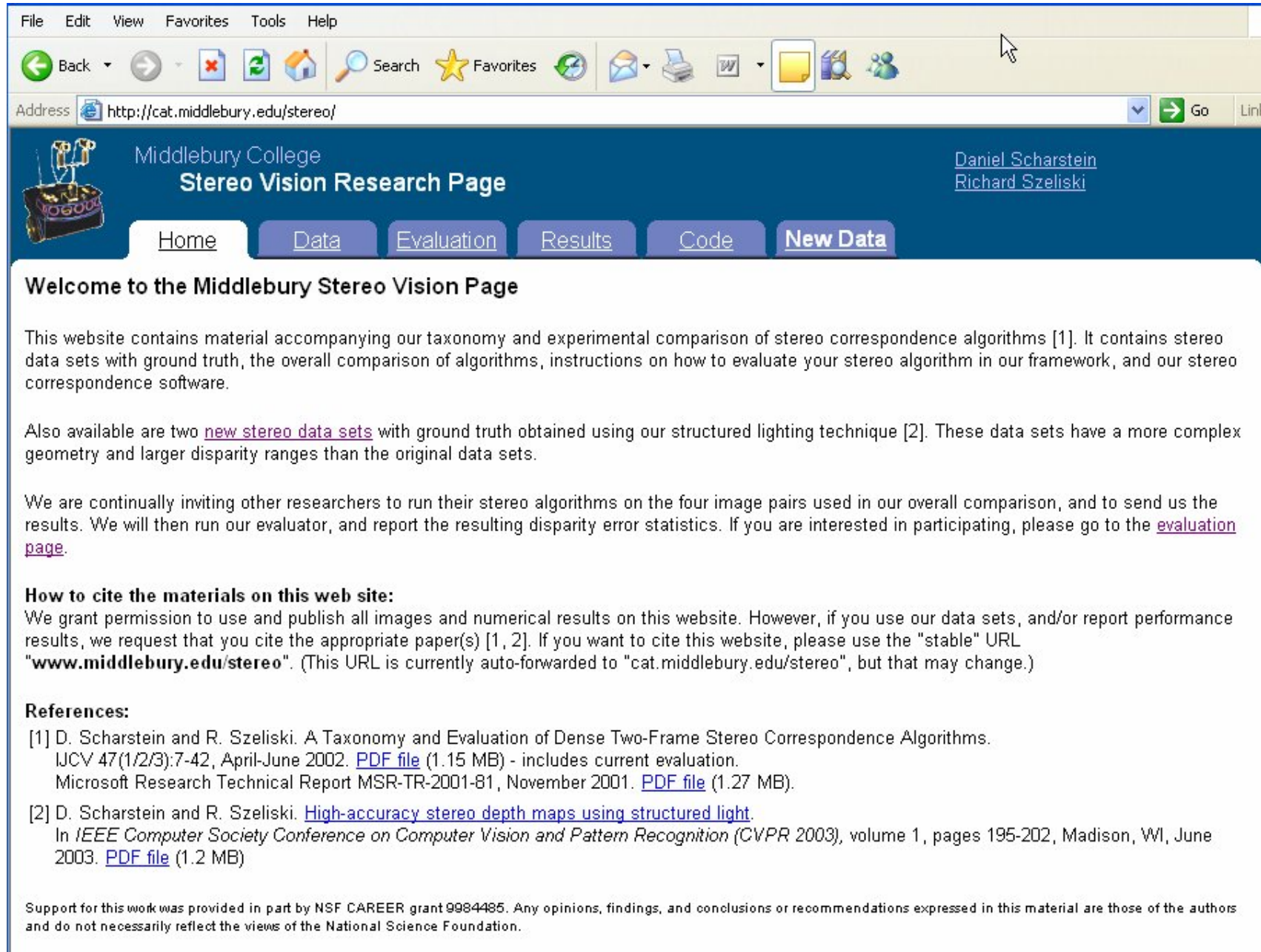
Example

Result of DP alg with occlusion filling.



Ground truth





The screenshot shows a web browser window with the address bar displaying `http://cat.middlebury.edu/stereo/`. The page header includes the Middlebury College logo and the title "Stereo Vision Research Page". Navigation buttons for "Home", "Data", "Evaluation", "Results", "Code", and "New Data" are visible. The main content area welcomes visitors to the Middlebury Stereo Vision Page and describes the website's purpose: providing material for taxonomy and experimental comparison of stereo correspondence algorithms. It also mentions the availability of new stereo data sets and invites researchers to participate in evaluations. A section titled "How to cite the materials on this web site:" provides guidelines for citing the website and its data sets. A "References:" section lists two papers by D. Scharstein and R. Szeliski. At the bottom, a disclaimer states that support for the work was provided by NSF CAREER grant 9984485 and that the opinions expressed are those of the authors.

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Address <http://cat.middlebury.edu/stereo/> Go Link

Middlebury College
Stereo Vision Research Page

[Daniel Scharstein](#)
[Richard Szeliski](#)

[Home](#) [Data](#) [Evaluation](#) [Results](#) [Code](#) [New Data](#)

Welcome to the Middlebury Stereo Vision Page

This website contains material accompanying our taxonomy and experimental comparison of stereo correspondence algorithms [1]. It contains stereo data sets with ground truth, the overall comparison of algorithms, instructions on how to evaluate your stereo algorithm in our framework, and our stereo correspondence software.

Also available are two [new stereo data sets](#) with ground truth obtained using our structured lighting technique [2]. These data sets have a more complex geometry and larger disparity ranges than the original data sets.

We are continually inviting other researchers to run their stereo algorithms on the four image pairs used in our overall comparison, and to send us the results. We will then run our evaluator, and report the resulting disparity error statistics. If you are interested in participating, please go to the [evaluation page](#).

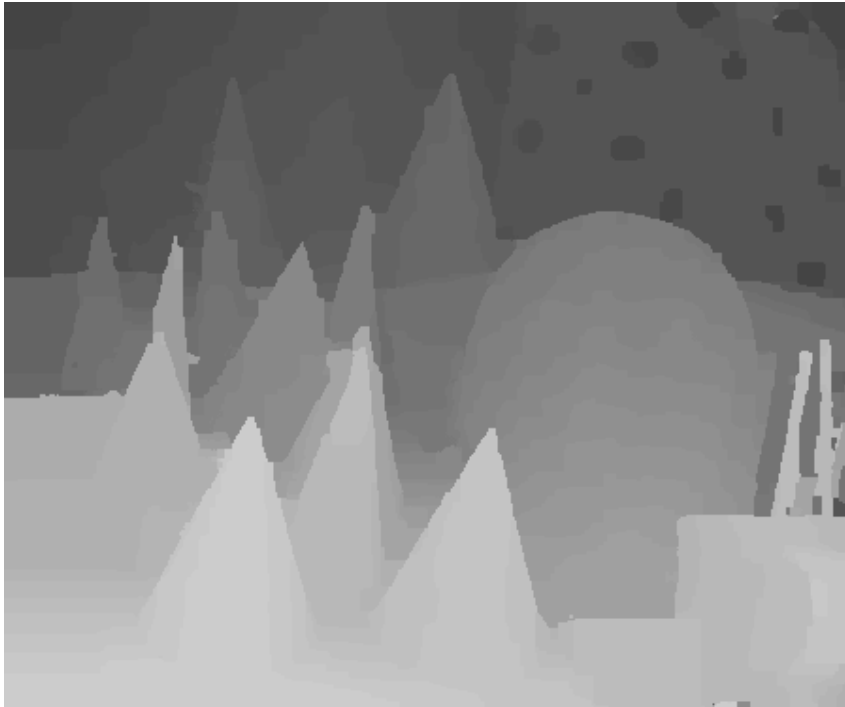
How to cite the materials on this web site:
We grant permission to use and publish all images and numerical results on this website. However, if you use our data sets, and/or report performance results, we request that you cite the appropriate paper(s) [1, 2]. If you want to cite this website, please use the "stable" URL "www.middlebury.edu/stereo/". (This URL is currently auto-forwarded to "cat.middlebury.edu/stereo", but that may change.)

References:

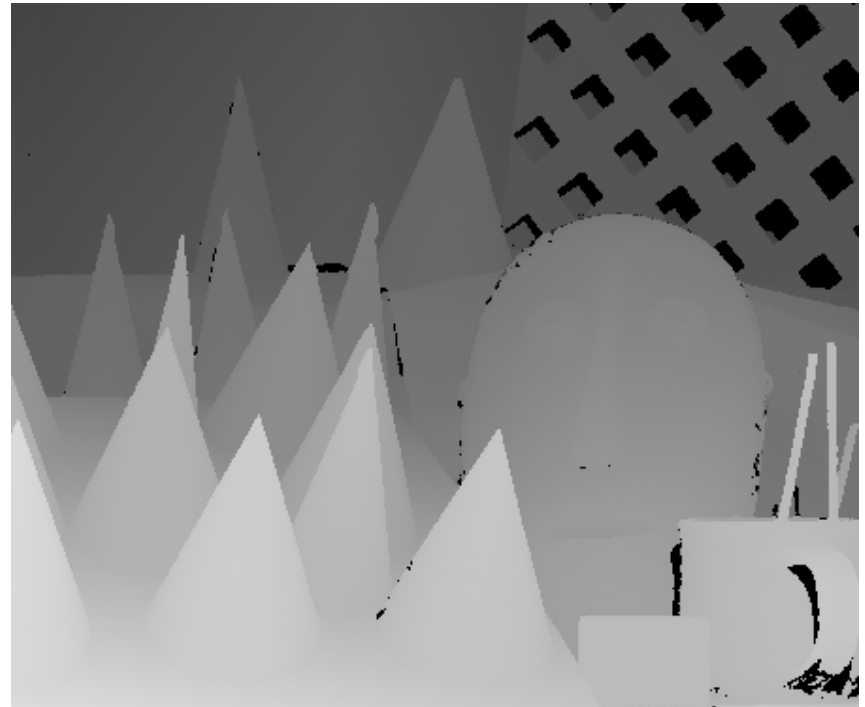
- [1] D. Scharstein and R. Szeliski. A Taxonomy and Evaluation of Dense Two-Frame Stereo Correspondence Algorithms. IJCV 47(1/2/3):7-42, April-June 2002. [PDF file](#) (1.15 MB) - includes current evaluation. Microsoft Research Technical Report MSR-TR-2001-81, November 2001. [PDF file](#) (1.27 MB).
- [2] D. Scharstein and R. Szeliski. [High-accuracy stereo depth maps using structured light](#). In *IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR 2003)*, volume 1, pages 195-202, Madison, WI, June 2003. [PDF file](#) (1.2 MB)

Support for this work was provided in part by NSF CAREER grant 9984485. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

State-of-the-Art Results



Algorithm Results



Ground truth

J. Sun, Y. Li, S.B. Kang, and H.-Y. Shum.
“Symmetric stereo matching for occlusion handling”.
IEEE Conference on Computer Vision and Pattern
Recognition, June 2005.