CS315: Principles of Database Systems
Query Optimization

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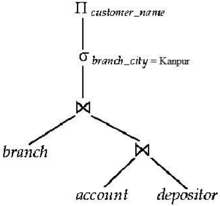
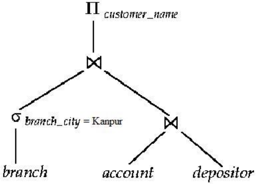
Tue, Fri 1530-1700 at CS101

Evaluation plan

- Equivalent expressions provide alternate ways of executing a query





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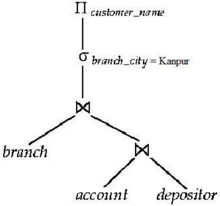
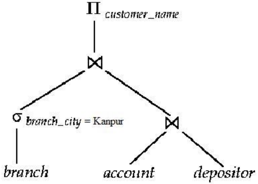
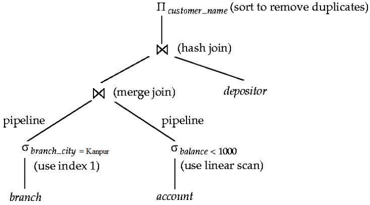




- An evaluation plan also specifies the algorithms



- Cost-based query optimization

Equivalent expressions

- Two relational algebra expressions are equivalent if they generate the same set of output tuples on every legal input relation
  - Order of tuples does not matter
  - For SQL, same multiset of output tuples
- Equivalence rule: specifies which expressions are equivalent
- Equivalent expressions are systematically generated by repeatedly applying equivalence rules and replacing one form by another
- Evaluation plans must account for all algorithms used
  - Merge join may be costlier than hash join, but since it provides a sorted output, a higher level aggregation will be faster

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11. $\sigma_{\theta_1 \wedge \theta_2}(E_1 \bowtie_{\theta} E_2) = (\sigma_{\theta_1}(E_1)) \bowtie_{\theta} (\sigma_{\theta_2}(E_2))$
    - θ_1 and θ_2 involve attributes from only E_1 and E_2 respectively

Equivalence rules (contd.)

12. $\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = \Pi_{L_1}(E_1) \bowtie_\theta \Pi_{L_2}(E_2)$
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18. $\Pi_L(E_1 \cup E_2) = \Pi_L(E_1) \cup \Pi_L(E_2)$

Banking example

- branch(bname, bcity, assets)
- customer(cname, cstreet, ccity)
- account(ano, bname, bal)
- loan(lno, bname, amt)
- depositor(cname, ano)
- borrower(cname, lno)

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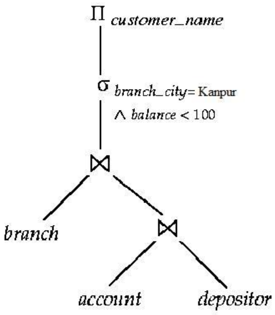
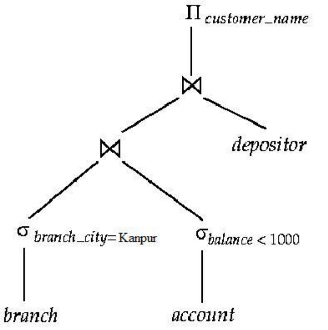


(a) Initial expression tree



(b) Tree after multiple transformations

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  - When $n = 2$, $C = 2$: $r_1 \bowtie r_2$ and $r_2 \bowtie r_1$
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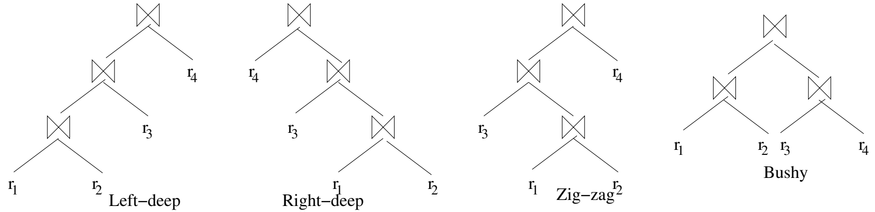
  - When $n = 3$, $N = 3$: $(r_1 \bowtie r_2) \bowtie r_3$, $r_1 \bowtie (r_2 \bowtie r_3)$ and $(r_1 \bowtie r_3) \bowtie r_2$

Heuristics

- Too many
- Use left-deep join tree
  - Right side of each join is a single relation, and not an intermediate result of join of two or more relations
- Similarly, right-deep join trees can be defined
- A tree where at least one child of an internal node is a single relation is called a zig-zag tree
- A general tree is also called a bushy tree



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Algorithm for join order

- Consider set S as the join of n relations
- S can be represented as $S_1 \bowtie (S - S_1)$ for any non-empty proper subset $S_1 \subset S$
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- Interesting sort order: Particular order of records that are useful later
  - Example: Merge join produces tuples in sorted order which makes later merge joins faster
  - Example: Sorted order makes later grouping and aggregation faster
- Algorithm should find the best subset for *each* interesting sort order
- Can extend the DP algorithm to handle this

Heuristics in query optimization

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- Perform semantic optimizations
  - Find all employees earning more than their manager
  - May use domain knowledge to return empty result directly

Statistics

- For each relation r
  - Number of tuples n_r
  - Number of blocks b_r
  - *Blocking factor*, i.e., number of tuples that fit in a block $f_r = \lfloor n_r / b_r \rfloor$
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- If $R \cap S$ is a foreign key in S referencing R, then it is exactly n_s
- Every tuple of r can join with $n_s/v_s(A)$ when joining attribute is A, thereby prducing $n_r.n_s/v_s(A)$ joined tuples
- Reversing r and s, estimate becomes $n_s.n_r/v_r(A)$
- Lower size is the better estimate
- Histograms can improve the estimates

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