6.2. You are going on a long trip. You start on the road at mile post 0. Along the way there are n hotels, at mile posts $a_1 < a_2 < \cdots < a_n$, where each a_i is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the final hotel (at distance a_n), which is your destination.

You'd ideally like to travel 200 miles a day, but this may not be possible (depending on the spacing of the hotels). If you travel x miles during a day, the *penalty* for that day is $(200 - x)^2$. You want to plan your trip so as to minimize the total penalty—that is, the sum, over all travel days, of the daily penalties.

Give an efficient algorithm that determines the optimal sequence of hotels at which to stop.

Scratch

Example milestones: 100, 200, 500, 950, 1800, 2250, 2500

Problem Formulation

T[i] = Minimum total penalty for reaching and stopping at T[i]

Recurrence

T[i] = Find one of the previous milestones such that the sum of minimum penalty to reach that milestone and penalty of reaching this milestone from that milestone is minimum.
i.e.
T[i] = $\min_j^{0->i-1}$(T[j] + (200 - (ai-aj))^2)

K[i] = $\text{argmin}_j^{0->i-1}$(T[j] + (200 - (ai-aj))^2), to track the previous milestone where we stopped for reaching i

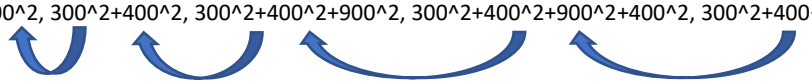
Base case

T[0] = 0, corresponding to the case when you have not travelled any mile yet.

K[0] = 0

Sample Run (rhymes with Temple Run)

S = 100, 200, 500, 900, 1800, 2200, 2500
T = 100, 0, 300^2, 300^2+400^2, 300^2+400^2+900^2, 300^2+400^2+900^2+400^2, 300^2+400^2+900^2+400^2+300^2



Return Value

The minimum penalty can be returned from last element in T i.e. T[n].

The optimal sequence of stops can be created by backtracking K array.

PseudoCode

```
T[0] = 0 ----------(1)
S[0] = 0 ----------(2)

for i = 1->n        ---------- (3)
        min = inf
        argmin = 0
        for j = 0->n-1      ----------(4)
                if (((S[i]-S[j])^2+T[j]) < min)
                        min = (S[i]-S[j])^2+T[j]
                        argmin = j

        T[i] = min
        K[i] = argmin

p = n

q = 1

While (p > 0)      ----------(5)
        B[q] = S[p]
        p = K[p]
        q = q+1

Return reverse(B), T[n]  ----------(6)
```

Runtime Complexity

Lines 1,2 take O(1) time
Lines 3,4 take O(n) time each and since loop 4 is nested in loop 3, hence collectively they take O(n^2) time.
Loop 5 takes O(n) in worst case if all milestone hotels are being used.
Line 6 takes O(n) time due to reverse array function.

Overall, it takes O(n^2) time.