Answer *all* questions

1. (a) What is meant by streamline flow of a fluid? Show that for non-viscous steady flow of a fluid

$$\frac{v^2}{2} + \int \frac{dp}{\rho} + gh = \text{constant}$$

at all points in the field of flow (symbols have their usual meaning). **(5 marks)**

(b) A tube of diameter 10 cm and 8 cm at the two ends is kept horizontally. An incompressible fluid flows through it in a streamline. If the pressure difference between the two ends is 25 cm of Mercury, find the velocity of the fluid at the two ends. (Density of the fluid is $1\,\text{gm/cm}^3$, Density of Mercury is $13.6\,\text{gm/cm}^3$, $g = 981\,\text{cm/sec}^2$) **(5 marks)**

2. (a) Consider a frame S', rotating with respect to another frame S about a fixed axis with angular velocity ω. Show that the acceleration vector in the two frames are related as (the symbols have their usual meaning),

$$\left(\frac{d^2\vec{r}}{dt^2}\right) = \left(\frac{d'^2\vec{r}}{dt^2}\right) + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + 2\vec{\omega} \times \left(\frac{d'\vec{r}}{dt}\right) + \left(\frac{d'\vec{\omega}}{dt}\right) \times \vec{r}$$

(5 marks)

(b) State and prove parallel axis theorem in estimating the moment of inertia of a body. Using this find the moment of inertia of a thin rod of length 1m and mass 1kg about an axis perpendicular to the rod and at a distance of 20cm from one end of the rod. **(5 marks)**

3. (a) In an elastic collision a mass 5kg, moving with speed 10m/sec , hits another stationary mass 5kg along a line. After collision the first mass moves at an angle $45°$ with the line of collision. Find the angle with respect to the line of collision at which the second mass moves? Also find the magnitude of the final velocities of the two masses **(5 marks)**

(b) A sinusoidal wave in one dimension is described by, $\phi(x, t) = 10 \sin(10x - 20t)$. Find its amplitude, frequency, wavelength and phase velocity.
What is meant by coherent sources ? Two sinusoidal waves superpose coherently. Show how the resulting energy is distributed in the form of an interference pattern on a screen. **(5 marks)**

4. (a) What is centre of mass of a system of particles, m_1, m_2, $\cdots$, m_n? Show that the location of the centre of mass of a system of particles is independent of the choice of origin of the coordinate system. **(5 marks)**

(b) Show that in absence of any external force the acceleration of the centre of mass of a collection of particles vanishes. **(5 marks)**

5. (a) Two simple harmonic motions of frequencies ω and $\omega + \Delta\omega$ are superposed. Plot the amplitude of the resulting oscillation as a function of time. **(5 marks)**

(b) Consider a periodic motion

$$y = (a/2) \quad \text{for} \quad 0 < t < (T/2)$$
$$= -(a/2) \quad \text{for} \quad (T/2) < t < T .$$

Decompose this motion into a Fourier series. **(5 marks)**