Indian Association for the Cultivation of Science
(Deemed to be University under the *de novo* category)

Master's/Integrated Master's-PhD Program/Integrated Bachelor's-Master's Program/PhD Course

Mid-Semester Examination-Autumn 2024

Subject: Introductory Mathematical Methods and Classical Mechanics | *Subject Code(s): PHS1101*

Full marks: 25 | *Time allotted: 2 hr*

Answer *all* questions

1. (a) Find a unit vector perpendicular to the plane defined by the following two vectors: $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. **(2 marks)**

   (b) If $\vec{A} + \vec{B} = 11\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{A} - \vec{B} = -5\hat{i} + 11\hat{j} + 9\hat{k}$, then what is the angle between the vectors $\vec{A}$ and $\vec{A} + \vec{B}$? **(2 marks)**

   (c) Two forces, $\vec{F_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ Newton and $\vec{F_2} = 4\hat{i} - 5\hat{j} - 2\hat{k}$ Newton acts on a particle to displace it from point $(20, 5, 0)$ m to $(0, 0, 7)$ m. What is the work done? **(1 mark)**

2. (a) For any vector field $\vec{A}$, show that $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$. **(2 marks)**

   (b) For any scalar field ϕ, show that $\vec{\nabla} \times (\vec{\nabla}\phi) = 0$. $\perp$ **(2 marks)**

   (c) Consider the surface, $\phi(x, y, z) = \cancel{x^2yz = 3}$ Find a unit vector ~~on~~ this surface at the point $(1, 1, 1)$. **(1 mark)**
   $x^2 + y + z = 3$

3. (a) Consider a one-dimensional damped harmonic oscillator of mass m. The restoring and the damping forces are $-k_1x$ and $-k_2(dx/dt)$, respectively. Find its position x at a time t choosing the origin at the centre of the force. Under what conditions the motion will not be oscillatory. **(3 marks)**

   (b) Show that for $k_2 \to 0$, the total energy of the oscillator will be conserved. **(2 marks)**

4. (a) Define a central force and show that for a central force $\vec{\nabla} \times \vec{F} = 0$. **(2 marks)**

   (b) Show that the work done by a central force is independent of path. Hence define the potential V of the force. How V is related to the force $\vec{F}$? **(2 marks)**

   (c) A particle moves in a central force. Show that the angular momentum about the centre of force is conserved. **(1 mark)**

5. (a) Show that the motion of a particle in an external gravitational field is confined to a two-dimensional plane. **(1 mark)**

(b) Show that for a motion confined in a plane, the components of acceleration are, $a_r = \ddot{r} - r\dot{\theta}^2$, and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, where symbols have their usual meaning. **(2 marks)**

(c) Equation of a conic section in polar coordinate is given as,

$$\frac{1}{r} = \frac{1}{\ell} + \frac{\epsilon}{\ell}\cos(\theta - \theta_i)$$

where, ℓ is the semi-latus rectum and ϵ is the eccentricity. Show that the motion of a particle under the influence of a central inverse square force is a conic section. **(2 marks)**