Indian Association for the Cultivation of Science

(Deemed to be University under the *de novo* category)

BS-MS Program

End-Semester Examination-2024 (Autumn Semester-I)

Subject: Mathematics I | *Subject Code(s): MAT 1101*

Full marks: 50 | *Time allotted: 3 hrs*

Answer question number **10** and any **6** of the remaining questions.

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1. Show that every sequence of real numbers has a monotone subsequence. [7]

2. Let $\{x_n\}$ be a bounded sequence. Suppose $s = \sup\{x_n,\ n \in \mathbb{N}\}$. Show that if $s \notin \{x_n,\ n \in \mathbb{N}\}$, there is a subsequence of $\{x_n\}$ that converges to s. [7]

3. Let $\{x_n\}$ be a Cauchy sequence. Show that $\{x_n\}$ is bounded. [7]

4. Show that the alternating harmonic series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

is convergent. Is it absolutely convergent? Justify your answer. [7]

5. Show that the series $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ is convergent. [7]

6. Let I be an interval in $\mathbb{R}$, $f : I \to \mathbb{R}$ and let $c \in I$. Suppose there exist constants K and l such that $|f(x) - l| \leq K|x - c|$ for $x \in I$. Show that $\lim_{x \to c} f(x) = l$. [7]

7. Let $f : \mathbb{R} \to \mathbb{R}$, and $f(x + y) = f(x) + f(y)$ for every $x, y \in \mathbb{R}$. Let $\lim_{x \to 0} f(x) = l$. Show that $l = 0$. Also prove that $\lim_{x \to c} f(x)$ exists for every $c \in \mathbb{R}$. [7]

8. Prove that if f and g are continuous, then so are $\max(f, g)$ and $\min(f, g)$. [7]

9. Let $[a\ b]$ be a closed and bounded interval in $\mathbb{R}$. Let $f : [a\ b] \to \mathbb{R}$ be a continuous function. Show that f is bounded. [7]

10. Let $f : [a\ b] \to [a\ b]$ be a continuous function. Show that there exists an $x \in [a\ b]$ such that $f(x) = x$. [8]