# TU-Delft Deep Learning course 2018-2019

03.backprop

20 Feb 2019

Delft University of Technology

Lecturer: Jan van Gemert
Several slides credit to Roger Grosse

# Learning goals

After this lecture you can:

- Understand what the goal of backpropagation is
- Explain the forward/backward pass
- Apply the calculus chain rule to compute derivatives
- Apply the backpropagation algorithm to efficiently compute derivatives
- Understand the modularity of network nodes

Book chapters: 4.3, 6.5

# Training a network

Chapter 4.3

Training set:

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|---|---|---|
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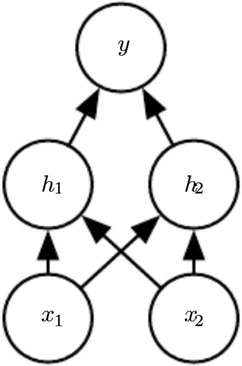
| 1 | 1 | 0 |
| 0 | 0 | 0 |
| 1 | 0 | 1 |

While not converged:

1. Present a training sample
2. Compare the results
3. Update the weights



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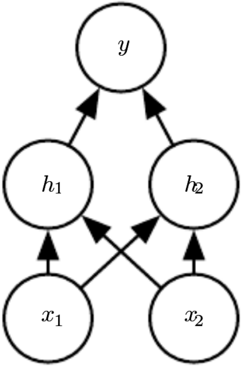

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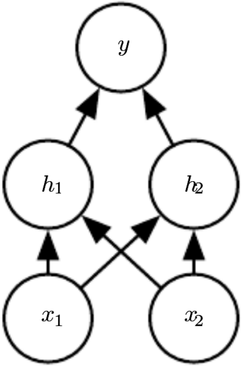

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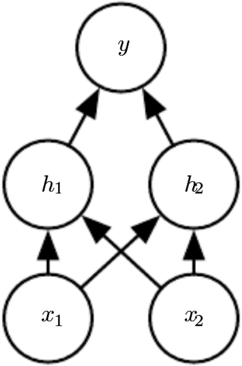
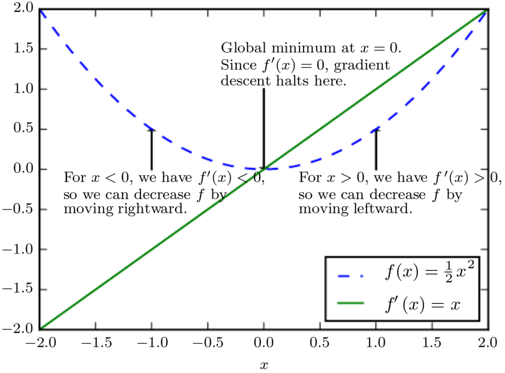


# Backward pass: Backpropagation

- Backpropagation: Efficient algorithm to compute gradients
- Used to train a huge majority of deep nets
- Engine that powers "End-to-End" optimization and representation learning
- Its "just" a clever application of the chain rule of calculus

# Updating the weights: Gradient descent




1-d gradient descent

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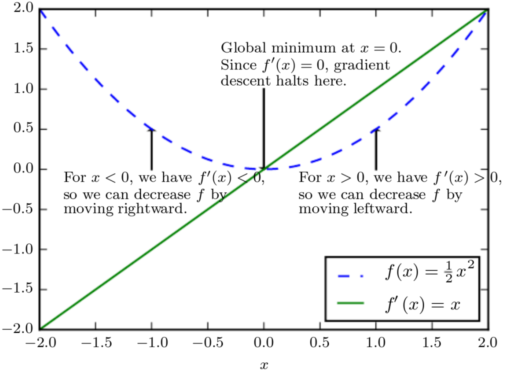


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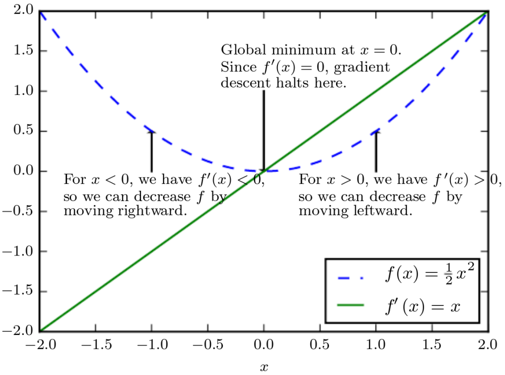


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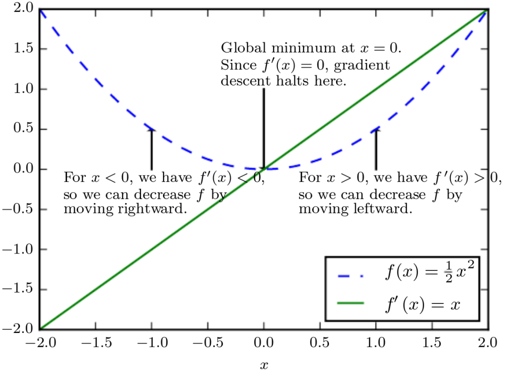


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- Q: How to get cost gradient from training samples?

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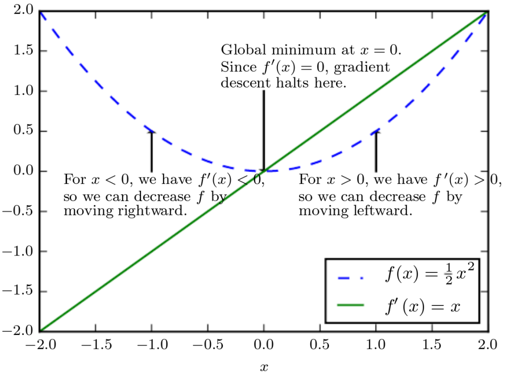


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- A: Average $\frac{d\mathcal{L}}{d\mathbf{w}}$ for loss $\mathcal{L}$ over all training samples

# Chain rule to compute derivatives

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- Let: $y = g(x)$, and $z = f(g(x)) = f(y)$.
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$$\frac{d}{dt}f(x(t), y(t)) = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$
- Interpretation: How much does output change if the input slightly changes

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Q: What to compute?

A: Derivative to $w$

$$\frac{\partial}{\partial w}\mathcal{L} = \frac{\partial}{\partial w}\left[\frac{1}{2}\left(\sigma(\mathsf{wx}+\mathsf{b})\text{-}\mathsf{t}\right)^2\right]$$

A: Derivative to $b$

$$\frac{\partial}{\partial b}\mathcal{L} = \frac{\partial}{\partial b}\left[\frac{1}{2}\left(\sigma(\mathsf{wx}+\mathsf{b})\text{-}\mathsf{t}\right)^2\right] =$$

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Q: What are the disadvantages of this approach?

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Model: $\mathcal{L} = \frac{1}{2}\left(\sigma(\mathsf{wx}+\mathsf{b})\text{-}\mathsf{t}\right)^2$

Decompose in: $z = wx + b, \quad y = \sigma(z), \quad \mathcal{L} = \frac{1}{2}(y - t)^2$

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Q: Dependency order of computation? A: Starts backwards.

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Expressions to evaluate

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Values already computed

[1] Bar notation credits to Roger Grosse

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Write:

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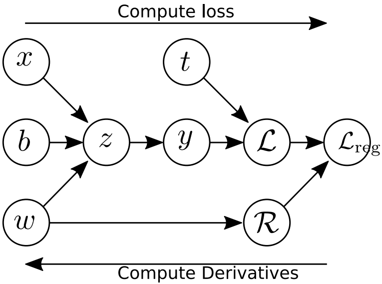
# Computational graph

Book: Section 6.5.1



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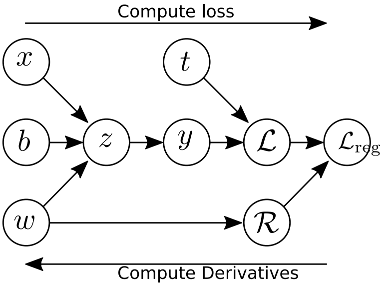
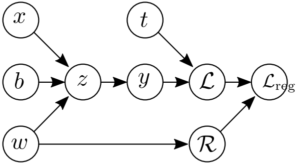



- A node is a variable: scalar, vector, matrix, tensor, other node, etc.
- An operation: simple function of 2 variables (often omitted)
- Operation have single output (possibly multiple entries)
- $x \to y$: y is computed by applying operation to x.

# Backpropagation algorithm

Book: Algorithms 6.1 and 6.2




Topological ordering:
Linear ordering of vertices so for every directed edge uv, vertex u comes before v in the ordering.

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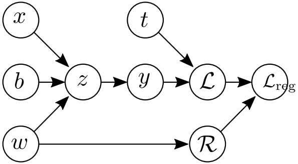


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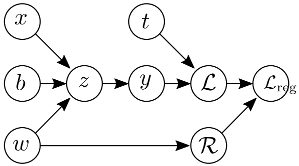


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Q: Topological ordering of graph?
A: $(w, b, x, z, y, t, \mathcal{L}, \mathcal{R}, \mathcal{L}_{\textsf{reg}})$

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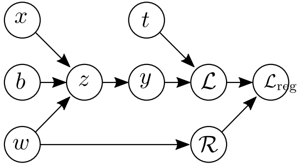


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Backpropagation algorithm to compute gradients of node $n_N$:

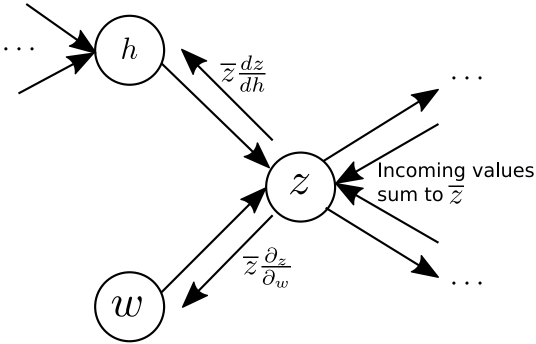
1. Create topological ordering of graph

Forward pass $= \{$
2. For $i = 1, 2, \ldots N$ :
   Evaluate $n_i$ using its function $f^{(i)}(n_i)$

Backward pass $= \{$
3. $\overline{n_N} = 1$ (gradient of a node wrt itself is 1; $\frac{df}{df} = 1$)
4. For $i = N-1, \ldots 1$ :
   $\overline{n_i} = \sum_{n_j \in \text{Children}(n_i)} \overline{n_j} \frac{\partial_{n_j}}{\partial_{n_i}}$

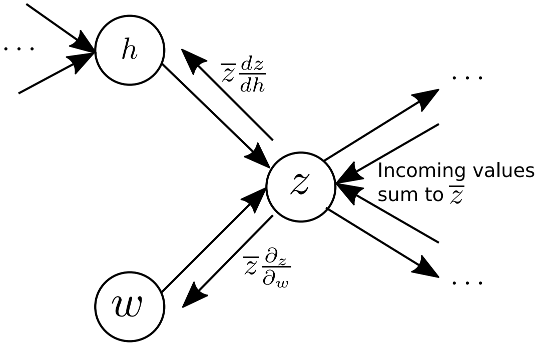
Note on step 4: node only gets gradients wrt children nodes

# Backprop as message passing



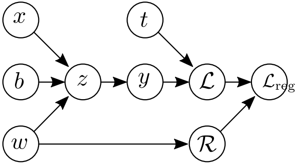

- Each node aggregates the error signal from its children
- Each node passes messages to its parents
- Q: Benefit from this node-centered point of view?

# Backprop as message passing




- Each node aggregates the error signal from its children
- Each node passes messages to its parents
- Q: Benefit from this node-centered point of view?
- A: Modularity: Only needs to compute derivatives wrt its arguments.

# Example: Univariate logistic least squares regression




Q: Do the backward pass:

Forward pass:

$$z = wx + b$$

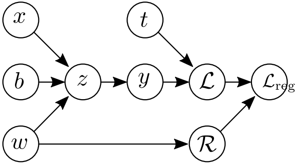
$$y = \sigma(z)$$

$$\mathcal{L} = \frac{1}{2}(y - t)^2$$

$$\mathcal{R} = \frac{1}{2}w^2$$

$$\mathcal{L}_{reg} = \mathcal{L} + \lambda\mathcal{R}$$

# Example: Univariate logistic least squares regression



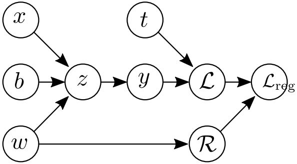

Forward pass:

$$
\begin{aligned}
z &= wx + b \\
y &= \sigma(z) \\
\mathcal{L} &= \frac{1}{2}(y - t)^2 \\
\mathcal{R} &= \frac{1}{2}w^2 \\
\mathcal{L}_{reg} &= \mathcal{L} + \lambda\mathcal{R}
\end{aligned}
$$

Q: Do the backward pass:

$$
\begin{aligned}
\overline{\mathcal{L}_{reg}} &= \\
\overline{\mathcal{R}} &= \\
\overline{\mathcal{L}} &= \\
\overline{y} &= \\
\overline{z} &= \\
\overline{w} &= \\
\overline{b} &=
\end{aligned}
$$

# Example: Univariate logistic least squares regression




Forward pass:

$$
\begin{aligned}
z &= wx + b \\
y &= \sigma(z) \\
\mathcal{L} &= \frac{1}{2}(y-t)^2 \\
\mathcal{R} &= \frac{1}{2}w^2 \\
\mathcal{L}_{reg} &= \mathcal{L} + \lambda\mathcal{R}
\end{aligned}
$$

Q: Do the backward pass:

$$
\begin{aligned}
\overline{\mathcal{L}_{reg}} &= 1 \\
\overline{\mathcal{R}} &= \overline{\mathcal{L}_{reg}}\frac{d\mathcal{L}_{reg}}{d\mathcal{R}} = \overline{\mathcal{L}_{reg}}\lambda \\
\overline{\mathcal{L}} &= \overline{\mathcal{L}_{reg}}\frac{d\mathcal{L}_{reg}}{d\mathcal{L}} = \overline{\mathcal{L}_{reg}} \\
\overline{y} &= \overline{\mathcal{L}}\frac{d\mathcal{L}}{dy} = \overline{\mathcal{L}}(y-t) \\
\overline{z} &= \overline{y}\frac{dy}{dz} = \overline{y}\sigma'(z) \\
\overline{w} &= \overline{z}\frac{\partial z}{\partial w} + \overline{\mathcal{R}}\frac{\partial \mathcal{R}}{\partial w} = \overline{z}x\overline{\mathcal{R}}w \\
\overline{b} &= \overline{z}\frac{\partial z}{\partial b} = \overline{z}
\end{aligned}
$$

# Learning goals

- Understand how backpropagation powers deep learning
- Explain about the forward/backward pass
- Apply the calculus chain rule to compute derivatives
- Apply the backpropagation algorithm to efficiently compute derivatives
- Understand the modularity of network nodes