TU-Delft Deep Learning course 2018-2019

05.optimization

6 Apr 2019

Delft University of Technology

Chapter 8

Lecturer: Jan van Gemert

Several slides inspired by Andrew Ng

Topic: Optimization

- Past gradient update statistics
- How to efficiently compute past statistics
- Momentum, RMSprop, Adam.
- Feature and Batch normalization

Book chapters: 8.3.1, 8.3.2, 8.5.2, 8.5.3, 8.7.1

Training a network

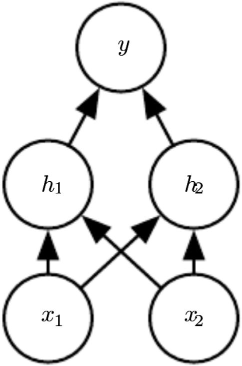
Chapter 4.3

Training set:

Data		Label
0	1	1
1	1	0
0	0	0
1	0	1

While not converged:

1. Present a training sample
2. Compare the results
3. Update the weights



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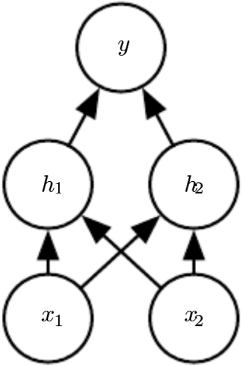

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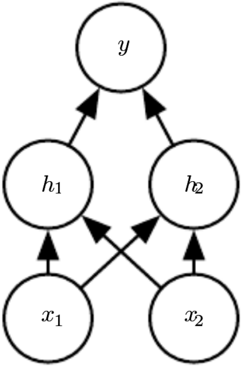

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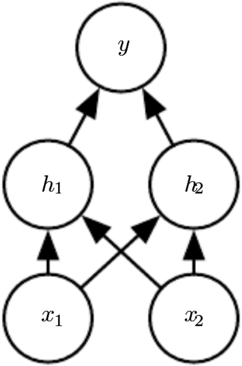
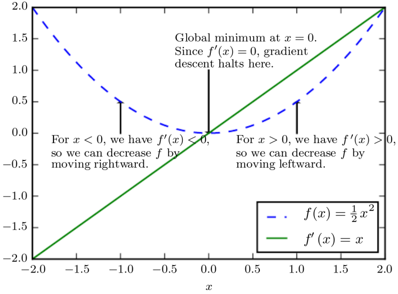


Stochastic Gradient Descent

Chapter 5.9 and 8.3.1



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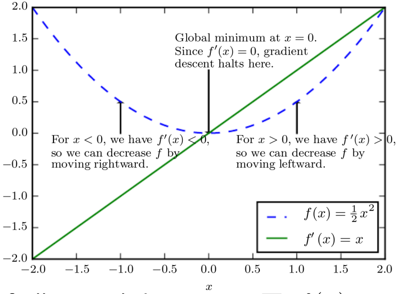

- Gradient: Vector of all partial derivatives $\nabla_x f(x)$

Stochastic Gradient Descent

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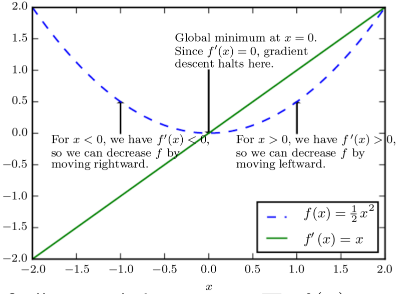

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Stochastic Gradient Descent

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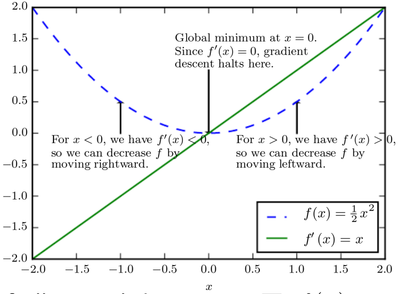

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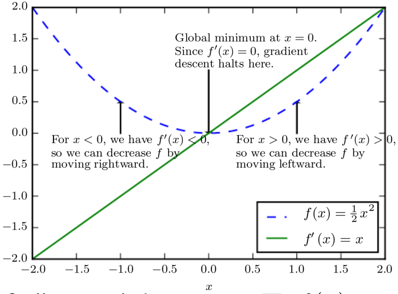

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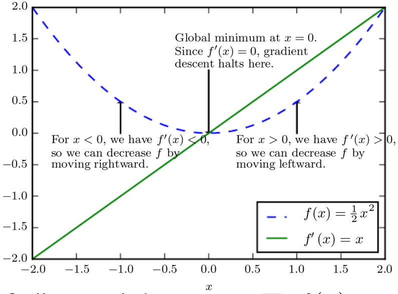

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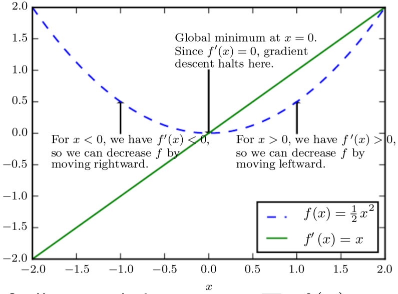

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SGD approximates gradient from small sample set:

Stochastic Gradient Descent

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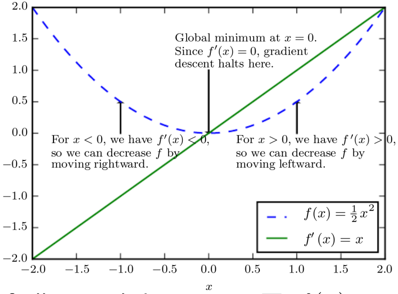

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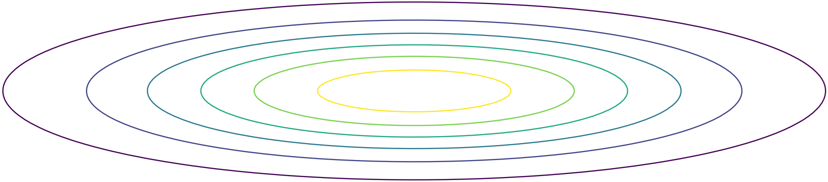
The learning rate is arguably the most important parameter to tune



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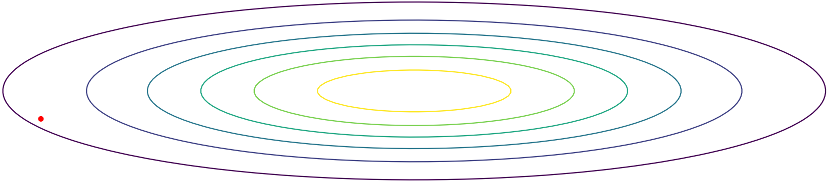

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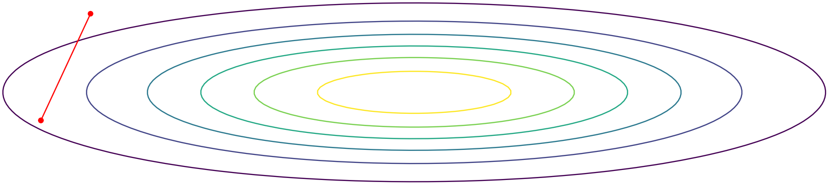

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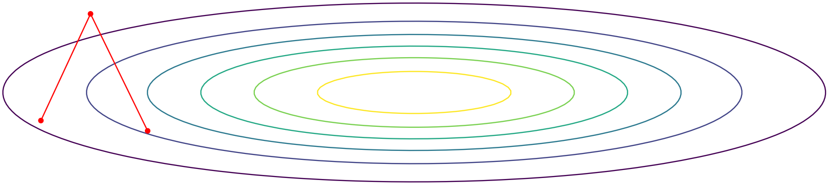

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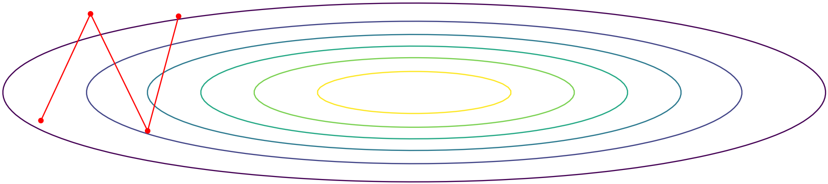

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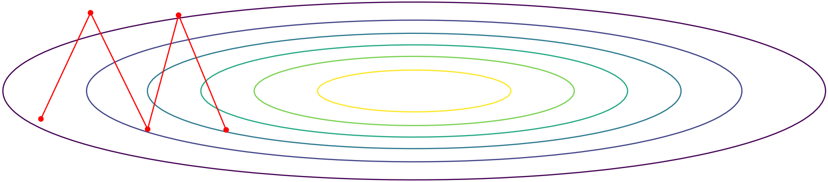

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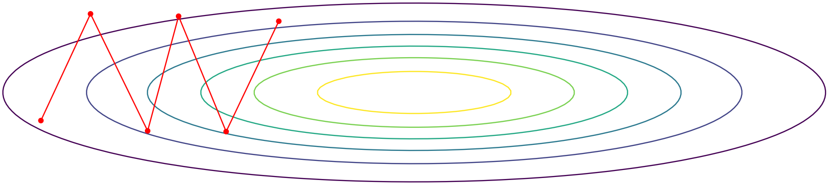

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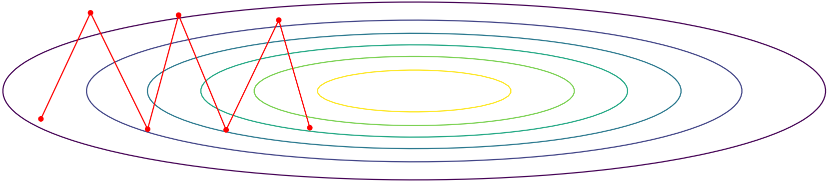

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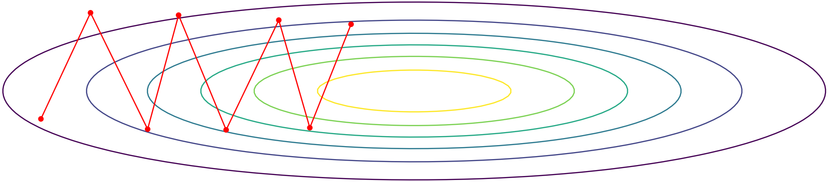

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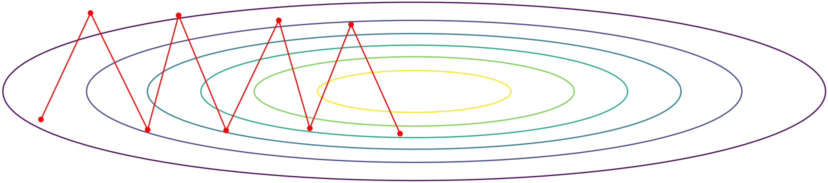

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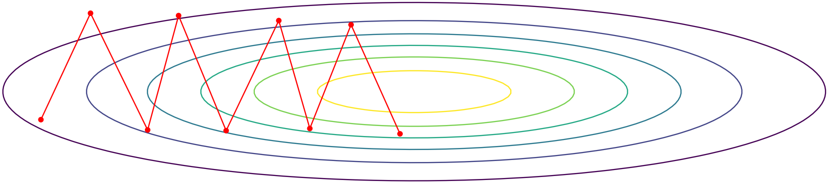

Q: What do you notice?
A: If LR too high it will never find a good instantiation

Tuning the learning rate; take 2.

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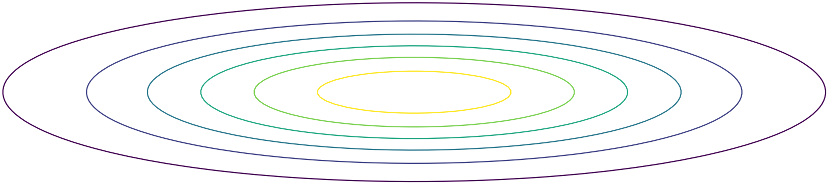

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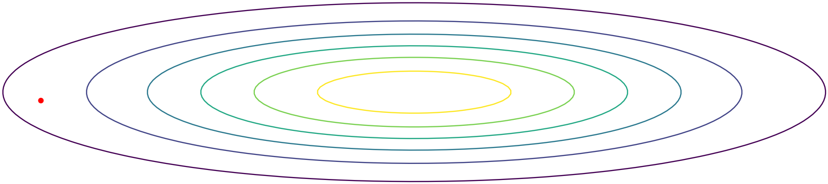

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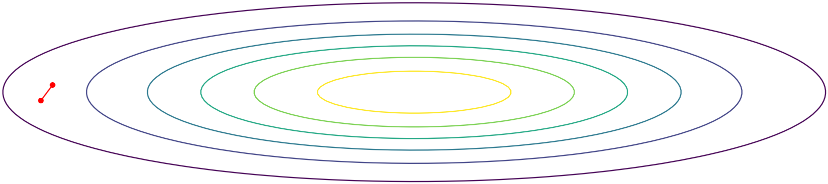

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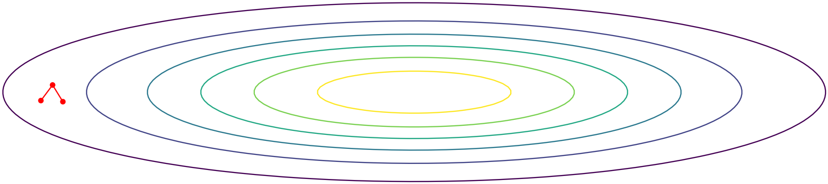

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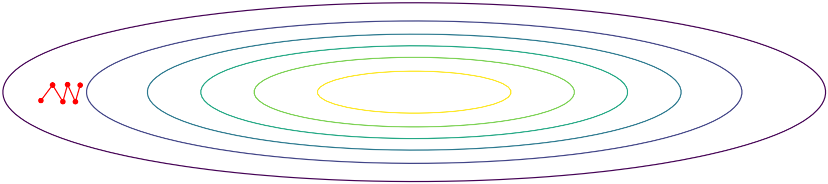

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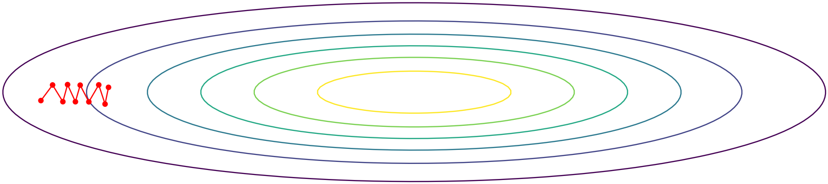

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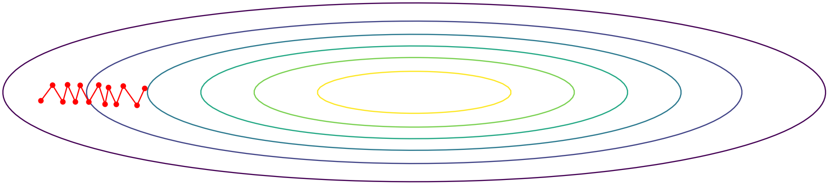

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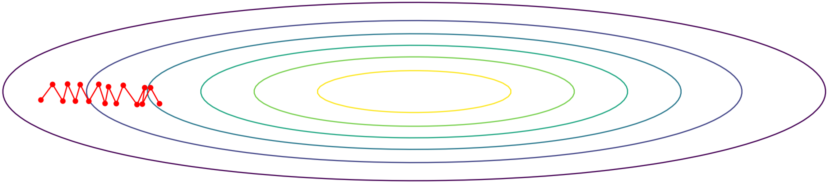

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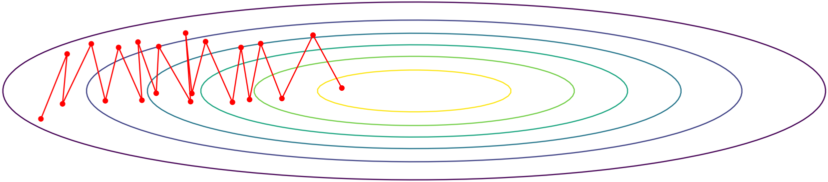

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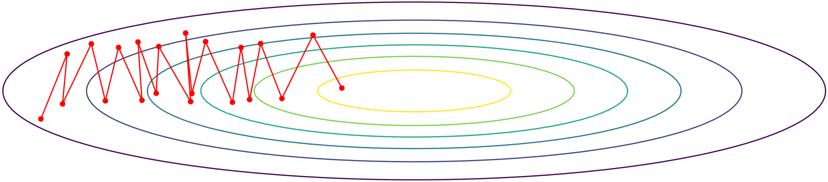

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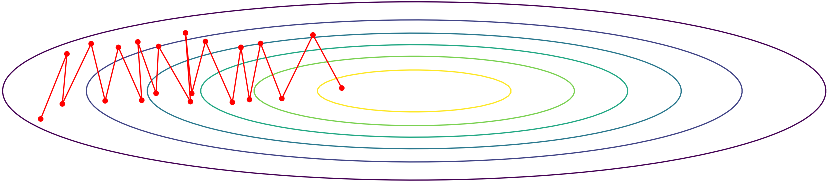

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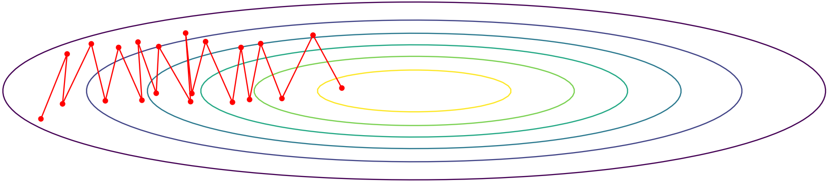

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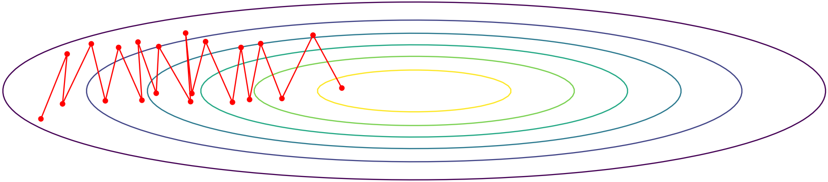

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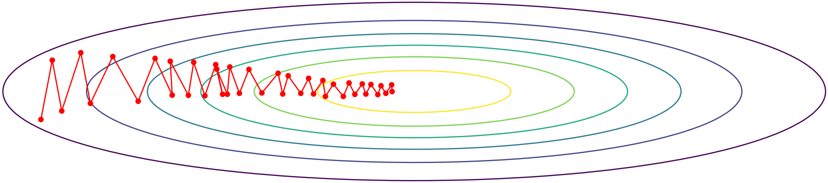

Learning rate: More art than science

Book, Chapter 8.3.1: "Most guidance should be regarded with some skepticism".

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There are some algorithms that seem to generally improve over SGD

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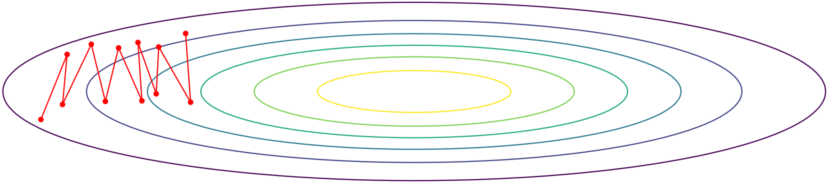

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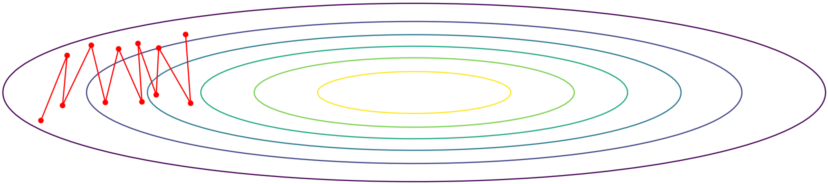

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A: Use different learning rates per parameter: Smaller for y-axis parameter.

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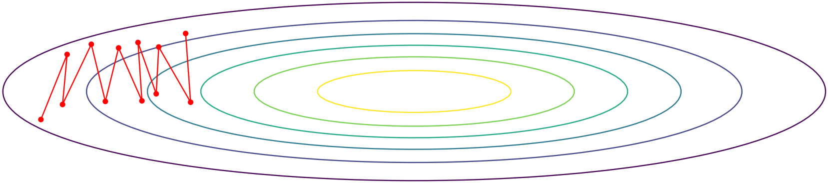

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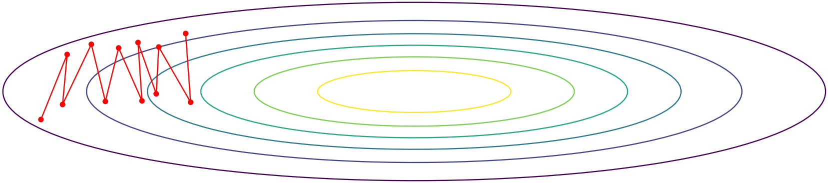

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- RMSprop: Variance in the wrong direction is high
- Adam: Combines momentum and RMSprop

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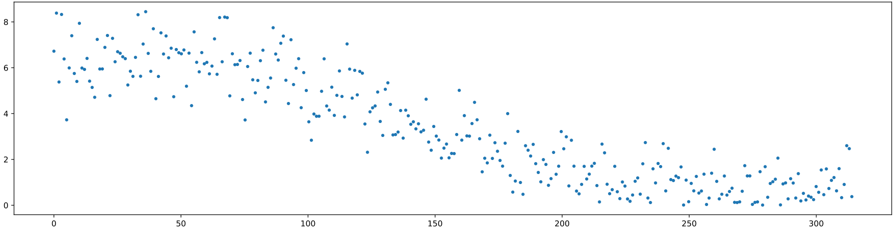
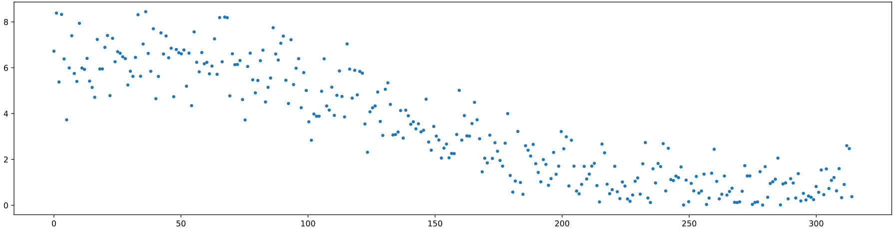
Next slides: How to efficiently keep track of such statistics.

Noisy time series



How to smooth out a noisy time series as values are coming in.

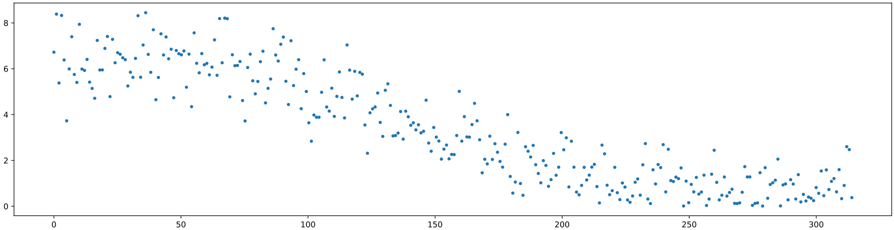
Noisy time series



How to smooth out a noisy time series as values are coming in.

- Q: Can we use convolution?

Noisy time series



How to smooth out a noisy time series as values are coming in.

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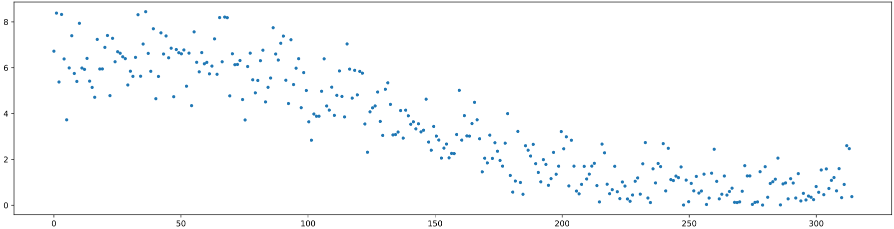
- Q: Keep recent values, and compute a weighted average?

Noisy time series



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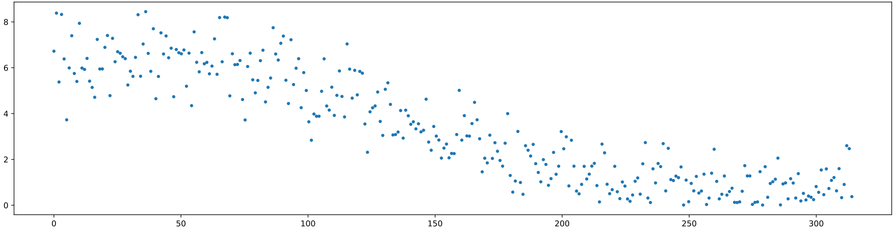

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An infinite impulse response filter (IIR) to the rescue:

Recursively compute online average.

Exponentially weighted moving average (EWMA)

For value y_t at time t, and $0 \leq \rho \leq 1$ and $S_{t-1} = 0,$ if $t = 1$.
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S_{100} &= \rho \quad S_{99} && + (1-\rho) y_{100} \\
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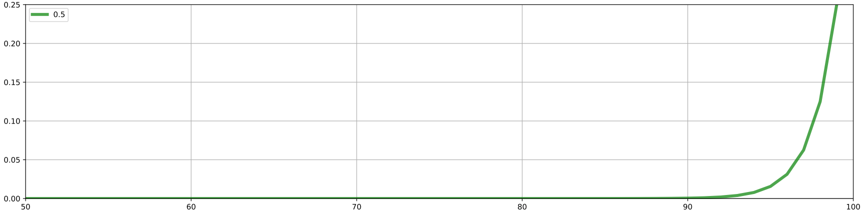

Example of the weights for $\rho = 0.5$

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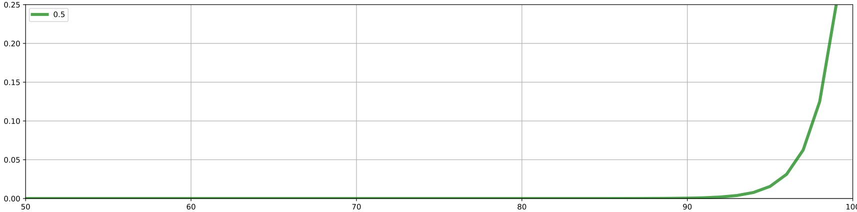
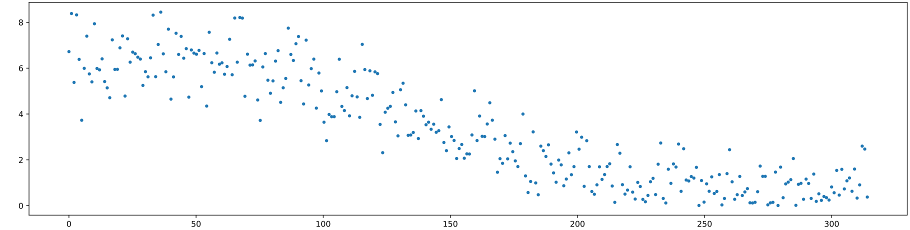
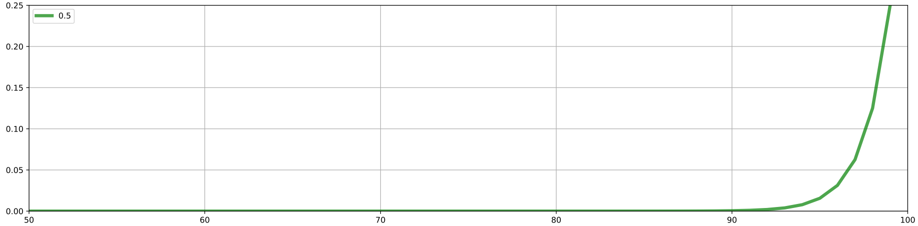


Example of the weights for $\rho = 0.5$
Q: How about $\rho = 0.9$? $\rho = 0.95$?

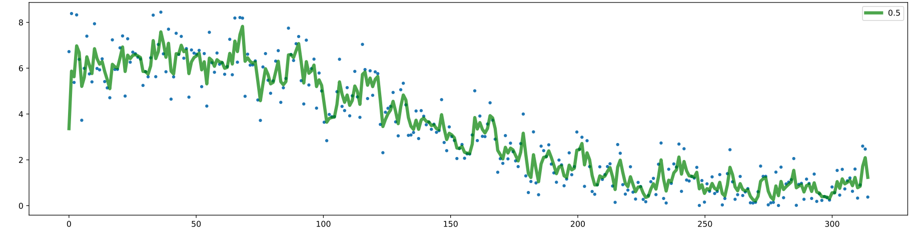
Exponentially weighted moving average (EWMA)

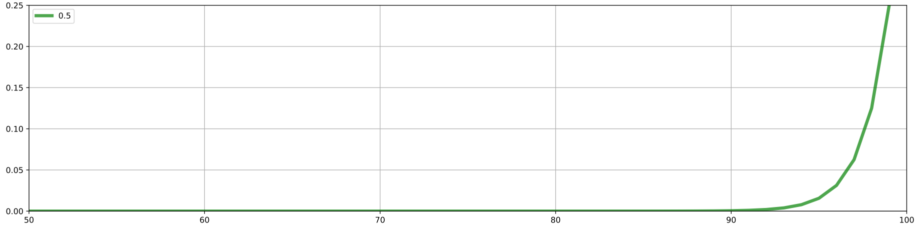




Example for $\rho = 0.5$, $\rho = 0.9$, $\rho = 0.95$

Exponentially weighted moving average (EWMA)





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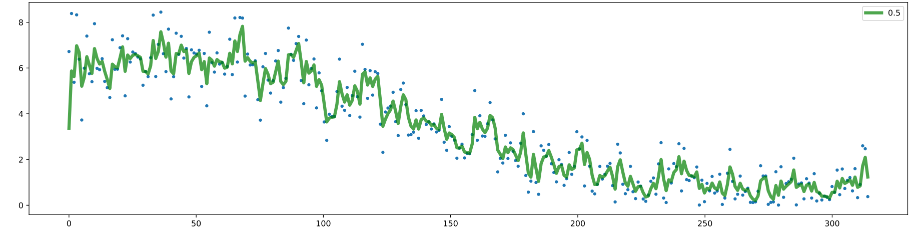

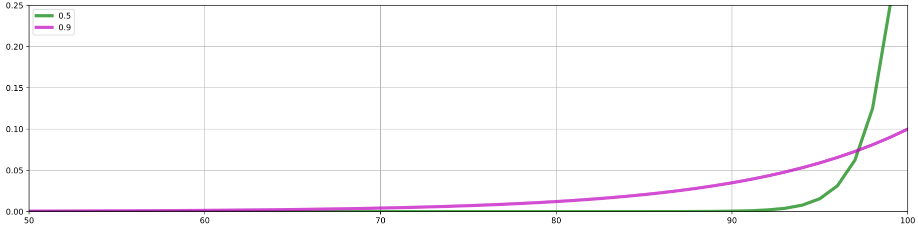

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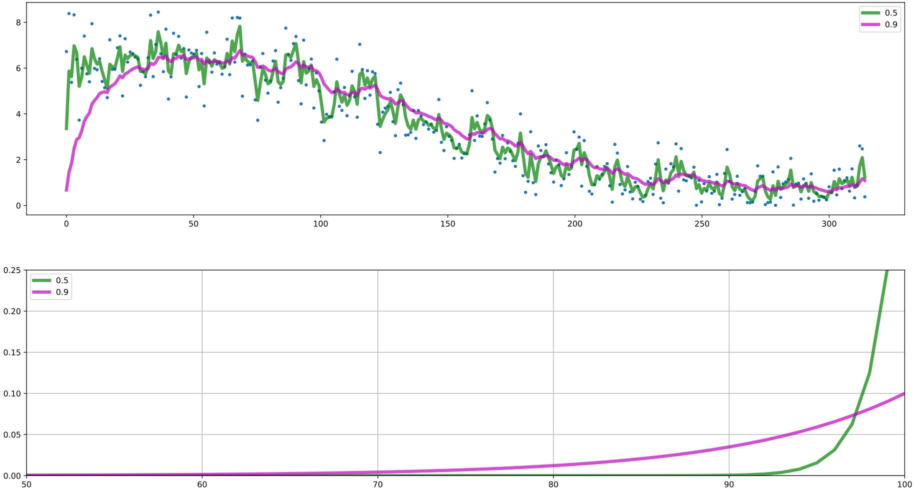

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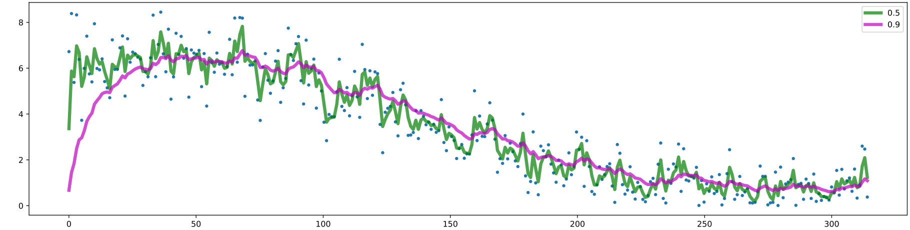

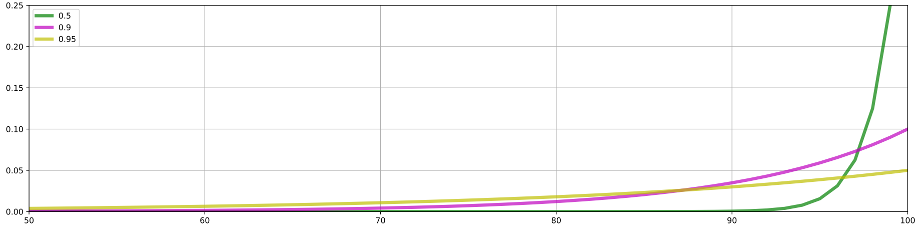

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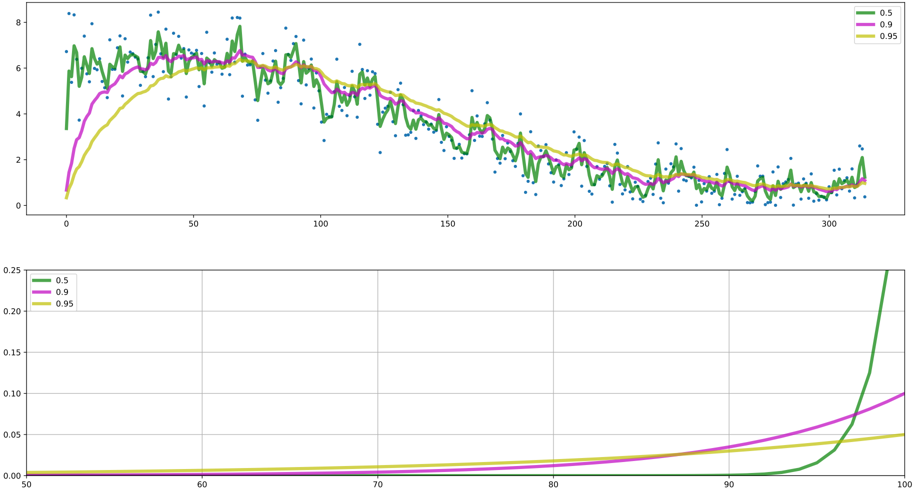
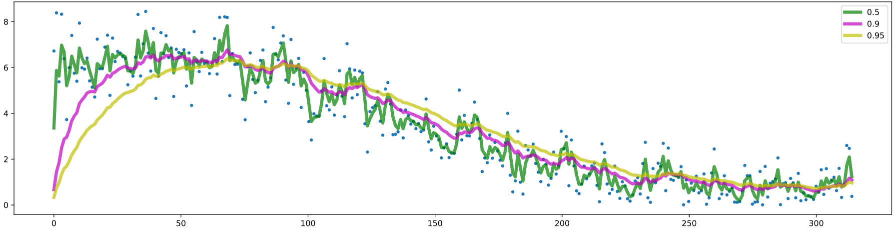


Example for $\rho = 0.5$, $\rho = 0.9$, $\rho = 0.95$

Bias



- Q: What do you notice about these curves?

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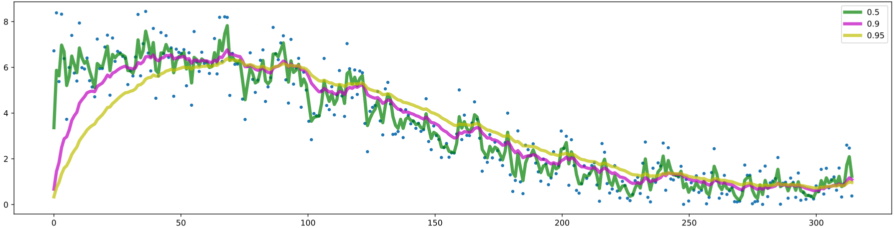

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- A: The larger ρ, the later it 'catches up'.

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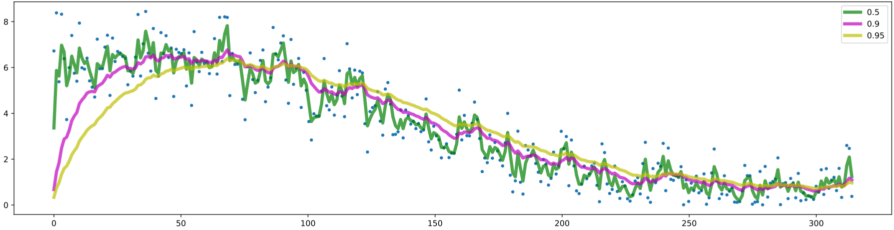

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- Q: Why is that?

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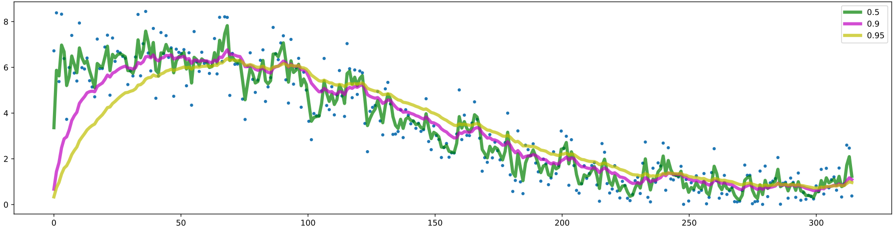
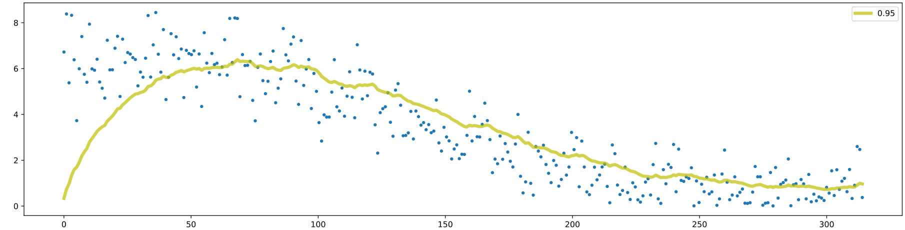


- Q: What do you notice about these curves?
- A: The larger ρ, the later it 'catches up'.
- Q: Why is that?
- A: Border effect: Unknown what happened before time.

Bias correction for $S_t = (\rho S_{t-1}) + (1-\rho)y_t$

Lets start an example at S_1 for $\rho = 0.95$



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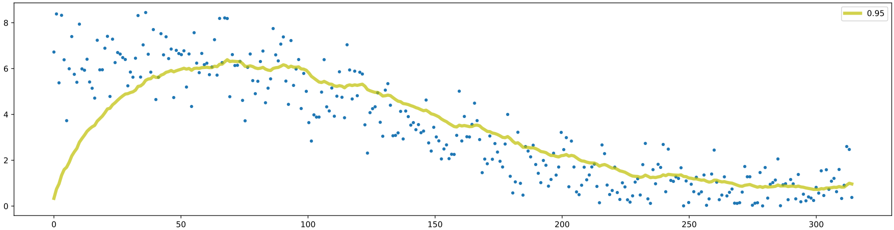

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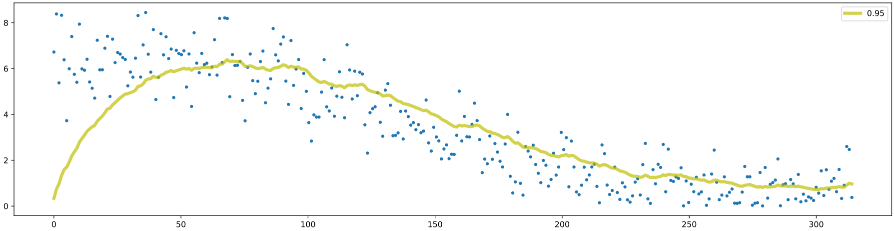
Indeed, very low values at the beginning.



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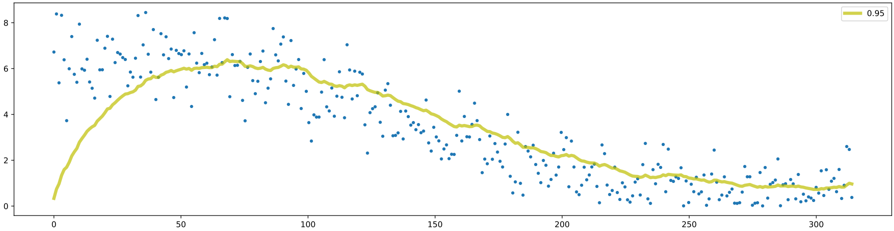

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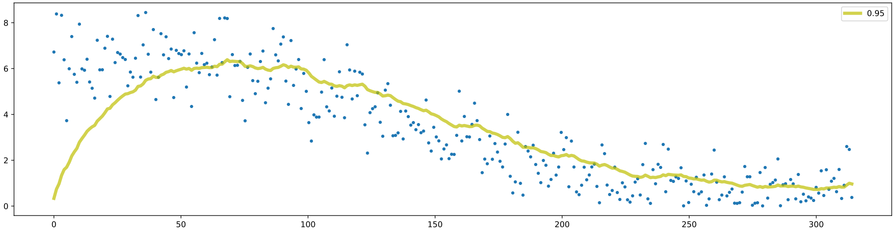
For $t = 2$, the value $1 - \rho^t = 0.0975$, so: $\frac{0.0475y_1 + 0.05y_2}{0.0975}$

Q: Notice?



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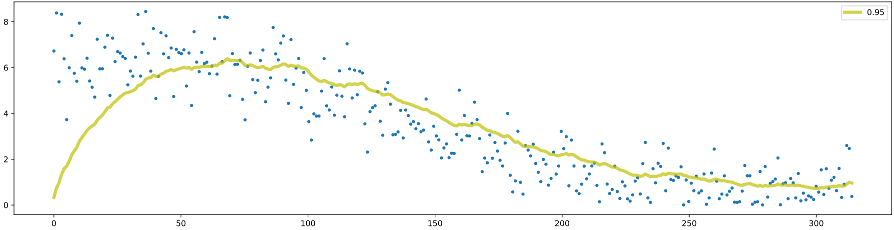

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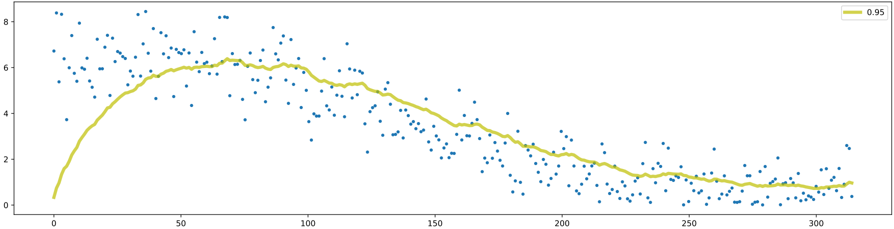

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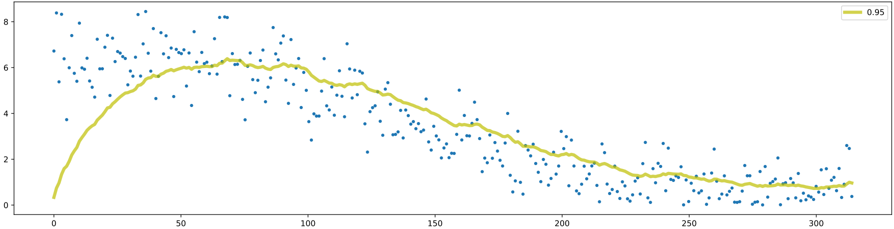
Q: What happens for large t ? $1 - \rho^t \approx 1$, so $\hat{S}_t \approx S_t$



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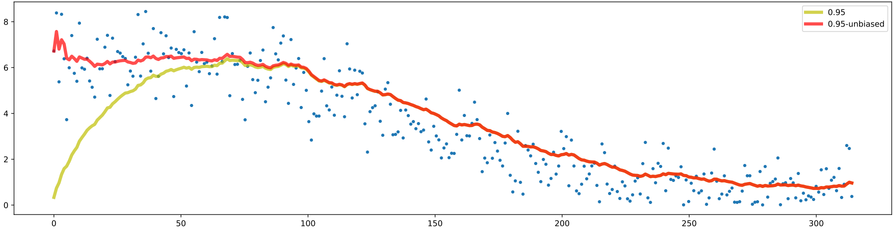

Questions?

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Useful algorithms:

- Momentum: On average in the good direction
- RMSprop: Variance in the wrong direction is high
- Adam: Combines momentum and RMSprop

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All these algorithms make use of past gradient update statistics.

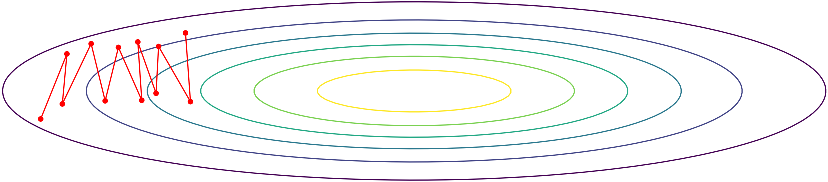
Now can efficiently keep track of such statistics.

Next slides: lets use them for gradient updates.

Stochastic Gradient Descent with momentum

Chapter 8.3.2

Momentum smooths the average of noisy gradients with EWMA



Hyper-parameters: EWMA: ρ; LR: ϵ; with ∇_θ for a mini-batch in iteration i:

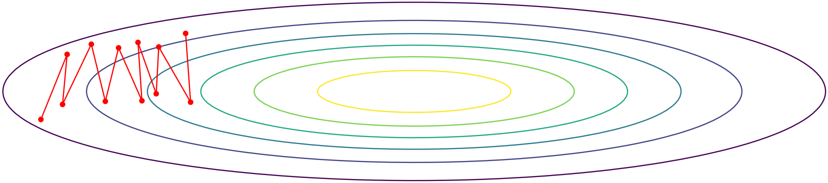
- $v_i = \rho v_{i-1} + (1-\rho)\nabla_\theta$
- $\theta' = \theta - \epsilon v_i$

Sometimes (book) written as $v_i = \rho v_{i-1} + \nabla_\theta$, where ϵ then needs to be re-tuned.
Often implemented without bias correction, as it does catch up quick.
Default setting $\rho = 0.9$

Stochastic Gradient Descent with RMSprop

Chapter 8.5.2

RMSprop smooths the zero-centered variance of noisy gradients with EWMA



Hyper-parameters: EWMA: ρ; LR: ϵ; with ∇_θ for a mini-batch in iteration i:

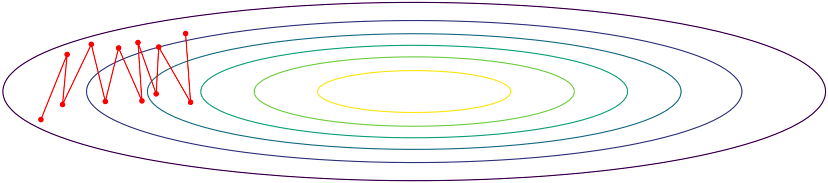
- $r_i = \rho r_{i-1} + (1-\rho)\nabla_\theta^2$
- $\theta' = \theta - \epsilon \frac{\nabla_\theta}{\sqrt{r_i}}$

To prevent dividing by 0, add a small $\delta \approx 10^{-6}$ to denominator: $r_i = \delta + r_i$
Often implemented without bias correction, as it does catch-up quick.
Default setting $\rho = 0.9$

Stochastic Gradient Descent with Adam

Chapter 8.5.3

Adam combines momentum with RMSprop



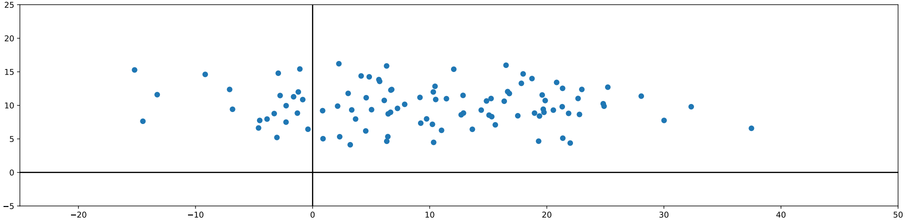
Hyper-parameters: EWMA: ρ_1, ρ_2; LR: ϵ; with ∇_θ for a mini-batch in iteration i:

- $v_i = \rho_1 v_{i-1} + (1-\rho_1)\nabla_\theta,$ $\qquad r_i = \rho_2 r_{i-1} + (1-\rho_2)\nabla_\theta^2$
- $\hat{v}_i = \frac{v_i}{(1-\rho_1^i)},$ $\qquad \hat{r}_i = \frac{r_i}{(1-\rho_2^i)}$
- $\theta' = \theta - \epsilon \frac{\hat{v}_i}{\sqrt{\hat{r}_i}}$

To prevent dividing by 0, add a small $\delta \approx 10^{-6}$ to denominator: $r_i = \delta + r_i$
Default settings: $\rho_1 = 0.9$ and $\rho_2 = 0.999$

Questions?

Effect of feature normalization on optimization



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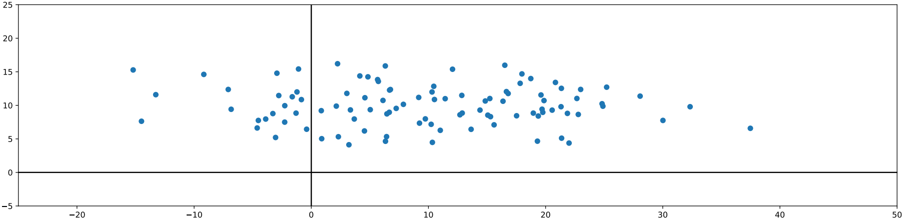

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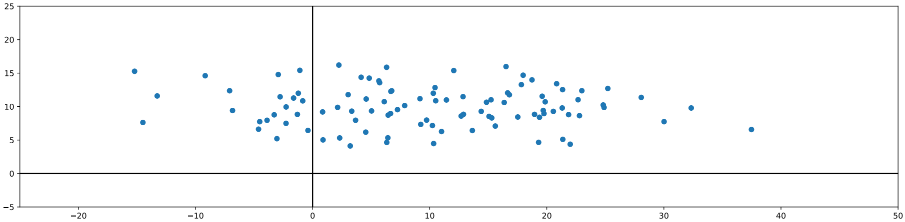

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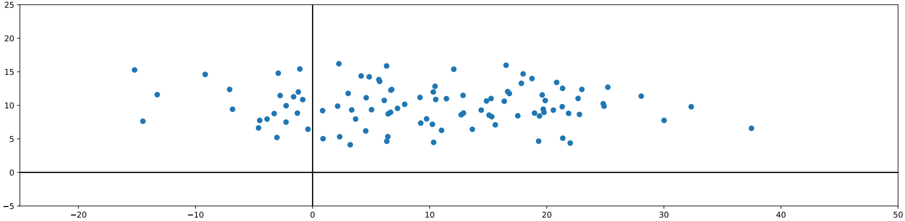

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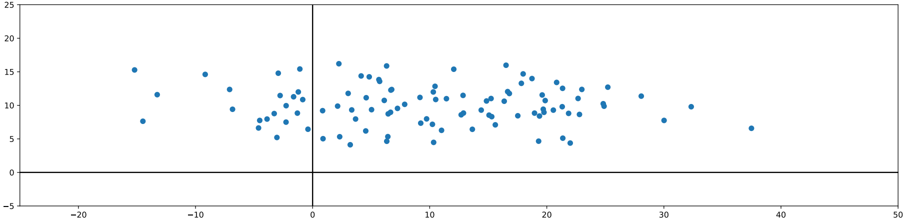

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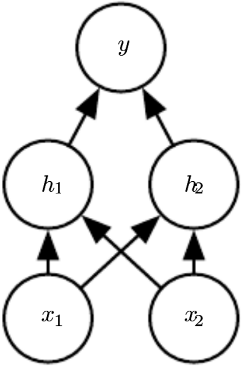
Batch norm: Normalizing learned features

Chapter 8.7.1

- A deep neural net learns many hidden feature representations.



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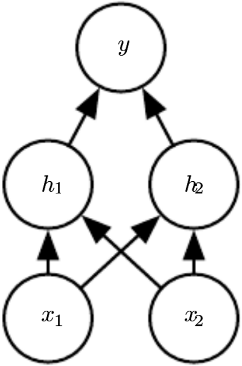

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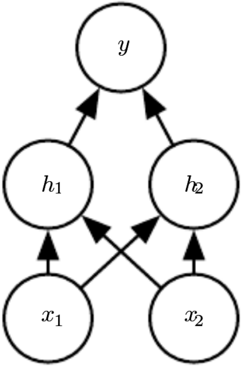

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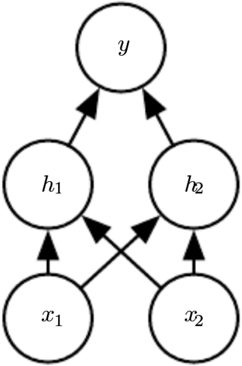

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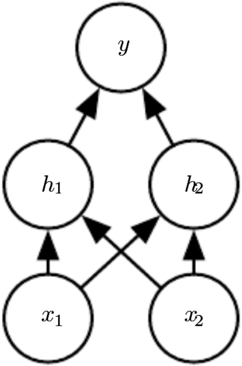

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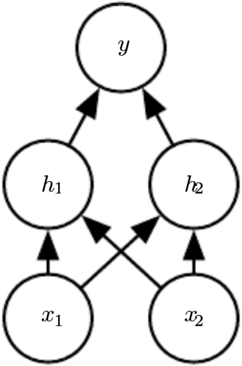

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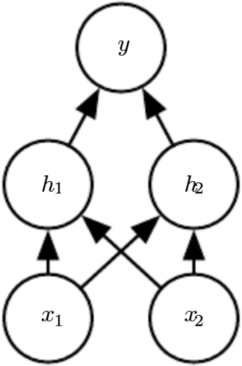

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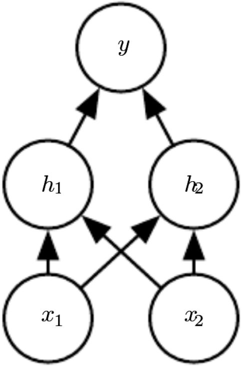

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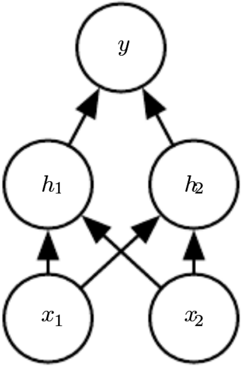

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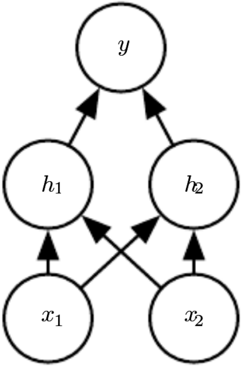

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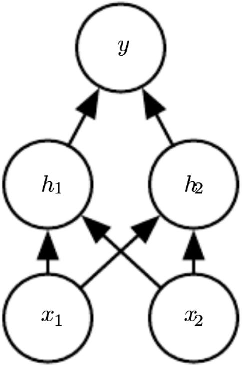

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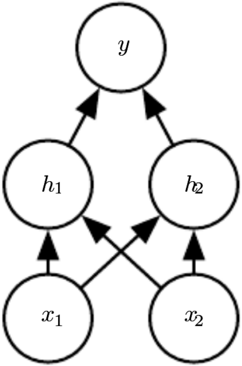

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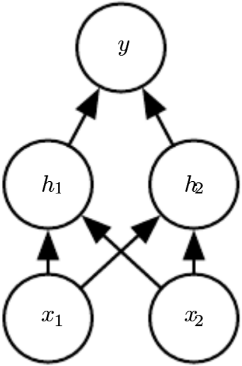

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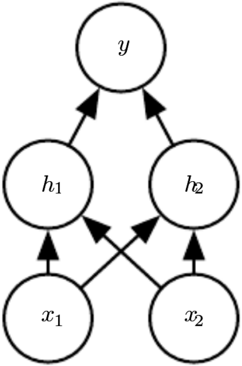

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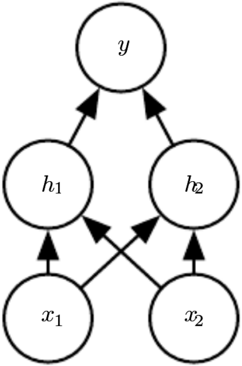

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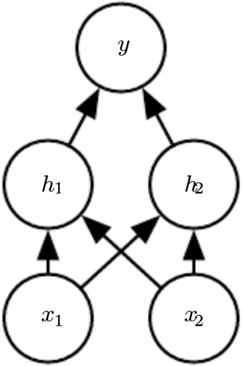

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- Use the weighted average for μ and σ^2 and the learned parameters γ and β to compute $\overline{z}_i^{\text{norm}} = \gamma_i z_i^{\text{norm}} + \beta_i$ for each test sample.

Questions?

Recap

- Stochastic Gradient Decent
- Learning rate
- Average and variance of past gradient update statistics help
- Exponentially weighted moving average
- Momentum: Average; RMSprop: variance; Adam: both.
- Feature and Batch normalization