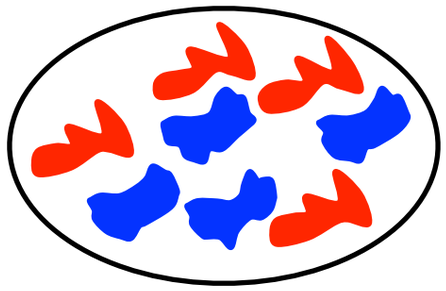
Classifier evaluation



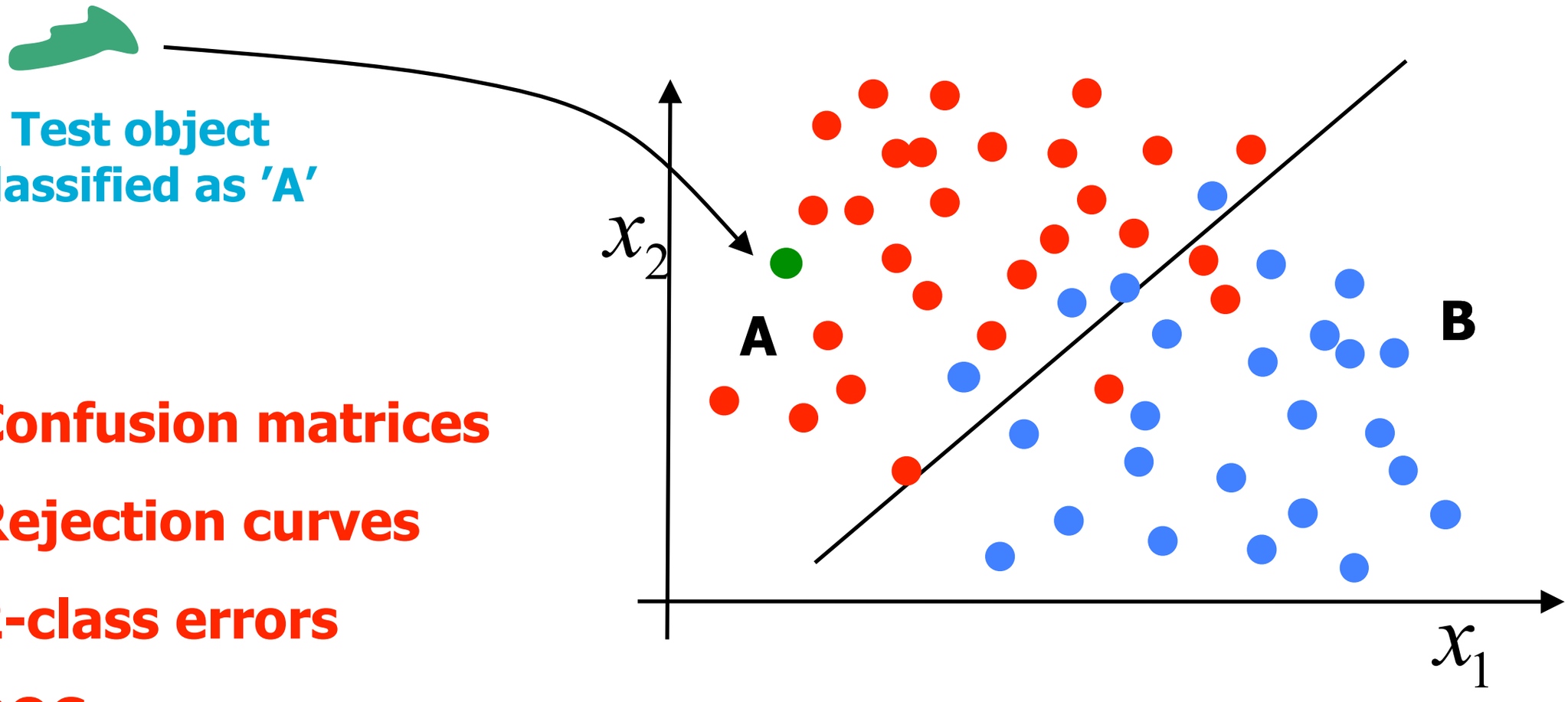
Sensor

Representation

Generalization

Feature Space

Classification



Confusion matrices

Rejection curves

2-class errors

ROC curves

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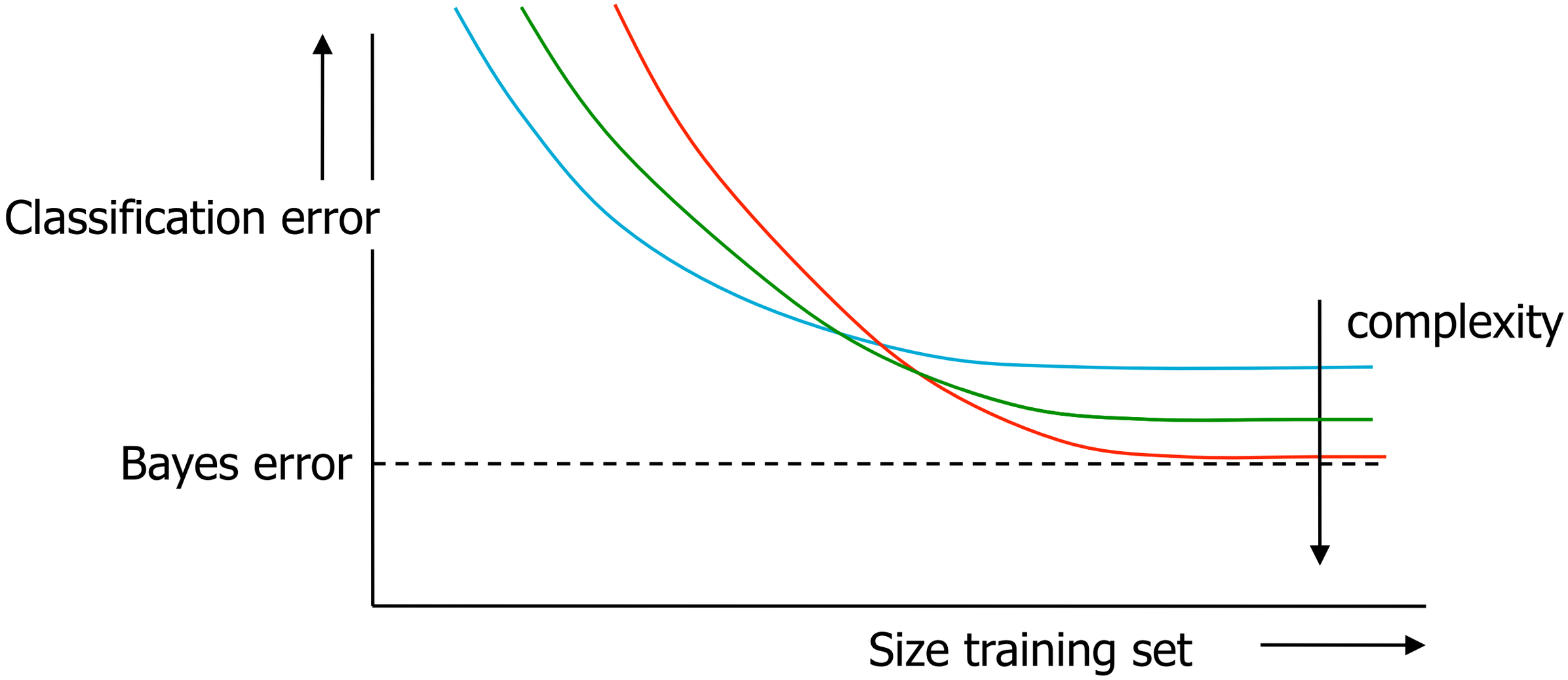
Classifier evaluation

- Last lecture:
  - Train and test set
  - Bootstrapping, cross-validation, leave-one-out
  - Learning curve
  - Classifier complexity and Bias-variance tradeoff
- This lecture:
  - Confusion matrices
  - Reject curve
  - ROC curve
  - Some final remarks
- Question hour

Different Classifier Complexity



Confusion Matrices

- Provides counts of class-dependent errors : How many object have been classified as A that should have been classified as B?

- Give a more detailed view than overall error rate

- Can be used to estimate overall cost for particular classifier

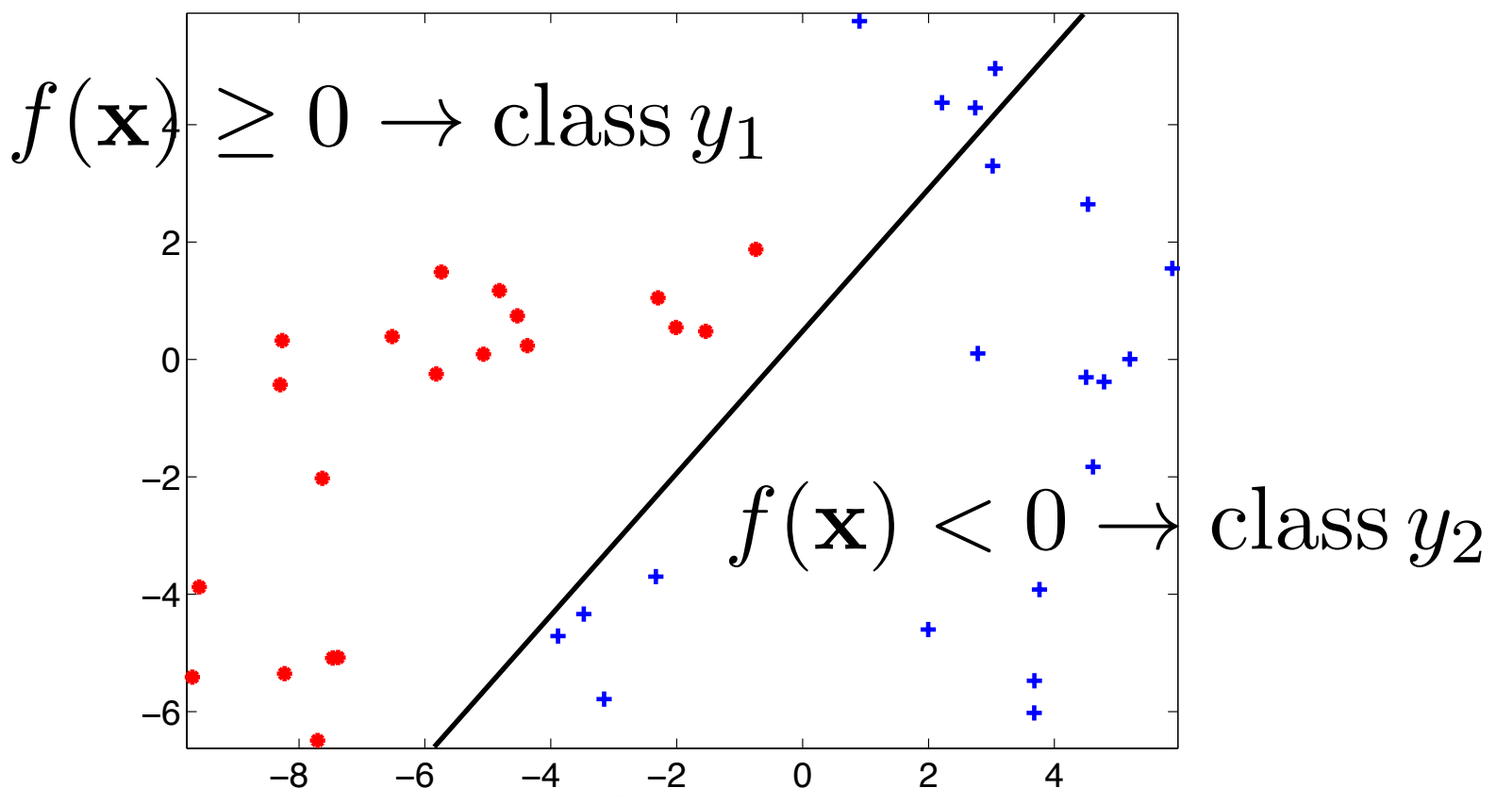
Confusion Matrix (1)

Real labels:

$$\Lambda = \begin{bmatrix} \lambda_1 \\ \dots \\ \lambda_N \end{bmatrix}$$

Predicted labels:

$$L = \begin{bmatrix} l_1 \\ \dots \\ l_N \end{bmatrix}$$



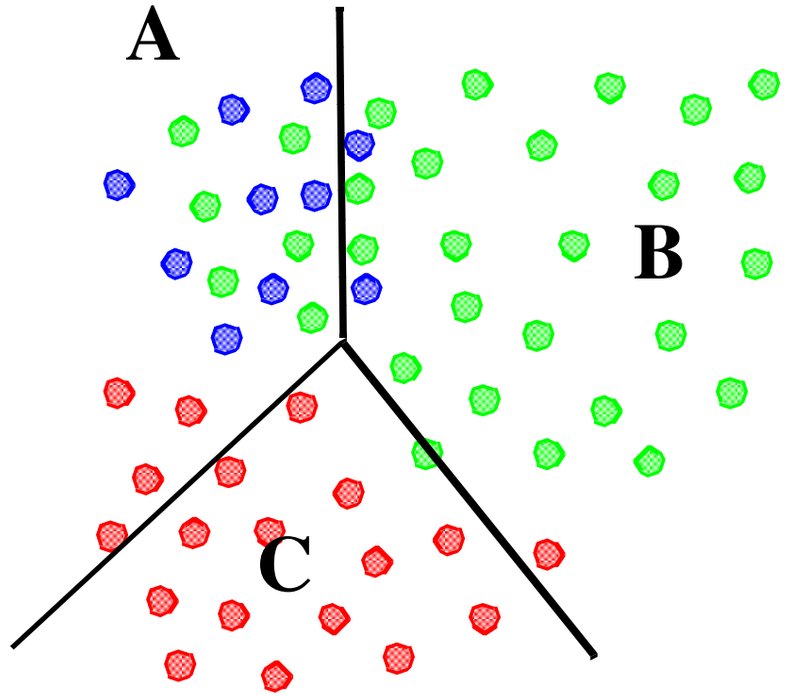
Confusion matrix:

$$C = \begin{bmatrix} c_{11} & \dots & c_{1K} \\ \dots & \dots & \dots \\ c_{K1} & \dots & c_{KK} \end{bmatrix}$$

$$c_{ij} = N \cdot P[l_j | \lambda_i]$$

Confusion Matrix (2)

$$N_A = 10,\ N_B = 30,\ N_C = 20$$



$$E = \frac{c_{12} + c_{13} + c_{21} + c_{23} + c_{31} + c_{32}}{N_A + N_B + N_C}$$

$$E = 14/60 = 0.2333$$

$$C = \text{confmat}(\Lambda, L)$$

Λ real labels

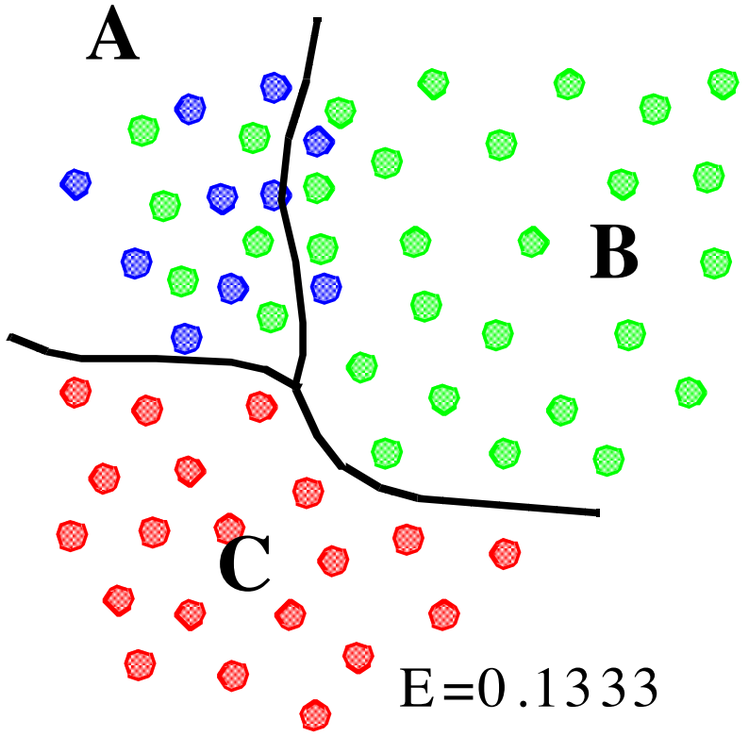
L obtained labels

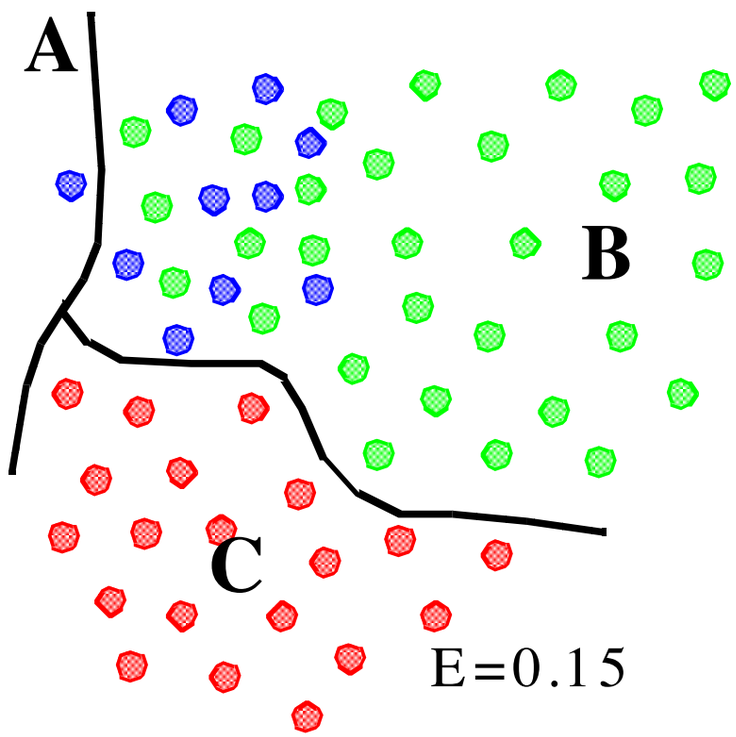
objects from	classified to A	B	C
class A	8	2	0
class B	6	23	1
class C	4	1	15

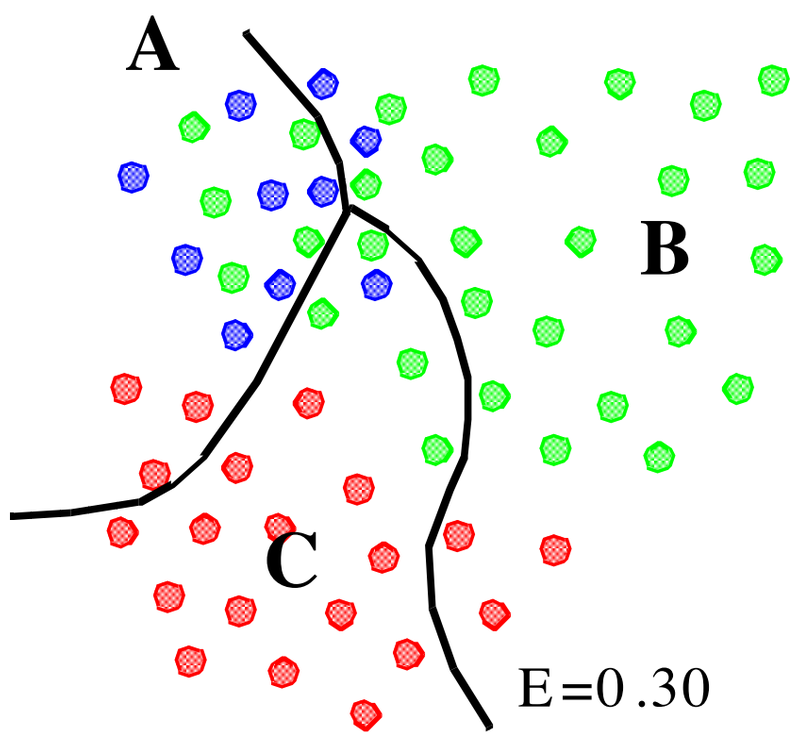
0.20 error in class A
0.23 error in class B
0.25 error in class C

0.228 averaged error

Confusion Matrix (3)







objects from

classified to

	A	B	C	
class A	8	2	0	0.20
class B	6	24	0	0.20
class C	0	0	20	0.00
				0.133

classified to

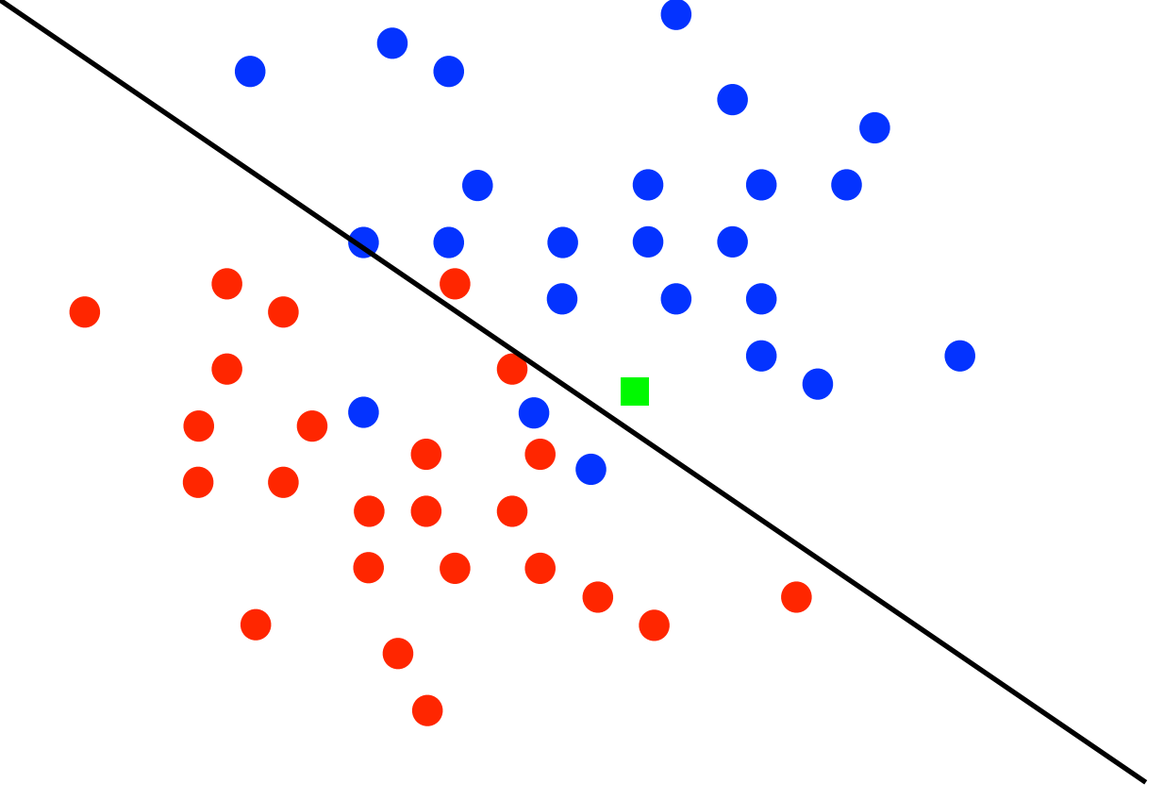
	A	B	C	
class A	1	9	0	0.9
class B	0	30	0	0.0
class C	0	0	20	0.0
				0.30

classified to

	A	B	C	
class A	7	2	1	0.30
class B	5	21	4	0.30
class C	3	2	15	0.30
				0.30

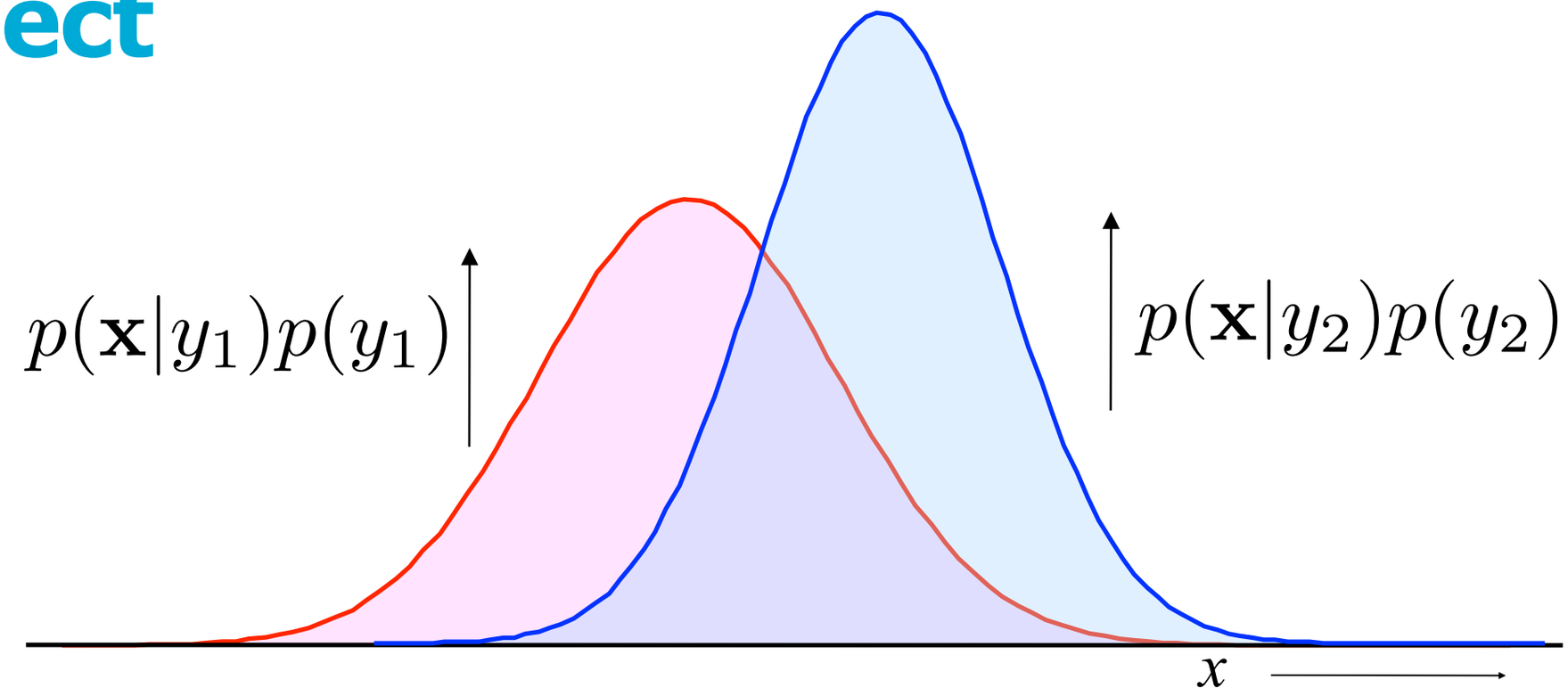
classification details are only observable in the confusion matrix!!

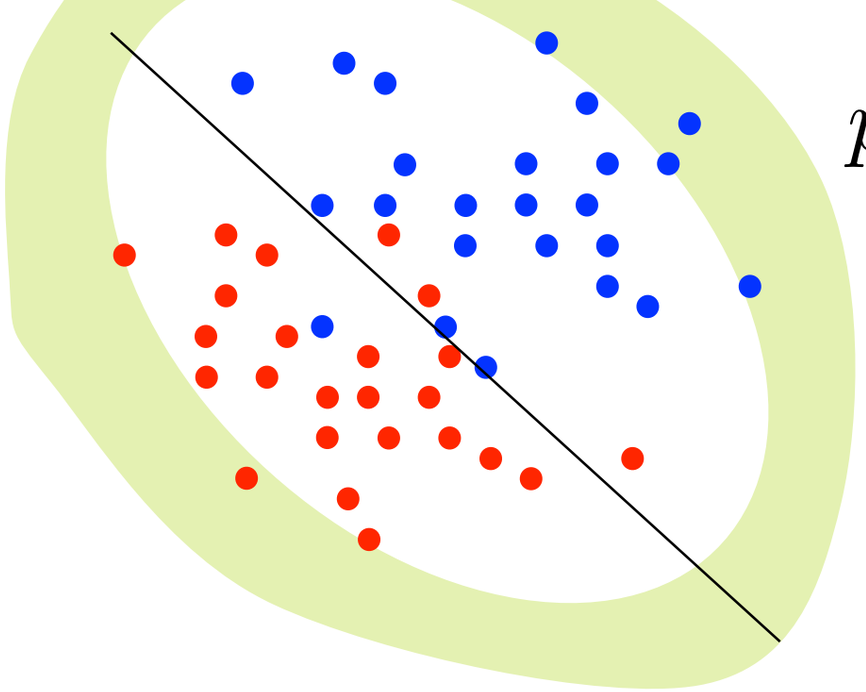
Rejection



Labels of Squares? How Sure Are We?

Outlier Reject

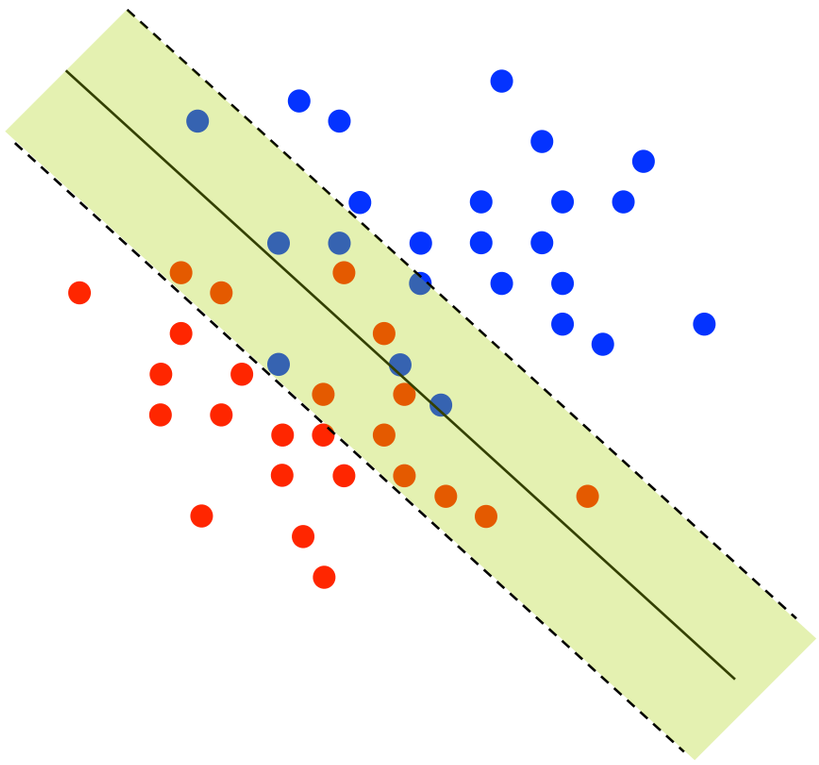




$$p(\mathbf{x}) = p(\mathbf{x}|y_1)p(y_1) + p(\mathbf{x}|y_2)p(y_2) \approx 0$$

Ambiguity Reject

- Reject objects for which classification is unsure, i.e., about equal posterior probabilities



$$p(y_1|\mathbf{x}) \approx p(y_2|\mathbf{x})$$

$$\frac{p(\mathbf{x}|y_1)p(y_1)}{p(\mathbf{x})} \approx \frac{p(\mathbf{x}|y_2)p(y_2)}{p(\mathbf{x})}$$

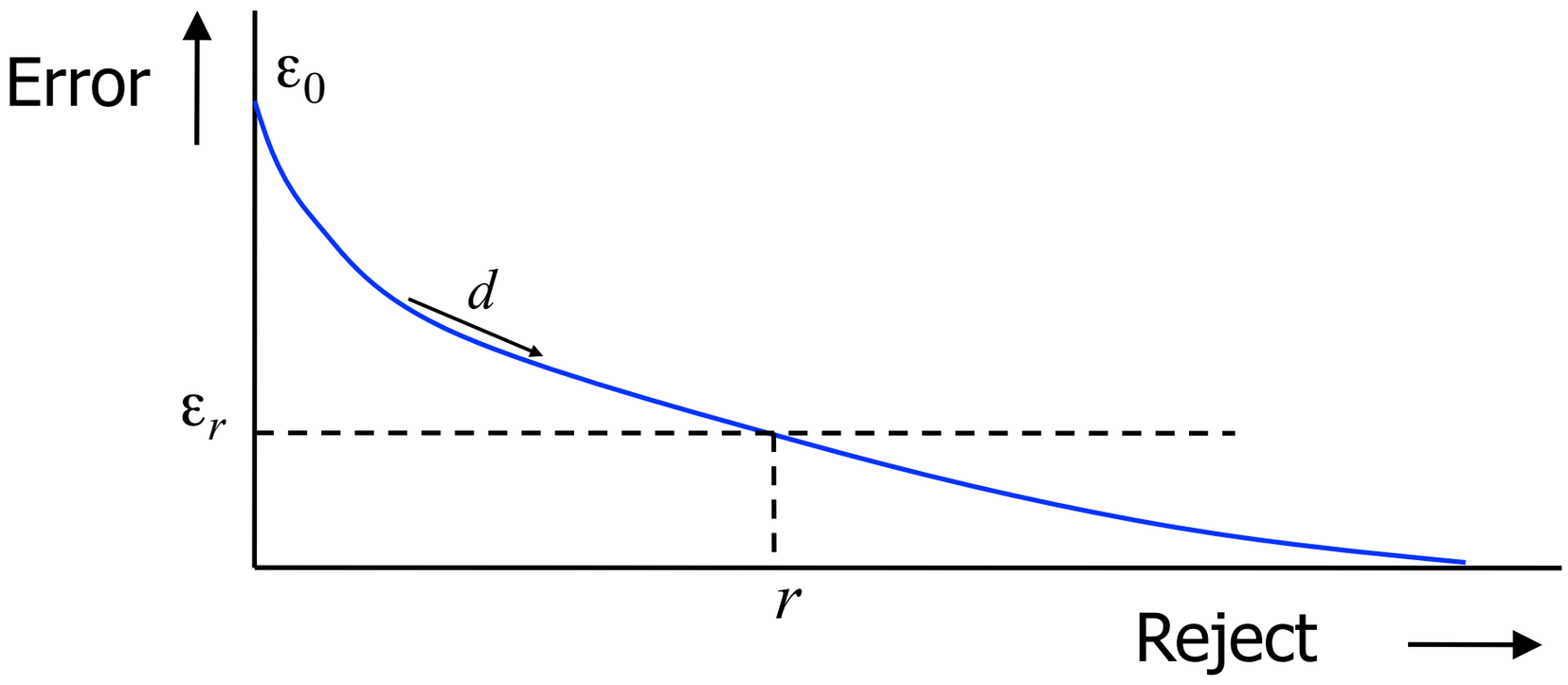
$$p(\mathbf{x}|y_1)p(y_1) - p(\mathbf{x}|y_2)p(y_2) \approx 0$$

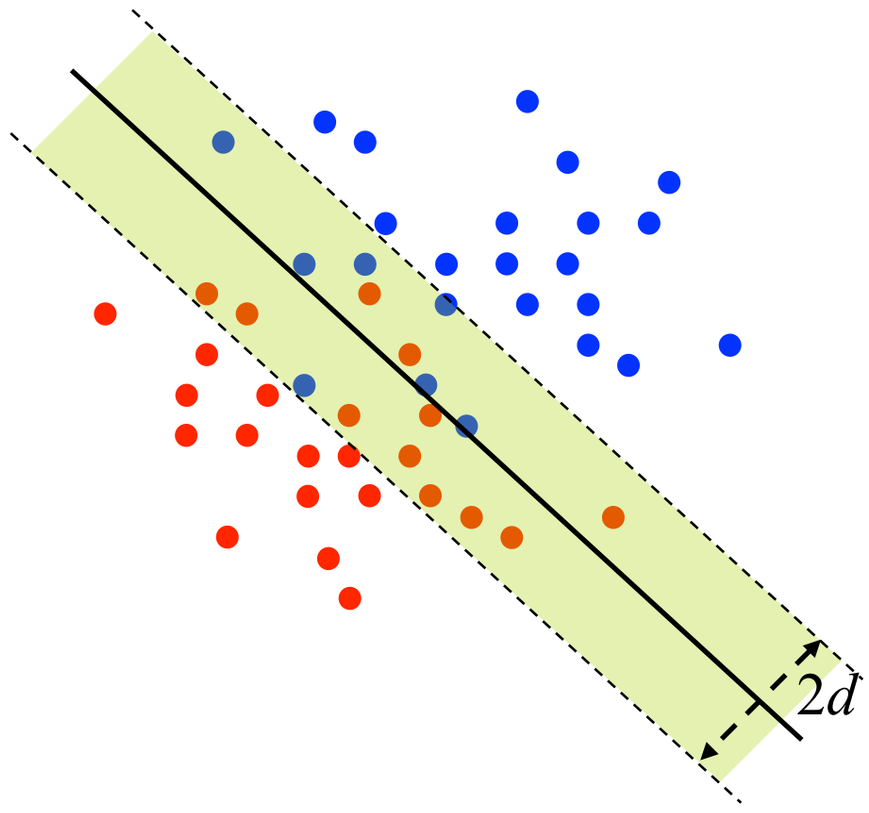
$$f(\mathbf{x}) \approx 0$$

Reject Curve

- Classification error ε_0 can be reduced to ε_r by rejecting a fraction r of the objects

- N.B. Rejection also costs!





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How much to reject?

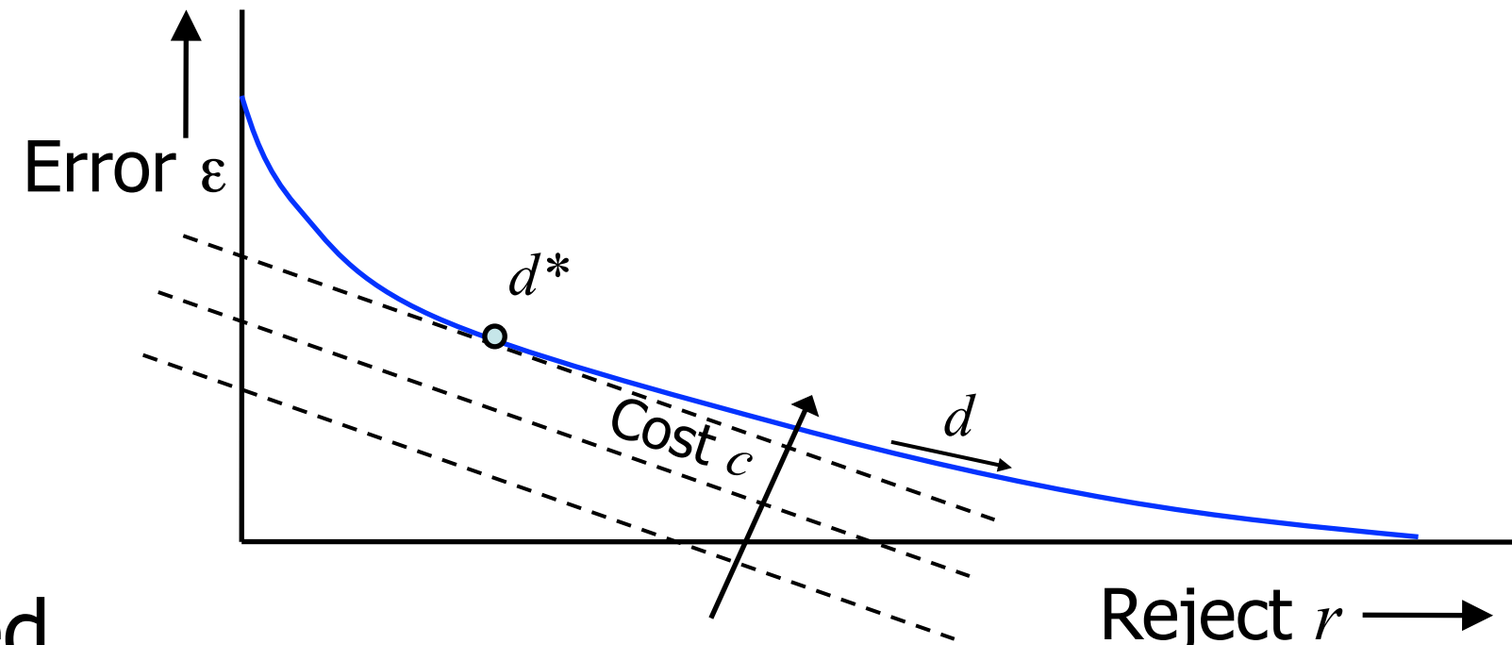
Compare the cost of a rejected object, c_r, with the cost of a classification error, c_ε:

$$c = c_r P(reject) + c_\varepsilon P(error)$$

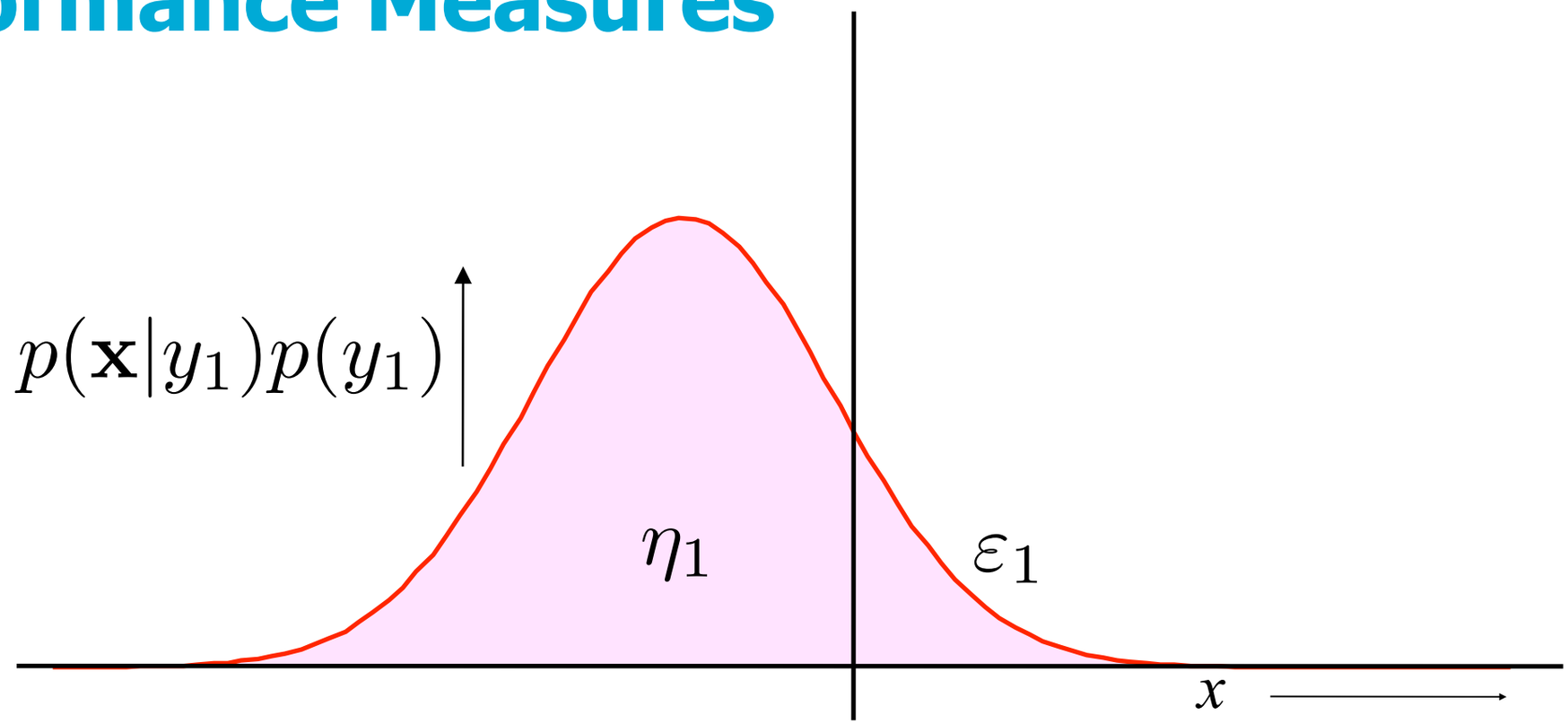
$$c = c_r r + c_\varepsilon \varepsilon$$

For given total cost c this is a linear function in the (r,ε) space.

Shift it until a possible operating point is reached.



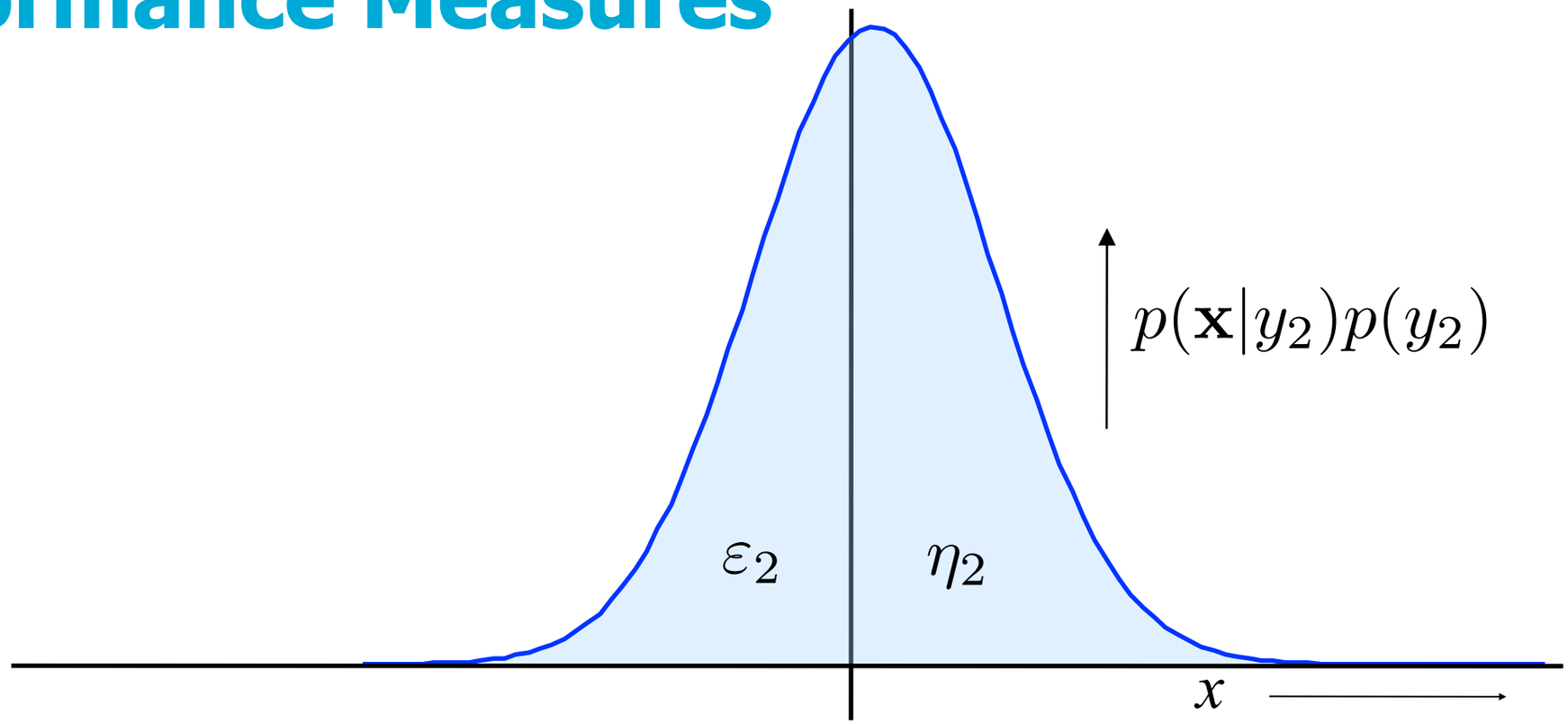
Error / Performance Measures



$S(x) > 0$ → Class 1 $S(x) = 0$ $S(x) \leq 0$ → Class 2

Objects of class 1, with prior $p(y_1)$, is classified by $S(x)$
in a part η_1, assigned to y_1, and a part ε_1, assigned to y_2.

Error / Performance Measures



$S(x) > 0$ → Class 1 $\quad$ $S(x) = 0$ $\quad$ $S(x) \leq 0$ → Class 2

Objects of class 2, with prior $p(y_2)$, is classified by $S(x)$
in a part η_2, assigned to y_2, and a part ε_2, assigned to y_1.

Error / Performance Measures

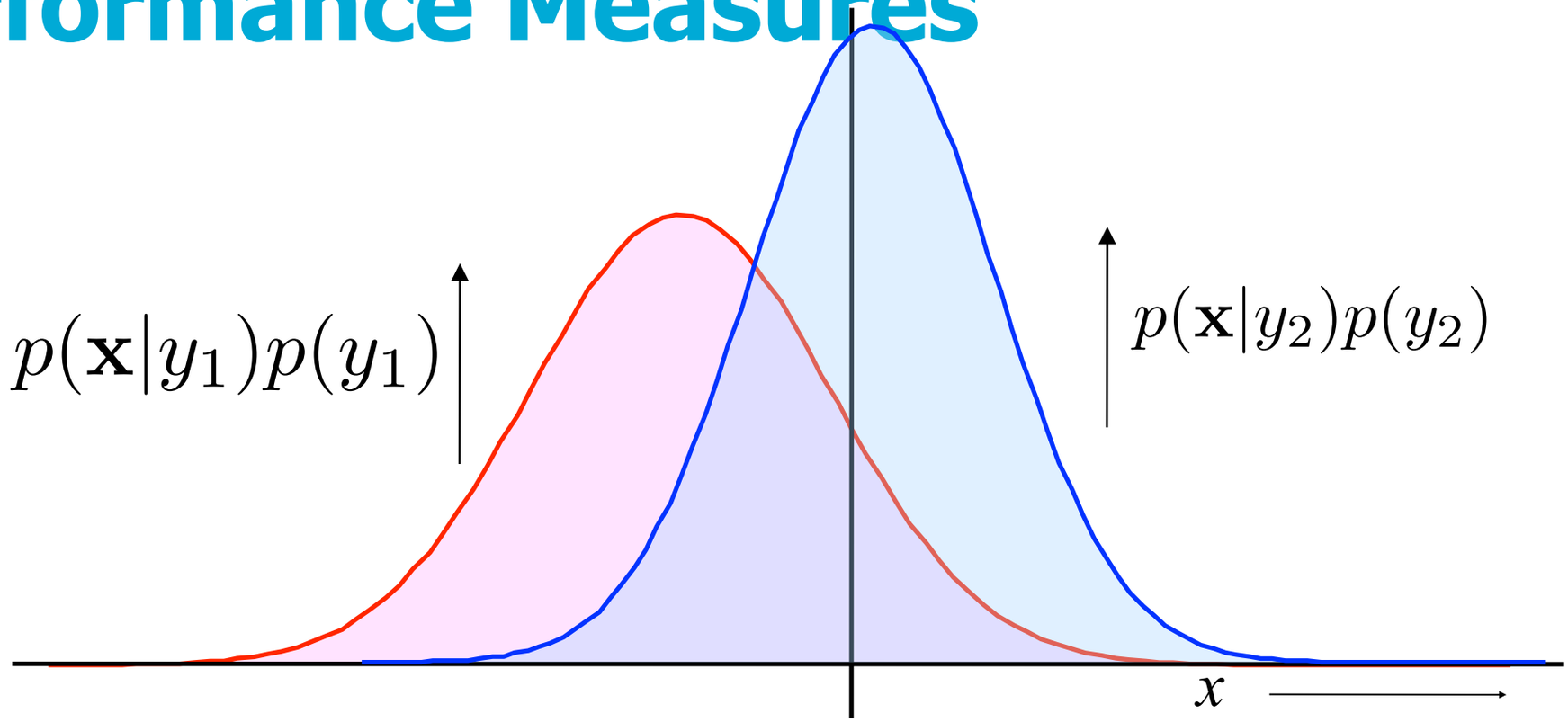
$\varepsilon = \varepsilon_1 + \varepsilon_2$

$\eta = \eta_1 + \eta_2$

$p(y_1) + p(y_2) = 1$

$p(y_1) = \eta_1 + \varepsilon_1$

$p(y_2) = \eta_2 + \varepsilon_2$



$S(x) > 0$ → Class 1 $S(x) = 0$ $S(x) \leq 0$ → Class 2

ε_1 : error contribution class A

ε_2 : error contribution class B

ε : classification error

η : performance / accuracy

$\varepsilon + \eta = 1$

$\varepsilon_A / p(y_1)$: error in class 1

$\varepsilon_B / p(y_2)$: error in class 2

$\eta_1 / p(y_1)$: sensitivity [recall] class 1

[what goes correct in class 1]

$\eta_1 / (\eta_1 + \varepsilon_2)$: precision class 1

[what belongs to A in what is classified as A]

Error / Performance Measures

- **Error** : probability of erroneous classifications
- **Performance / accuracy** : 1 – error
- **Sensitivity** of a target class [e.g. diseased patients] : performance for objects from that target class
- **Specificity** : performance for all objects outside target class
- **Precision** of a target class : fraction of correct objects among all objects assigned to that class.
- **Recall** : fraction of correctly classified objects; identical to sensitivity when related to particular class
- **True positive rate** : identical to sensitivity
- **False positive rate** : error for all objects outside target

Combining the two errors

- There is a fundamental tradeoff between the two errors/performances of the two classes

- Standard classification error: $\varepsilon = \varepsilon_1 p(y_1) + \varepsilon_2 p(y_2)$

- Weighted classification error:

$$\varepsilon = \lambda_{12}\varepsilon_1 p(y_1) + \lambda_{21}\varepsilon_2 p(y_2)$$

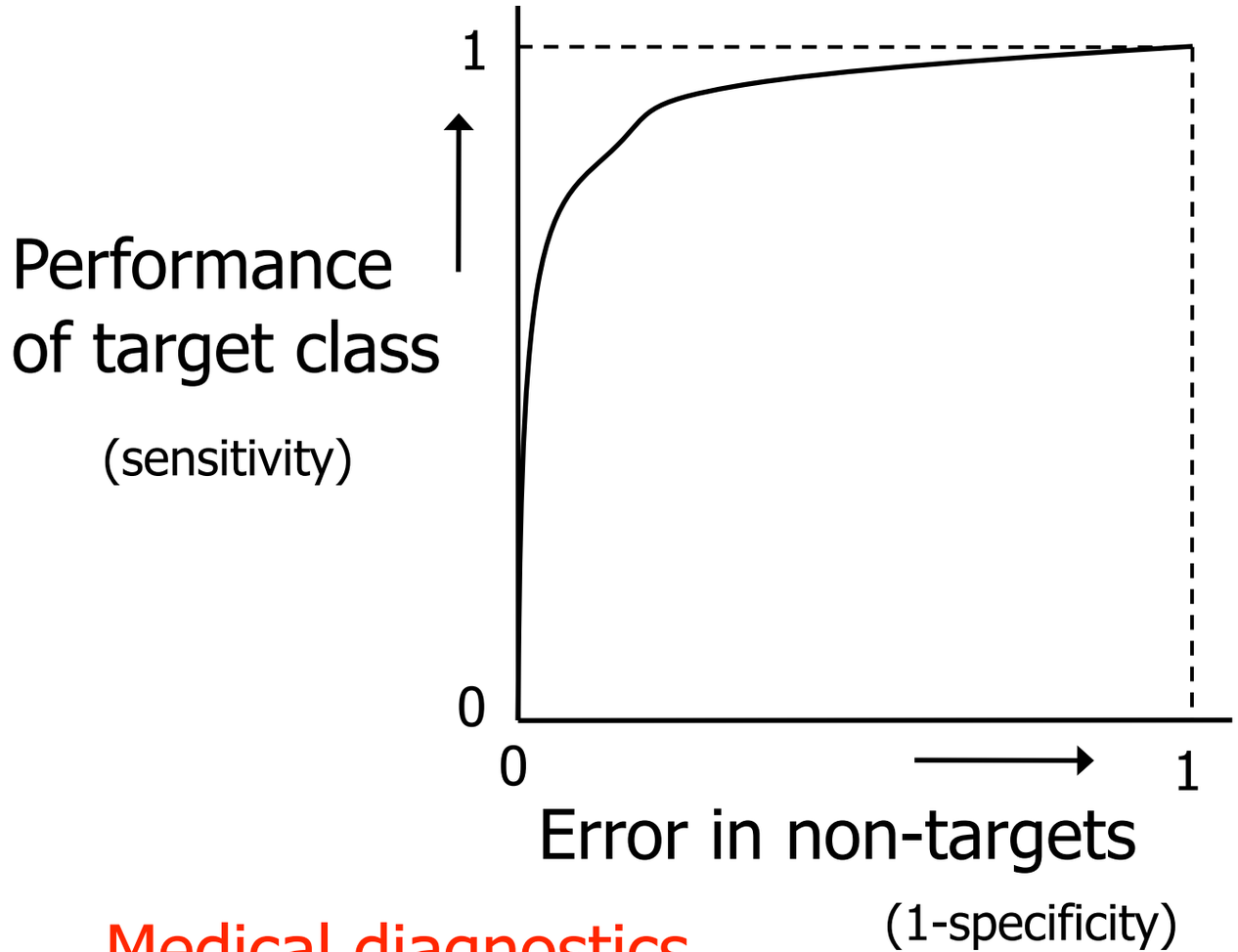
- F1-score (harmonic mean):

$$F_1 = 2\frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

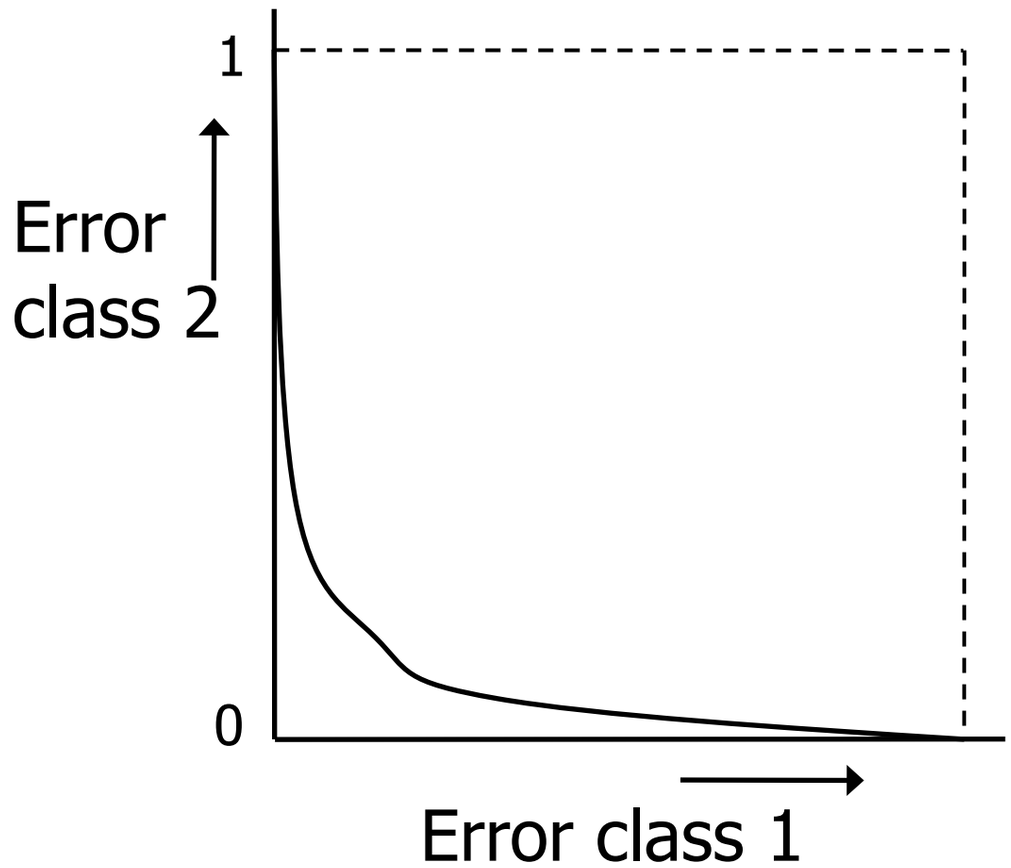
- ...

ROC Analysis

ROC: Receiver-Operator Characteristic (from communication theory)



Medical diagnostics
Database retrieval



2-class pattern recognition

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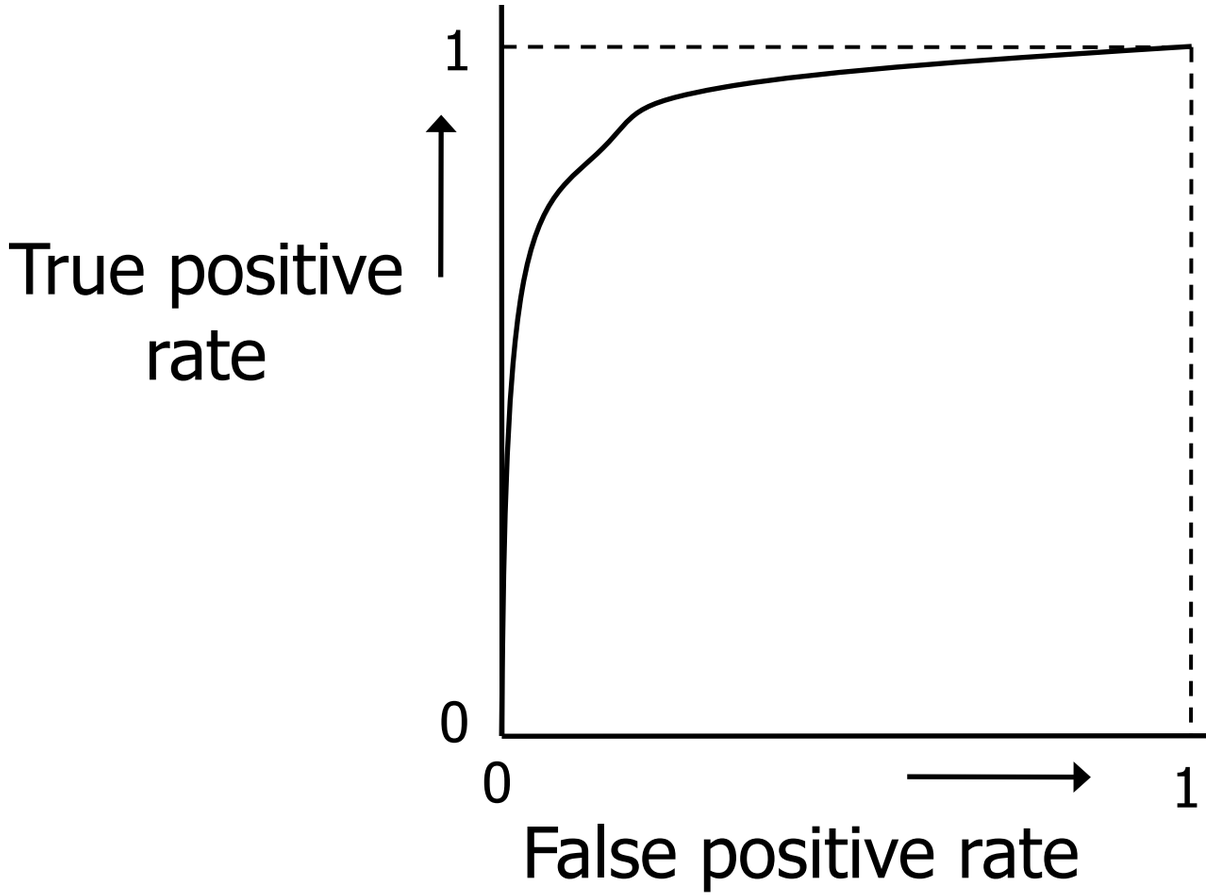
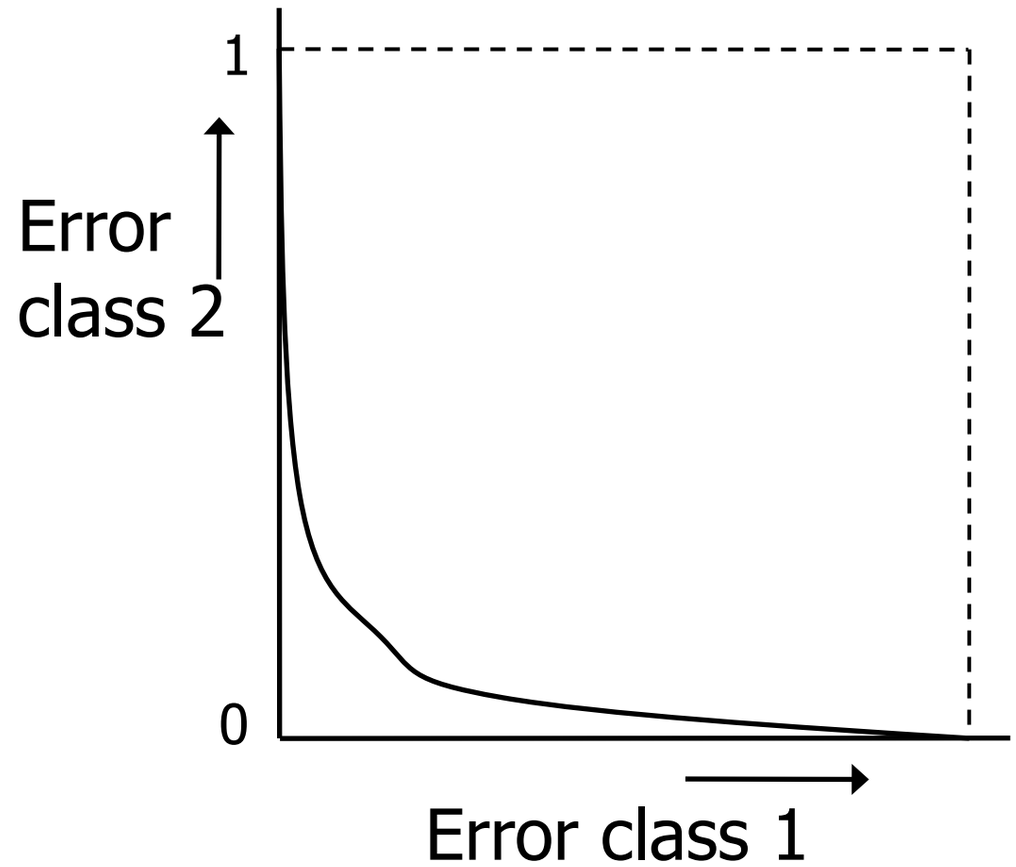
ROC Analysis

ROC: Receiver-Operator Characteristic (from communication theory)



Medical diagnostics
Database retrieval



2-class pattern recognition

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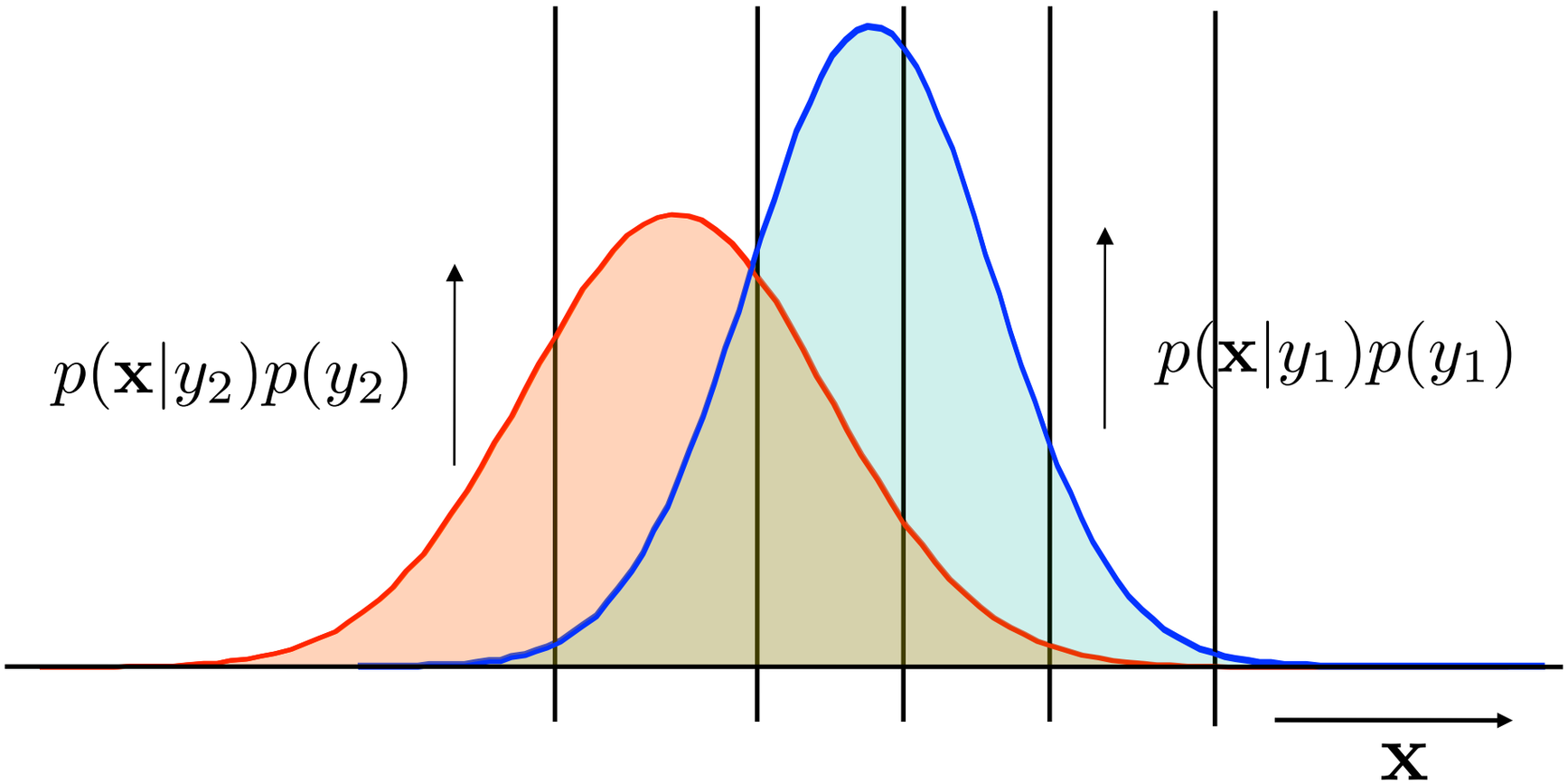
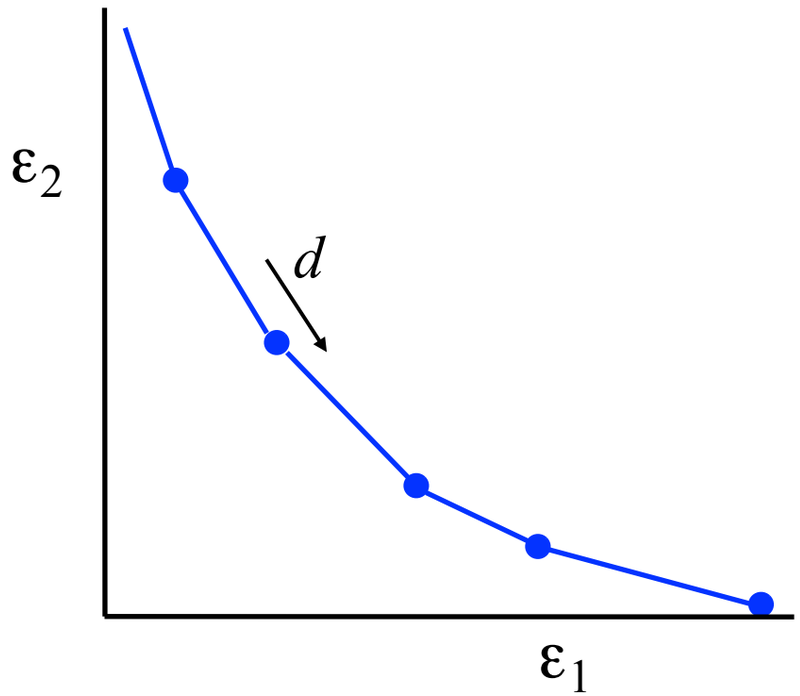
ROC Curve

- Curve is obtained by varying classifier threshold d





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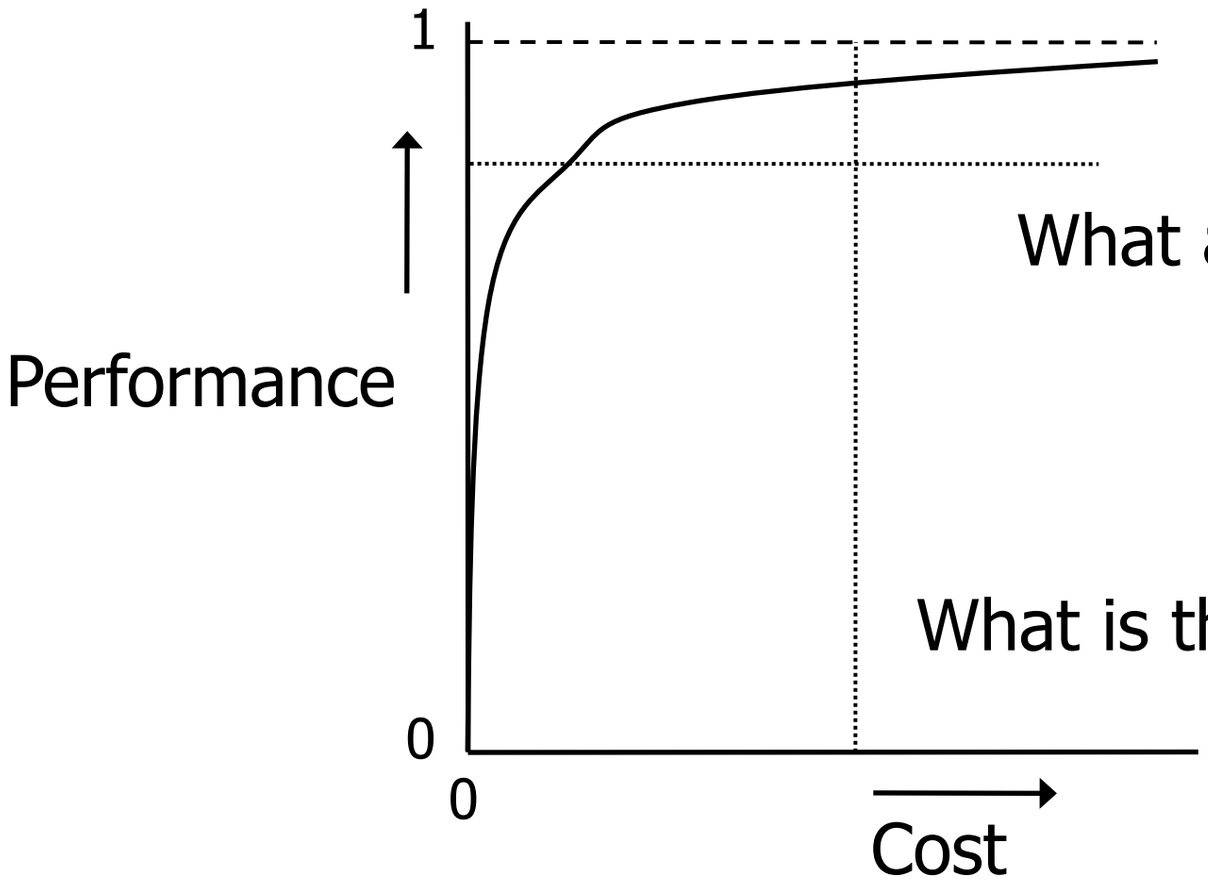
When are ROC Curves Useful?

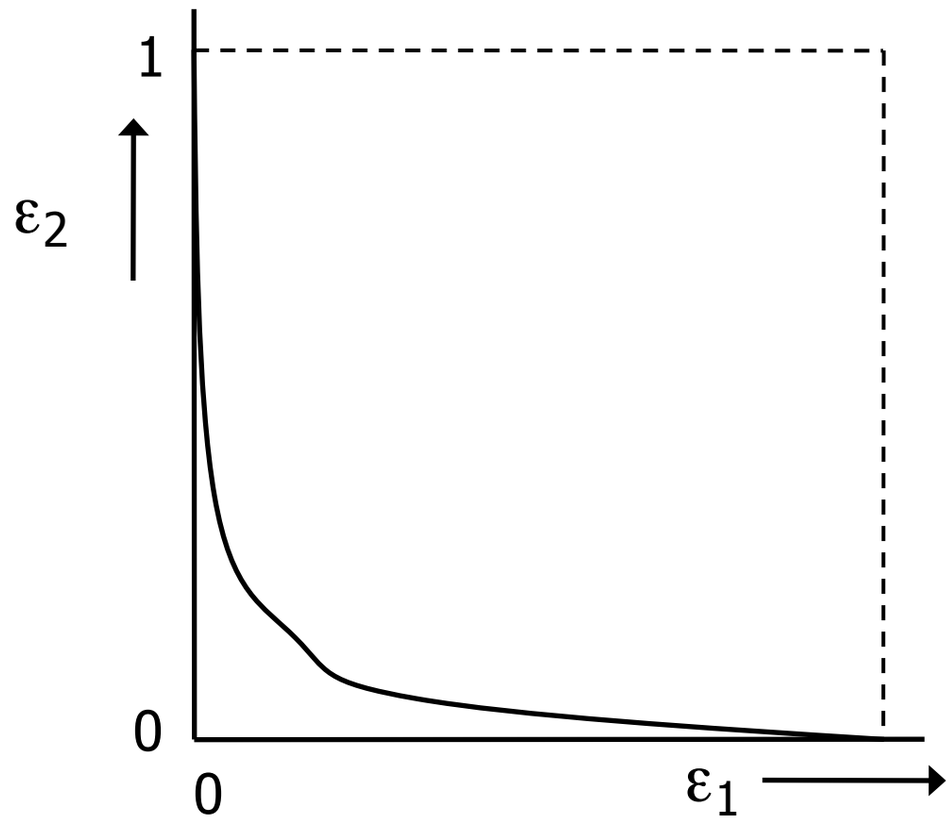


What are the cost of a preset performance?

What is the performance for given cost?

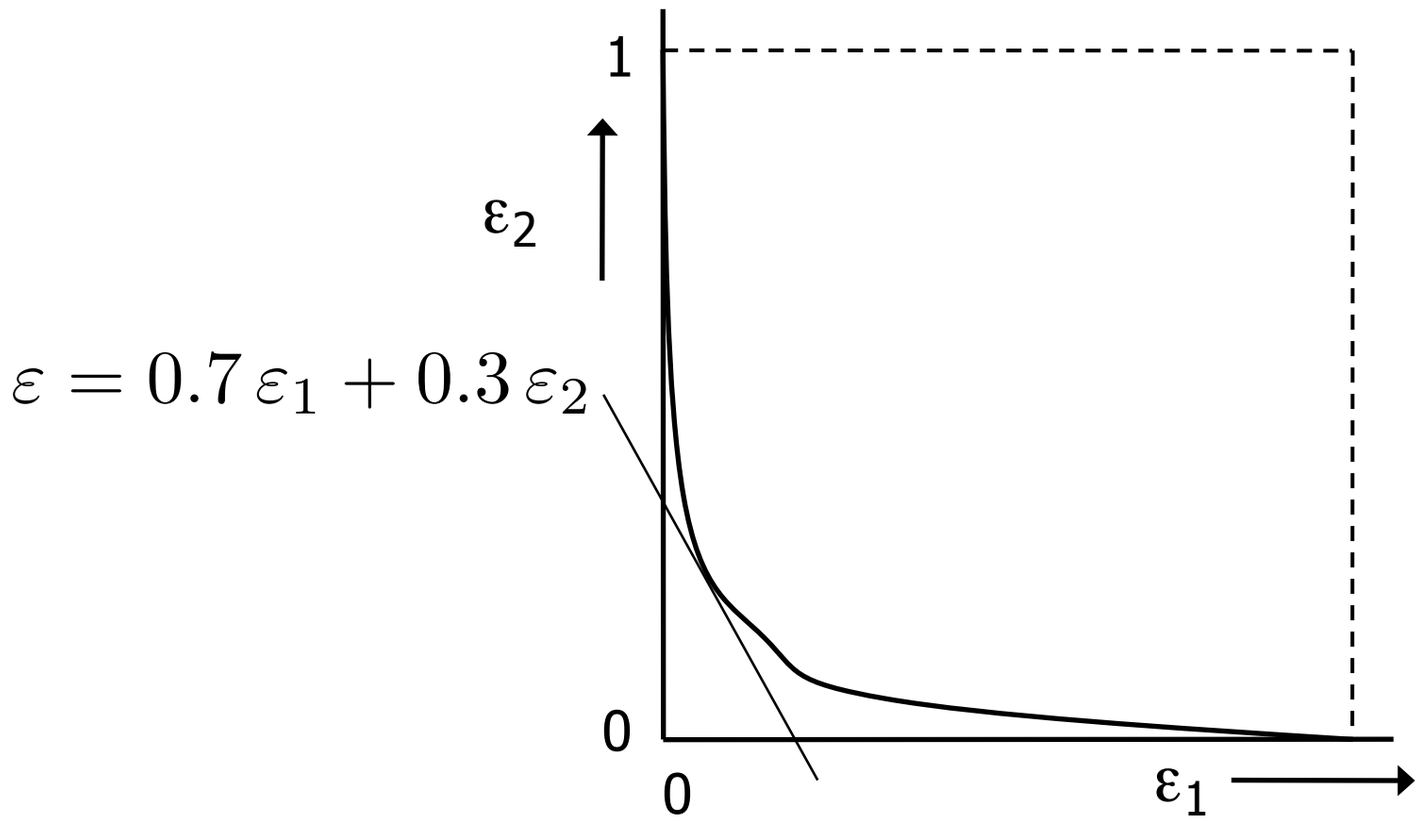
Trade-off between cost and performance.

Effect of Changing Priors/Costs



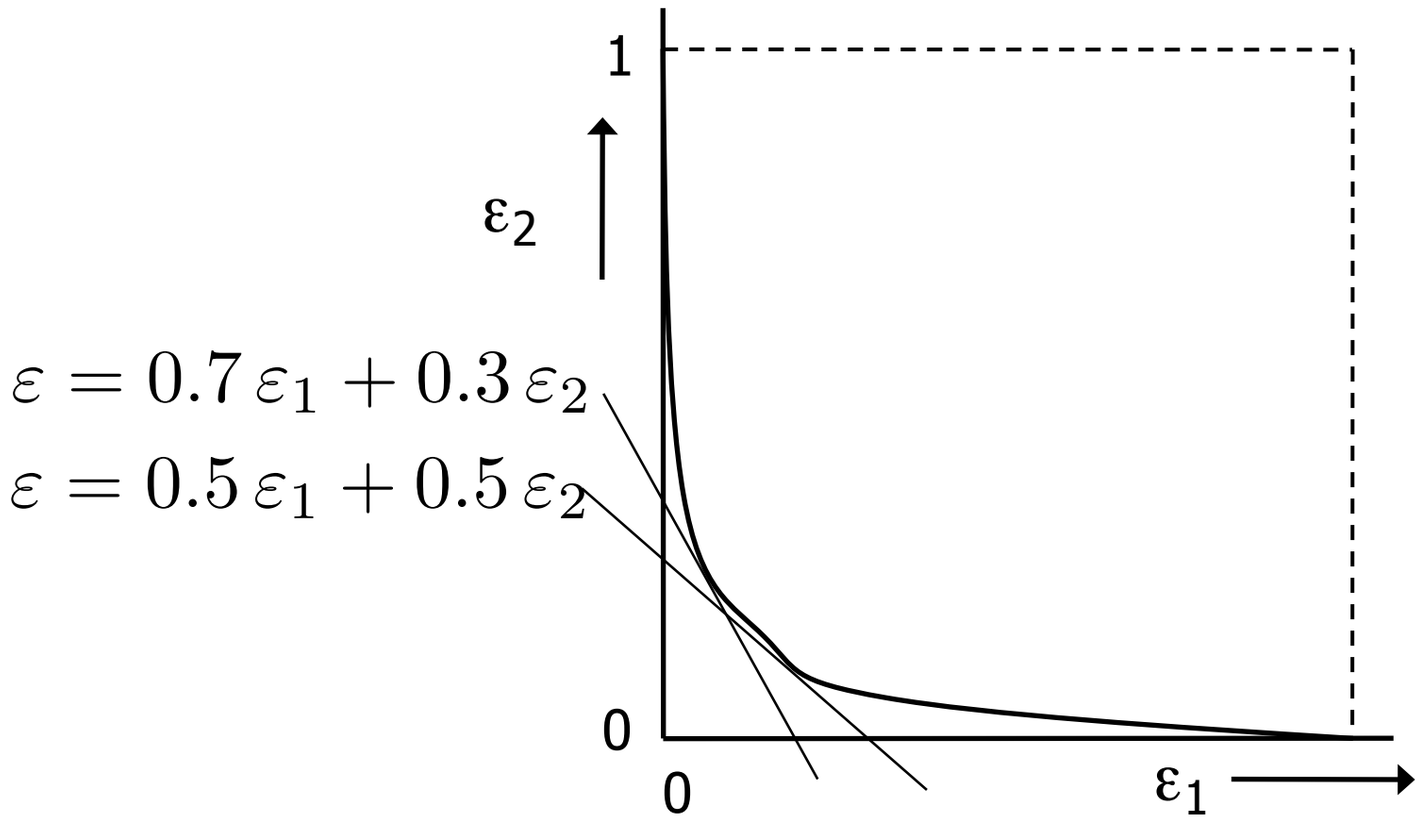
$$\varepsilon = p(y_1)\,\varepsilon_1 + p(y_2)\,\varepsilon_2$$

Effect of Changing Priors/Costs



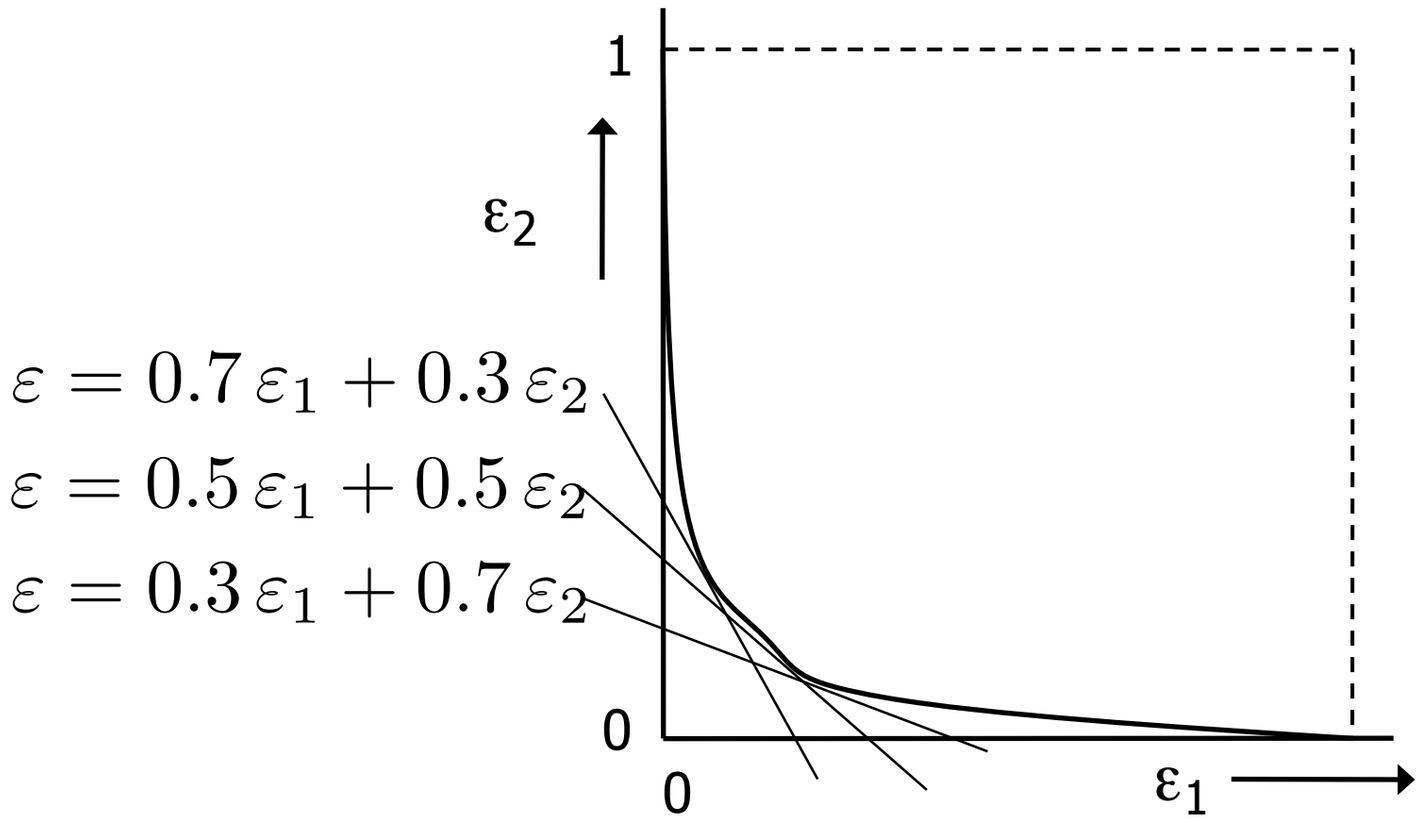
$$\varepsilon = p(y_1)\,\varepsilon_1 + p(y_2)\,\varepsilon_2$$

Effect of Changing Priors/Costs



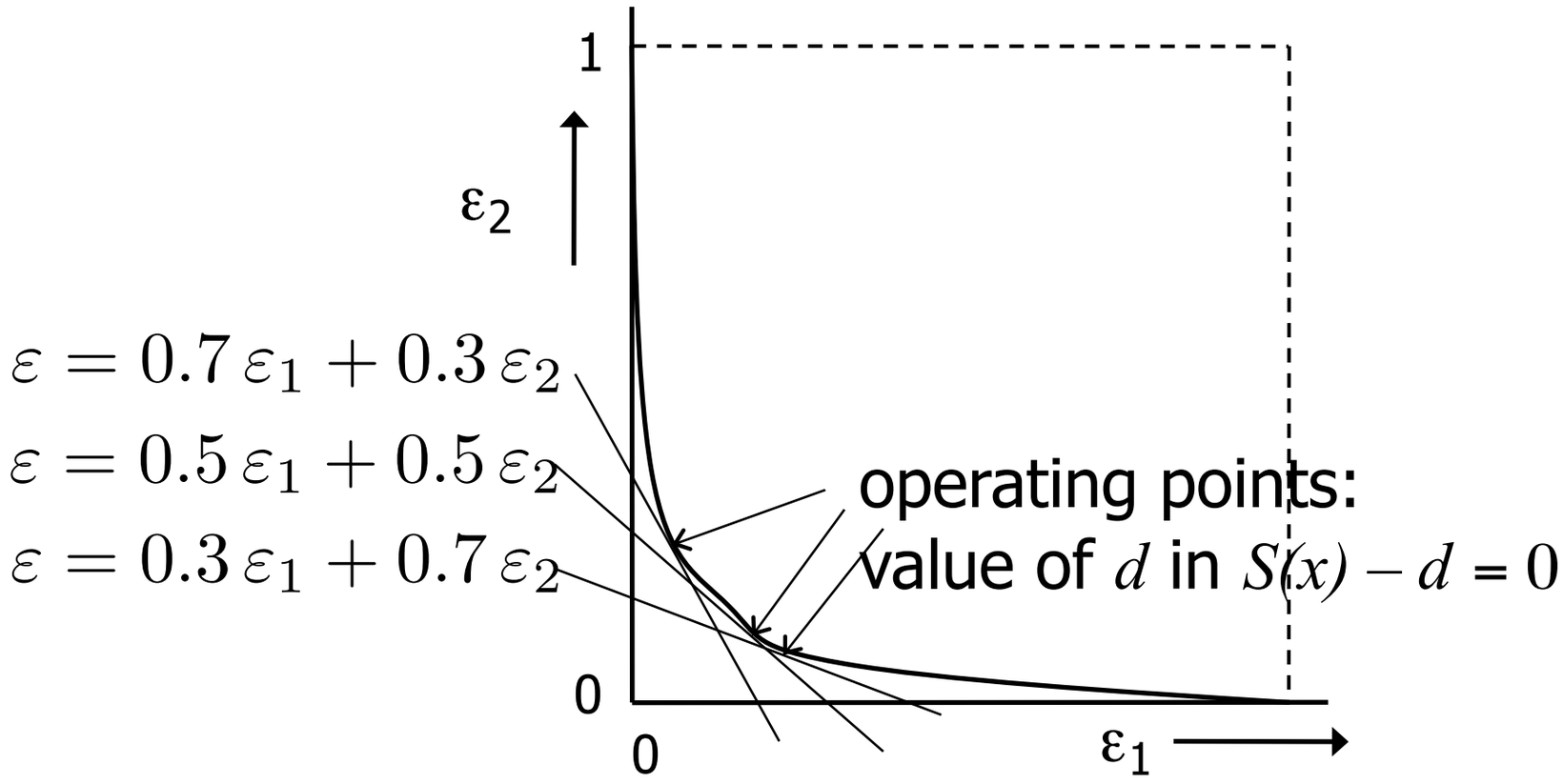
$$\varepsilon = p(y_1)\,\varepsilon_1 + p(y_2)\,\varepsilon_2$$

Effect of Changing Priors/Costs



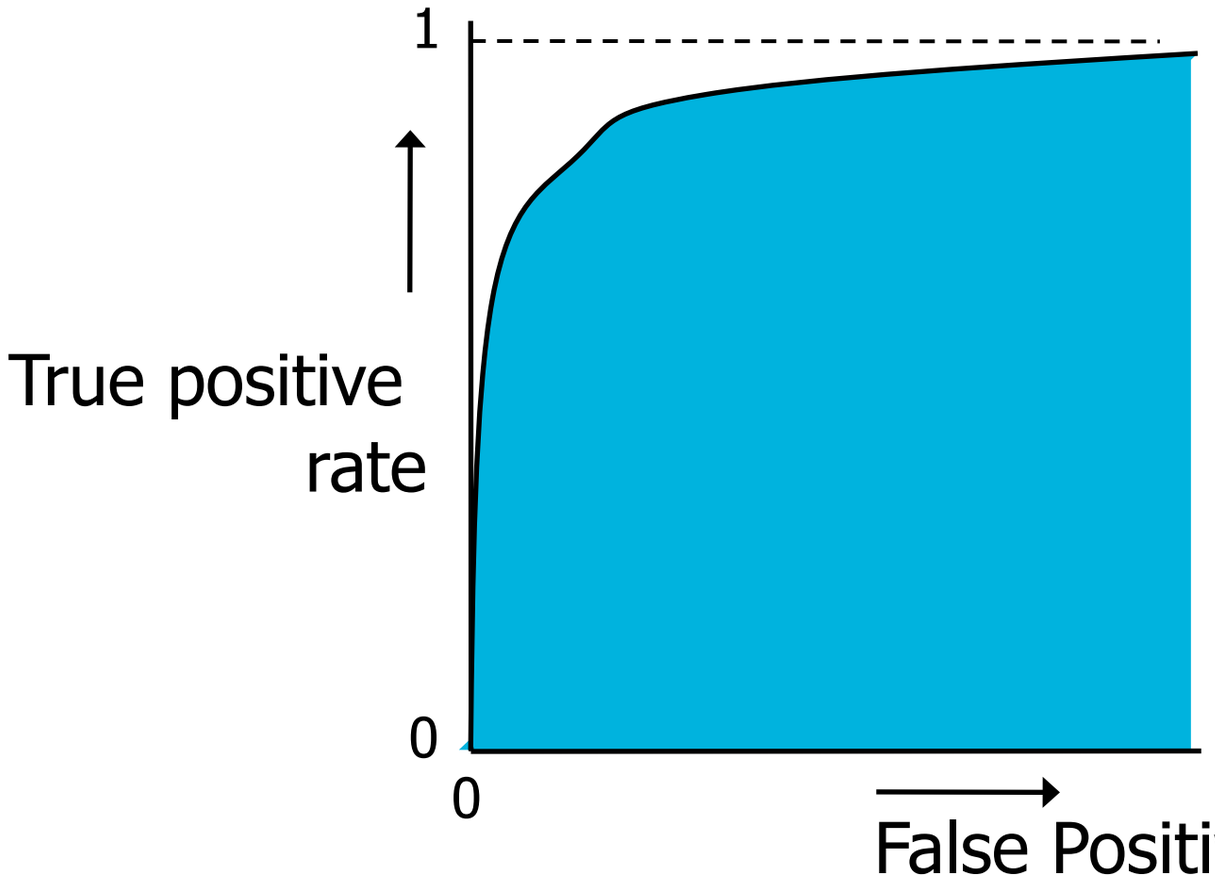
$$\varepsilon = p(y_1)\,\varepsilon_1 + p(y_2)\,\varepsilon_2$$

Effect of Changing Priors/Costs



$$\varepsilon = p(y_1)\,\varepsilon_1 + p(y_2)\,\varepsilon_2$$

Area under the ROC curve: AUC



- Integrate the area: perfect classifier gives AUC=1.0
- Random classifier gives AUC=0.5
- Performance measure that is insensitive to class priors

Example to compute ROC curve

- Assume I trained a classifier, and testing it on a test set:
- Four objects of class 1:

$$\hat{p}(y_1|\mathbf{x}_1) = 0.3$$
$$\hat{p}(y_1|\mathbf{x}_2) = 0.8$$
$$\hat{p}(y_1|\mathbf{x}_3) = 0.9$$
$$\hat{p}(y_1|\mathbf{x}_4) = 0.55$$

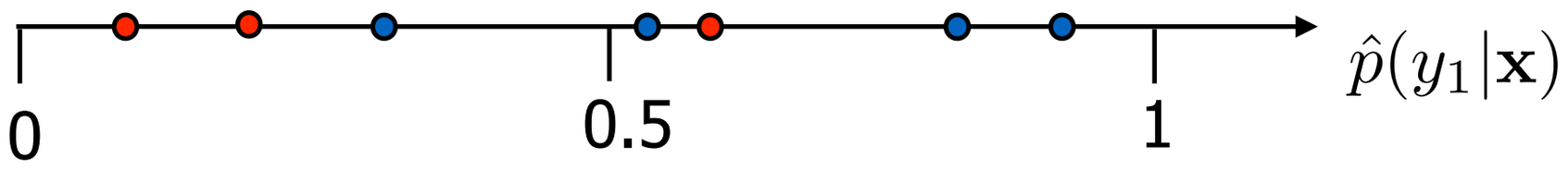
- Three objects of class 2:

$$\hat{p}(y_1|\mathbf{x}_5) = 0.2$$
$$\hat{p}(y_1|\mathbf{x}_6) = 0.1$$
$$\hat{p}(y_1|\mathbf{x}_7) = 0.6$$

- How does the ROC curve look like? ε_2 vs. ε_1

Example to compute ROC curve

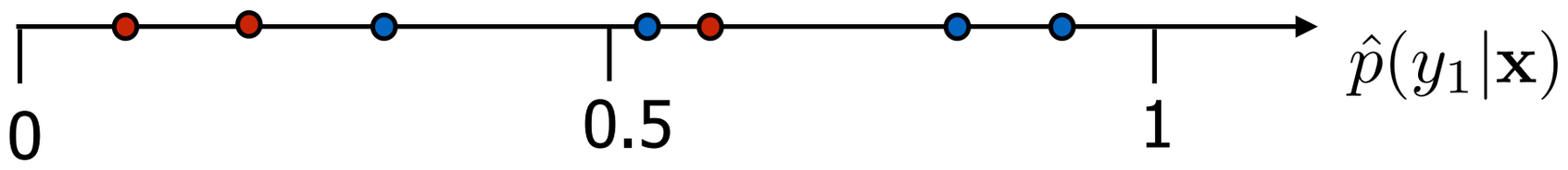
- Find out how obj's are classified for different thresholds



- For example, what are the errors for $d = 0.5$?

Example to compute ROC curve

- Find out how obj's are classified for different thresholds

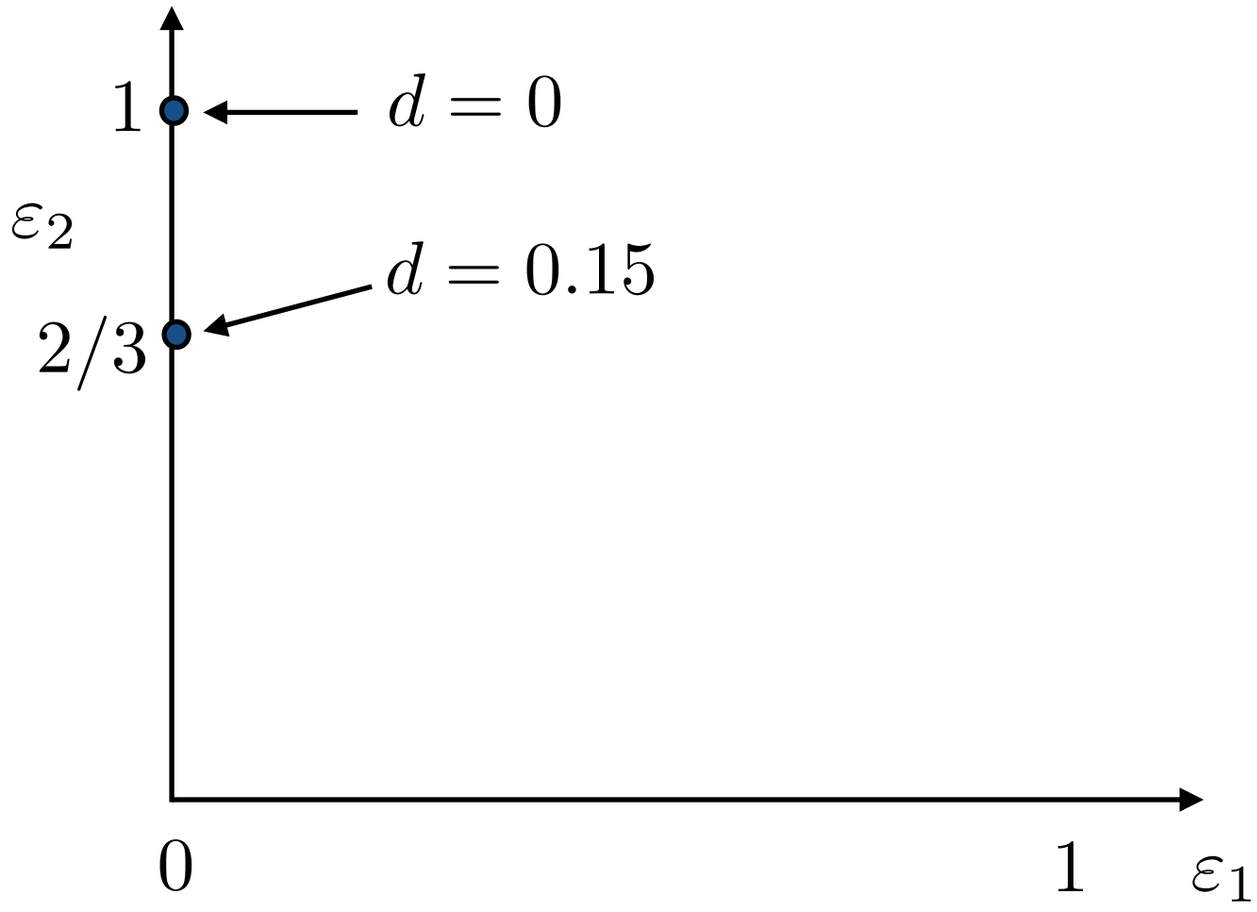


- For example, what are the errors for $d = 0.5$?

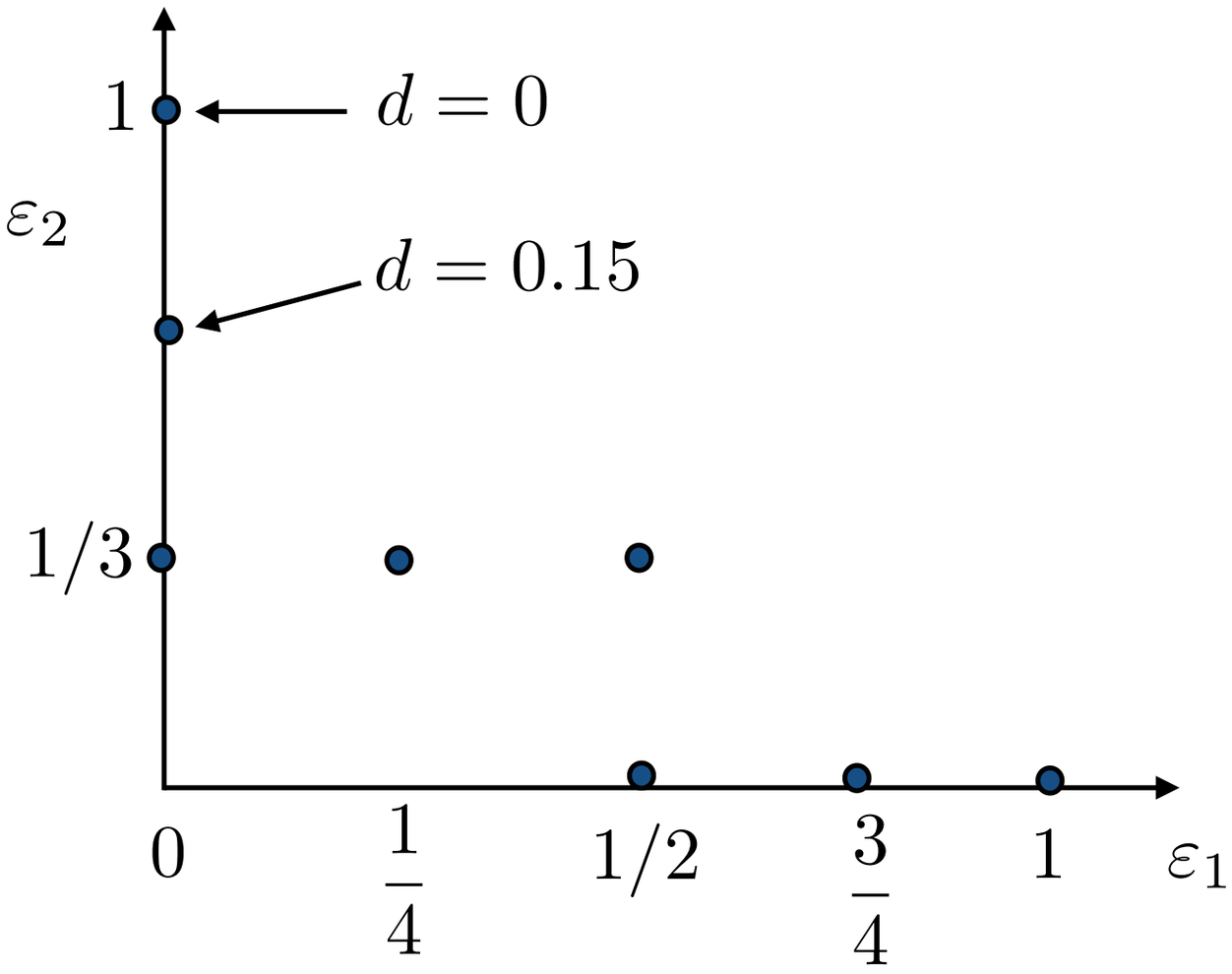
$$\varepsilon_1 = 1/4$$

$$\varepsilon_2 = 1/3$$

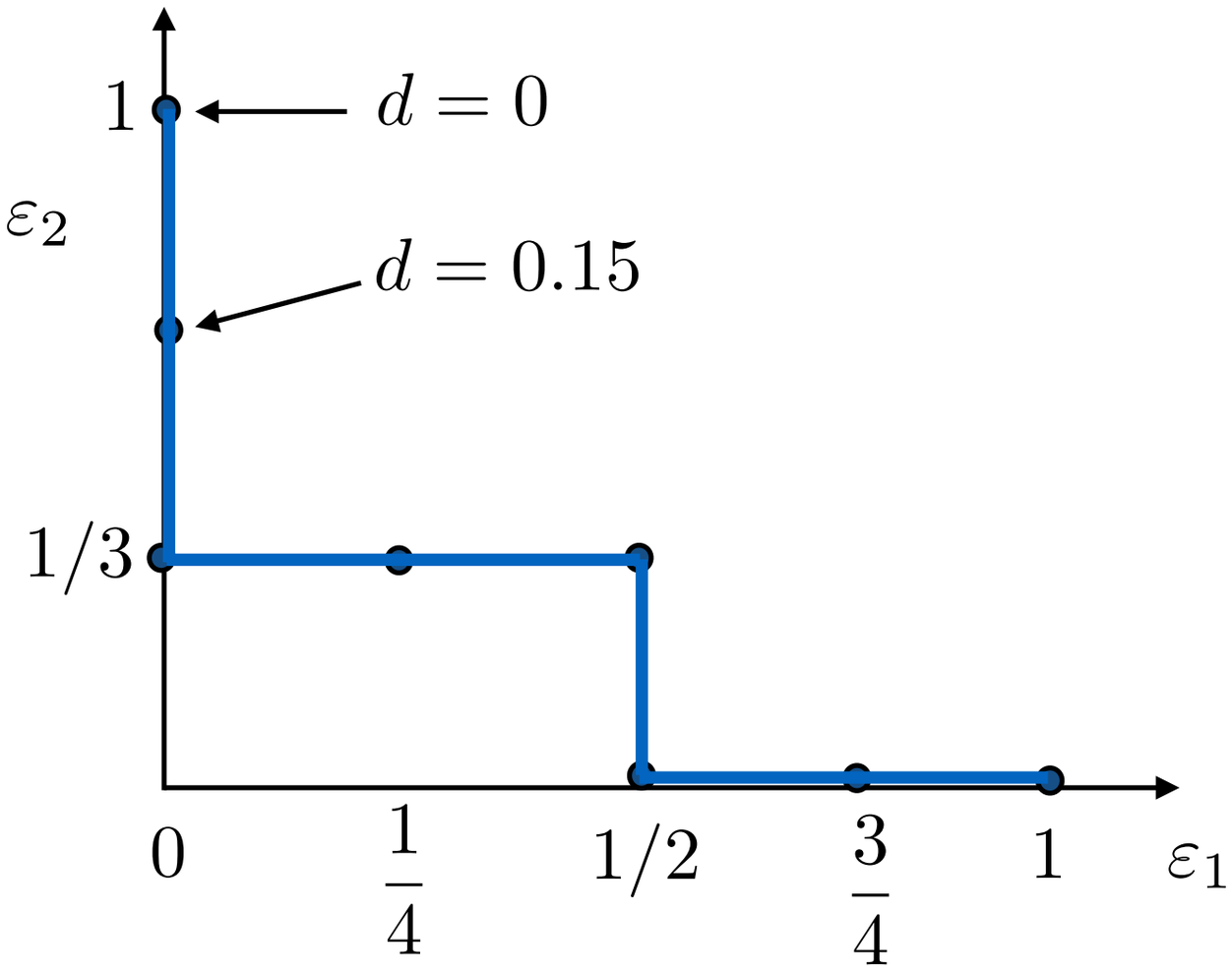
Example to compute ROC curve



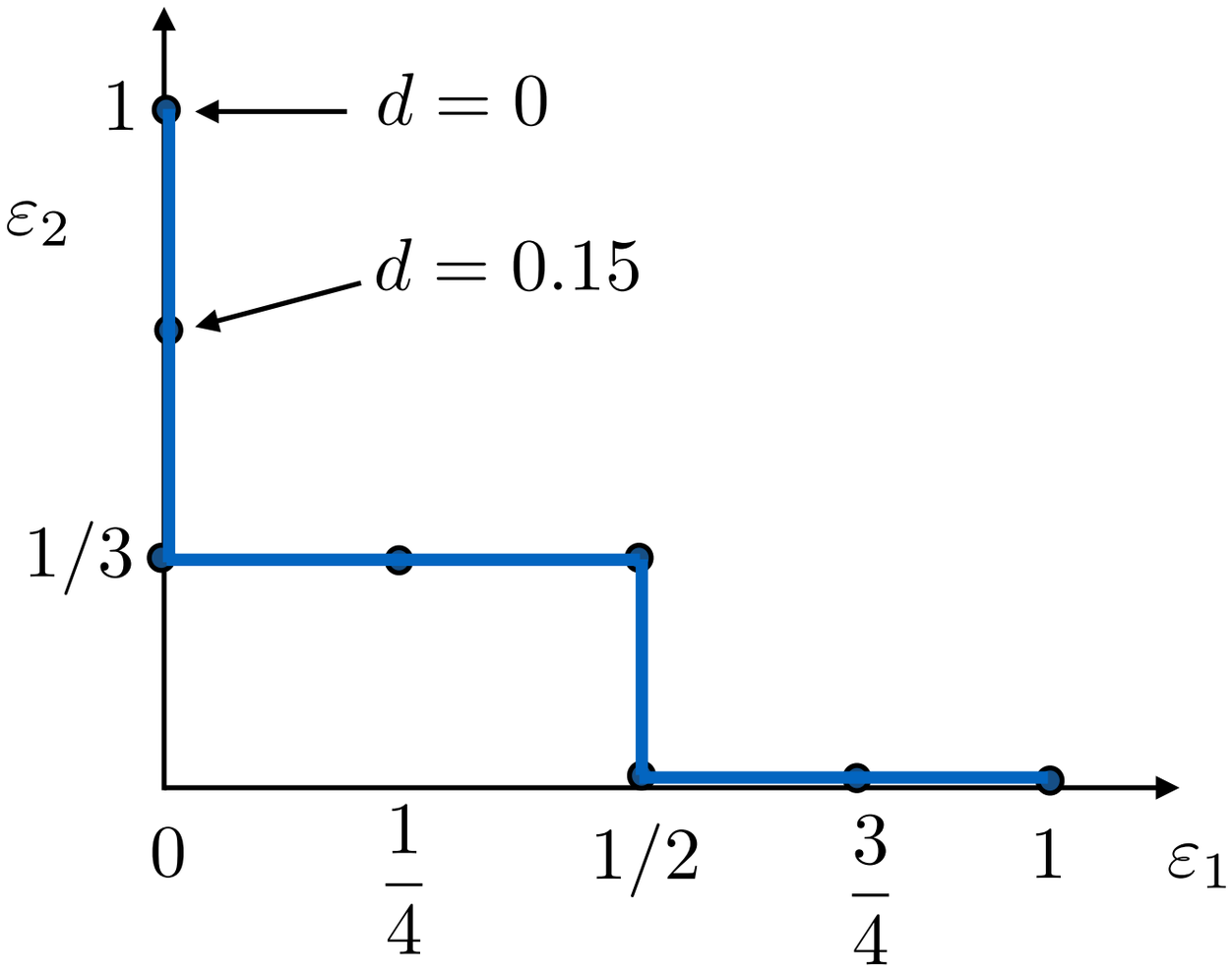
Example to compute ROC curve



Example to compute ROC curve



Example to compute ROC curve



Summary Reject and ROC

- Reject for solving ambiguity : reject objects close to the decision boundary → lower costs
- Reject option for coping with outliers
- ROC analysis in case of unknown or varying priors
- ROC analysis for comparing [and combining] classifiers

Presenting results

- If you present performance results, also give the standard deviation of the results!

method	error
A	0.124
B	0.125

- NO! Do:

method	error
A	0.124 (0.001)
B	0.125 (0.003)

- Also when you plot a graph!

Presenting results

- If you present performance results, mind the number of significant digits!

method	error
A	$0.124522197\ (0.00145)$
B	$0.1584423210006\ (0.003562245)$

- NO! Do:

method	error
A	$0.124\ (0.001)$
B	$0.158\ (0.004)$

Final conclusions

- It is possible to learn from examples!
- You need features, a model and a loss function

- No model is ultimately the best:
  depending on the amount of examples, more/less flexible models can be learned

- When your model is ‘well-biased’, less examples are needed to learn well