(1) Euclidean length: square root of the inner product with itself
In 2D plane that is the distance between two vectors

$$||\boldsymbol{x}|| = (\boldsymbol{x}'\boldsymbol{x})^{1/2} = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

Representing equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1,n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2,n}x_n &= b_2 \\ \vdots \qquad \vdots \qquad & \vdots \quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{n,n}x_n &= b_n \end{aligned}$$

These equations can be written in the more compact, matrix no

$$\boldsymbol{Ax} = \boldsymbol{b}$$

As a numerical example, suppose $\boldsymbol{b} = \{9, 7, 5, 3\}$ and $\boldsymbol{A}$ is a 4×4 matrix of random values that were generated uniformly between 0 and 1.

```
> (b <- c(9, 7 5, 3)
[1] 9 7 5 3
> (A <- matrix(runif(16),c(4,4)))
          [,1]      [,2]      [,3]      [,4]
[1,] 0.6692485 0.5644291 0.4361838 0.7869338
[2,] 0.9458449 0.4710188 0.8120588 0.5741354
[3,] 0.3366968 0.1971159 0.3774943 0.3954844
[4,] 0.1841985 0.9128133 0.3773530 0.2363638
> solve(A,b)
[1] -1.9162283 -0.5453288  2.7192532 11.9503542
```

(2x)Solving linear equations in R

(3) Inverse of a 3x3 matrix

The inverse of the 3×3 matrix

$$\boldsymbol{B} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

is given by

$$\boldsymbol{B}^{-1} = \frac{1}{\mathrm{Det}(\boldsymbol{B})} \begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix},$$

where $\mathrm{Det}(\boldsymbol{B})$ is the determinant given in (4.6) and

$$\begin{array}{lll} A = ei - fh & B = ch - bi & C = bf - ce \\ D = fg - di & E = ai - cg & F = cd - af \\ G = dh - eg & H = bg - ah & I = ae - bd \end{array}$$

Just to check, we can multiply the matrix **x** by its inverse:

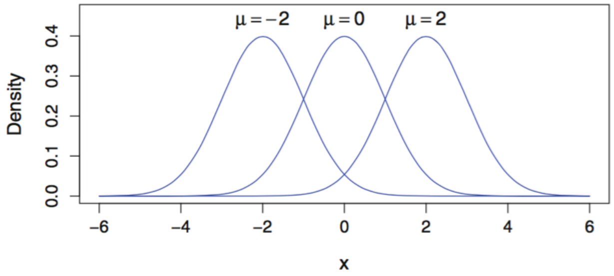
```
> x %*% solve(x)
              [,1]          [,2]         [,3]          [,4]          [,5]
[1,]  1.000000e+00 -6.661338e-16 1.110223e-16  4.163336e-17 -1.110223e-16
[2,]  1.665335e-16  1.000000e+00 5.551115e-17  2.775558e-17  0.000000e+00
[3,]  0.000000e+00 -2.220446e-16 1.000000e+00  1.387779e-16 -1.110223e-16
[4,]  1.110223e-16 -2.220446e-16 0.000000e+00  1.000000e+00 -1.110223e-16
[5,] -5.551115e-17  1.110223e-16 1.110223e-16 -1.387779e-17  1.000000e+00
```

Due to numerical error, identity matrix have non-zero value

An $n \times n$ matrix will have n eigenvalues. Just as a polynomial may have multiple roots, some of the eigenvalues may be repeated, corresponding to multiple roots[4] of $p(\lambda) = 0$. A polynomial may have complex roots, and the eigenvalues may be *complex numbers*. Complex numbers are expressible as $\alpha + \beta i$ where $i^2 = -1$. Complex roots will appear as *complex conjugates*. That is, if $\alpha + \beta i$ is a complex root of a polynomial with real-valued coefficients, then $\alpha - \beta i$ will also be a root of this polynomial.

The density function of the *standard normal distribution* is

$$\phi(x) = (2\pi)^{-1/2} \exp(-x^2/2) .$$

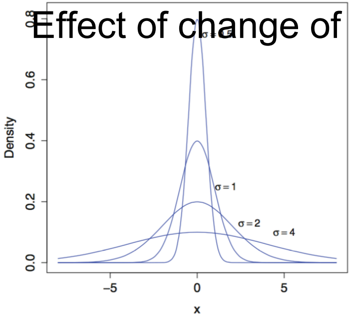


Normal distribution density function

This more general and useful form of the distribution of $Z = \mu + \sigma X$ defined for $\sigma > 0$ has density function

$$\phi(z \mid \mu, \sigma) = \sigma^{-1}(2\pi)^{-1/2} \exp\{-(z-\mu)^2/2\sigma^2\} . \qquad (5.2)$$

Effect of change of sigma



Lesser value of sigma is steeper

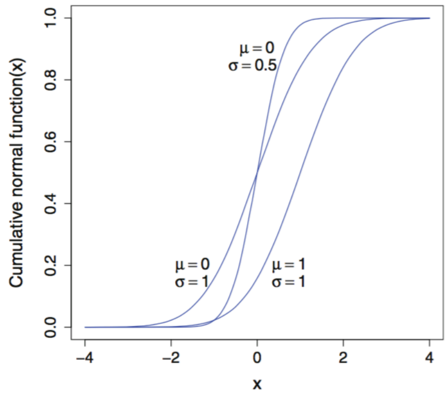
Cumulative distribution

$$\Phi(x) = \int_{-\infty}^{x} \phi(t)\, dt$$

denote the cumulative probability up to the value x.

The tail areas are symmetric so that

$$\Phi(-x) = 1 - \Phi(x)$$



The cumulative normal distribution function (5.3) for means μ and standard deviations σ as given

We would expect that these ordered, standardized values to obey

$$\Phi(x_1) = 1/(n+1), \quad \Phi(x_2) = 2/(n+1), \quad \cdots \quad , \Phi(x_n) = n/(n+1)\,. \tag{5.6}$$

theoretical, expected sample values should be spread out evenly with all n observations having equal cumulative

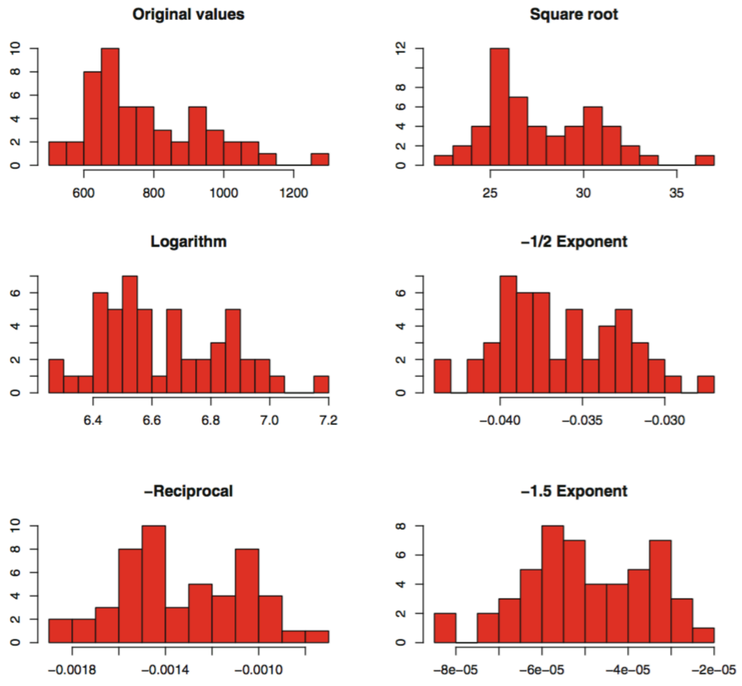


Figure 5.5: Histograms of the median apartment rents in Table 1.1. The data has been transformed using the power transformation (5.5)

in the sample. Specifically, the expected quantiles for the QQ plot are

$$\widehat{x}_1 = \Phi^{-1}(1/(n+1)), \quad \widehat{x}_2 = \Phi^{-1}(2/(n+1)), \quad \ldots \quad \widehat{x}_n = \Phi^{-1}(n/(n+1))\,. \tag{5.7}$$

Evaluation of qq plot for normal and transformed values