(1) Euclidean length: square root of the inner product with itself
In 2D plane that is the distance between two vectors

$$||\boldsymbol{x}|| = (\boldsymbol{x}'\boldsymbol{x})^{1/2} = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

Representing equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1,n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2,n}x_n &= b_2 \\ \vdots \qquad\qquad \vdots \qquad\qquad &\vdots \quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{n,n}x_n &= b_n \end{aligned}$$

These equations can be written in the more compact, matrix no

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$$

As a numerical example, suppose $\boldsymbol{b} = \{9,7,5,3\}$ and $\boldsymbol{A}$ is a 4×4 matrix of random values that were generated uniformly between 0 and 1.

```
> (b <- c(9, 7 5, 3)
[1] 9 7 5 3
> (A <- matrix(runif(16),c(4,4)))
          [,1]      [,2]      [,3]      [,4]
[1,] 0.6692485 0.5644291 0.4361838 0.7869338
[2,] 0.9458449 0.4710188 0.8120588 0.5741354
[3,] 0.3366968 0.1971159 0.3774943 0.3954844
[4,] 0.1841985 0.9128133 0.3773530 0.2363638
> solve(A,b)
[1] -1.9162283 -0.5453288  2.7192532 11.9503542
```

(2x)Solving linear equations in R

(3) Inverse of a 3x3 matrix

The inverse of the 3×3 matrix

$$\boldsymbol{B} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

is given by

$$\boldsymbol{B}^{-1} = \frac{1}{\mathrm{Det}(\boldsymbol{B})} \begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix},$$

where $\mathrm{Det}(\boldsymbol{B})$ is the determinant given in (4.6) and

$$\begin{array}{lll} A = ei - fh & B = ch - bi & C = bf - ce \\ D = fg - di & E = ai - cg & F = cd - af \\ G = dh - eg & H = bg - ah & I = ae - bd \end{array}$$

Just to check, we can multiply the matrix **x** by its inverse:

```
> x %*% solve(x)
              [,1]          [,2]         [,3]          [,4]          [,5]
[1,]  1.000000e+00 -6.661338e-16 1.110223e-16  4.163336e-17 -1.110223e-16
[2,]  1.665335e-16  1.000000e+00 5.551115e-17  2.775558e-17  0.000000e+00
[3,]  0.000000e+00 -2.220446e-16 1.000000e+00  1.387779e-16 -1.110223e-16
[4,]  1.110223e-16 -2.220446e-16 0.000000e+00  1.000000e+00 -1.110223e-16
[5,] -5.551115e-17  1.110223e-16 1.110223e-16 -1.387779e-17  1.000000e+00
```

Due to numerical error, identity matrix have non-zero value

An $n \times n$ matrix will have n eigenvalues. Just as a polynomial may have multiple roots, some of the eigenvalues may be repeated, corresponding to multiple roots[4] of $p(\lambda) = 0$. A polynomial may have complex roots, and the eigenvalues may be *complex numbers*. Complex numbers are expressible as $\alpha + \beta i$ where $i^2 = -1$. Complex roots will appear as *complex conjugates*. That is, if $\alpha + \beta i$ is a complex root of a polynomial with real-valued coefficients, then $\alpha - \beta i$ will also be a root of this polynomial.