State Estimation Using IMU Data

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In this document, we estimate the position, velocity, and orientation of an object using data from an Inertial Measurement Unit (IMU). The IMU provides acceleration and angular velocity data, from which we derive the object's motion in three-dimensional space. The main components of the estimation process are: - **Position Estimate**: $\mathbf{p}_{\text{est}}$ - **Velocity Estimate**: $\mathbf{v}_{\text{est}}$ - **Orientation Estimate**: $\mathbf{q}_{\text{est}}$ (a quaternion)

1 Definitions

- **Position Estimate**: $\mathbf{p}_{\text{est}} \in \mathbb{R}^3$, representing the 3D coordinates of the object in space.
- **Velocity Estimate**: $\mathbf{v}_{\text{est}} \in \mathbb{R}^3$, representing the rate of change of position.
- **Orientation Estimate**: $\mathbf{q}_{\text{est}} \in \mathbb{R}^4$, a quaternion representing the rotation of the object.
- **Gravity Vector**: $\mathbf{g} = [0, 0, -9.81]^\top$, representing Earth's gravitational force in the z-direction.
- **Time Step**: Δt, the time difference between consecutive IMU measurements.
- **Rotation Matrix**: $\mathbf{R}(\mathbf{q})$, derived from the quaternion $\mathbf{q}$, used to transform body frame measurements to the world frame.

2 Initialization

Before updating the state, we initialize the estimates with ground truth (GT) values, if available:

$$
\begin{aligned}
\mathbf{p}_{\text{est}} &= \mathbf{p}_{\text{gt}} \\
\mathbf{v}_{\text{est}} &= \mathbf{v}_{\text{gt}} \\
\mathbf{q}_{\text{est}} &= \mathbf{q}_{\text{gt}}
\end{aligned}
$$

Here, $\mathbf{p}_{\text{gt}}$, $\mathbf{v}_{\text{gt}}$, and $\mathbf{q}_{\text{gt}}$ are the ground truth values for position, velocity, and orientation, respectively. The quaternion $\mathbf{q}_{\text{gt}}$ can be derived from Euler angles using known conversion formulas.

3 Update Equations

At each time step k, we update the state estimates using the following equations based on the IMU data.

3.1 Position Update

The position estimate is updated using the basic kinematic equation of motion, which accounts for the current velocity and the acceleration:

$$\mathbf{p}_{\text{est}}[k] = \mathbf{p}_{\text{est}}[k-1] + \Delta t \cdot \mathbf{v}_{\text{est}}[k-1] + \frac{1}{2}\Delta t^2 \cdot (\mathbf{R}(\mathbf{q}_{\text{est}}[k-1]) \cdot \mathbf{a}_{\text{imu}}[k-1] + \mathbf{g}) \tag{1}$$

Where:

- $\mathbf{a}_{\text{imu}}[k-1]$ is the specific force measured by the IMU at time step $k-1$.
- $\mathbf{R}(\mathbf{q}_{\text{est}}[k-1])$ is the rotation matrix, derived from the current orientation quaternion, which transforms IMU data from the body frame (sensor's perspective) to the world frame (global reference).

3.2 Velocity Update

The velocity is updated using the following equation, which integrates the acceleration data over the time interval Δt:

$$\mathbf{v}_{\text{est}}[k] = \mathbf{v}_{\text{est}}[k-1] + \Delta t \cdot (\mathbf{R}(\mathbf{q}_{\text{est}}[k-1]) \cdot \mathbf{a}_{\text{imu}}[k-1] + \mathbf{g}) \tag{2}$$

The term $\mathbf{R}(\mathbf{q}_{\text{est}}) \cdot \mathbf{a}_{\text{imu}}$ transforms the IMU's acceleration data from the sensor's body frame into the world frame, where it can be combined with the gravity vector $\mathbf{g}$.

3.3 Orientation Update

The orientation of the object is updated using angular velocity data from the IMU. This angular velocity is used to calculate the change in orientation over the time step Δt. The orientation is represented as a quaternion, which avoids the singularities and complexities of other representations like Euler angles.

The update is given by:

$$\mathbf{q}_{\text{est}}[k] = \mathbf{q}_{\Delta} \cdot \mathbf{q}_{\text{est}}[k-1] \tag{3}$$

Where:

- **Rotation Quaternion**: $\mathbf{q}_{\Delta}$, represents the rotation due to the angular velocity of the IMU over the time interval Δt. It is calculated as:

  $$\mathbf{q}_{\Delta} = \text{Quaternion}(\text{axis\_angle} = \boldsymbol{\omega}_{\text{imu}}[k-1] \cdot \Delta t) \tag{4}$$

  Here, $\boldsymbol{\omega}_{\text{imu}}[k-1]$ is the angular velocity measured by the IMU in radians per second at the previous time step. The axis-angle representation is used to convert this angular velocity into a quaternion that describes the rotation.

The quaternion multiplication $\mathbf{q}_{\Delta} \cdot \mathbf{q}_{\text{est}}[k-1]$ updates the object's orientation by applying the rotation due to the angular velocity to the previous orientation.

4 Conclusion

This framework allows continuous estimation of an object's state (position, velocity, and orientation) using data from an IMU. This is critical in many applications such as robotics, aerospace navigation, and augmented reality systems, where precise motion tracking is necessary.