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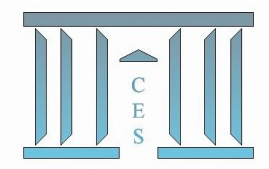

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A Practical Approach to Financial Crisis Indicators Based on Random Matrices

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Abstract

The aim of this work is to build financial crisis indicators based on market data time series. After choosing an optimal size for a rolling window, the market data is seen every trading day as a random matrix from which a covariance and correlation matrix is obtained. Our indicators deal with the spectral properties of these covariance and correlation matrices. Our basic financial intuition is that correlation and volatility are like the heartbeat of the financial market: when correlations between asset prices increase or develop abnormal patterns, when volatility starts to increase, then a crisis event might be around the corner. Our indicators will be mainly of two types. The first one is based on the Hellinger distance, computed between the distribution of the eigenvalues of the empirical covariance matrix and the distribution of the eigenvalues of a reference covariance matrix. As reference distributions we will use the theoretical Marchenko Pastur distribution and, mainly, simulated ones using a random matrix of the same size as the empirical rolling matrix and constituted of Gaussian or Student-t coefficients with some simulated correlations. The idea behind this first type of indicators is that when the empirical distribution of the spectrum of the covariance matrix is deviating from the reference in the sense of Hellinger, then a crisis may be forthcoming. The second type of indicators is based on the study of the spectral radius and the trace of the covariance and correlation matrices as a mean to directly study the volatility and correlations inside the market. The idea behind the second type of indicators is the fact that large eigenvalues are a sign of dynamic instability.

Keywords : Quantitative Finance, Econometrics, Mathematical Methods, Statistical Simulation Methods, Forecasting and Prediction Methods, Large Data Sets Modeling and Analysis, Computational Techniques, Simulation Modeling, Financial Crises, Random Matrix Theory

JEL Classification : B16, C01, C02, C15, C53, C55, C58, C63, G01

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1 Introduction

There is large literature on financial crisis indicators, especially works by Didier Sornette [8] [9] or Dominik Maltritz [10], that aim at predicting the precise date of the next crisis. This isn't going to be our approach and our aim is merely to detect a heightened risk of a crisis happening, not to predict its actual occurrence. In other words, we find it extremely difficult if not impossible when there is a gas leak to predict whether and when a random spark will cause an actual explosion. Our approach is closer to the work of Leonidas Sandoval Junior who proved in his 2012 paper [3], using random matrix theory techniques, that high volatility in financial markets is intimately linked to strong correlations between them or the work of Jean-Philippe Bouchaud and Marc Potters [7] who apply random matrix theory and principal component analysis to the financial context. The work of Ajay Singh and Dinghai Xu [6] and of Malgorzata Snarska [5] about the dynamics of the covariance matrix in a random matrix theory framework was also inspirational to us. Our ultimate goal is to provide a probability of occurrence of a crisis, not to predict its actual occurrence. In other words, that means to measure the gas concentration rather than predicting the moment it will blow-up. In this sense, our indicators will be able to say whether this probability of occurrence has increased, yet remaining at an absolute rather low level, meaning that there is still a high probability that nothing will happen. The indicators that we are trying to build are therefore, in essence, closer to being "instability" indicators. From a practitioner's point of view, the information that the probability of a crisis occurring in the near future has risen from, say, 0.1% to 10% has tremendous value, even though there is still a 90% chance of nothing happening: the implication for a practitioner is a timely and wise use of hedging instruments such as put options and/or CDS. For us, a financial crisis indicator is a mathematical tool that makes use of publicly available data to determine whether the conditions (we will talk in this paper about correlations and volatility) are ripe in the market for a crisis event to happen, like a brutal and persistent fall in the value of many individual securities and indices or a cascade of major firm failures. False positives are inevitable and in a way, they are healthy counterparts to false negatives: when dealing with financial stability, safety must be paramount.

The mathematical methods used in this paper are universal and are applied to very simple and intuitive principles of financial economics : when correlations between asset returns increase and develop abnormal patterns, when volatility goes up, then something is wrong in the market and major trouble might be around the corner. Any kind of market data can be used in that regard. The kind of data used (one country only, one region, equity indices, forex data, commodities data, sector indices, individual stocks, etc...) and its frequency can be adapted to reflect the kind and scope of the financial crises that one is trying to forecast, but the underlying mathematical tools and financial principles remain the same. It would be like adapting the spectrum band and resolution on a radio-telescope in an attempt to detect various kinds of astronomical phenomena.

In this paper, we chose to interest ourselves mainly with global financial crises, most of which are well known to the general public, and the data has been selected accordingly with the study of a global event in mind. The data we use in this paper has been collected from Bloomberg and Yahoo Finance. The code has been done using Matlab and its various optional toolboxes. The complete code is stored online and is available upon request: please send me an email at the electronic address provided [1] if you wish to access it. The reader is very much encouraged to apply the methods developed in this paper to their own datasets and to verify the reproducibility of their forecasting power to various kinds and scopes of financial crises. We look forward to

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feedback and comments.

2 The Data

The data is constituted every trading day of daily log-returns (log-returns at five and twenty days were also considered but gave us harder to interpret experimental results) computed from open or close prices. We have done our study on daily data because of easy access and numerical handling. Further studies may explore higher frequency data. Our model requires the choice of a rolling window in order to compute the financial crisis indicators. In order to limit averaging effects and to have financial crisis indicators with enough reactivity to provide useful information to a practitioner, we restrict the size of the rolling windows to at most 150 days in the past. Larger rolling windows will mean that our covariance matrix will be degenerate sometimes (there will be more observations than assets) but that isn't going to be a problem because for the first type of indicators, the Hellinger distance will be computed with regard to reference distributions that have been truncated around zero for reasons that will be apparent in the next section and for the second type of indicators, the presence of zero, even quite a lot of them, in the spectrum will not change anything for our computation of the trace and spectral radius.

Seven datasets, each designed with its own unique properties and composition will be considered :

- The first dataset (Dataset1) is constituted of eleven stock indices representative of the Asian, European and American financial markets in order to obtain a picture of the global financial system. It is a pure equity dataset that is designed to capture contagion between major financial markets as a way to forecast financial crises. It contains the Nikkei225 (NKY, Japan), Hang Seng (HSI, Hong-Kong/China), Taiwan Stock Exchange Weighted Index (TWSE, Taiwan) for the Asian market, the DAX30 (DAX, Germany), FTSE100 (UKX, U.K), IBEX35 (IBX, Spain) for the European market, the SP500 (SPX, U.S.A), Russel3000 (RAY, U.S.A), NASDAQ (CCMP, U.S.A), Dow Jones Industrial Average (INDU, U.S.A), SP/TSX Composite Index (SPTSX, Canada) for the North American market. Dataset1 spans from January 7th 1987 to February 5th 2015. In order to avoid contaminating the data with time differentials which might create bias and spurious correlations, we matched at a same date-point t the close price in Asia at t, the close price in Europe at t and the open price in America (East Coast) at t. In this absence of intraday data, this appeared to be a reasonable solution. We considered only the trading days and because of the different holidays specific to each of the three markets considered (Asian, European and North American) and the requirement to keep only the trading days that were common to all the markets, the 252 trading days a year have been reduced to around 200 date-points. Comparison with the other datasets (particularly Dataset3 and Dataset4 below which do have around 250 data-points a year since they are exclusively American and European, respectively) shows that this isn't a major issue in practice.
- The second datastet (Dataset2) is constituted of sixteen assets. It contains all of the indices of Dataset1, some commodity indices and some safe haven or cash equivalent securities toward which investors tend to turn in a time of crisis or impending crisis. It spans the same period as Dataset1, from January 7th 1987 to February 5th 2015. The treatment of the data with regard to time differential between geographical regions and non-trading days is the same. On top of the content of Dataset1, Dataset2 includes : The London Gold Market

Fixing Index (GOLDLNPM, U.K), the Philadelphia Stock Exchange Gold and Silver Index (XAU, U.S.A), Oppenheimer Limited-Term Government Fund Class A (OPGVX, U.S.A) equity price, Sugar Generic Future Contract (SB1, U.S.A) commodity price, generic First Crude Oil WTI (CL1, U.S.A) commodity price. The inclusion of precious metal indices, cash equivalent short-government monetary funds, representative agricultural as well as energy commodities (in the form of investable futures) is supposed to provide a longer fuse to our financial crisis indicators. As a matter of fact, when the market starts to overheat, investors may liquidate some of their equity positions but they will have to re-invest the cash somewhere and those cash equivalent security are here to account for that. Since those safer, cash equivalent securities are in Dataset2, our anticipation is that the risk of a crisis happening will be detected sooner. Moreover, when the market is becoming unstable, one typically witnesses an increase in the correlations between commodity and energy securities (typically large oil companies stocks included in the indices). Since we included some investable commodity futures (like oil futures) in Dataset2, we expect to capture that effect which is indicative of the appearance inside the financial market of the right conditions for a crisis to happen.

- The third dataset (Dataset3) contains twelve assets which are the SP500 index and its ten sector sub-indices (consumer discretionary, consumer staples, energy, financials, health care, industrials, information technology, materials, telecommunication services, utilities) plus a small capitalization index (the Russel 2000). This dataset should provide information about the inner workings of the SP500 and enable us to detect "American" crises (for example the sub-prime crisis of 2007) sooner and with a higher precision than Dataset1 or Dataset2 which are global by design and include information about the contagion between the three largest financial markets (Asia, Europe, North America). However, since the North American market still leads the world of finance, it is to be expected that the actual crises anticipated by the use of either three of Dataset1, Dataset2 or Dataset3 will be roughly the same. The inclusion in the mix of a small capitalization index is to try to take advantage of the fact that in the times leading up to a financial crisis, the small caps tend to overheat and form speculative bubbles while they become more and more correlated between themselves and stocks with larger market capitalization. Dataset3 spans from September 13rd 1989 to December 27th 2013.

- The fourth dataset (Dataset4) is the European counterpart of Dataset3. It contains eleven assets : the Bloomberg European 500 Index (BE500) and its ten sector sub-indices, which are the same as for the SP500 (consumer discretionary, consumer staples, energy, financials, health care, industrials, information technology, materials, telecommunication services, utilities). As we didn't find any European-wide equivalent to the Russel 2000, it doesn't include small caps however. It should enable us to better and sooner detect "European" crises like the E.U Sovereign Debt Crisis of 2010 while still containing enough information to detect all the other global financial crises. It spans from January 1st 1987 to December 27th 2013.

- The fifth dataset (Dataset5) is designed with the financial concept of "flight to quality" in mind. Indeed, in the times preceding a financial crisis, the anxiety of market agents is building up and they tend to abandon equity positions in favor of safer investment grade treasury or corporate bonds. In that regard, the usual observed phenomenon is a positive correlation between equity and bonds in a bull market and a negative correlation between equity and bonds in a bear market. When the correlation between equity and bonds is becoming too high, this may be a sign that the bull market is about to burn itself out,

that a bubble is about to burst, heralding the start of a financial crisis. Dataset5 is built with the detection of that phenomenon in mind. It contains all of the data of Dataset3 (SP500 index, its 10 sector indices and the Russel 2000 as a small capitalization index) plus a number of funds based on investment grade sovereign or corporate bonds. Much like Dataset3, Dataset5 is U.S market oriented and is therefore more suited to anticipate crises that originate from or directly affect the North American market. For the long government bonds we have : Wasatch-Hoisington U.S. Treasury Fund (WHOSX) and Thornburg Limited Term U.S. Government Fund Class A (LTUCX). For the corporate bonds we have selected Lord Abbett Bond Debenture Fund Class A (LBNDX) and Vanguard Long-Term Investment-Grade Fund Investor Shares (VWESX) which have both enough AUM (Assets Under Management) to be systemically significant and have existed for long enough to be historically relevant. Dataset5 contains therefore 16 assets and spans from September 13rd 1989 to December 27th 2013.

- The sixth dataset (Dataset6) is constituted of 226 individual components of the SP500 index. Because of the evolution over time in the composition of the index, a balance had to be found between keeping a sufficient number of components and having enough historical data. It spans from January 17th 1990 to May 15th 2015. The Apple Inc (AAPL) stock was chosen as the reference with regard to filtering out non-trading days and whenever another element of data was unavailable (on rare seemingly random days it appears that some individual stocks weren't traded or the data was unavailable) we carried over the last value while assuming that this manipulation wouldn't compromise the overall quality of the data. Besides those considerations, a few stocks like for example Range Resources Corporation (RRC UN) and The Charles Schwab Corporation (SHCW UN) presented significant data gaps and were removed from the dataset. Since building a dataset with exactly 500 components of the SP500, taking in account the evolution in the composition of the index over time, proved an impossible task due to its complexity and the availability of the data (mergers, corporate spin-offs and private equity acquisitions would have had to be taken into considerations as well!), we are aware of the fact that Dataset6, especially when used in conjunction with financial crises indicators might suffer from survivorship bias. As a matter of fact, especially in the times leading up to a crisis, the failing companies drop below the capitalization threshold or are acquired by others while new healthier firms enter the index. We built Dataset6 because, as we will see in the empirical section, working with whole indices and/or limited number of individual securities like in all the previous datasets we created (especially Dataset3 which resembles a scaled down version of Dataset6), tends to have an averaging effect on the correlations and renders the correlation signal too noisy and blurred to be useful as a crisis detection method. For reference, the Bloomberg tickers of all the stocks inside Dataset6 are provided in appendix. Besides the daily close price that is contained in all our other datasets, Dataset6 also includes daily volumes and market capitalization. Those extra variables will enable us to add appropriate weights to the individual stocks in order to refine the computation of our indicators.

- The seventh dataset (Dataset7) is constituted of the SP500, the Russel 2000 index and ten indices from emerging markets : Buenos Aires Stock Exchange Merval Index (MERVAL, Argentina), Ibovespa Brasil Sao Paulo Stock Exchange Index (IBOV, Brasil), Mexican Stock Exchange Index (MEXBOL, Mexico), Moscow Exchange Composite Index (MICEX, Russia), Hong Kong Hang Seng Index (HSI, Hong Kong/China), Shanghai Stock Exchange Composite Index (SHCOMP, China), Jakarta Stock Exchange Composite Index (JCI, Indonesia), National Stock Exchange CNX Nifty Index (NIFTY, India), FTSE/JSE Africa All Share Index (JALSH, South Africa), Borsa Instabul 100 Index (XU100, Turkey). It

spans from September 22th 1997 to May 12th 2015. The relatively shallow depth of this dataset, which in particular may render the study of the Asian crisis of the late 1990' more difficult, is due to gaps in data availability, especially for the Russian index that we decided to keep anyway due to its importance for the global commodity and energy markets. All those emerging indices were expressed in the local currency on Bloomberg and were therefore converted into U.S dollars. This conversion was very important when dealing with emerging economies where the the local currency's exchange rate against the U.S dollar can fluctuate wildly and violently especially in the times leading up to, and during a financial crisis. Unlike in advanced economies (we didn't convert the European and Japanese indices into U.S dollars in Dataset1 for example), the position of emerging countries' currencies against the dollar is also highly correlated to the health of the local real economy. This is the point that John Hawkins and Marc Klau, working for the Bank of International Settlements develop in their 2000 paper : in emerging markets, financial crises are often preceded by overvalued exchange rates and inadequate international monetary reserves.[11].

The code for data importation into Matlab is provided online as well : one asset corresponds to one line and one column to one observation. In order to increase the readability of our results and to limit too frequent occurrences of false positives that would defeat the purpose of an indicator, we considered at first applying denoising techniques to the time-series of the log-returns. However, such an approach proved unworkable because the denoised log-return datasets obtained using Matlab's Wavelets Toolbox made use of interval-dependent denoising techniques using the Haar wavelet, which implied considering at each date t a neighborhood of the date-point, therefore making use of log-return data for dates strictly greater than t. Such an approach would have violated the measurability of our indicators with respect to the natural time filtration. Using the denoised version of the datasets, the indicators would have used at each date some data from the future in a manner that couldn't be controlled or averaged using historical data, so this approach was mothballed as it had become useless from a practitioner's point of view.

With regards to the selection of the financial crisis events with global (Dataset1 and Dataset2) or at least regional (Dataset3, Dataset4, Dataset5, Dataset6 and Dataset7) of the last 30 years, we compiled the following table (**Table1**) which has no ambition of being exhaustive. Summary historical context will be discussed in the empirical results section when needed. While categorizing the various kinds of financial crises goes far beyond the scope of this paper, we strove to consider a wide selection in the kinds of crises. There are stock market crashes like Black Monday in 1987 and the NASDAQ Crash in 2000. There are financial crises that are rooted into a deep structural fragility of some parts of the real economy, like the real estate sector in the case of the Japanese Asset Price Bubble of the early 1990' and the Subprime crisis that started in America during the summer of 2007 or the automobile industry in the case of the bankruptcy of General Motors in June 2009 four years after Delphi Corporation, which was General Motors' main supplier of automotive parts. There are financial crises which main trigger was a sovereign debt default like the Russian crisis in 1998, the Argentine crisis in 2001 or the Eurozone crisis, triggered by the Greek haircut in 2010. There are monetary crises as well, like Black Wednesday in 1992 when the British government was forced to withdraw the Pound Sterling from the European Exchange Rate Mechanism (ERM) or the Mexican crisis triggered by the devaluation of the peso against the U.S Dollar. Since none of our datasets include foreign exchange data, we do not expect that any of our indicators will perform well when it comes to anticipating monetary crises, however. There are banking crises as well such as the S&L crisis in America that spanned from the mid-1980 to the mid-1990' and during which almost one third of all American savings and loans associations (financial institutions that are allowed to accept savings deposits and to make

loans) failed, including hundreds of banks of all sizes and systemic significance. The dates chosen may seem somewhat arbitrary but choices had to be made, especially for crises that, unlike Black Monday that played out mostly within a few days of extreme market distress, took place over many months or even years of sustained drop like the NASDAQ in early 2000 which took nearly four months to lose almost two fifth of its March 10th peak. Most financial crises do not happen in one day and instead result from a long process of instability buildup inside the market, the kind of which our indicators are detecting. When a crisis is best described by a clear explosion, then the date of that event was chosen (Black Monday, the day Lehman Brothers failed, etc...). When a date for a crisis spanning months or years had to be chosen for this study, we considered either the date of the most marking event (the day the NASDAQ peaked, the day General Motors filled for Chapter 11 bankruptcy, the day the Greek haircut was announced, etc...) or a date roughly situated in the middle of the crisis process like January 1st 1990 for the S&L crisis.

Date (Y/M/D)	Matlab Serial	Name
1987-10-19	726029	Crisis 1 : Black Monday
1990-01-01	726834	Crisis 2 : S&L Crisis
1990-08-01	727046	Crisis 3 : Japanese Asset Prices Bubble Burst
1991-09-19	727460	Crisis 4 : Scandinavian Banking Crisis
1992-09-16	727823	Crisis 5 : Black Wednesday
1994-12-20	728648	Crisis 6 : Mexican Crisis
1997-07-25	729596	Crisis 7 : Asian Crisis
1998-08-17	729984	Crisis 8 : Russian Crisis
2000-03-10	730555	Crisis 9 : NASDAQ Crash (dot-com Bubble)
2001-02-19	730901	Crisis 10 : Turkish Crisis
2001-09-11	731105	Crisis 11 : 911 Attacks
2001-12-27	731212	Crisis 12 : Argentine Default
2005-10-08	732593	Crisis 13 : Delphi (G.M) Bankruptcy
2007-07-01	733224	Crisis 14 : Subprime Crisis
2008-09-15	733666	Crisis 15 : Lehman Brothers Collapse
2009-06-01	733925	Crisis 16 : General Motors Bankruptcy
2010-04-23	734251	Crisis 17 : European Sovereign Crisis
2011-08-05	734720	Crisis 18 : US Sovereign Credit Degradation
2014-12-16	735949	Crisis 19 : Russian Financial Crisis

3 Methodology

In this section we will present the tools we will use to develop our financial crisis indicators. After many trial and errors we decided to consider a rolling window T of 150 days in the past. That value experimentally gave us the best results with a good balance of signal readability versus averaging.

Using the data we talked about in the previous section, we construct at each date t a rolling window $ROL(t)$ of length T. We then compute the rolling covariance matrix $CV(t)$ and the rolling correlation matrix $CR(t)$. Using Matlab's way of indexing vectors, we can write for all dates t and for all lines (i.e assets) j:

$$ROL^*(t)(j,:) = ROL(t)(j,:) - mean(ROL(t)(j,:))$$

$$CV(t) = \frac{1}{T} \times ROL^*(t) \times (ROL^*(t))'$$

$$CR(t)(j,:) = \frac{ROL^*(t)(j,:)}{\sqrt{var(ROL(t)(j,:))}}$$

While working with a covariance matrix instead of a correlation matrix, we will of course have to rescale the eigenvalues of $CV(t)$. We will do this either by remarking that the standard deviation of financial log-returns is typically in the order of magnitude of a few percents ($a \in [0.01, 0.03]$) and therefore multiplying the eigenvalues by $\frac{1}{a^2}$ which is in the order of magnitude of $[1000, 10,000]$ or by computing the mean of the variances of all the complete time series in advance and rescale by the inverse of that value (for example, we find a rescaling factor of 3410 for Dataset2). This is what we decided to do but it shouldn't be considered as a violation of the measurability of our indicators with respect to the natural time filtration (i.e knowledge of the future). It is just a practical way of rescaling by choosing the most appropriate value and it could just as well have been obtained from historical data predating our sample.

With regard to the reference distributions we will use for the first type of indicators, there will be three of them :

- $\Theta 1$: the theoretical Marchenko Pastur distribution. It is derived from Marchenko Pastur's theorem (1967) [2]. Let X be a $l \times c$ random matrix of i.i.d normal $N(0, \sigma^2)$ coefficients (in our study, each line represents an asset and each column represents an observation at a date t), then when $l, c \longrightarrow \infty$ and the aspect ratio of the matrix, $l/c \longrightarrow \gamma < \infty$, then the distribution of the eigenvalues of the covariance matrix $Y = \frac{1}{c}(X \times X')$ is the Marchenko Pastur distribution with the density below. This formula is supposed to be valid for $0 < \gamma < 1$, otherwise in the degenerate case an atom at zero has to added, but since we intend to truncate the computation of the Hellinger distance to exclude very small eigenvalues, as we will explain in the next section, this is the formula we wil use anyway for simplicity.

  $$\lambda^+ = \sigma^2(1 + \sqrt{\gamma})^2$$

  $$\lambda^- = \sigma^2(1 - \sqrt{\gamma})^2$$

  $$f(x) = \frac{1}{2\pi\sigma^2\gamma} \frac{\sqrt{(\lambda^+ - x)(x - \lambda^-)}}{x} 1_{[\lambda^-, \lambda^+]}$$

  The Marchenko Pastur distribution will also provide thresholds λ^+ and λ^- that we will use even when working with other simulated reference distributions. Because of the stringent theoretical requirements of Marchenko Pastur's theorem, that will never be even remotely satisfied by real financial data, the Marchenko Pastur distribution has no vocation to be the best reference for the computation of the Hellinger distance.

- $\Theta 2$: the distribution of the eigenvalues of the covariance matrix of a simulated random matrix made of Gaussian $\mathcal{N}(0, 1)$ coefficients that present some correlation to one another. Z_0 is following a Gaussian $\mathcal{N}(0, 1)$ law, each of the coefficients $X_{i \in l \times T}$ (we have l assets and T observations) of the random matrix is computed in the following manner :

  $$X_i = \rho Z_0 + \sqrt{(1 - \rho^2)} Z_i$$

  Where $Z_i \sim \mathcal{N}(0, 1)$. ρ is chosen as the mean of the long term correlation coefficients between all the assets of the whole sample contained in the chosen dataset. Like when we had to decide on a rescaling coefficient for the spectum, this choice of ρ shouldn't be

considered as knowledge from the future, as it could just as well have been obtained from historical data. As a matter of fact, we find something very close to 50 % for all the datasets, which is what we had expected.

- $\Theta 3$: the distribution obtained using the same blueprint as $\Theta 2$ but where all the Gaussian $\mathcal{N}(0,1)$ distribution have been replaced by Student (t=2) distributions.

As an illustration, we draw in **Figure1** the three reference distributions computed for Dataset1 and a rolling window of 150 days:

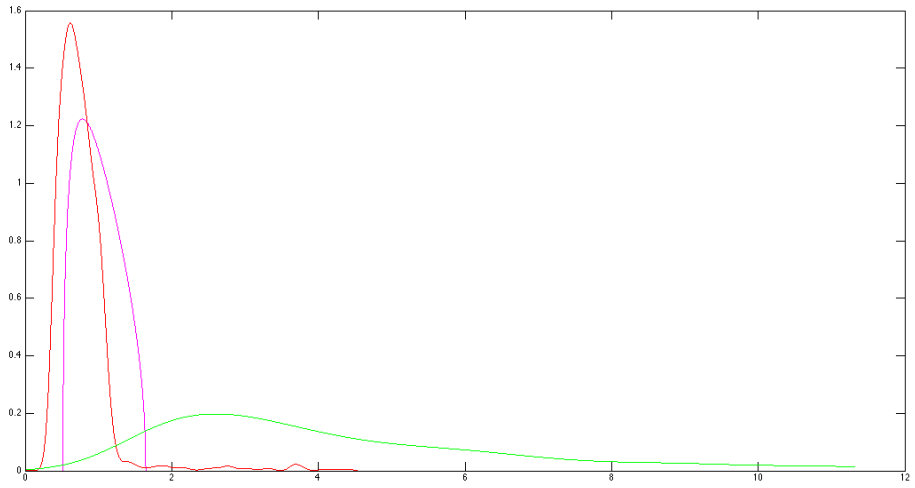

Fig.1 (purple:$\Theta 1$, red:$\Theta 2$, green:$\Theta 3$)

4 Financial Crisis Indicators

4.1 The Hellinger Distance

We recall that the Hellinger distance $\mathbb{D}$ between two probability distributions densities $P(x)$ and $Q(x)$ which are both known at a number of points X_i is computed by the formula below :

$$\mathbb{D} = \sum_{i=1}^{N} (\sqrt{P(X_i)} - \sqrt{Q(X_i)})^2$$

The distance of Hellinger is at first glance a good candidate as a financial crisis indicator. We consider a reference distribution and compute its Hellinger distance to the empirical distribution of the eigenvalues of the covariance matrix. Our assumption is that the further away in the sense of the Hellinger distance the empirical distribution drifts from the reference, the more likely the market is about to experience a crisis because such a drift indicates a build-up of correlation and volatility inside the market. There is however no way to study those two effects separately in the Hellinger distance approach.

Since all datasets except Dataset6 only have a small number of assets and will therefore give us only a small number of eigenvalues at each date, we combine at each date t the spectra obtained from the last 20 days in order to have enough values to derive a distribution. We then compute the Hellinger distance to the reference distribution on a sufficiently large support in order to capture all of the spectral distribution. In empirical studies such as H.E. Stanley, P. Gopikrishnan, V. Plerou, L.A.N. Amaral's 2000 paper [4] , eigenvalues of the covariance matrix have been observed to grow as large as twenty five times the critical λ^+ of Marchenko Pastur's distribution which means that we decided to consider 25 times its support $[\lambda^-, \lambda^+]$ in order to account for all of the empirical spectrum.

The indicators of our first series will be the following :

- **Indicator A1** : Hellinger distance between the empirical distribution of the eigenvalues of the covariance matrix and the theoretical distribution of Marchenko Pastur $\Theta 1$. For a given dataset, this indicator will measure at each date by how much the assumptions of Marchenko Pastur's theorem (normal i.i.d coefficients of variance equal to 1) are violated. Since the finite size of the rolling covariance matrix doesn't change over time, it won't be responsible for any dynamical variations of the Hellinger distance although it does certainly account for part of the distance between the theoretical asymptotic distribution of Marchenko Pastur and the empirical distribution. This indicator will lump together the apparition of non-normality, correlations and volatility in the log-return time series, it cannot differentiate between all those effects but it is still very useful : the apparition of any of those phenomena, which effects aren't expected to compensate one another, can be interpreted as a warning that a crisis might be around the corner so our assumption will be that the further away the empirical distribution becomes from the reference Marchenko Pastur distribution in the sense of the Hellinger distance, the more likely a crisis is going to happen.

  Indicator A1, as well as all our indicators based on the Hellinger distance, needs also to be adapted to filter out the effects of the following parasitic phenomenon (**Figure 2**) :

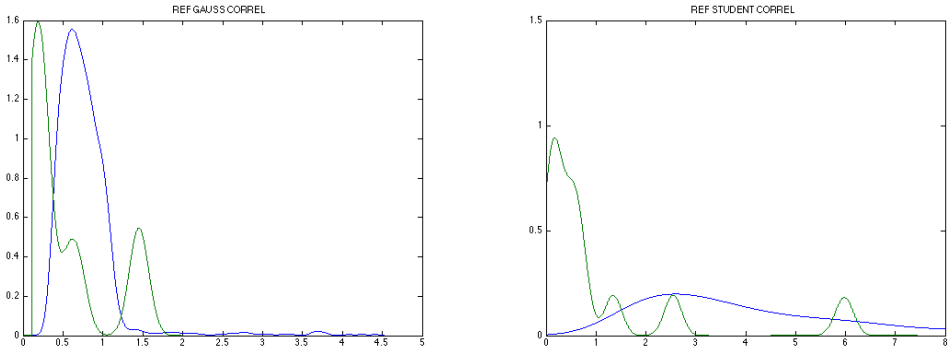

Fig.2: Accumulation of small eigenvalues for $\Theta 2$ and $\Theta 3$

In those two arbitrary examples, which are very typical of the situations we encounter in practice for a given date t while using our datasets, we see the reference distribution (in blue) and we see the empirical distribution of the eigenvalues of the covariance matrix (in green). The empirical distribution can differ from the reference distribution in two ways. It can overflow to the right toward the higher eigenvalues : that's the kind of behavior

that we are looking for in order to detect a financial crisis and it can also unfortunately accumulate itself, sometimes most of the mass is even there, closer to zero toward the very small eigenvalues. Such a behavior of the distribution of the eigenvalues of the covariance matrix is more indicative of the prevalence of risk free combinations of assets which equates to a very calm and diversified market. In other words, such as we have defined it until now, Indicator A1 tends to spike in time of crisis and in time of great calm and the result is an extremely agitated profile that holds little predictive value.

The solution was to adapt A1 and the A-series indicators in the following way : instead of computing the distance of Hellinger between the empirical distribution $\mathscr{E}$ and the Marchenko Pastur reference $\Theta 1$, we compute the distance of Hellinger between $\Theta 1$ and the distribution $\mathscr{E}^*$ defined in the following way :

$$\mathscr{E}^*(x) = min(\mathscr{E}(x), \Theta 1(x)), x < \frac{\lambda^+}{10}$$

$$\mathscr{E}^*(x) = \mathscr{E}(x), x > \frac{\lambda^+}{10}$$

Therefore :

$$A1 = \mathbb{D}\{\mathscr{E}^*, \Theta 1\}$$

Like we said earlier, using the Marchenko Pastur distribution for a given aspect ratio as a reference distribution might not be optimal because it's an asymptotic result and we deal with finite size matrices and because even a perfectly calm financial market might be better modeled by random matrices of coefficients with some natural correlations.

As an illustration, we represented below (**Figure 3**) the entire profile for Dataset2 and the same zoomed on the day of Lehman's Brothers failure (September 15th 2008) for the reference $\Theta 2$ when the above procedure isn't applied : the profiles are much too agitated and noisy.

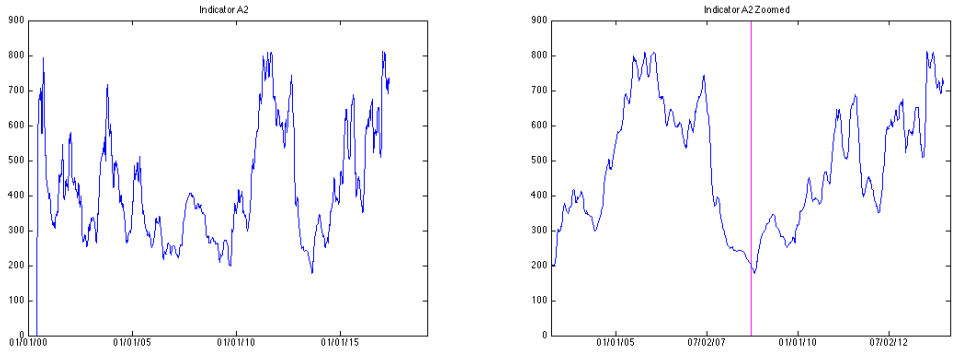

Fig.3 Indicator A2 without empirical spectrum truncation (too agitated).

- **Indicator A2** : Hellinger distance between the empirical distribution of the eigenvalues of the covariance matrix and the simulated distribution $\Theta 2$. Since correlated Gaussian coefficients are supposed to better model the market situation, we expect $\Theta 2$ to provide a better reference from which to measure a drift in the times leading up to a financial crisis. Like with Indicator A1, we will work on 25 times the support of the theoretical Marchenko

Pastur distribution and the same issues of accumulation of the empirical distribution toward the small eigenvalues in time of market calm presented itself. There is no closed form formula for the $\Theta 2$ and we can't assume that its support is bounded like the support of Marchenko Pastur's distribution is bounded by λ^- and λ^+ so we decided to keep $\lambda^* = \frac{\lambda^+}{10}$ as a threshold such that an abundance of very small eigenvalues wouldn't make the Hellinger distance explode. Therefore, we compute the Hellinger distance between $\Theta 2$ and $\mathscr{E}^*$ such that :

$$\mathscr{E}^*(x) = min(\mathscr{E}(x), \Theta 2), x < \frac{\lambda^+}{10}$$

$$\mathscr{E}^*(x) = \mathscr{E}(x), x > \frac{\lambda^+}{10}$$

Therefore :

$$A2 = \mathbb{D}\{\mathscr{E}^*, \Theta 2\}$$

- **Indicator A3** : Hellinger distance distance between the empirical distribution of the eigenvalues of the covariance matrix and the simulated distribution $\Theta 3$. We included very fat tails (coefficients that follow a Student (t=2) distribution) as a way to model crisis conditions, therefore Indicator A3 is an inverted indicator: its red flags happen when it's getting small which means that the empirical distribution of the eigenvalues of the covariance matrix is getting very close to $\Theta 3$ with is extremely heavy tailed and represents a spectrum entirely shifted toward the large eigenvalues. When the market goes from a calm state to a crisis state, our modeling of it goes from a Gaussian to a Student (t=2) distribution. As a remark, we didn't include skew in the random coefficients from which we derive our reference distributions because financial log-returns do not typically present persistent skewness, especially over the time periods considered for the rolling window, as demonstrated in the work of Singleton and Wingender [1]. We retain for A3 the same method of computation as in the other ones : the 25 times the support of the theoretical Marchenko Pastur distribution and the threshold $\lambda^* = \frac{\lambda^+}{10}$ to filter out the very small eigenvalues. Indicator A3 then computes at each date t the Hellinger distance between $\Theta 3$ and $\mathscr{E}^*$ such that :

$$\mathscr{E}^*(x) = min(\mathscr{E}(x), \Theta 3), x < \frac{\lambda^+}{10}$$

$$\mathscr{E}^*(x) = \mathscr{E}(x), x > \frac{\lambda^+}{10}$$

Therefore :

$$A3 = \mathbb{D}\{\mathscr{E}^*, \Theta 3\}$$

4.2 The Spectral Radius and the Trace

At each date t, the centered rolling matrix $ROL^*(t)$ contains two components: a volatility component and a correlation component. Our indicators are based on those two components. As we will see, both components are important but the relative strength of their signal will greatly depend on the choice of dataset we use. We build at each date t our three indicators of the second type in the following way :

- **Indicator B1** : the spectral radius of the covariance matrix $CV(t)$. It measures a mixed signal depending on both volatility and correlations in the market. A larger value for the spectral radius is indicative of dynamical instability and increased correlations in the system but it also takes the volatility effect into account since we are working with a covariance

instead of a correlation matrix. This indicator takes the effects of both volatility and correlations into account and those two effects aren't supposed to compensate each others, on the contrary they are expected to evolve in the same direction in the times leading up to a financial crisis as it was demonstrated by Sandoval [3].

- **Indicator B2** : the trace of the covariance matrix $CV(t)$. It measures the volatility signal alone. While it may seem at first that B2 lacks a very important aspect of what is happening inside the market, we will see that it is still a very good indicator and it is also very easy and fast to compute since obtaining the spectrum of the covariance matrix isn't even needed.

- **Indicator B3** : the spectral radius of the correlation matrix $CR(t)$. It measures the correlation signal alone. Indicator B3's usefulness will greatly depend on the choice of the dataset. Only with Dataset6 which contains a large number of assets (which are individual stocks components of an index) does indicator B3 realize its full potential, indeed there is too much averaging effect inside an index and that smothers the correlation signal. The potential of Indicator B3 is great however because unlike with the study of volatility, the study of correlation may be the only way to give our indicators real predictive power. Since Dataset6 also features daily volume and daily market capitalization data, we also build the following variations of indicator B3 for it :

  - **Indicator B3A** : the spectral radius of the matrix $CR_1(t)$ which coefficients are those of $CR(t)$ which have been weighted at each date t by the market capitalization ($cap(t)$) expressed in dollars, in the following way for a dataset containing F assets. $\forall (i,j) \in [1,F]^2$:

$$CR_1(t)(i,j) = CR(t)(i,j).\frac{cap(t)(i).cap(t)(j)}{\sum_{k=1}^{F} cap(t)(k)^2}$$

  - **Indicator B3B** : the spectral radius of the matrix $CR_2(t)$ which coefficients are those of $CR(t)$ which have been weighted at each date t by the volume of stocks exchanged ($volu(t)$) expressed in dollars, in the following way for a dataset containing F assets. $\forall (i,j) \in [1,F]^2$:

$$CR_1(t)(i,j) = CR(t)(i,j).\frac{volu(t)(i).volu(t)(j)}{\sum_{k=1}^{F} volu(t)(k)^2}$$

  - **Indicator B3C** : Since indicator B3B will prove useful but will also produce a very agitated signal, B3C is computed at each date t as a moving average (we chose to average on 150 days which is also the length T of the rolling window) of B3B :

$$B3C(t) = \frac{\sum_{k=1}^{T} B3B(t-k)}{T}$$

5 Empirical Results

5.1 Global Study

In this section we will look at the global profiles produced by our indicators for our seven datasets and look at what happens around the crises that we presented in **Table1**. Starting with the indicators of the series A, we obtain the following results, the crisis events in **Table 1** have

been added as vertical purple lines and A1 is in blue, A2 in green and A3 in red. As a general remark, we can say that although some global structures do clearly appear, all the the profiles also appear to be very noisy and somewhat anarchic. We can't miss the collapse of Lehman Brothers (Crisis15) for example and it is a fact that most other crises are accompanied by very noticeable patterns in the value of the Hellinger distance as well as an increased volatility of the Hellinger distance, but there are also many false positives that blur the message of the indicators of the A-series. A1 and A2 always produce profiles that are similar, which is explained by the close resemblance of the reference distribution $\Theta 1$ and $\Theta 2$ (as seen in **Figure1**) while the inverted indicator A3 produces radically different profiles: many times we see A1 and A2 climb just before a truly major crisis, while A3 plummets. Indeed, the distribution $\Theta 3$, which was derived from the covariance matrix obtained from a random matrix made of sums of very heavy tailed and correlated Student (t=2) coefficients is a good reference for a market that is in the process of dislocation with the preeminence of very large eigenvalues in the covariance matrix that are indicative of dynamical instability. With that point of view, the correct way to read the profiles is: if A1 and A2 go up, then we are moving away in the sense of Hellinger from a distribution that is characteristic of a calm market, danger might be around the corner. If on top of that A3 is going down, then we are moving closer in the sense of Hellinger to a distribution of the eigenvalues of the covariance matrix that is characteristic of a market in distress. If those two effects are happening at the same time, then the probability of a truly major market event is getting dangerously high.

- For Dataset1, the pure international equity, we get **Figure4** below. We observe elevated levels of A1 and A2 in the aftermath of Black Monday and during the build-up toward the S&L crisis and Japanese Asset Price Bubble of 1990. Then there is a relative period of calm in the early 1990'. Monetary crises like Black Wednesday aren't going to be visible using a dataset that doesn't contains any FX data because, despite causing a lot of pain, especially in the U.K, its long lasting influence on the global financial system remained limited. Then there is a a sharp increase of A1 and A2 accompanied by a sudden characteristic drop of A3 just before and during the terrible blow of the 1997 Asian Crisis (Crisis7). The 2000 NASDAQ crash isn't very visible on those profiles, even though the NASDAQ is part of Datasert1. Maybe this is due to the fact that it remained primarily an "American crisis" that isn't going to be very apparent in a dataset that focuses primarily on contagion between international markets. The bullish market period of the early 2000 is characterized by mostly flat profiles of A1, A2 and A3 indicating a globally stable market structure. Before the Lehman Brothers collapse of 2008, we see again that pattern of an increase in A1 and A2 accompanied by a drop of A3.

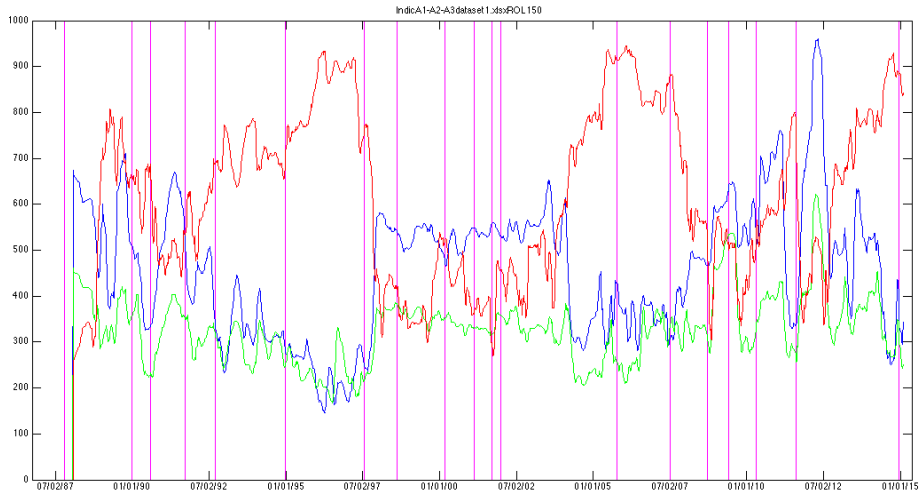

Fig.4 (A1 blue, A2 green, A3 red)

- For Dataset2, which is Dataset1 augmented with commodities and safe, cash equivalent securities, we obtain the profiles below (**Figure5**). Those profiles structurally resemble those obtained from Dataset1 but they are much tamer, probably because of the presence of safe haven securities inside the dataset which provide a way for market agents to re-invest their money as they liquidate equity positions in the times leading to and during financial crisis. Since Dataset2 includes commodities, we observe a very noticeable buildup in A1 and A2 leading up to the 2014 Russian crisis (crisis19). It is again accompanied by an ominous drop in A3. The increased correlation between commodity and energy securities, which are represented in our equity indices by the major U.S oil companies, in the times leading up to a financial crisis, creates a strong build-up of market instability which provides valuable beforehand information that a crisis is becoming more likely.

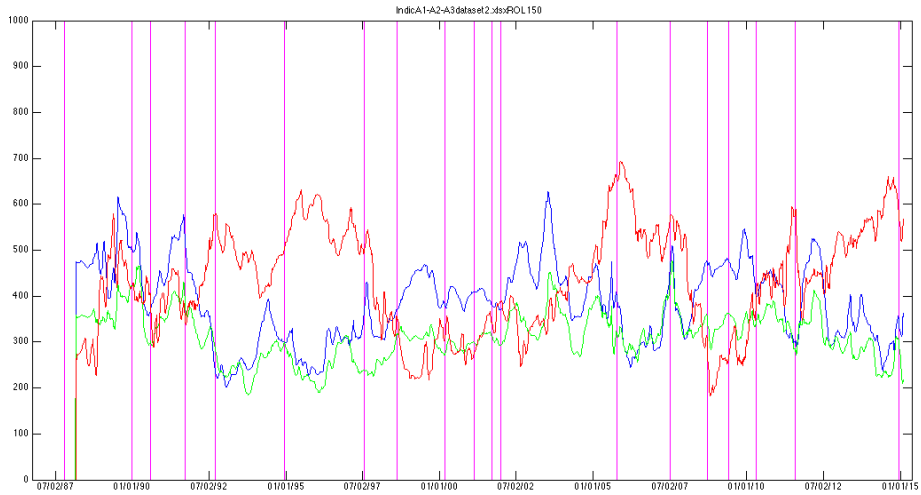

Fig.5 (A1 blue, A2 green, A3 red)

- For Dataset3, which is the "American" dataset, we get the following profiles (**Figure6**). Those profiles emphasize primarily, as anticipated, the events when the U.S market is overheating. The Savings and Loans crisis (Crisis2) is anticipated a few months in advance by a sharp increase in A1 and A2. The indicators are mostly unresponsive during the Asian crisis of 1997 but the NASDAQ crash of March 2000 is this time accompanied by a spectacular spike of A1 and A2 accompanied by a depression in A3. This very good anticipation of the NASDAQ crash for this dataset which contains the sector components of the SP500 could be explained in part by the fact that the information technology sector component of the SP500 is correlated at over 90% with the NASDAQ. In the wake of the dot-com bubble, the same pattern reproduces itself around the 9-11 attacks. The same phenomenon happens again in the times leading up to the subprime crisis of August 2007, although the drop in À3 is less noticeable and on an even grander scale around the time of the Lehman Brothers collapse. The bankruptcy of General Motors on June 1st 2009 and the U.S sovereign credit degradation of August 5th 2011, when Standard & Poor's reduced the country's rating from AAA (outstanding) to AA+ (excellent), are also anticipated by a spike in A1 and A2 while the behavior of A3 is less easy to interpret in those instances.

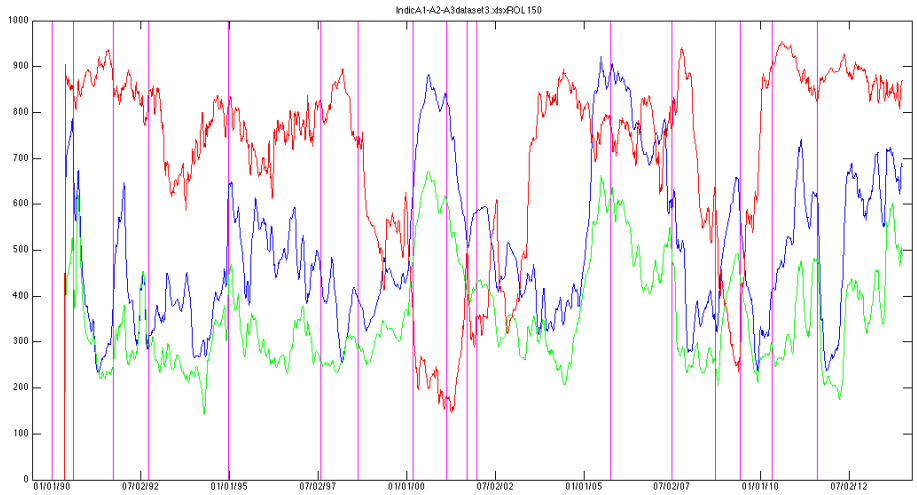

Fig.6 (A1 blue, A2 green, A3 red)

- For Dataset4, which is the "European" dataset, we get the following profiles (**Figure7**).

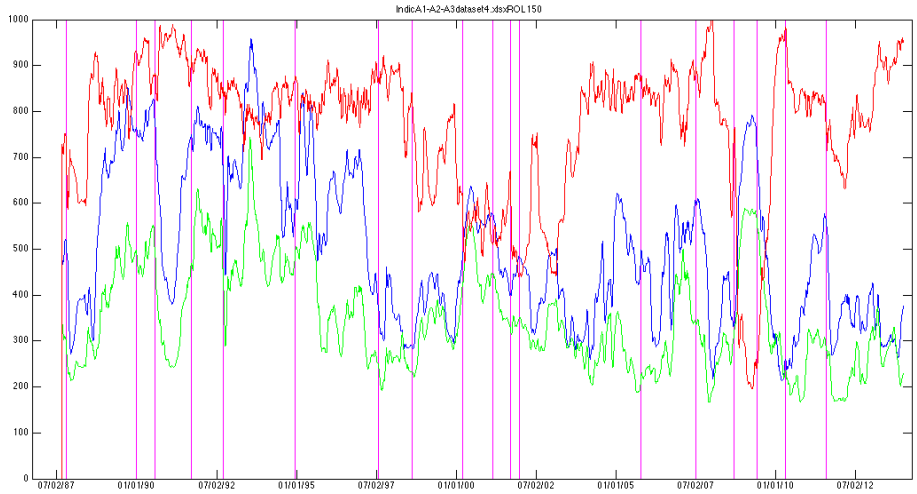

Fig.7 (A1 blue, A2 green, A3 red)

Since the U.S financial market still leads the world, the global structure of the profiles of A1, A2 and A3 is somewhat similar to the structure of the profiles we had obtained for Dataset1 and Dataset3. There are however some very interesting specificities which are characteristic of the European nature of Dataset4. Events like the S&L, the NASDAQ bubble burst and even subprime crisis of 2007 that preceded the Lehman Brothers collapse of 2008 are much less visible while the European sovereign debt crisis of April 23rd 2010 (Crisis 17) is spectacularly well anticipated with a huge spike in A1 and A2 accompanied

by the ominous drop in A3. The U.S sovereign credit degradation of 2011 is also very well anticipated. That could be explained by the fact that the sovereign credit degradation of several leading European countries (France was degraded as well from AAA to AA1 by Moody's on November 19th 2012) was also being discussed by the media and anticipated by the financial markets.

- For Dataset5, the "flight to quality" dataset which exploits the increasing correlation between equity and bonds in the times leading up to a financial crisis, we obtain the A1, A2 and A3 profiles below (**Figure8**). In a way they seem even tamer and noisier than those of Dataset2 which included, besides the commodities, some safe haven securities adding elements of "flight to quality" to its design as well. In Dataset5, the profiles of the indicators of series A present few remarkable features, besides the obvious ones that all the others possess like the spike in A1, A2 and the drop in A3 around the failure of Lehman Brothers in 2008. Unfortunately it seems that the inclusion of high quality sovereign and corporate bonds into the equity mix produced a dataset that is a little too resistant to most financial crises and is therefore, for the indicators of A-series at least, of limited interest as a way to anticipate crisis events.

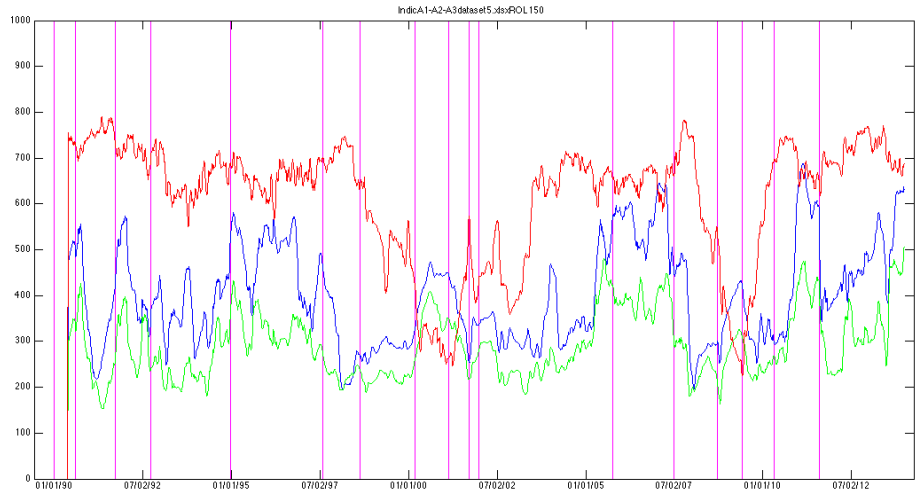

Fig.8 (A1 blue, A2 green, A3 red)

- Dataset6, our largest dataset containing 226 individual components of the SP500 index, produces the following high quality profiles when used with the indicators of the A-series (**Figure9**). Black Monday is outside of the span of Dataset6 and the S&L crisis is surprisingly not visible (maybe it's because the affected componies dropped out of the index which, like all composite stock indices, present some survivorship bias). The buildup to the 2000 NASDAQ crash is spectacular and characterized by the usual climb of A1 and A2 and fall of A3 as the correlations and volatility simmer inside the market and the spectrum of the covariance matrix is shifting to the right, toward the larger eigenvalues, away from $\Theta 1$ and $\Theta 2$ and toward $\Theta 3$. This pattern started well in advance of the actual crisis event (on March 10th 2000 when the NASDAQ started its sharp fall) and the indicators did provide an early warning in that instance. Then we observe the period of bullish market

in the early 2000' and again a slow buildup of correlations and volatility inside the market characterized by an increase in A1 and A2 (but not a drop in A3 that seems to only accompany truly catastrophic events) culminating during the subprime crisis of August 2007 which sets into motion the pattern of A1 and A2 spiking while A3 plummets leading to the collapse of Lehman Brothers. Then there is some recovery in the market before the 2011 U.S sovereign credit rating degradation which is anticipated by an increase in A1 and A2. Finally the Russian financial crisis of December 2014, which hit very hard some of the largest firms inside the SP500 which are energy/oil companies because of the fall in the price of crude and gas, is anticipated by an increase in A1 and A2 but we don't observe much on A3 at the same time.

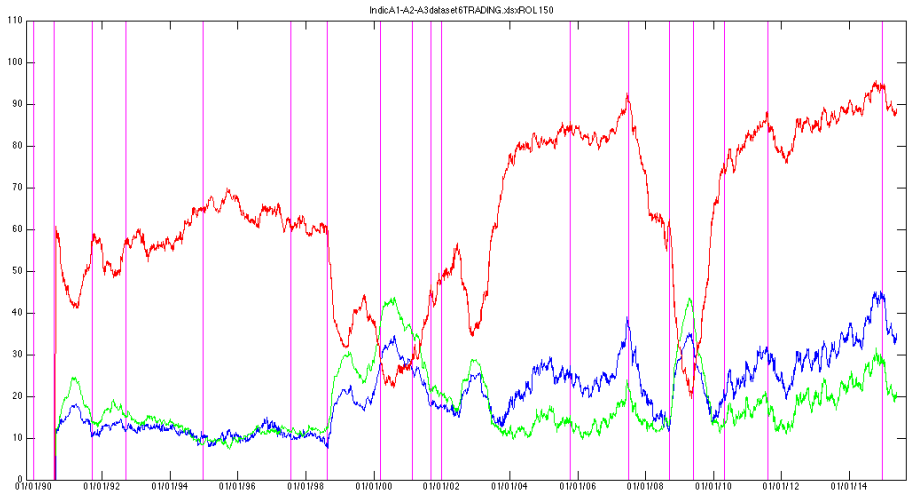

Fig.9 (A1 blue, A2 green, A3 red)

- The A-series profile for Dataset7 are represented below (**Figure10**). This dataset contains international indices from emerging economies converted from the local currency into U.S dollars. The usual features that dominated all the other A-series profiles for all the other datasets look a bit diluted in the case of Dataset7 : the collapse of Lehman Brothers is one unremarkable bump among dozens of others and the NASDAQ crash isn't visible for example. The Delphi bankruptcy of late 2005 ("Red October") did trigger an economic crisis in many emerging countries due to the closure or expected closure of many of the overseas factories of the American automotive parts giant and this event is indeed anticipated by a rise in A1 and A2 but it is difficult to differentiate it from the many false positives. The Argentine sovereign default of late 2001 is surprisingly not visible although many South American indices (and the MERVAL itself) are included in Dataset7. The Russian crisis of 2014 is however much more visible and better anticipated now than with the previous datasets and we observe a large spike in A1 and A2 accompanied by a noticeable drop in A3.

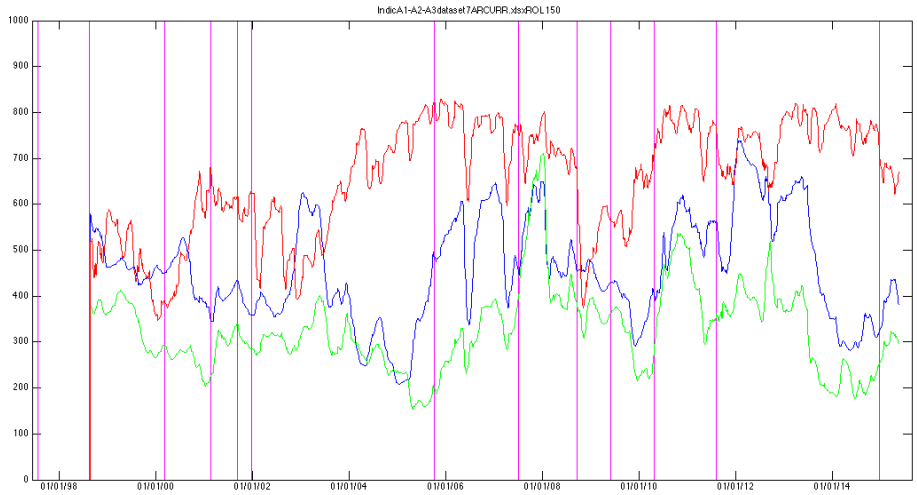

Fig.10 (A1 blue, A2 green, A3 red)

We now switch our attention to the indicators of the B-series. On all the profiles below, the crises of **Table1** are again represented as vertical purple lines. The spectral radius of the covariance matrix (mix volatility and correlation signal), Indicator B1, is in green. The trace of the covariance matrix (volatility signal), Indicator B2, is in red. The spectral radius of the correlation matrix (correlation signal), Indicator B3, is in blue and is not represented on the same scale as the others for better readability (we multiplied it by 20). For Dataset6, the correlation signal for the assets weighted by the market capitalization and volume traded, as defined in the previous section, will also be studied. We first remark that all the profiles of the B-series feature many false positives as those of the A-series and the spikes are located, with specificities depending on the dataset used, in the vicinity of the crisis events of **Table1** for B1 and B2, while the structure of B3 is somewhat harder to interpret but may still hold some valuable information.

For most datasets and most crises, B1 and B2 produce very similar profiles. When the profiles of B1 and B2 get closer to one another (ie. for the covariance matrix, the trace becomes close to the spectral radius) it means that correlations are increasing inside the financial market because one eigenvector's direction (the direction of the spectral radius) is becoming dominant over all the other ones. We do not however generally observe on our graphs that B3 is increasing when B1 and B2 are getting closer and that is due to the fact that B3 is only taking correlations into account while B1 is a mixed signal of volatility and correlation. The correlation component of B1 is at each date an average correlation weighted by the the volatility of the assets constituting the dataset. In order to compare the relative position of B1 and B2 to the behavior of B3, we should have weighted the coefficients of the correlation matrix by the volatility of the assets which would have defeated our goal to study the correlations alone. In other words, when the relative position of B1 and B2 isn't compatible with the behavior of B3, then it means that the assets are becoming correlated or uncorrelated depending on their volatility. For example when B1 and B2 are getting closer (correlations are increasing) but B3 isn't increasing in a clear manner, it means that the high volatility stocks are becoming uncorrelated while the low volatility stocks are becoming correlated.

- For Dataset1, we obtain the following results (**Figure11**). This international equity dataset produces profiles of B1 and B2 which are very similar with spikes in the vicinity of the major international crises like the NASDAQ crash and the failure of Lehman Brothers. The relaxation in both volatility and correlations after a crisis event is also much more noticeable than with the profiles based on the Hellinger distance, especially after the NASDAQ crash and the onset of the bullish market period. The B1 and B2 signals are almost always co-monotonic, it is what we expected and coherent with the works of Leonidas Sandoval Junior [3] who proved, using data which was very different from ours, that high volatility in financial markets usually accompany a high level of correlations.

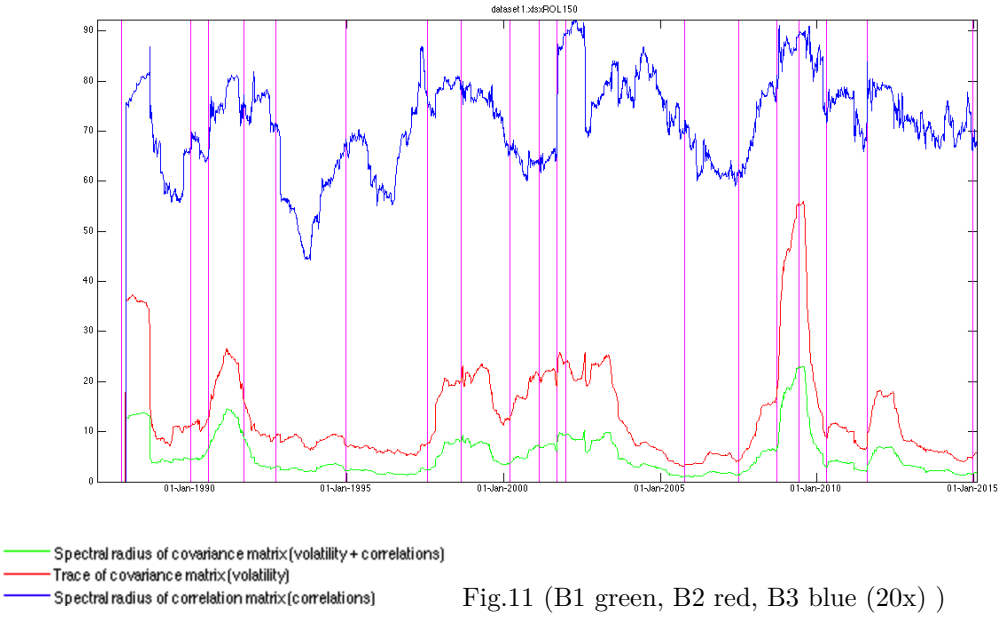

Fig.11 (B1 green, B2 red, B3 blue (20x))

- For Dataset2, we obtain the following results (**Figure12**). The profiles are very similar to those of Dataset1, with less prominent features because of the inclusion of safe haven securities in the dataset. The elevation in both volatility and correlations is a little more noticeable around the Russian sovereign crisis of 1998 which may result from the inclusion of energy related commodities in this dataset and highlight the characteristic increased correlations between energy (like oil companies stocks) and commodity securities in the times leading up to a financial crisis.

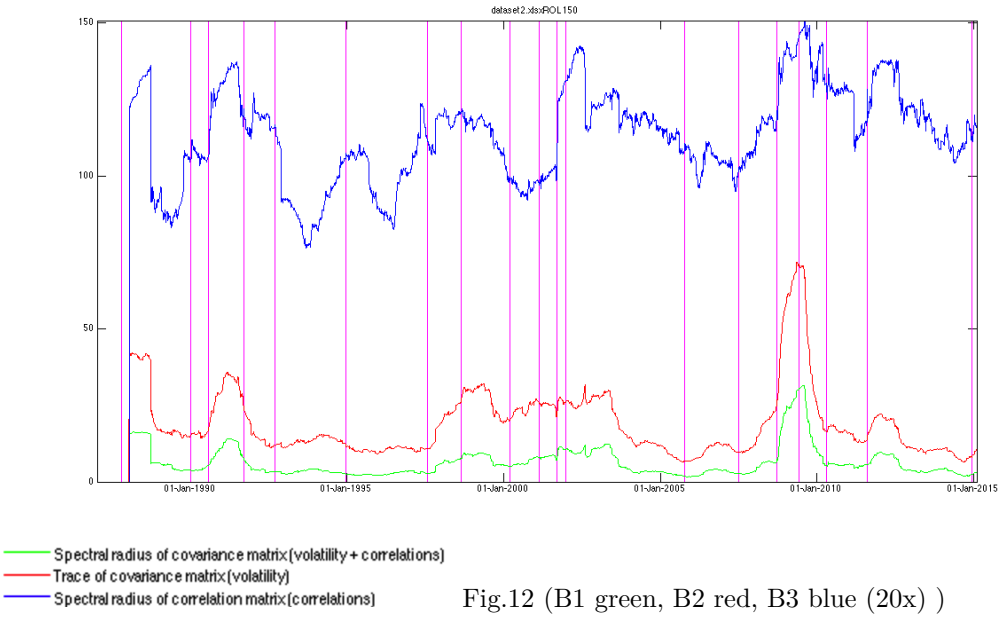

Fig.12 (B1 green, B2 red, B3 blue (20x))

- For Dataset3, we obtain the following results (**Figure13**). The profiles of B1 and B2 highlight the American orientation of Dataset3 with especially visible features for the NASDAQ crisis, the 2008 global financial crisis and the U.S sovereign credit degradation. The profile of B3 is again somewhat difficult to interpret. In particular there is an apparent massive drop in correlations following the NASDAQ crisis that isn't accompanied by a similar drop in volatility.

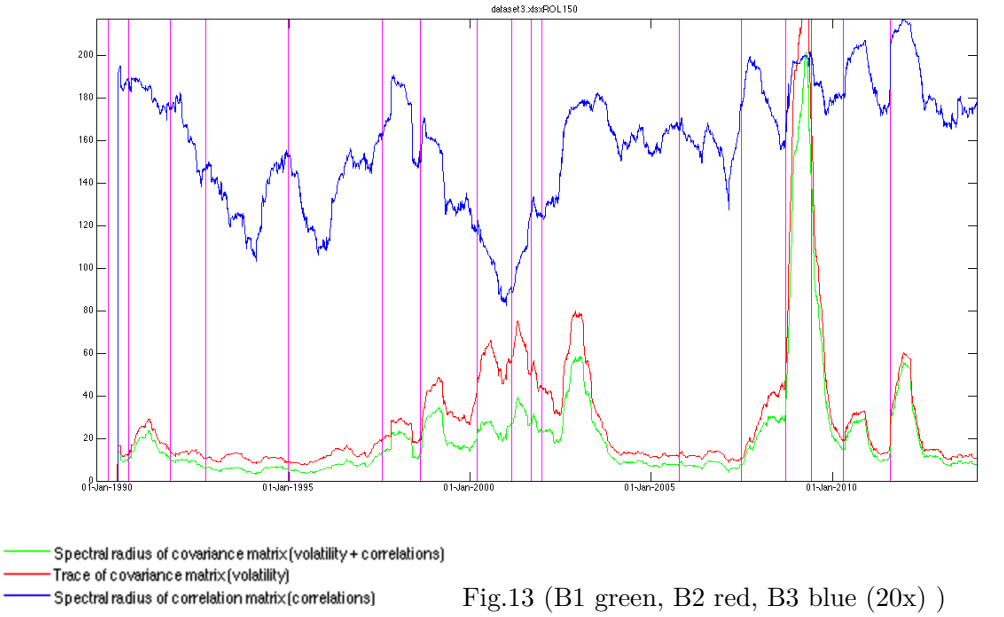

Fig.13 (B1 green, B2 red, B3 blue (20x))

- For Dataset4, we obtain the following results (**Figure14**). It is the European counterpart of Dataset3 and produces better detection of crisis that are mostly or originally European in nature like the Eurozone sovereign debt crisis.

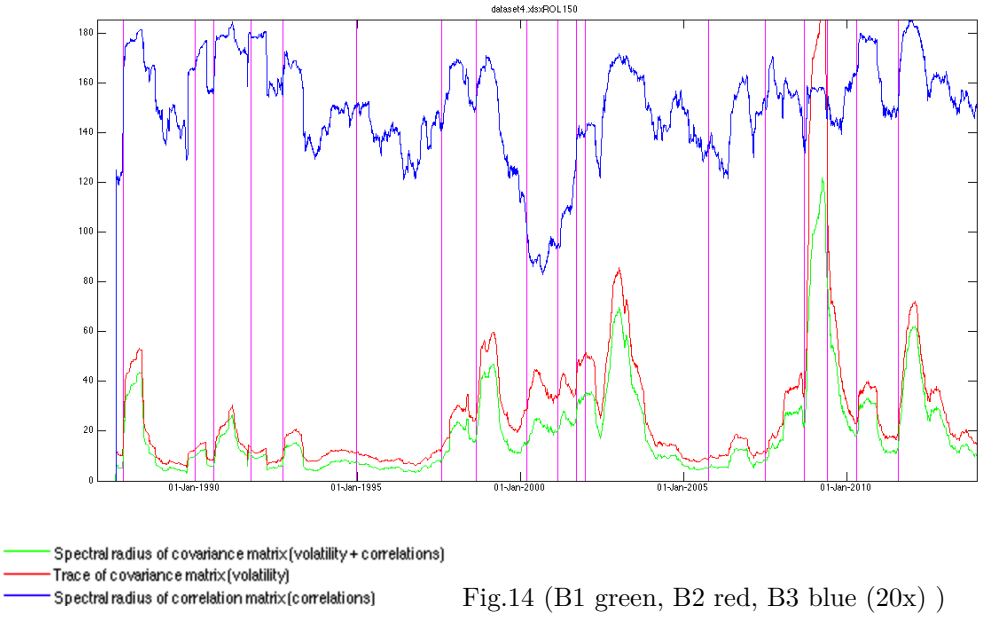

Fig.14 (B1 green, B2 red, B3 blue (20x))

- For Dataset5, we obtain the following results (**Figure15**). The inclusion of high quality bonds to model the "flight to quality" effect in the times leading up to a financial crisis has the effect, like for the A-series indicators, to produce very tame profiles of B1 and B2. Only the truly momentous events like the NASDAQ crash and the 2008 crisis are visible but all the other events are difficult to see, even the Asian crisis of 1997 which is surprising. The volatility signal B3 is still very noisy but some structure is starting to emerge with big spikes in the vicinity of known crises and large drops afterwards when the market is entering a post-crisis relaxation phase.

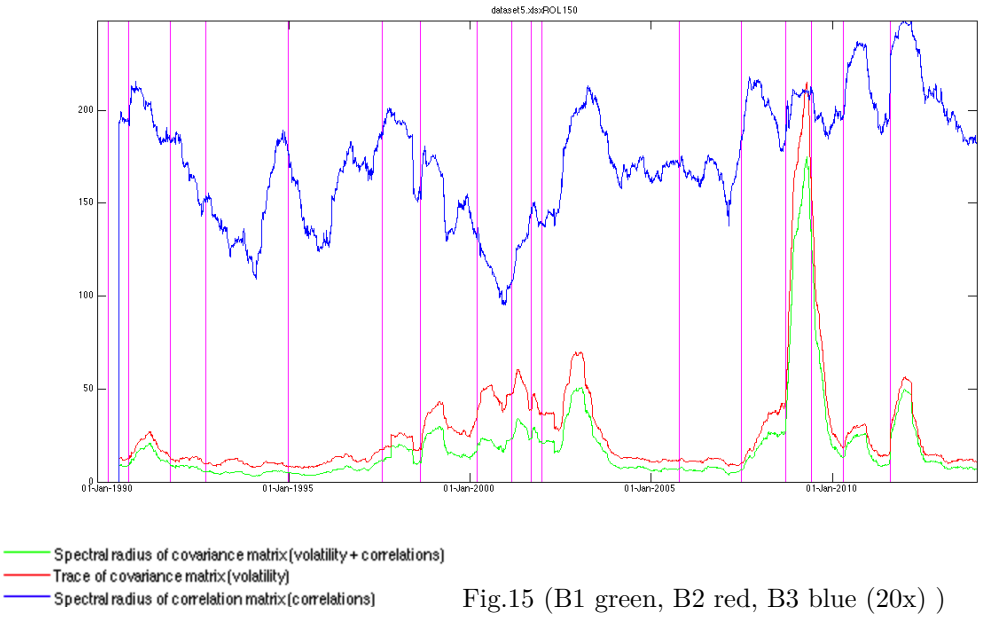

Fig.15 (B1 green, B2 red, B3 blue (20x))

- For Dataset6, we obtain the following results (**Figure16,17,18,19**).

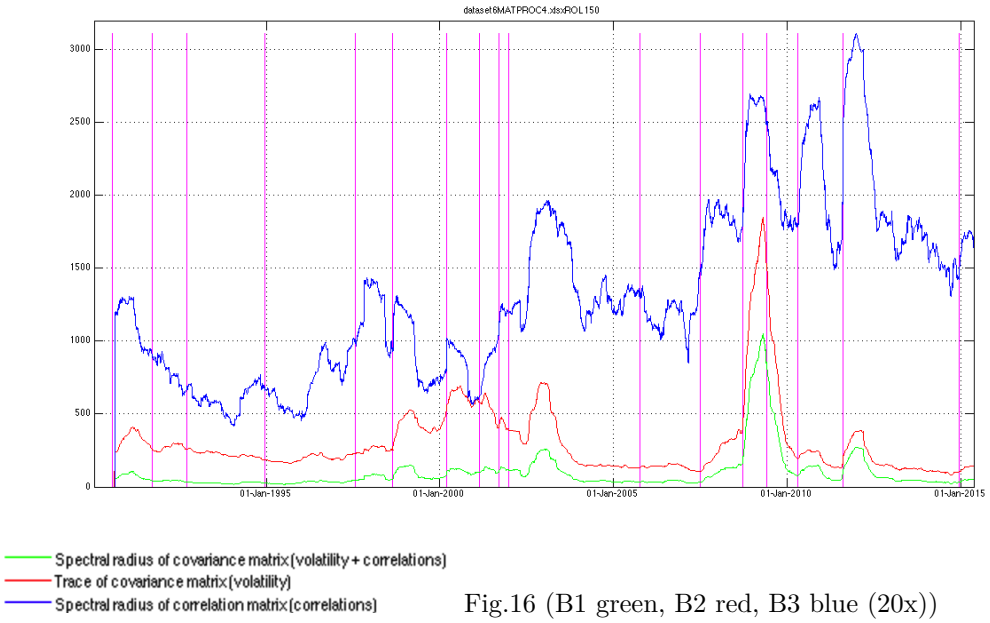

Fig.16 (B1 green, B2 red, B3 blue (20x))

The profiles for B1 and B2 are globally similar to those we obtained for Dataset1 and Dataset3 with the buildup of leverage during the subprime crisis more clearly visible but this time the correlation signal B3 in blue is much more interesting and contains a lot of usable information. Indeed, Dataset6 is constituted a large number of individual stocks instead of indices and there is no averaging effect on the correlations.

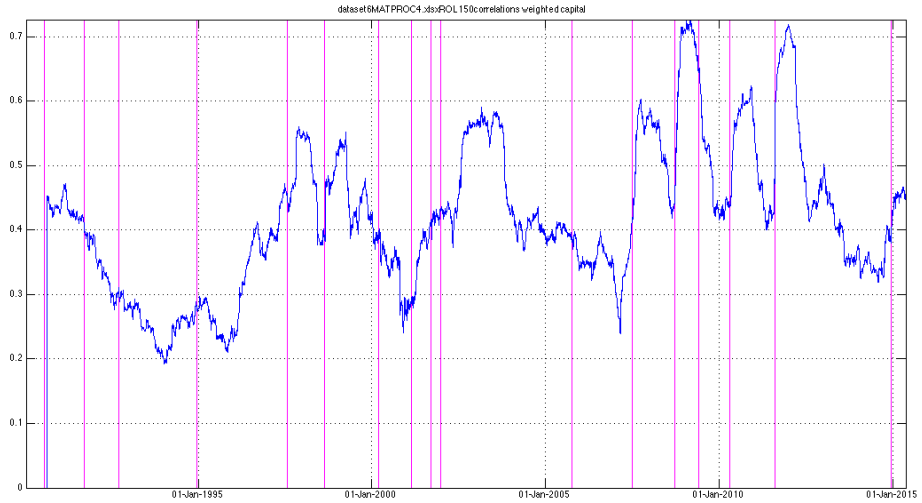

Fig.17 (B3A)

Here we have the correlation signal weighted by the capital of the companies corresponding to the stocks. We see very interesting patterns emerge and a possible increase of the power of prediction for this indicator.

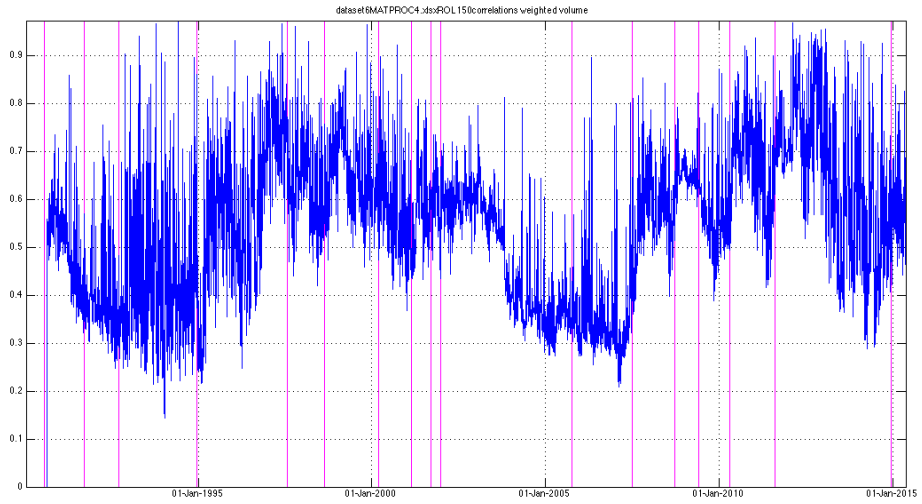

Fig.18 (B3B)

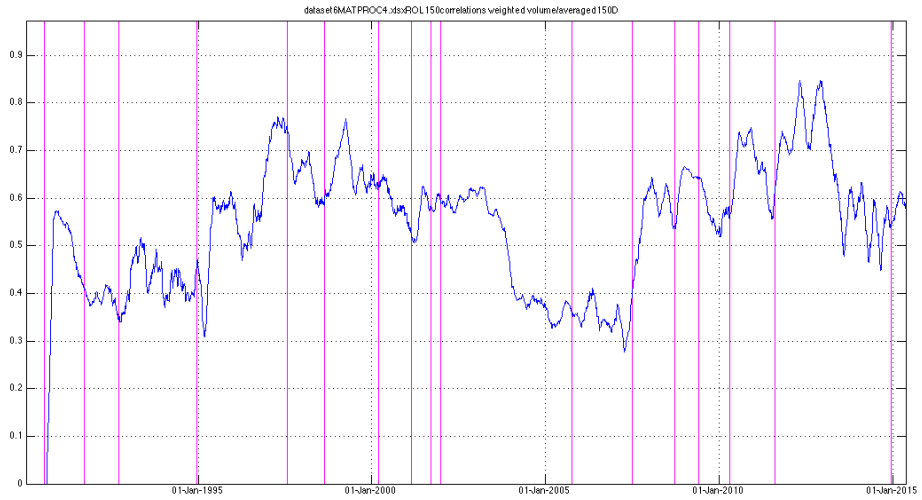

Fig.19 (B3C)

We represented the correlation signal B3 weighted by the volume traded in dollars and an averaged version for better readability. Again, patterns clearly emerge and the power of this indicator to preempt rather than merely confirm the crises, as it was likely the case for most of the financial crisis indicators we have studied until now, seems to have increased.

- For Dataset7, we obtain the following results (**Figure20**).

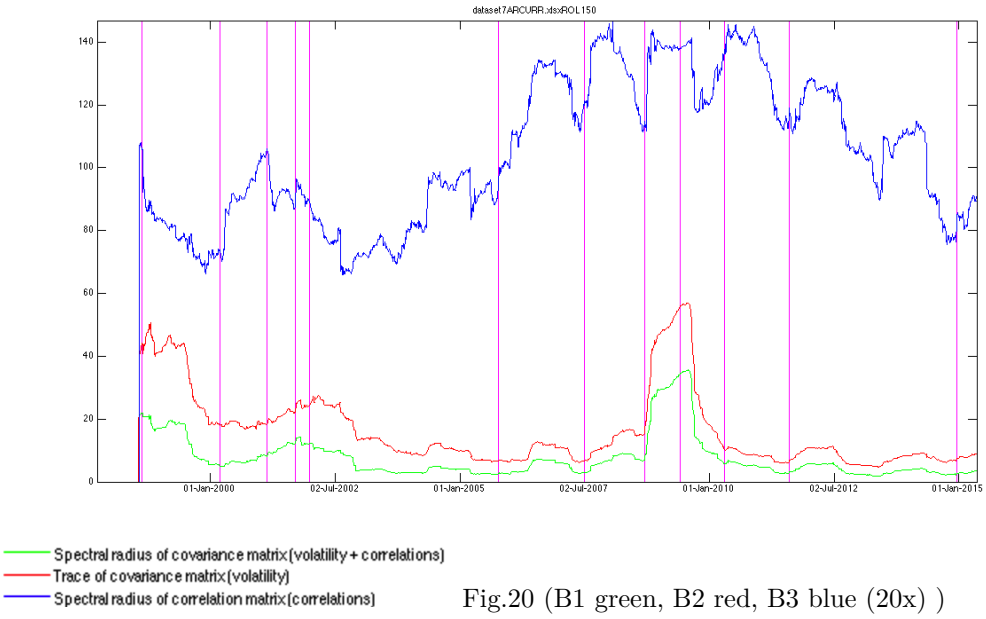

Fig.20 (B1 green, B2 red, B3 blue (20x))

The profiles are again a bit disappointing and few truly noticeable features appear besides the obvious ones in B1 and B2. The Asian crisis of 1997 (at the very edge of our dataset unfortu-

nately) seems to have been much more visible when using Dataset7 than when using the other datasets. Here we can only see the aftermath of the crisis.

5.2 Predictive Power

In this section we are going to study the predictive power of indicators B3B and B3C, those that are based on the correlations weighted by market volume, using Dataset6. We will rely on the Max Draw Down (MDD) at horizon $H = 100$ days which is commonly used by professionals in the financial industry. The reference asset price R for which this quantity will be computed at each date will be the SP500.

$$MDD_H(t) = max_{t \leq x \leq y \leq t+H}(1 - \frac{R(y)}{R(x)})$$

We work with the scatter plots $[B3B(t), MDD_H(t)]$ and $[B3C(t), MDD_H(t)]$ for all dates t covering the span of Dataset6. We notice that the structures of those scatter plots is dominated by a double threshold. We are going to exploit that fact in order to build a trigger for our indicators : at a given time t, when the value of B3B (resp.B3C) is between those twin threshold, we say that we are in the "danger zone", which is where the probability of a crisis happening within 100 days is the highest. This makes also a lot of sense from a theoretical point of view. As a matter of fact, when the weighted correlations are low, then the probability of a crisis happening (ie. experiencing a very high MDD over the next 100 days) is low, but when it's extremely high that means that we are already right in the middle of a crisis and the expected MDD at 100 days is low as well because the market would be likely out of the crisis and in full recovery.

A crisis C from **Table1** happening at time t_0 will be considered predicted ex-post by B3B (resp. B3C) if at least 60% of the points $[B3B(t), MDD_H(t)]$ (resp. $[B3C(t), MDD_H(t)]$) such that $t \in [t_0 - 100, t_0]$ are in the danger zone of the indicator. The false positive ratio is defined as the number of points inside the danger zone that belong to one of the crisis of **Table1** over the total number of points inside the danger zone.

To define the danger zone, we separate Dataset6 into two periods: one calibration period between January 17th 1990 and December 31st 1999 and a forecast period where the power of prediction of B3B and B3C will be put to the test. For both indicators, the scatter plot restricted to the calibration period and the scatter plot covering the whole sample are represented below (**Figure 21,22**). The scatter plots have roughly the same global shape in both periods.

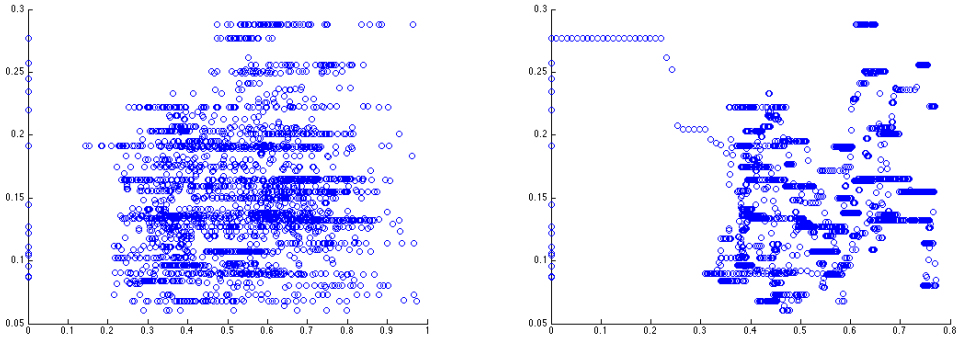

Fig.21 (Calibration period / left : B3B, right : B3C)

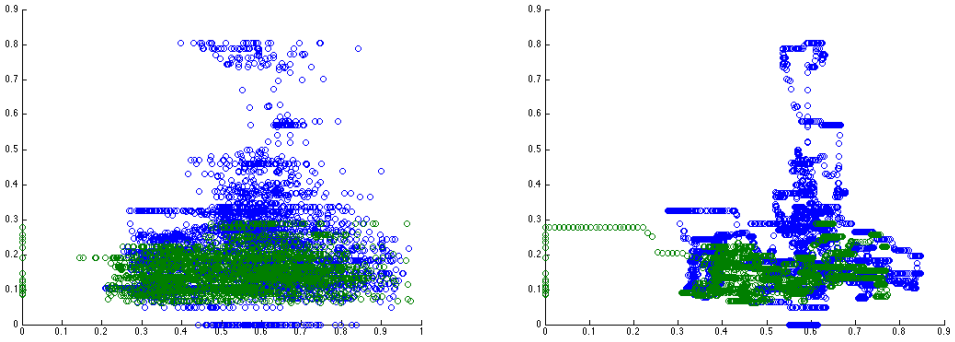

Fig.22 (Calibration period : green, Whole sample : blue ; left : B3B, right : B3C)

Working on the calibration period, we consider four thresholds of MDD (15%, 20%, 25%, 30%), and after choosing arbitrary values for the twin thresholds of the indicator, we plot the proportion of days for which the actual MDD has been greater than a given MDD threshold against the number of points inside the danger zone for a 100 days window before each date.

The goal of this procedure is to manipulate the values for the twin thresholds of the indicator until the plots are globally increasing (ie. the more points inside the 100 days window before each date, the greater the probability than the MDD threshold is going to be exceeded) or at least present spikes toward the higher numbers of points. Getting some help from Figure 21, we obtained experimentally the best possible plots (by using increments of 0.1) which are represented below in **Figure 23**.

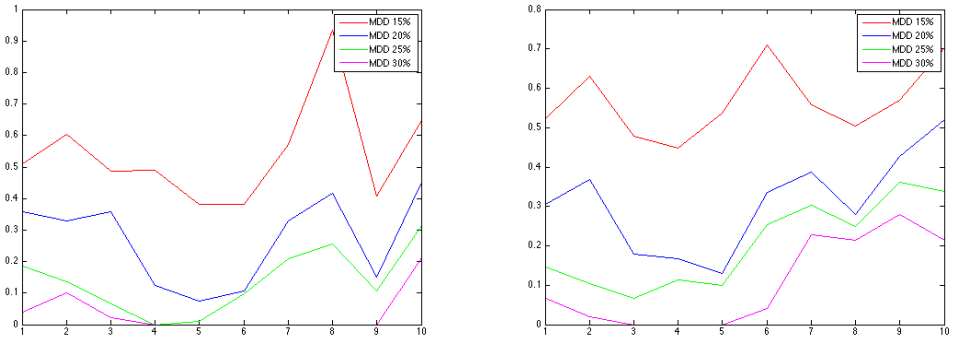

Fig.23 Calibration plots (X-axis : number of points inside the danger zone (x10) ; Y-axis: proportion of days for which the actual MDD has been greater than a MDD threshold).

We obtained the optimal twin thresholds for the danger zone as : [0.41,0.8] for B3B and [0.50,0.70] for B3C. Using those values, we are now able to count for each crisis of **Table1** inside the forecast period the number of dates in a 100 days window preceding each crisis for which the value of B3B (resp. B3C) was inside the danger zone as well as the global proportion of false positives.

We obtain the following results (**Table2**) in which the crises inside the calibration period have been put in italic (crisis1 and crisis2 are located before the start of Dataset6).

	B3B	B3C
crisis1	NA	NA
crisis2	NA	NA
crisis3	*0*	*0*
crisis4	*51*	*18*
crisis5	*22*	*0*
crisis6	*41*	*0*
crisis7	*74*	*0*
crisis8	*98*	*100*
crisis9	93	100
crisis10	94	100
crisis11	98	100
crisis12	100	100
crisis13	13	0
crisis14	15	0
crisis15	98	100
crisis16	99	100
crisis17	97	100
crisis18	96	100
crisis19	90	78
False Positive (%)	73.38	71.30

We observe that using those indicators with their proper calibration method would have enabled us to truly anticipate almost all the crises of **Table1** inside the forecast period : there are very few false negatives. Only the Bankruptcy of Delphi (crisis13), which effects weren't instantaneous on the U.S economy in general and on the SP500 in particular and the subprime crisis (crisis17) which was a months long process starting in August 2007 and on which we had to pin an arbitrary date, aren't properly forecasted. The proportion of false positives, while still relatively high, is unlikely to represent an insurmountable obstacle for a practitioner as the information at a given date t that there is around a 30% chance of a crisis happening (and a 70% chance of nothing happening) in the next 100 days is already an extremely valuable piece of information and a much more precise one than what we had expected in the introduction where we talked about a 10% chance of a crisis happening.

6 Conclusion

As a general conclusion, we could start by saying that all our indicators do perform quite well as a crisis confirmation method and present interesting results depending on the datasets used. On top of that we demonstrated that Indicators B3B and B3C do possess, after proper calibration, a real power of prediction in estimating the probability of a financial crisis happening at a given horizon in the future. Each financial crisis is a unique event happening at a given point in time and in unique world-wide conditions, so even if we could create an indicator that was able to react perfectly in advance of every single past crisis within the restrictions of the dataset it used, it would by no means guarantee its ability to predict the next crisis but it would make that detection highly probable. This is enough for us in the real world. Since even mathematics has to bow down before causality, the best way to "prove" the predictive power of our financial crisis indicators would be therefore to apply them in real time to the market through algorithmic trading strategies directly derived from them. That approach will be the topic of a future paper.

If practitioners were able to make money from those indicators then, for all intent and purposes, they would be right and true... maybe not forever, maybe not on every market, but right now and in a given situation. That is the best we could hope for when doing applied mathematical finance with the goal to build something that is useful in practice.

7 Appendix

7.0.1 Composition of Dataset6

AA, AAPL, ABT, ADM, ADSK, AEP, AFL, AIG, ALTR, AMAT, AMGN, AON, APA, APC, APD, ARG, AVY, AXP, BA, BAC, BAX, BBY, BCR, BDX, BEN, BHI, BK, BLL, BMY, C, CAG, CAT, CB, CCE, CELG, CI, CINF, CL, CLX, CMA, CMCSA, CMI, CMS, CNP, COP, CPB, CSC, CSX, CTAS, CTL, CVS, CVX, D, DD, DE, DHR, DIS, DOV, DOW, DTE, DUK, EA, ECL, ED, EFX, EIX, EMC, EMR, EOG, EQT, ES, ETN, ETR, EXC, EXPD, F, FAST, FDO, FDX, FISV, FITB, FMC, GAS, GCI, GD, GE, GIS, GLW, GPC, GPS, GWW, HAL, HAR, HBAN, HCP, HD, HES, HOG, HON, HOT, HP, HPQ, HRB, HRL, HRS, HSY, HUM, IBM, IFF, INTC, IP, IPG, IR, ITW, JCI, JEC, JNJ, JPM, K, KLAC, KMB, KO, KR, KSU, L, LB, LEG, LEN, LLTC, LLY, LM, LNC, LOW, LRCX, LUK, LUV, MAS, MCD, MDT, MHFI, MMC, MMM, MO, MRK, MSFT, MSI, MUR, NBL, NEE, NEM, NI, NOC, NSC, NTRS, NUE, NWL, OKE, OXY, PAYX, PBCT, PBI, PCAR, PCG, PCL, PCP, PEG, PEP, PFE, PG, PGR, PH, PHM, PKI, PNC, PNW, PPG, PPL, PVH, R, ROST, RTN, SCG, SHW, SIAL, SLB, SNA, SO, SPLS, STI, SWK, SWN, SYMC, SYY, T, TE, TEG, TGT, THC, TIF, TJX, TMK, TMO, TROW, TRV, TSO, TSS, TXT, TYC, UNM, UNP, USB, UTX, VAR, VFC, VMC, VZ, WEC, WFC, WHR, WMB, WMT, WY, XEL, XOM, XRAY, XRX,

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