Investment Strategies Based on Financial Crisis Indicators

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Optimal Investment Strategies Based on Financial Crisis Indicators

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- **Professor Raphael Douady (Stony Brook University)**, my PhD adviser.
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- Financial crises cannot be *predicted*. What can be done is to evaluate the *probability* of a crisis happening in the near future by measuring whether the right conditions are present in a market for a random spark to ignite a crisis.
- Correlations between asset returns and volatility are the two market vital signs that we study in order to build financial crisis indicators.
- We build two types of financial crisis indicators :
  - Those that compute the Hellinger distance between the empirical distribution of the eigenvalues of the covariance or correlation matrix and a distribution of reference.
  - Those that rely on spectral properties (spectral radius, trace,Frobenius norm) of the covariance or correlation matrix.

Introduction

- Using the forecasting power of our indicators, we build optimal active investment strategies.
- The basic idea is to anticipate market movements : liquidate our positions before prices start to drop and buy them back before prices begin to recover.
- The data is constituted of the stock components of major equity indices.

The Method and the Data

- Data frequency is daily but our approach can be easily adapted to intraday data.
- For each dataset constituted of L assets, we consider at each date t a rolling window $ROL(t)$ of the log-returns of the components. This is a matrix with L rows and T columns. $T = E[110\% \times L]$, the number of observations.

The Method and the Data

- T is chosen large enough ($T > L$) to be able to obtain the whole spectrum of the $L \times L$ covariance and correlation matrices using singular value decomposition (SVD).
- T isn't chosen too large in order to avoid giving our indicators a too long memory and preserve their responsiveness to market events.
- Using SVD enabled me to significantly reduce computation time and eliminated the symmetry defects (and hence the nuisance negative and imaginary parts of the eigenvalues) introduced by obtaining the covariance and correlation matrices by transposition and multiplication of the rolling window.

The Method and the Data

- At a given date t, $ROL^*(t)$ (centered) and $ROL^{**}(t)$ (centered and normalized) are used respectively to obtain the singular values of the covariance and correlation matrices.
  $\forall j \in [1, L], \forall k \in [1, T]$

$$ROL^*(t)(j,k) = \frac{1}{\sqrt{T}}(ROL(t)(j,k) - mean(ROL(t)(j,:)))$$

$$ROL^{**}(t)(j,k) = \frac{ROL^*(t)(j,k)}{\sqrt{var(ROL(t)(j,:))}}$$

- $ROL^{**}_{weight}(t)$ is also used to obtain a weighted version of the covariance matrix designed to give more importance to stocks from companies possessing a given characteristic, which can be: most traded, highest market capitalization, highest financial leverage (debt to market capitalization ratio).

- At a given date t, for a given weight (volume, market capitalization, leverage) $\forall j \in [1, L], \forall k \in [1, T]$

$$ROL^{**}_{weight}(t)(j,k) = \frac{weight_j(t)}{\sum_{l=1}^{L} weight_l(t)}$$

- With the $L \times T$ matrix $A(t)$ representing $ROL^{*}(t)$, $ROL^{**}(t)$ or $ROL^{**}_{weight}(t)$, we perform the SVD :

$$A = U\Sigma V^T; \Sigma = \left[\begin{array}{cccc|ccc} \sigma_1 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_L & 0 & \dots & 0 \end{array}\right]$$

- The eigenvalues λ_i of AA^T are obtained from the singular values σ_i found in Σ : $\forall i \in [1, L], \lambda_i = \sigma_i^2$

Indicators Based on the Hellinger Distance

- Those indicators are based on comparing the empirical spectrum of one of our matrices (covariance, correlation, correlation weighted) at time t and a distribution of reference representing either a calm or agitated state of the market.
- The metric used is the Hellinger distance $\mathbb{D}$ measuring the distance between two discretized probability distributions, both known at the points X_i :

$$\mathbb{D}^2 = \sum_{i=1}^{N} (\sqrt{P(X_i)} - \sqrt{Q(X_i)})^2$$

- A spectrum shifted toward the higher eigenvalues is indicative of dynamical instability of the market: excess correlation and volatility (unless normalized).

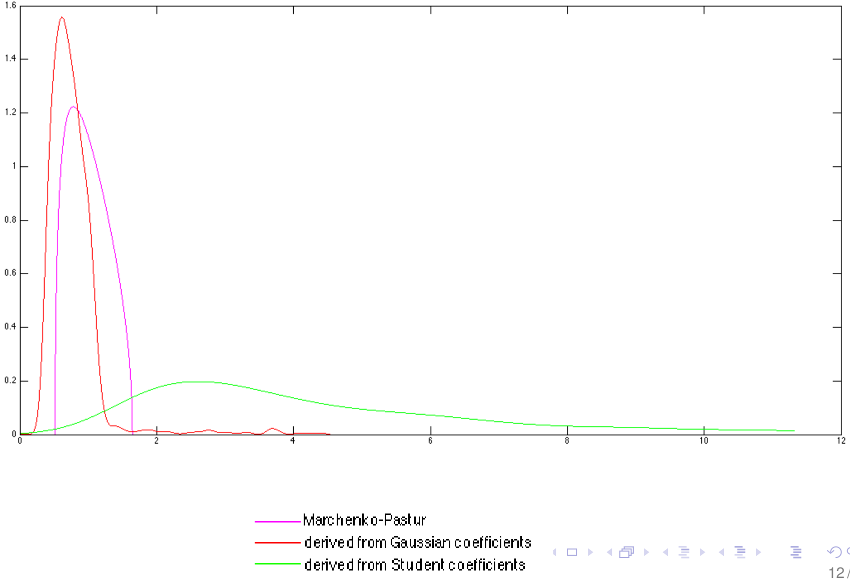
Indicators Based on the Hellinger Distance

- Reference A1: Marchenko-Pastur's distribution (Gaussian without correlations). Reference : calm market
- It is the theoretical distribution of the eigenvalues of the covariance matrix of the limit of a sequence of matrices constituted of i.i.d Normal Gaussian coefficients $N(0,1)$. The matrices in the sequence are increasing in size toward infinity but keep a finite aspect ratio.

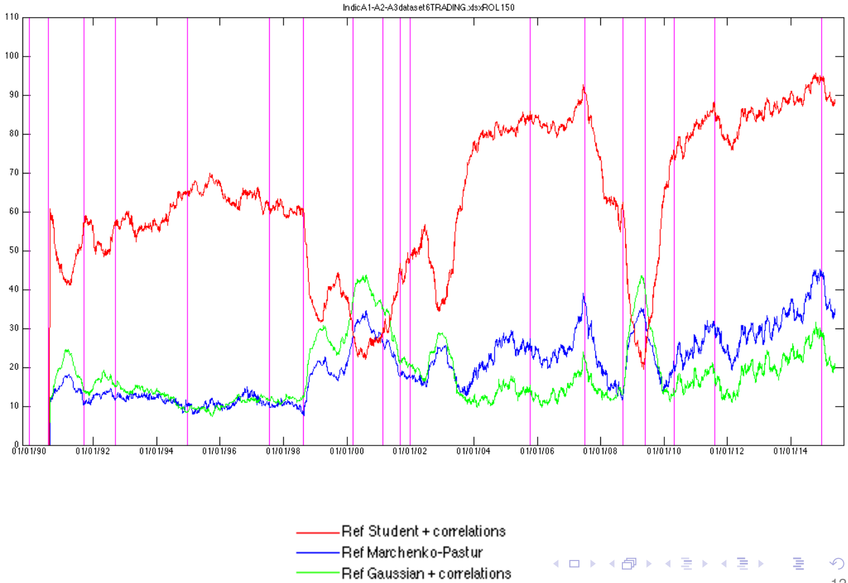
Indicators Based on the Hellinger Distance

- Reference A2: Distribution of the eigenvalues of the covariance matrix of a random matrix obtained from Normal Gaussian $N(0,1)$ coefficients with simulated correlations. Reference : calm market
- Reference A3: Distribution of the eigenvalues of the covariance matrix of a random matrix obtained from Student (t=2) coefficients with correlations. Reference : distressed market

Indicators Based on the Hellinger Distance



Indicators Based on the Hellinger Distance



Indicators Based on the Hellinger Distance

- Dataset used is constituted of 226 individual components of the SP500. Some known crises are represented as vertical lines. The covariance matrix is used to compute the spectrum.
- A1 and A2 tend to spike before and during a crisis. As for A3, it plummets around truly large systemic events (dot-com bubble, 2008 Lehman Brothers failure, etc...).
- For a given dataset, at a given date t, we compute 15 indicators of that type: we can use three reference distributions (A1, A2, A3) and five matrices : covariance, correlation, correlation weighted by volume, correlation weighted by market capitalization and correlation weighted by financial leverage.

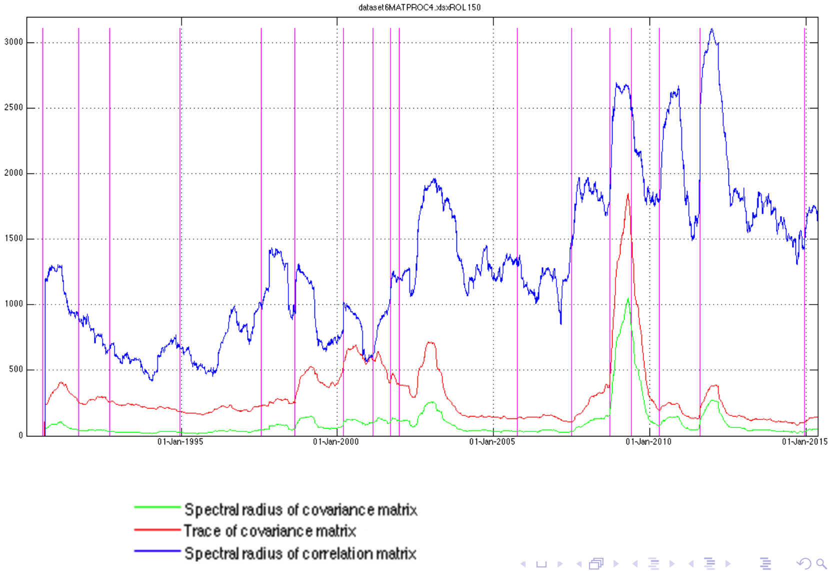
Indicators Based on the Spectral properties

- We use 3 spectral properties ; the trace, the spectral radius and the Frobenius norm.
- We have 5 matrices (covariance, correlation, corelation weighted by volume, correlation weighted by market capitalization and correlation weighted by financial leverage)
- This gives us 14 indicators at each date t (nb: the trace of the correlation matrix is constant and equal to L, the number of assets).

Indicators Based on the Spectral properties

- A high spectral radius indicates dynamic instability in the system (some assets are becoming correlated to all the others) and, for the covariance matrix, a spike in volatility as well.
- A high trace (and a trace much bigger than the spectral radius) indicates that the whole spectrum is shifting toward the higher eigenvalues and that correlation and volatility are spiking in the market.
- A high Frobenius norm similarly indicates a shift of the spectrum toward the larger eigenvalues. Sensitivity of this indicator may be increased by its summing of the squared eigenvalues.

Indicators Based on the Spectral properties



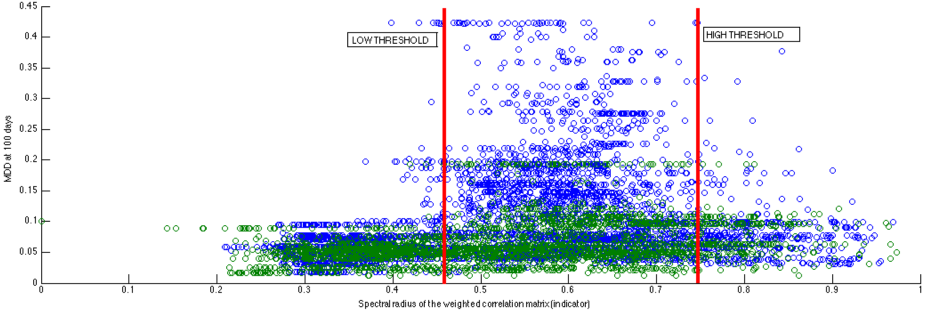
Indicators Based on the Spectral properties

- Dataset is constituted of 226 individual components from the SP500, well known crises are represented by vertical purple lines.
- Correlations and volatility (20x, not on the same scale) spike before and during crisis events.
- Pattern especially visible around the Dot-Com crash and its aftermath and the 2008 Lehman Brothers failure.

Financial Crisis and Calibration

- For all of our 29 indicators, we define a calibration period. It has to be strictly larger than the rolling window and preferably contain at least 100 calibration dates.
- At each date t of the calibration period, we retrospectively compute the *maximum draw down* (MDD) for the price $X(t)$ of the index at a given time horizon T, which will be taken at 100 days.
- Over the calibration period, we plot the scatter graph of MDD vs. Indicator Value and we typically obtain something which has the following structure (whole period in blue and the calibration period in green):

Financial Crisis and Calibration



- Considering the points above a chosen ***MDD Threshold*** and given a chosen ***Danger Zone Span***, we fit the low and high thresholds by maximizing the points between them.
- In order not to use any information from the future, active trading can start 100 days after the last date of the calibration period.
- At each date t_0 of the active trading period, for each of our 29 indicators, we look back 100 days in the past and count the number of times each indicator was in its danger zone.
- We define the ***Indicator Sensitivity*** as the third parameter of out trading strategies. A given indicator returns a red flag at t_0 if the number of times it was in its danger zone during $[t_0 - 100, t_0]$ is greater or equal to the Indicator Sensitivity.

Financial Crisis and Calibration

- Given a dataset, choosing the appropriate values for the three parameters of our model is vital for our active trading strategies to be successful.
- They are determined through trial and error. The skill of the users and their knowledge of the index history are very important.
- We compute every day the number of red flags $\Gamma \in [0, 29]$. Our trading decision (buy/stay/sell) will be taken every day according to a set of rules about that number.

Active Trading Strategy

- Once we have a profile of Γ, building an active trading strategy means assigning an action (buy/stay/sell and in what quantities) to every possible value of Γ.
- Typically, since we usually choose high indicator sensitivities, we use at each date t the following rules :
  - $\Gamma \in [0, 1]$: buy 10% more shares or no more cash
  - $\Gamma = 2$: do nothing
  - $\Gamma > 2$: sell 10% of the shares or no more share
- Blue Investor is passive, Green investor is active. Both start with 10,000 shares of the index and \$10M in cash.

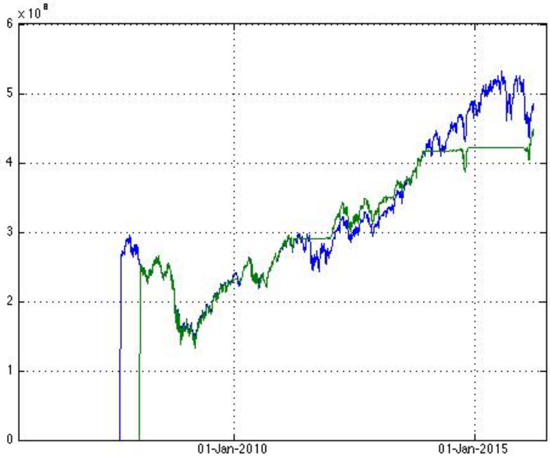
Active Trading Strategy

CAC40 index : success. Threshold 10%; Sensitivity 75%; Span 10

Active Trading Strategy

NASDAQ index : fail. Threshold 10%; Sensitivity 60%; Span 10

Active Trading Strategy

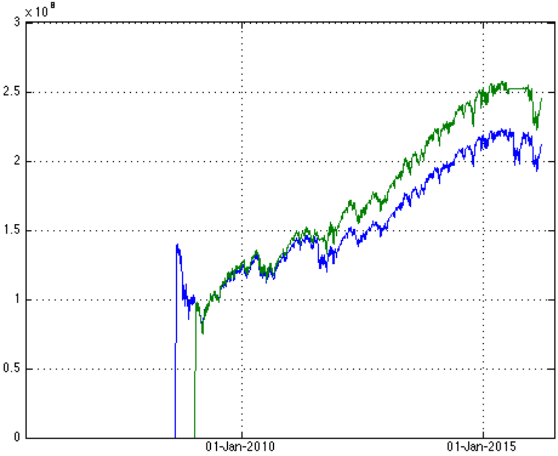


NASDAQ index : fail. Threshold 15%; Sensitivity 70%; Span 10

Active Trading Strategy

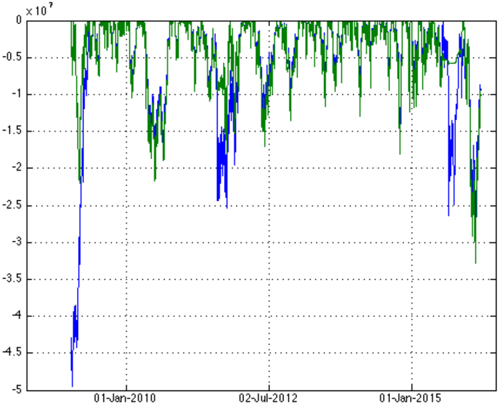
NASDAQ index : success. Threshold 15%; Sensitivity 80%; Span 10. Increasing the sensitivity and threshold makes red flags harder to achieve, the strategy is less inclined to sell and follows the index more.

Active Trading Strategy



SP500 index : success. Threshold 25%, Sensitivity 90%, Span 10.
Good strategy : steady profit while never being completely un-invested.

Active Trading Strategy



Daily draw down. More than the actual profit, this is what many investors are interested in. A lesser draw down is more desirable. The green curve is most of the time under the blue one which is a good result.

Active Trading Strategy

Difference in value between the passive strategy portfolio and the active strategy portfolio. Good steady increase.

Conclusions

- While exploiting the power of forecast of our financial crisis indicators, we intend to prove that, in a very modest way limited to our framework and data, it may be possible sometimes to "beat the market" in a reproducible fashion and that active trading strategies can be successful as a result of skill and not only of pure luck.
- More work is needed to devise more elaborate strategies. In particular, we would like to avoid lengthy un-invested periods of time.

Thank you for your attention :-)