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Analysing correlations after the financial crisis of 2008 and multifractality in global financial time series

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Abstract. We apply random matrix theory (RMT) to investigate the structure of cross-correlation in 20 global financial time series after the global financial crisis of 2008. We find that the largest eigenvalue deviates from the RMT prediction and is sensitive to the financial crisis. We find that the components of eigenvectors corresponding to the second largest eigenvalue changes sign in response to the crisis. We show that 20 global financial indices exhibit multifractality. We find that the origin of multifractality is due to the long-range correlations as well as broad probability function in the financial indices, with the exception of the index of Taiwan, as in all other indices the multifractal degree for shuffled and surrogate series is weaker than the original series. We fit the binomial multifractal model to the global financial indices.

Keywords. Random matrix theory; correlations; financial crisis; multifractality.

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1. Introduction

The financial markets are complex systems in which various approaches and concepts from physics have been used to investigate the deterministic mechanisms in trends of prices in financial time series. The availability of a huge amount of financial data has opened a new path where various techniques from physics and nonlinear dynamics can be applied so that a systemic comparison between theory and real data can be made. Random matrix theory (RMT) has a long history since 1950s and is very useful for the physicists, mathematicians and other scientists [1–3]. It deals with the statistics of eigenvalues and eigenvectors of cross-correlation matrix of the complex many-body system. The RMT approach has been successfully used to investigate cross-correlations in financial time series [4,5]. In the case of financial markets, a few largest eigenvalues are found to deviate significantly from the RMT prediction. These deviations are used to analyse the underlying interactions in the financial time series that are different from a randomly interacting system. The largest eigenvalue and components of its eigenvectors indicate a collective response in the global financial time series. Thus, the RMT method has been widely used in the financial time series [6–21]. The analysis of crisis in financial markets is necessary to develop methods for prevention and control. The global financial crisis of 2008 is the worst financial crisis since the great depression of 1930s. The global financial crisis of 2008 has been investigated by using RMT and results before and during the crisis of 2008 have been compared [22]. It has been shown that the structure of cross-correlation changes during the period of financial crisis. The largest eigenvalue calculated from the cross-correlation matrix of 20 global financial markets are found to deviate significantly from the RMT prediction. It is found that components of eigenvector corresponding to the second largest eigenvalue are associated with the formation of clusters in the global financial indices. The structure of these clusters changes before and during the period of crisis of 2008 [22].

The multifractal detrended fluctuation analysis (MF-DFA) [23] is a robust and powerful technique which is used to identify and quantify the multiple scaling exponents within the time series. It has been successfully applied in different and heterogeneous scientific fields [24–30]. We use this technique to investigate the multifractal properties in the 20 global financial time series. In order to study the origin of multifractality in time series, one can compare the MF-DFA results for the original time series with those of shuffled and surrogated time series [23,31–33]. There are two types of multifractality in the series: (i) multifractality due to a broad probability density function for the values of the time series, this type of multifractality cannot be removed by shuffling the series, (ii) multifractality due to different long-range correlations for small and large fluctuations, here the corresponding shuffled series will exhibit non-multifractal scaling, because all long-range correlations are destroyed by the shuffling procedure. If both types of multifractality are present in a given series then the shuffled series will show weaker multifractality than the original one.

In this paper, we use RMT approach to investigate the effect of financial crisis of 2008 in the 20 global financial time series. We present the results obtained after the global financial crisis of 2008. We study the multifractal properties and investigate the origin of multifractality in 20 global financial indices. This paper is organized as follows: in §2, the financial data is discussed. Section 3 describes the RMT approach and results. The MF-DFA method and its application to global financial indices are discussed in §4. Finally we conclude in §5.

2. Data

We analyse the daily closing prices of 20 global financial indices for the period January 2010 to June 2011. This period is after the crisis period and is chosen on the basis of volatility which is a measure of fluctuations in financial markets. Countries and their indices whose data have been studied are as follows: Argentina (MERV), Brazil (BVSP), Egypt (CCSI), India (BSESN), Indonesia (JKSE), Malaysia (KLSE), Mexico (MXX), South Korea (KS11), Taiwan (TWII), Australia (AORD), Austria (ATX), France (FCHI), Germany (GDAXI), Hong Kong (HSI), Israel (TA100), Japan (N225), Singapore (STI), Switzerland (SSMI), UK (FTSE) and US (GSPC). The daily data for the multifractal analysis are from 2 January 1997 to 1 June 2009. The data have been accessed from [34] and filtered manually [22].

3. Random matrix theory approach

We use RMT to extract important information from the cross-correlation matrix constructed from global financial time series. Let $S_i(t)$ and $R_i(t)$ denote the daily closing prices and returns of indices i at time t ($i = 1, 2, ..., N; t = 1, 2, ..., L$), respectively [22,35,36]. The logarithmic returns $R_i(t)$ can be defined as

$$R_i(t) \equiv \ln(S_i(t + \Delta t)) - \ln(S_i(t)), \tag{1}$$

where $\Delta t = 1$ day. The normalized returns for indices i is defined as

$$r_i(t) \equiv \frac{R_i(t) - \langle R_i \rangle}{\sigma_i}, \tag{2}$$

where $\sigma_i \equiv \sqrt{\langle R_i^2 \rangle - \langle R_i \rangle^2}$ is the standard deviation of R_i and $\langle \cdots \rangle$ denotes the time average over the period studied. We then compute the equal-time cross-correlation matrix C with elements

$$C_{ij} \equiv \langle r_i(t) r_j(t) \rangle \,. \tag{3}$$

The elements of C_{ij} are limited to the domain $-1 \leq C_{ij} \leq 1$, where $C_{ij} = 1$ defines perfect positive correlations, $C_{ij} = -1$ corresponds to perfect negative correlations and $C_{ij} = 0$ corresponds to no correlation. If N time series of length T are mutually uncorrelated, the resulting cross-correlation matrix is termed as a Wishart matrix. Statistical properties of such random matrices are known [3]. When $N \to \infty$, $L \to \infty$, such that $Q \equiv L/N \geq 1$, the probability distribution $P_{rm}(\lambda)$ of the eigenvalue λ is given by

$$P_{rm}(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{\max}^{\text{rand}} - \lambda)(\lambda - \lambda_{\min}^{\text{rand}})}}{\lambda}, \tag{4}$$

for λ within the bounds $\lambda_{\min}^{\text{rand}} \leq \lambda_i \leq \lambda_{\max}^{\text{rand}}$, where $\lambda_{\min}^{\text{rand}}$ ($\lambda_{\max}^{\text{rand}}$) are the lower (upper) bound given by

$$\lambda_{\max(\min)}^{\text{ran}} = [1 \pm (1/\sqrt{Q})]^2. \tag{5}$$

Recently we have applied the RMT method to 20 global financial indices before and during the financial crisis of 2008 [22]. In this paper, we present the RMT results after the crisis of 2008. The daily closing prices before, during and after the crisis are shown in figures 1a, 1b and 1c, respectively. The periods for the analysis have been chosen on the basis of volatility which is a measure of market fluctuations. It is quantified as the local average of the absolute value of daily returns of indices. In figure 1d, we show that the volatility of global financial indices decreases after the crisis of 2008. We construct the cross-correlation matrix from the daily normalized returns after the crisis period. The probability density of C_{ij} is found to be asymmetric and centred around a positive mean value, shown in figure 2a, which indicate that positive correlation is larger than the negative correlation. The value of average correlation coefficient $\langle C_{ij} \rangle$ decreases to 0.415 after

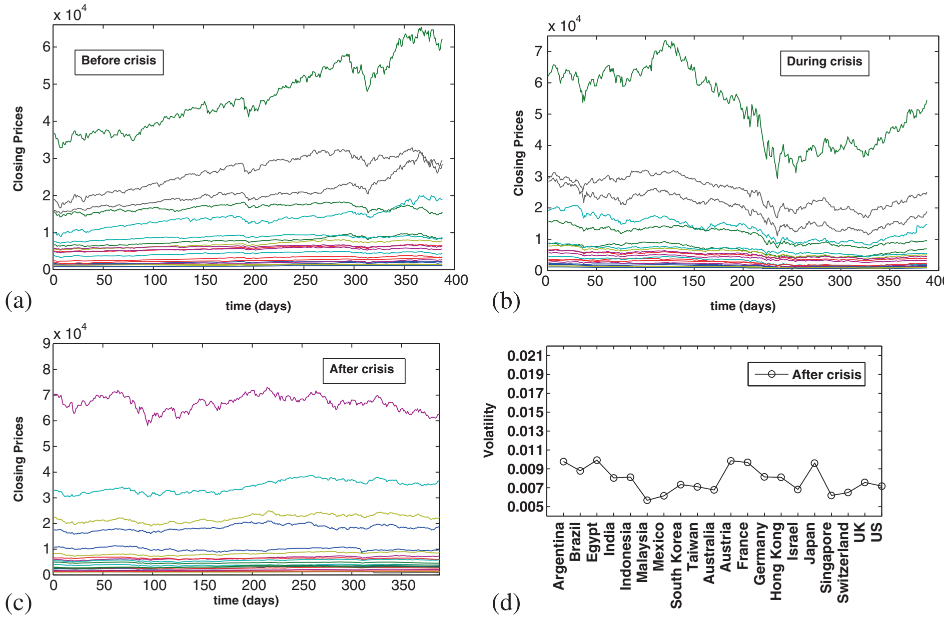

Figure 1. Daily closing prices of 20 global financial indices: (**a**) before the crisis, (**b**) during the crisis and (**c**) after the crisis, of 2008. (**d**) Volatilities after the crisis period.

the crisis of 2008 (the value before and during the crisis was 0.4353 and 0.4634 [22,37]). We study the probability distribution of eigenvalues after the crisis period. In figure 2b, we find that a few of the largest eigenvalues deviate significantly from the RMT predictions. The eigenvalues contained in the RMT bound do not reflect any useful information. Thus, to get information about the correlation, the properties of C has been compared with those of a random correlation matrix. For no correlations in financial time series the eigenvalues should lie within RMT prediction, i.e., $\lambda_{\min}^{\text{rand}} = 0.597$ and $\lambda_{\max}^{\text{rand}} = 1.506$ [22]. But we find $\lambda_{\min}^{\text{real}} = 0.0505$ and $\lambda_{\max}^{\text{real}} = 8.977$ for the financial cross-correlations after the crisis of 2008. This shows strong correlations in financial time series. Also, we find that there is a decrease in the largest eigenvalue after the crisis of 2008. Thus, we observe that the largest eigenvalue is sensitive to the financial crisis. Its value increases during the period of crisis as compared to the periods before and after the crisis of 2008 [22]. We study the components of eigenvectors corresponding to the largest eigenvalues as shown in figure 2c and we find that all components of the eigenvectors are positive after the crisis. This reflects a common global financial market mode. These components are found to remain positive in the periods before, during and after the crisis of 2008. We study the components of eigenvectors corresponding to the second largest eigenvalue after the crisis of 2008. The positive components (Egypt, India, Indonesia, Malaysia, South Korea, Taiwan, Australia, Hong Kong, Japan, Singapore) and negative components (Argentina, Brazil, Mexico, United States, Austria, France, Germany, Switzerland, UK) of eigenvectors corresponding to second largest eigenvalues are shown in figure 2d. We find that components changed their sign again after the crisis. These are, slightly more or less, the

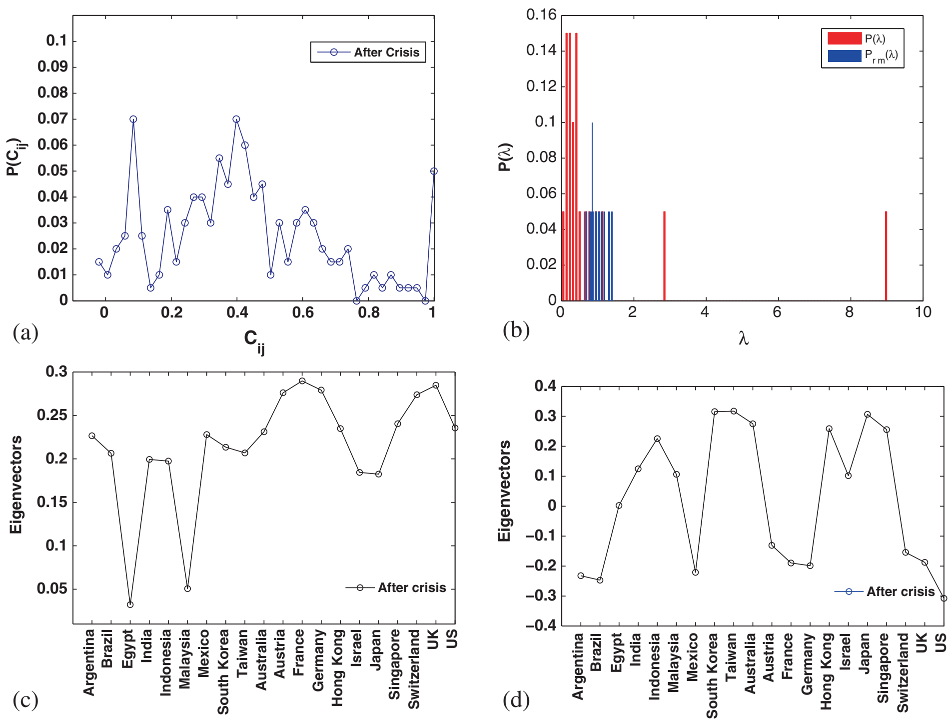

Figure 2. (**a**) Probability distribution of C_{ij} after the crisis of 2008, (**b**) comparison of probability distribution of eigenvalues with RMT predictions, (**c**) components of eigenvector corresponding to the first largest eigenvalue after the crisis, (**d**) components of eigenvector corresponding to the second largest eigenvalue after the crisis of 2008.

same as were in the period before the crisis [22]. Thus, the study of these components are important to analyse their sensitivity to the crisis.

4. Multifractal analysis

We define the normalized logarithmic returns

$$g_t = \frac{\log S(t+1) - \log S(t)}{\sigma}$$

of length N, where $S(t)$ denotes the daily closing price of the index and σ is the standard deviation of the logarithmic returns. In order to study the multifractal properties of 20 financial time series, we use the MF-DFA method [23,28] which consists of five steps.

Step 1: Calculate the 'profile',

$$Y(i) \equiv \sum_{k=1}^{i} [g_k - \langle g \rangle], \quad i = 1, ..., N,$$

where N is the length of the series and $\langle g \rangle$ is the time average of normalized log-returns g_t.
Step 2: Divide the profile $Y(i)$ into $N_s \equiv \text{int}(N/s)$ non-overlapping segments of equal length s. As the length of the series is often not a multiple of the considered time-scale s, a short part of the series remains, the same procedure is repeated starting from the opposite end, thereby, $2N_s$ segments are obtained altogether.
Step 3: Calculate the local trend for each of the $2N_s$ segments by a least-square fit of the time series. Then determine the variance

$$F^2(s,\nu) \equiv \frac{1}{s}\sum_{i=1}^{s}\{Y[(\nu-1)s+i] - y_\nu(i)\}^2$$

for each segment ν, $\nu = 1, ..., N_s$ and

$$F^2(s,\nu) \equiv \frac{1}{s}\sum_{i=1}^{s}\{Y[N-(\nu-N_s)s+i] - y_\nu(i)\}^2$$

for $\nu = N_s + 1, ..., 2N_s$. Here, $y_\nu(i)$ is the fitting polynomial in segment ν.
Step 4: Average over all segments to obtain the qth order fluctuation function,

$$F_q(s) \equiv \left\{\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}[F^2(s,\nu)]^{q/2}\right\}^{1/q},$$

where the variable q can take any real value except zero [23].
Step 5: Determine the scaling behaviour of the fluctuation functions by analysing log–log plot of $F_q(s)$ vs. s for each value of q. If the time series g_t are long-range power-law correlated, $F_q(s)$ increases for large value of s, as a power-law $F_q(s) \sim s^{h(q)}$. The family of scaling exponents $h(q)$ can be obtained by observing the slope of the log–log plot of $F_q(s)$ vs. s. $h(q)$ is the generalization of the Hurst exponent $H(\equiv h(2))$. The monofractal time series are characterized by a single exponent over all time-scales, i.e., $h(q)$ is independent of q, whereas for multifractal time series, $h(q)$ varies with q. Obviously, richer multifractality corresponds to higher variability of $h(q)$. Then, the multifractality degree can be quantified by $\Delta h = h(q_{\min}) - h(q_{\max})$.

We study the multifractal properties of 20 financial indices for the period 2 July 1997 to 1 June 2009 (due to the large number of figures for the multifractal scaling exponents these results are not shown here; instead we compare multifractal degrees in table 1). To find the origin of multifractality in the financial time series, we have shuffled the financial time series. In the shuffling procedure the data are put in random order. So, all temporal correlations are destroyed without affecting the probability density function. In order to quantify the influence of the fat-tail distribution, we generate the surrogate time series from original series by using the Schreiber method [38]. This algorithm for generating the surrogate data is based on a simple iteration scheme called iterated amplitude-adjusted Fourier transform (IAAFT), which is an improved version of the phase randomization algorithm [39]. In table 1, we compare the multifractal degrees for original, shuffled and surrogated financial time series, respectively. We find that there is a contribution of long-range correlation as well as broad probability density function in the multifractality of all financial indices as the multifractal degree for shuffled and surrogate series are weaker than those of original series, except the Taiwan index. In table 1, we find that financial indices corresponding to the American indices (Argentina, Brazil, Mexico, United States) and European indices (Austria, France, Germany, Switzerland, UK) almost lie in the same range of degrees of multifractality. India, South Korea, Hong Kong are found to be near the degrees of multifractality of indices corresponding to the American and European indices. A large variation in degrees of multifractality in Egypt, Indonesia, Malaysia, Taiwan and Singapore is observed. We do not find any relation among them due to large variation in their multifractal degrees and volatilities.

We compare multifractal results of financial indices with the binomial multifractal model (BMFM) [23,40,41]. A binomial multifractal series of $N = 2^{n_{\max}}$ numbers k with $k = 1, ..., N$ is defined by

$$x_k = a^{n(k-1)}(1-a)^{n_{\max}-n(k-1)}. \tag{6}$$

Here $0.5 < a < 1$ is a parameter and $n(k)$ is the number of digits equal to 1 in the binary representation of the index k, e.g., $n(19) = 3$ because 19 corresponds to binary 10011. We generate binomial multifractal series with $n_{\max} = 12$ and different values of a, then compare their multifractal results with the financial indices. We find that at $a = 0.4125$, the BMFM fits well with the indices corresponding to the American, European and Australian, i.e., these indices exhibit a common multifractal behaviour as compared to the other indices. Other indices also fit with BMFM for different values of

Table 1. Degrees of multifractality for original, shuffled and surrogated time series of 20 global financial indices and their volatilities.

Country	Δh_{orig}	Δh_{shuf}	Δh_{sur}	Volatility
Argentina	0.492	0.266	0.306	0.0153
Brazil	0.415	0.240	0.223	0.0161
Egypt	2.318	0.377	0.494	0.0055
India	0.335	0.236	0.241	0.0123
Indonesia	1.230	0.319	0.321	0.0119
Malaysia	0.760	0.335	0.358	0.0090
Mexico	0.417	0.263	0.312	0.0115
South Korea	0.546	0.310	0.219	0.0145
Taiwan	0.153	0.267	0.237	0.0115
Australia	0.447	0.230	0.267	0.0067
Austria	0.465	0.291	0.326	0.0093
France	0.459	0.238	0.252	0.0108
Germany	0.465	0.256	0.255	0.0117
Hong Kong	0.522	0.254	0.254	0.0122
Israel	0.386	0.327	0.324	0.0090
Japan	0.361	0.223	0.217	0.0111
Singapore	0.920	0.217	0.267	0.0101
Switzerland	0.448	0.283	0.216	0.0093
UK	0.445	0.243	0.246	0.0090
US	0.463	0.247	0.221	0.0091

the parameter a as follows: India, Japan, and Israel at 0.45, South Korea and Hong Kong at 0.6, Indonesia at 0.725, Malaysia at 0.65, Singapore at 0.675 and Egypt at 0.85.

5. Conclusion

We have applied the RMT method to investigate the structure of cross-correlation in global financial time series after the global financial crisis of 2008. We found that the largest eigenvalue, representing the collective information about the correlation in financial time series, deviated from the RMT prediction. We found that the largest eigenvalue was sensitive to the crisis and its value decreased in the period after the financial crisis of 2008. In the eigenvectors analysis, we found that the components of eigenvector corresponding to second largest eigenvalue changed their sign after the crisis. We added this results with our previous finding and found that there was a change in the sign of components of eigenvectors corresponding to the second largest eigenvalue when the crisis of 2008 occurred. The structure of components before and after the crisis is found to be almost the same. This analysis can be helpful for studying other crises.

We apply the MF-DFA method to 20 global financial time series from 2 July 1997 to 1 June 2009. We found that all financial indices under the study exhibit multifractality. We investigated multifractal properties and found the origin of multifractality in the financial time series. We compared the multifractal degrees for original, shuffled and surrogated time series, respectively and found that there was a contribution of long-range correlation as well as broad probability function in the multifractality of financial indices, except the index of Taiwan, as multifractal degree for shuffled and surrogate series are weaker than those of the original series. We fit the binomial multifractal model to 20 global financial indices.

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