Assignment 3
Game Theory and Auctions

Q1.
Roger has invited Caleb to his party. Roger must choose whether or not to hire a clown. Simultaneously, Caleb must decide whether or not to go the party. Caleb likes Roger but he hates clowns - he even hates other people seeing clowns! Caleb's payoff from going to the party is 4 if there is no clown, but 0 if there is a clown there. Caleb's payoff from not going to the party is 3 if there is no clown at the party, but 1 if there is a clown at the party. Roger likes clowns - he especially likes Caleb's reaction to them - but does not like paying for them. Roger's payoff, if Caleb comes to the party, is 4 if there is no clown, but 8x if there is a clown (x is the cost of a clown). Roger's payoff if Caleb does not come to the party is 2 if there is no clown, but 3x if there is a clown there.

(a) Write down the payoff matrix of this game.
(b) Suppose x = 0. Identify any dominated strategies. Find the pure strategy Nash equilibrium. What are the equilibrium payoffs?
(c) Suppose x = 2. Identify any dominated strategies. Is there a pure strategy Nash equilibrium? If not find the mixed strategy Nash equilibrium. What are the equilibrium payoffs?

Q2.
Player 1 is a constable who must decide whether to patrol the market or relax in the police station. His payoff from relaxing in the police station is 10, while his payoff from patrolling the market depends on whether he catches a pickpocket, who is player 2. If pickpocket prowls the market, then the constable(if he is patrolling the market) will catch him and obtain a payoff of 30. If the pickpocket stays home, the constable's payoff from patrolling the market is 0. The pickpocket must choose between staying home or prowling the market. If he stays at home, then his payoff is 0, while if he prowls the market, his payoff is -15 if the constable is prowling the market and 10 if the constable is relaxing in the police station. Find all(pure strategy and mixed strategy) Nash equilibria of this game.

Q3.
Two identical firms — let's call them Firm 1 and Firm 2 — must decide simultaneously and independently whether to enter a new market and what product to produce if they do enter the market. Each firm, if it enters, can develop and produce either product A or product B. If both firms enter and produce product A they each lose ten million dollars. If both firms enter and both produce product B, they each make a profit of five million dollars. If both enter and one produces A while the other produces B, then they each make a profit of ten million dollars. Any firm that does not enter makes a profit of zero. Finally, if one firm does not enter and the other firm produces A it makes a profit of fifteen million dollars, while if the single entering firm produces B it makes a profit of thirty million dollars.
You are the manager of Firm 1 and you have to choose a strategy for your firm.

(a) Set this situation up as a game with two players, firms 1 and 2, and three strategies for each firm: produce A, produce B or do not enter. Write down the payoff matrix for this game.
(b) One of your employees argues that you should enter the market (although he is not sure what product you should produce) because no matter what firm 2 does, entering and producing product B is better than not entering. Evaluate this argument.
(c) Another employee agrees with the person in part (b) and argues that as strategy A could result in a loss (if the other firm also produces A) you should enter and produce B. If both firms reason this way, and thus enter and produce product B, will their play of the game form a Nash equilibrium? Explain.
(d) Find all the pure strategy Nash equilibria of this game.

Q4.
A seller will run a second-price, sealed-bid auction for an object. There are two bidders, a and b, who have independent, private values vi which are either 0 or 1. For both bidders the probabilities of $v_i = 0$ and $v_i = 1$ are each 1/2. Both bidders understand the auction, but bidder b sometimes makes a mistake about his value for the object. Half of the time his value is 1 and he is aware that it is 1; the other half of the time his value is 0 but occasionally he mistakenly believes that his value is 1. Let's suppose that when b's value is 0 he acts as if it is 1 with probability 1/ 2 and as if it is 0 with probability 1/ 2 . So in effect bidder b sees value 0 with probability 1/4 and value 1 with probability 3/4 . Bidder a never makes mistakes about his value for the object, but he is aware of the mistakes that bidder b makes. Both bidders bid optimally given their perceptions of the value of the object. Assume that if there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x.

(a) Is bidding his true value still a dominant strategy for bidder a? Explain
(b) What is the seller's expected revenue? Explain briefly.

Q5.
A seller announces that he will sell a case of rare wine using a sealed-bid, second-price auction. A group of I individuals plan to bid on this case of wine. Each bidder is interested in the wine for his or her personal consumption; the bidders' consumption values for the wine may differ, but they don't plan to resell the wine. So we will view their values for the wine as independent, private values. You are one of these bidders; in particular, you are bidder number i and your value for the wine is v_i.
How should you bid in each of the following situations?

(a) You know that a group of the bidders will collude on bids. This group will chose one bidder to submit a "real bid" of v and the others will all submit bids of 0. You are not a member of this collusive group and you cannot collude with any other bidder.
(b) You, and all of the other bidders, have just learned that this seller will collect bids, but won't actually sell the wine according to the rules of a second-price auction. Instead, after collecting the bids the seller will tell all of the bidders that some other fictional bidder actually submitted the highest bid and so won the auction.

This bidder, of course, doesn't exist so the seller will still have the wine after the auction is over. The seller plans to privately contact the highest actual bidder and tell him or her that the fictional high bidder defaulted (he didn't buy the wine after all) and that this bidder can buy the wine for the price he or she bid in the auction. You cannot collude with any bidder.