Foundation of Machine Learning

Assignment-5

In [15]:
```
import numpy as np
import matplotlib.pyplot as plt
```

Generate the datasets A and B in R2 with each of them consisting 2000 data points from normal distribution. The dataset A and B has been drawn from the N (μ1, Σ1)

and N(μ2, Σ2) . Let us fix

the μ1 = [-1,1] and μ2 = [1,1].

In [16]:
```
mue1 = np.array([-1,1])
mue2 = np.array([1,1])
```

a. Find the optimal decision boundary for the classification of the dataset A and B using Σ1 = Σ2= 0.6 0 0 0.6 . Plot the dataset A and B with different colors and plot the obtained optimal decision boundary. Comment on the characteristics of obtained decision boundary.

In [17]:
```
cov = [[0.6,0],[0,0.6]]
A = np.random.multivariate_normal(mue1, cov, 2000)
B = np.random.multivariate_normal(mue2, cov, 2000)
A.shape
```

Out[17]:
```
(2000, 2)
```

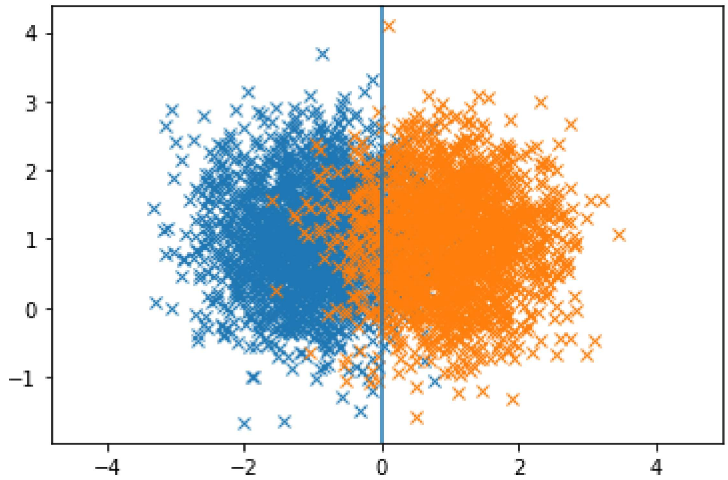
In [18]:
```
'''### Another way

x = 1/2 * (mue1+mue2)
y = np.array([[x[0],i] for i in np.arange(-2,5)])
'''
w1=(1/0.6)*mue1
w2=(1/0.6)*mue2
w10=-0.5*(1/0.6)*(mue1.T).dot(mue1)
w20=-0.5*(1/0.6)*(mue2.T).dot(mue2)

x=(w20-w10) / (w1.T-w2.T)
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:11: RuntimeWarning: inva
lid value encountered in true_divide
  # This is added back by InteractiveShellApp.init_path()
```

In [19]:
```
plt.plot(A[:, 0].reshape(-1,1), A[:, 1].reshape(-1,1), 'x')
plt.plot(B[:, 0].reshape(-1,1), B[:, 1].reshape(-1,1), 'x')
plt.axvline(x[0], ymin=-2,ymax=4) # if you have choose another way plt.plot(y[:,0].res
plt.axis('equal')
plt.show()
```



In this case, I get point from var x which will be the point of line. Then by keeping value of x component of var x same but we'll have use different value of y component of var x to generate line on that point.

b. Find the optimal decision boundary for the classification of the dataset A and B using Σ1 = Σ2= 0.7 0 0 0.3 . Plot the dataset A and B with different colors and plot the obtained optimal decision boundary. Comment on the characteristics of obtained decision boundary.

In [20]:

```
u1 = np.array([-1,1])
u2 = np.array([2,2])
cov2 = np.array([[0.7,0],[0,0.3]])

A = np.random.multivariate_normal(u1, cov2, 2000)
B = np.random.multivariate_normal(u2, cov2, 2000)
```

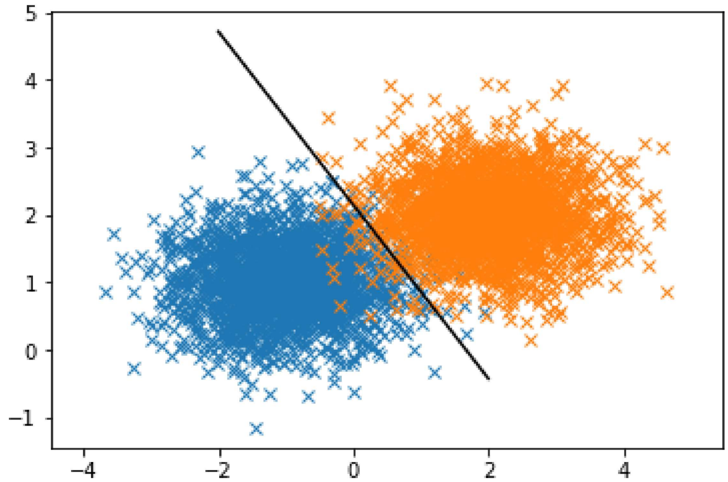
In [21]:

```
cov2_inv = np.linalg.inv(cov2)
w1=cov2_inv@ u1
w2=cov2_inv @ u2
w10= -(1/2) * (u1.T @ cov2_inv @ u1)
w20= -(1/2) * (u2.T @ cov2_inv @ u2)
vals = np.linspace(-2,2,A.shape[0])
w_diff = w1 - w2
w0_diff = w10 - w20

x= ((-w0_diff) - (w_diff[0] * vals )) / (w_diff[1])
```

In [22]:

```
plt.plot(A[:, 0].reshape(-1,1), A[:, 1].reshape(-1,1), 'x')
plt.plot(B[:, 0].reshape(-1,1), B[:, 1].reshape(-1,1), 'x')
plt.plot(vals.reshape(-1,1),x.reshape(-1,1), c = "black")
# plt.axvline(x[0], ymin=-2,ymax=4) # if you have choose another way plt.plot(y[:,0].r
plt.axis('equal')
plt.show()
```



In this case, we get point from var x which will be the point of line. Then by keeping value of x component of var x same but we'll have use different value of y component of var x to generate line on that point.

c. Find the optimal decision boundary for the classification of the dataset A and B using Σ1 = Σ2= 0.6 0.25 0.25 0.4

. Plot the dataset A and B with different colors and plot the obtained optimal decision boundary. Comment on the characteristics of obtained decision boundary.

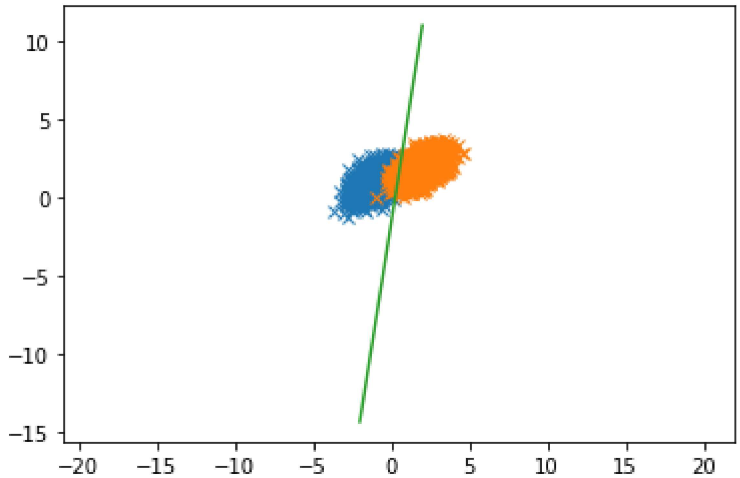
In [23]:
```
cov3 = np.array([[0.6,0.25],[0.25,0.4]])
A = np.random.multivariate_normal(u1, cov3, 2000)
B = np.random.multivariate_normal(u2, cov3, 2000)
```

In [24]:
```
w1_minus_w2 = w1 - w2
```

In [25]:
```
cov3_inv = np.linalg.inv(cov3)
w1=cov3_inv @ u1
w2=cov3_inv @ u2
w10= -(1/2) * (u1.T @ cov3_inv @ u1)
w20= -(1/2) * (u2.T @ cov3_inv @ u2)
vals = np.linspace(-2,2,A.shape[0])
w1_minus_w2 = w1 - w2
w10_minus_w20 = w10 - w20

x = ((-w10_minus_w20) - (w1_minus_w2[0] * vals)) /(w1_minus_w2[1])
```

In [26]:
```
plt.plot(A[:, 0].reshape(-1,1), A[:, 1].reshape(-1,1), 'x')
plt.plot(B[:, 0].reshape(-1,1), B[:, 1].reshape(-1,1), 'x')
plt.plot(vals.reshape(-1,1), x.reshape(-1,1))
# plt.axvline(A[:,0], x)
plt.axis('equal')
plt.show()
```



In this case, we covariance matrix are same so first term ie. X.T @ (W1-W2) @ X will be cancel out, so at the end only ((-w10_minus_w20) - (w1_minus_w2[0] * vals)) /(w1_minus_w2[1]) will remains, and due to that we wont get non linear boundry. To generate boundry line, we have to generate values and put into the equation, later I stored it in var x.