#### **Q1** Team name

0 Points

Cipherberg

### **Q2** Commands

10 Points

List the commands used in the game to reach the ciphertext.

go, enter, pluck, c, c, back, give, back, back, thrnxxtzy, read

# **Q3** Analysis

50 Points

Give a detailed analysis of how you figured out the password? (Explain in less than 500 words)

After giving the mushrooms, we were presented a screen which had some hints and equations related to multiplicative groups.

This gave us the idea to use Modular Arithmetic in further analysis.

Given, the prime modulus p = 19807040628566084398385987581

Using the equations given in the hint,

 $wg^{324} \equiv 11226815350263531814963336315(say,y_1)$  (mod

p) -----eqn 1

 $wg^{2345} \equiv 9190548667900274300830391220(say,y_2)$  (mod

p) -----eqn 2

 $wg^{9513} \equiv 4138652629655613570819000497(say, y_3)$  (mod

p) -----eqn 3

where w is the password.

Divide eqn. 2 with eqn. 1, 
$$g^{2021} \equiv y_2 * y_1^{-1}$$
 (mod p) -----eqn 4

Divide eqn. 3 with eqn. 1, 
$$g^{9189} \equiv y_3 * y_1^{-1}$$
 (mod p) -----eqn 5

Divide eqn. 3 with eqn. 2, 
$$g^{7168} \equiv y_3 * y_2^{-1}$$
 (mod p) -----eqn 6

To perform modular division, we need to find the modular inverse of the denominators  $(y_1 \text{ and } y_2)$ , if it exists. From properties of Multiplicative group of integers modulo n, we know that,  $\gcd(y_1, p) = 1$  and  $\gcd(y_2, p) = 1$  i.e.  $y_1$  and  $y_2$  are coprime to p, therefore, modular inverse of  $y_1$  and  $y_2$  exists under modulo p. Modular inverse of  $y_1$  is a number x such that  $(y_1 * x) \% p = 1$ .

Since p is a prime number, therefore we can use Fermat's little theorem.

Therefore, Using Fermat's little theorem,  $a^{p-1} \equiv 1 \pmod{\mathrm{p}}$  Multiplying both sides with  $a^{-1}$  and rearranging,

$$\implies a^{-1} \equiv a^{p-2} \pmod{p},$$

Using the above equation, we get

$$y_1^{-1} = x \equiv y_1^{p-2} \pmod{p} \equiv 17983774594023309985368857902 \pmod{p}.$$

Therefore using eqn 4,

$$g^{2021} \equiv (y_2 * x) ext{ (mod p)}$$
  $g^{2021} \equiv$ 

$$g^{2021} \equiv 7021284369301638640577066679$$
 (mod p) ------eqn 7

Similarly, by solving equation 5 and 6, we get,  $g^{9189}\equiv 3426347385144995225825016781$  (mod p) ------eqn 8  $g^{7168}\equiv 6339248851737327508924059257$  (mod p) ------eqn 9

Multiplying both sides of equation 9 by inverse of  $\left(g^{2021}\right)^3$  gives,  $g^{7168}*\left(\left(g^{2021}\right)^3\right)^{-1} \pmod{\mathrm{p}} \equiv 6339248851737327508924059257*$   $\left(\left(g^{2021}\right)^3\right)^{-1} \pmod{\mathrm{p}}$   $g^{1105} \equiv 1332524359715193692493602650 \pmod{\mathrm{p}}$ 

Similarly the following calculations can be carried out:

$$g^{349} \equiv g^{9189} * \left( \left( g^{1105} \right)^8 \right)^{-1} \pmod{p} \equiv$$

$$9054846785544512610175699226 \pmod{p}$$

$$g^{73} \equiv \left( g^{349} \right)^6 * \left( g^{2021} \right)^{-1} \pmod{p} \equiv$$

$$2748579083294760009905704356 \pmod{p}$$

$$g^{16} \equiv \left( g^{73} \right)^5 * \left( g^{349} \right)^{-1} \pmod{p} \equiv$$

$$10610366411880988999637482966 \pmod{p}$$

$$g^3 \equiv \left( g^{16} \right)^{22} * \left( g^{349} \right)^{-1} \pmod{p} \equiv$$

$$5924011030095759455963670302 \pmod{p}$$

$$g \equiv g^{16} * \left( \left( g^3 \right)^5 \right)^{-1} \pmod{p} \equiv$$

$$192847283928500239481729 \pmod{p}$$

Also, in the hints mentioned on the panel, it is written that g is  $1\_\_4\_2\_\_0\_94\_\_9$  with some values missing. The result obtained for g from our computation agrees with this hint. Therefore, we can surely say that g is 192847283928500239481729.

To compute the password put value of g in eqn. 1,  $wg^{324}\equiv 11226815350263531814963336315 \ ({\rm mod\ p})$  Multiply both sides by inverse of  $g^{324}$  ,

$$w*g^{324}*(g^{324})^{-1}\equiv \\ \left(11226815350263531814963336315*(g^{324})^{-1}\right) \text{ (mod p)} \\ \text{From properties of Multiplicative group of integers modulo p, we know that } a*a^{-1}=1, \\ \text{Therefore,} \\ w\equiv \left(11226815350263531814963336315*\\ 7280920143223660694435112264\right) \text{ (mod p)} \\ \\$$

where the modular inverse $(g^{324})^{-1}$  is again obtained using

Fermat's little theorem. Therefore, w=3608528850368400786036725

Hence the password is 3608528850368400786036725

The python code for above computations and for finding the modular inverse using Fermat's little theorem is attached.

### **Q4** Password

10 Points

What was the final command used to clear this level?

```
3608528850368400786036725
```

## **Q5** Codes

0 Points

Upload any code that you have used to solve this level.

```
▲ Download
▼ crypto_ass3.ipynb
     In [15]:
                  def gcd(x, y):
                      if (x == 0):
                           return y
                      return gcd(y % x, x)
                  def power(a, b, m):
                      if (b == 0):
                          return 1
                      p = power(a, b // 2, m) % m
                      p = (p * p) % m
                      if (b % 2 == 0):
                          return p
                      else:
                           return ((a * p) % m)
                  def modInverse(a, m):
                      g = gcd(a, m)
                      if(g==1):
                           z=power(a, m - 2, m)
                          print("The modular multiplicative
                  inverse is ",z)
                           return z
                      else:
                          print("Oops! The inverse does not
```

```
exist")
             y1 = 11226815350263531814963336315
             y2 = 9190548667900274300830391220
             y3= 4138652629655613570819000497
             p = 19807040628566084398385987581
             y1_inverse = modInverse(y1, p)
             y2_inverse= modInverse(y2,p)
             The modular multiplicative inverse is 17983774594
             The modular multiplicative inverse is 14487011570
 In [16]:
             g_2021= (y2 * y1_inverse)%p
             g 2021
Out [16]:
             7021284369301638640577066679
 In [17]:
             g_7168= (y3 * y2_inverse)%p
             g_7168
Out [17]:
             6339248851737327508924059257
 In [18]:
             g_9189= (y3 * y1_inverse) % p
             g_9189
Out [18]:
             3426347385144995225825016781
 In [19]:
             g_1105= (g_7168 *
             modInverse(power(g_2021,3,p),p)) % p
             g_1105
             The modular multiplicative inverse is 37593003101
Out [19]:
             1332524359715193692493602650
 In [20]:
             g_349= (g_9189 *
             modInverse(power(g_1105,8,p),p)) % p
             g_349
             The modular multiplicative inverse is 14402582163
Out [20]:
             9054846785544512610175699226
 In [21]:
             g_{349,6,p} *
             modInverse(g 2021,p)) % p
```

```
g_73
             The modular multiplicative inverse is 16586880129
Out [21]:
             2748579083294760009905704356
 In [22]:
             g_16= (power(g_73,5,p) * modInverse(g_349,p))
             % p
             g_16
             The modular multiplicative inverse is 98360082593
Out [22]:
             10610366411880988999637482966
In [23]:
             g_3 = (power(g_16, 22, p) * modInverse(g_349, p))
             % p
             g_3
             The modular multiplicative inverse is 98360082593
Out [23]:
             5924011030095759455963670302
 In [24]:
             g= (g_16 * modInverse(power(g_3,5,p),p)) % p
             The modular multiplicative inverse is 16621723109
Out [24]:
             192847283928500239481729
 In [25]:
             password= (y1 * modInverse(power(g, 324,
             p),p))%p
              password
             The modular multiplicative inverse is 72809201432
Out [25]:
             3608528850368400786036725
```

Assignment 3	GRADED
GROUP SAMBHRANT MAURYA DEEKSHA ARORA SHRUTI SHARMA  View or edit group	
TOTAL POINTS	
70 / 70 pts	
QUESTION 1	
Team name	<b>0</b> / 0 pts
QUESTION 2	
Commands	<b>10</b> / 10 pts
QUESTION 3	
	<b>EQ</b> / <b>EQ</b> ntc
Analysis	<b>50</b> / 50 pts
QUESTION 4	
Password	<b>10</b> / 10 pts
QUESTION 5	
Codes	<b>0</b> / 0 pts