

CS685: DATA MINING DATA REDUCTION

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Data Reduction

- *Data reduction* decreases data
- Benefits of data reduction

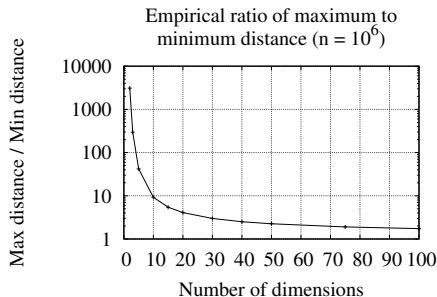
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 - Simpler model
 - Less number of rules
 - Less complex rules, i.e., involving less number of attributes
 - Faster algorithms
 - Easier visualization
 - Avoids *overfitting*

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- Important ways of data reduction
 - Dimensionality reduction
 - Numerosity reduction
 - Data discretization
 - Data modeling
 - Feature selection

Dimensionality Reduction

- Dimensionality reduction reduces the number of dimensions
- New dimensions are generally different from original ones
- **Curse of dimensionality**
 - Data becomes too sparse as dimensions increase
 - Data is mostly at the boundaries
 - Classification: Not enough data to create good models or methods
 - Clustering: Density becomes irrelevant and distance between points becomes similar



Singular Value Decomposition (SVD)

- **Singular value decomposition** is factorization of a matrix

$$A = U\Sigma V^T$$

- If A is of size $m \times n$, then U is $m \times m$, V is $n \times n$ and Σ is $m \times n$
- Columns of U are *eigenvectors of AA^T*
 - Left singular vectors
 - $UU^T = I_m$ (orthonormal)
- Columns of V are *eigenvectors of $A^T A$*
 - Right singular vectors
 - $V^T V = I_n$ (orthonormal)
- σ_{ii} are the **singular values**
 - Σ is *diagonal*
 - Singular values are *positive square roots of eigenvalues of AA^T or $A^T A$*
- $\sigma_{11} \geq \sigma_{22} \geq \dots \geq \sigma_{nn}$ (assuming n singular values)

Transformation using SVD

- Transformed data

$$T = AV = U\Sigma$$

- V is called SVD transform matrix
- Essentially, T is just a rotation of A
- Dimensionality of T is n
- n different basis vectors than the original space
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- U shows how each *dimension* can be represented as a linear combination of other dimensions
- Lengths of vectors are preserved

$$||\vec{a}_i||_2 = ||\vec{t}_i||_2$$

SVD of Real Symmetric Matrix

- A is real symmetric of size $n \times n$
- $A = A^T$
- $U = V$ since $A^T A = A A^T = A^2$

$$A = Q \Sigma Q^T$$

- Q is of size $n \times n$ and contains eigenvectors of A^2
- This is called **spectral decomposition** of A
- Σ contains n singular values
- Eigenvectors of $A =$ eigenvectors of A^2
- Eigenvalues of $A =$ square root of eigenvalues of A^2
- Eigenvalues of $A =$ singular values of A

Example

$$A \begin{bmatrix} 2 & 4 & 1 \\ 1 & 3 & 0 \\ 5 & 2 & 1 \\ 0 & 0 & 7 \\ 3 & 3 & 3 \end{bmatrix} = U \begin{bmatrix} -0.41 & 0.29 & 0.49 & -0.41 & -0.56 \\ -0.23 & 0.27 & 0.48 & 0.77 & 0.18 \\ -0.48 & 0.36 & -0.71 & 0.23 & -0.25 \\ -0.47 & -0.83 & 0.02 & 0.18 & -0.19 \\ -0.55 & 0.05 & 0.01 & -0.37 & 0.73 \end{bmatrix} \\ \times \Sigma \begin{bmatrix} 9.30 & 0 & 0 \\ 0 & 6.47 & 0 \\ 0 & 0 & 2.91 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times V^T \begin{bmatrix} -0.55 & 0.44 & -0.70 \\ -0.53 & 0.45 & 0.70 \\ -0.63 & -0.77 & 0.01 \end{bmatrix}^T$$

Transformed Data

$$\begin{aligned} T = AV = U\Sigma &= \begin{bmatrix} 2 & 4 & 1 \\ 1 & 3 & 0 \\ 5 & 2 & 1 \\ 0 & 0 & 7 \\ 3 & 3 & 3 \end{bmatrix} \times \begin{bmatrix} -0.55 & 0.44 & -0.70 \\ -0.53 & 0.45 & 0.70 \\ -0.63 & -0.77 & 0.01 \end{bmatrix} \\ &= \begin{bmatrix} -3.89 & 1.93 & 1.44 \\ -2.16 & 1.80 & 1.42 \\ -4.47 & 2.36 & -2.08 \\ -4.45 & -5.39 & 0.08 \\ -5.18 & 0.38 & 0.05 \end{bmatrix} \\ \text{Lengths} &= [4.58, 3.16, 5.47, 7.00, 5.19] \end{aligned}$$

Compact Form

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- If A is of size $m \times n$, then U is $m \times n$, V is $n \times n$ and Σ is $n \times n$
- Works because there at most n non-zero singular values in Σ

Dimensionality Reduction using SVD

$$A = U\Sigma V^T = \sum_{i=1}^n (u_i \sigma_{ii} v_i^T)$$

- Use only k dimensions
- Retain first k columns for U and V and first k values for Σ
- First k columns of V give the basis vectors in reduced space

$$A_k \approx \sum_{i=1}^k (u_i \sigma_{ii} v_i^T) = U_{1\dots k} \Sigma_{1\dots k} V_{1\dots k}^T$$

$$T_k \approx AV_{1\dots k}$$

Reduced Dimensionality

$$\begin{aligned} A \approx A_k &= U_k \begin{bmatrix} -0.41 & 0.29 \\ -0.23 & 0.27 \\ -0.48 & 0.36 \\ -0.47 & -0.83 \\ -0.55 & 0.05 \end{bmatrix} \times \Sigma \begin{bmatrix} 9.30 & 0 \\ 0 & 6.47 \end{bmatrix} \times V^T \begin{bmatrix} -0.55 & 0.44 \\ -0.53 & 0.45 \\ -0.63 & -0.77 \end{bmatrix}^T \\ &= \begin{bmatrix} 3.01 & 2.97 & 0.98 \\ 2.00 & 1.98 & -0.01 \\ 3.52 & 3.48 & 1.02 \\ 0.05 & -0.05 & 6.99 \\ 3.03 & 2.96 & 2.99 \end{bmatrix} \\ T \approx T_k &= AV_k = U_k \Sigma_k = \begin{bmatrix} -3.89 & 1.93 \\ -2.16 & 1.80 \\ -4.47 & 2.36 \\ -4.45 & -5.39 \\ -5.18 & 0.38 \end{bmatrix} \end{aligned}$$

Reduced Lengths = [4.34, 2.82, 5.06, 6.99, 5.19]

Length Ratios = [0.95, 0.89, 0.92, 1.00, 1.00]

Best Approximation

- **Frobenius norm** of a matrix C of size $n \times m$ is

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- Consider any rank- k approximation A_k of A
- SVD produces A_k^* that minimizes the Frobenius norm of the difference
 - Best in terms of sum squared error

$$A_k^* = \arg \min_{A_k: \text{rank}=k} \|A - A_k\|_F$$

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- In the above example, $k = 1$ (resp. $k = 2$) retains 63 % (resp. 94 %) of the energy
- Running time: $O(m.n.r)$ for A of size $m \times n$ and rank r

Principal Component Analysis (PCA)

- Way of identifying patterns in data
 - How input basis vectors are correlated for the given data
- A transformation from a set of (possibly) correlated axes to another set of uncorrelated axes
- Orthogonal linear transformation (i.e., rotation)
- New axes are **principal components**
- First principal component produces projections that are best in the squared error sense
- Optimal least squares solution

Algorithm

- Mean center the data (optional)
- Compute the *covariance matrix* of the dimensions
- Find eigenvectors of covariance matrix
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- Assume data matrix is B of size $m \times n$
- For each dimension, compute mean μ_i
- Mean center B by subtracting μ_i from each column i to get A
- Compute covariance matrix C of size $n \times n$
 - If mean centered, $C = A^T A$
- Find eigenvectors and corresponding eigenvalues (V, E) of C
- Sort eigenvalues such that $e_1 \geq e_2 \geq \dots \geq e_n$
- Project step-by-step onto the principal components $\vec{v}_1, \vec{v}_2, \dots$, etc.

Example

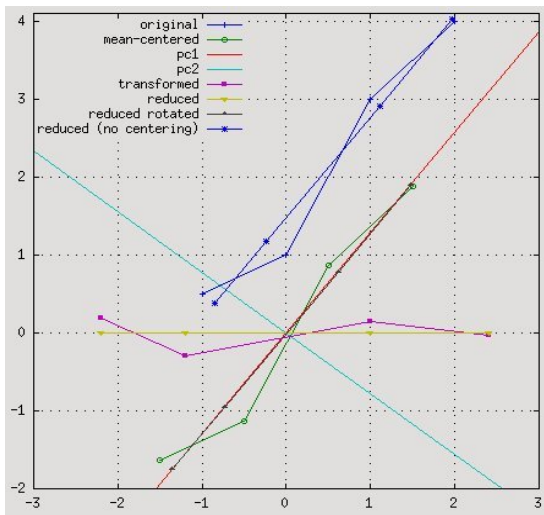
$$B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \\ 0 & 1 \\ -1 & 0.5 \end{bmatrix}; \mu(B) = \begin{bmatrix} 0.500 & 2.125 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1.5 & 1.875 \\ 0.5 & 0.875 \\ -0.5 & -1.125 \\ -1.5 & -1.625 \end{bmatrix} \text{ and, } C = A^T A = \begin{bmatrix} 5.000 & 6.250 \\ 6.250 & 8.187 \end{bmatrix}$$

$$\text{Eigenvectors } V = \begin{bmatrix} 0.613 & -0.789 \\ 0.789 & 0.613 \end{bmatrix}; \text{ eigenvalues } E = \begin{bmatrix} 13.043 \\ 0.143 \end{bmatrix}$$

$$\text{Transformed data } T = AV = \begin{bmatrix} 2.400 & -0.034 \\ 0.997 & 0.142 \\ -1.195 & -0.295 \\ -2.203 & 0.187 \end{bmatrix}$$

Visual Example



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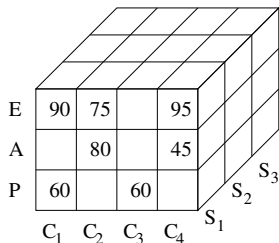
Numerosity Reduction

- Numerosity reduction reduces the *volume* of data
- Reduction in number of data objects
- Compression
- Modeling
- Discretization

Aggregation

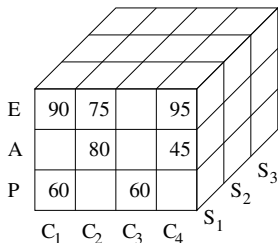
- Considers a set of data objects having some similar attribute(s)
- **Aggregates** some other attribute(s) into single value(s)
- Example: sum, average
- Benefits of aggregation
 - Aggregate value has **less variability**
 - **Absorbs individual errors**
 - **Reduces noise**

Data Cube



- For multi-dimensional datasets, aggregation can happen along different dimensions
- Data cubes are essentially multi-dimensional arrays
- Each cell or face or lower dimensional surface represents a certain *projection* operation

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- Aggregation can also happen along different resolutions in each dimension

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- Sampling approaches

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- For different types of objects, **stratified sampling**
 - **Picks equal** or representative **number of objects from each group**
- Sample size
 - Sample should have **enough data to capture variability**
- **Progressive sampling** or **adaptive sampling**
 - **Start with a small sample size**
 - **Keep on increasing till it is acceptable**

Histograms

- Method of discretizing the data
- Mostly useful for one-dimensional data
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- **MaxDiff histograms**
 - Values are first sorted
 - To get b bins, the largest $b - 1$ differences are made bin boundaries

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 - Mean: may be weighted
 - Median: “middle” value
 - Mode: dataset may be *unimodal* or *multimodal*
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Data Summarization (contd.)

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 - Example: mean
- **Holistic** measures
 - Cannot be computed by partitioning the dataset
 - Example: median
- Graphical measures help in **data visualization**

- Histogram:

Data Visualization

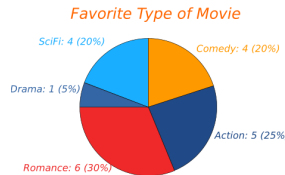
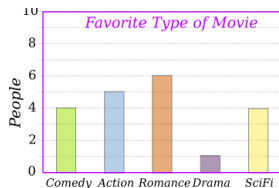
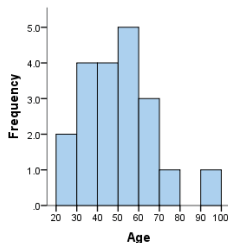
- **Histogram**: frequency versus grouped values
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Data Visualization

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- **Pie chart**:

Data Visualization

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- **Bar chart:** histograms where bins are categorical
- **Pie chart:** relative frequencies shown as sectors in a circle



Data Plots

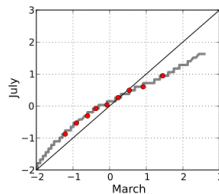
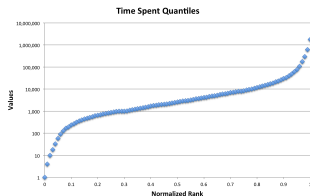
- Quantile plot:

Data Plots

- Quantile plot: quantiles against value
- Quantile-quantile plot (q-q plot):

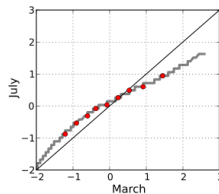
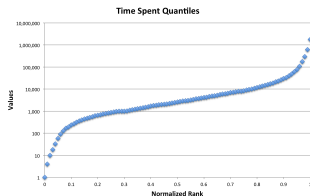
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- **Quantile-quantile plot (q-q plot):** quantiles against quantiles



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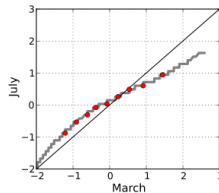
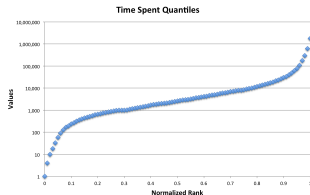
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- **Scatter plot:**

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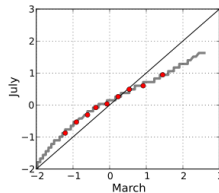
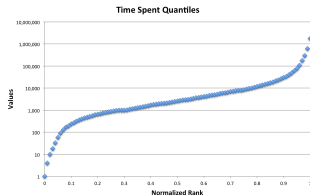
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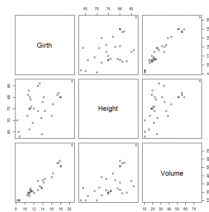
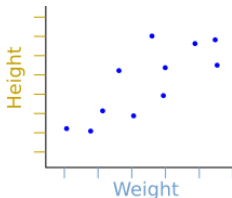
- **Scatter plot:** values of one variable against another
- **Scatter plot matrix:**

Data Plots

- **Quantile plot:** quantiles against value
- **Quantile-quantile plot (q-q plot):** quantiles against quantiles



- **Scatter plot:** values of one variable against another
- **Scatter plot matrix:** n^2 scatter plots for n variables



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- Can predict missing values
- The general form of regression model is

$$Y = f(X) + \epsilon$$

- The function f can be chosen using domain knowledge
- For *linear regression*, f is linear
- ϵ encodes the *error* (includes *noise*) terms associated with each observation

Linear Regression

- The function f is chosen to be linear
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- W can be solved through *least squares method*

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- Feature selection or attribute selection is different from dimensionality reduction, as it retains original attributes
- Aims to remove redundant and irrelevant features

Feature Selection

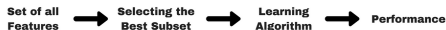
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- *Feature weighting*: Variant of feature selection
 - SVM does it naturally

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 - Features chosen before the algorithm
 - These use criteria that are independent of the algorithm
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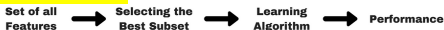


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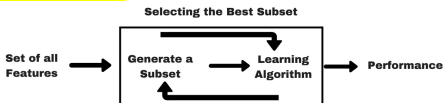
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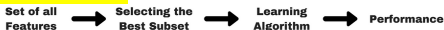


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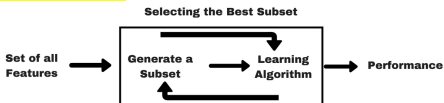
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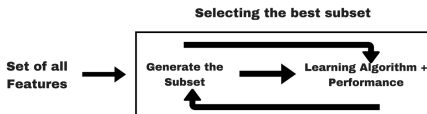
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- **Embedded**

- Algorithm has built-in feature selection strategy
- Example: decision trees | LASSO regression



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