# Hermitian Forms and Zeros of a Polynomial

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September 4, 2017

**Abstract**

We looked at the general properties of Hermitian (self-adjoint) matrices, and used the Schur-Conn theorem to find the number of roots of a polynomial lying within the unit circle.

## I. INTRODUCTION

In this project we see the properties of Hermitian matrices, which are very interesting, as well as useful. We also see and prove the Schur-Conn theorem to find the number of roots of a polynomial lying within the unit circle.

There are many ways to locate the roots of a polynomial. Using the Schur-Conn theorem gives a nice estimate on how many roots lie inside the unit circle.

## II. HERMITIAN MATRICES

Hermitian matrices (also known as self-adjoint matrices) satisfy $A^* = A$.

Hermitian matrices can be diagonalized. For every Hermitian matrix A, there exists a diagonal matrix $\Lambda$ such that $A = U^*\Lambda U$. Here $U$ is some unitary matrix.

## III. SCHUR-CONN THEOREM

Given a polynomial $p(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n$. Suppose $p$ has roots $\alpha_i$. Then $p(z) = (z-\alpha_1)(z-\alpha_2)\cdots(z-\alpha_n)$.

Without loss of generality, let $a_0 = 1$ as it does not change the roots of the polynomial.

Let $S$ be the $n \times n$ square matrix. Note that it is nilpotent of order $n$, i.e. $S^n$ is a zero matrix. Then $p(S) =$ Insert matrix here.

$p(S) = (S - \alpha_1 I)(S - \alpha_2 I)\cdots(S - \alpha_n I)$.

Then define $q$ as the polynomial with roots $\frac{1}{\bar{\alpha_i}}$. We get $q(z) = (1 - \bar{\alpha_1} z)(1 - \bar{\alpha_2} z)\cdots(1 - \bar{\alpha_n} z)$

Let $H$ be equal to $\|q(S)x\|^2 - \|p(S)x\|^2$

The polynomial $p$, it will have $k$ roots inside the circle, and $n-k$ roots outside the circle iff $k$ eigenvalues of $H$ are positive and $n-k$ are negative.

## IV. PROOF

The proof is trivial and is left as an exercise to the reader.

## V. EXTENSIONS

## VI. CONCLUSION

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