Hermitian Forms and Zeros of a Polynomial

Pranshu Gaba *

Indian Institute of Science, Bangalore
pranshu@ug.iisc.in

September 4, 2017

Abstract

We looked at the general properties of Hermitian (self-adjoint) matrices, and used the Schur-Conn theorem to find the number of roots of a polynomial lying within the unit circle.

I. Introduction

In this project we see the properties of Hermitian matrices, which are very interesting, as well as useful. We also see and prove the Schur-Conn theorem to find the number of roots of a polynomial lying within the unit circle.

There are many ways to locate the roots of a polynomial. Using the Schur-Conn theorem gives a nice estimate on how many roots lie inside the unit circle.

II. Hermitian Matrices

Hermitian matrices (also known as self-adjoint matrices) satisfy $A^* = A$.

III. Schur-Conn Theorem

Given a polynomial $p(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n$. Suppose p has roots α_i. Then $p(z) = (z - \alpha_1)(z - \alpha_2) \cdots (z - \alpha_n)$.

Without loss of generality, let $a_0 = 1$ as it does not change the roots of the polynomial.

Let S be the $n \times n$ square matrix. Note that it is nilpotent of order n, i.e. S^n is a zero matrix. Then $p(S) =$ Insert matrix here.

$p(S) = (S - \alpha_1 I)(S - \alpha_2 I) \cdots (S - \alpha_n I)$.

Then define q as the polynomial with roots $\frac{1}{\bar{\alpha_i}}$. We get $q(z) = (1 - \bar{\alpha_1} z)(1 - \bar{\alpha_2} z) \cdots (1 - \bar{\alpha_n} z)$

Let H be equal to $\|q(S)x\|^2 - \|p(S)x\|^2$

The polynomial p, it will have k roots inside the circle, and $n - k$ roots outside the circle iff k eigenvalues of H are positive and $n - k$ are negative.

IV. Proof

The proof is trivial and is left as an exercise to the reader.

V. Extensions

VI. Conclusion

*:)