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ECE320H1F: Fields and Waves
Laboratory 3: Design of a Double Stub Matching Network

1 Objective

The input impedance and the reflection coefficient of a transmission line are investigated. The transformation between the two is introduced with graphical aids. The measurement and interpretation of the voltage standing wave ratio (VSWR) is discussed.

2 References

[1] Your lecture notes.

[2] F. T. Ulaby and U. Ravaioli, *Fundamentals of Applied Electromagnetics*, 7th ed. Upper Saddle River, NJ: Pearson, 2015.

3 Background

3.1 De-embedding of Measured Data

A device-under-test (DUT) is often measured in a test fixture, as illustrated in Fig. 1. Vector Network Analyzer (VNA) measurements are made at the *calibration plane*, which is typically the end of the cables attached to the VNA ports. Sometimes, there is an additional section of transmission line (e.g an extra cable) between the ends of the cables and the output ports of the DUT, and as a result the VNA measurement data includes the effects of the connecting feed lines. Even assuming this extra section of cable to be lossless, any length of transmission line causes a rotation of the impedance on the Smith chart, and so this rotation must be "undone." The process of *de-embedding* removes these effects and allows the accurate characterisation of the DUT.

Although circuits or mathematical models can be used to account for the effects of the test fixture, the de-embedding procedure you will use for this experiment is straightforward: an additional length of transmission line will be added "virtually" to the measured data as shown in Fig. 1b. Therefore, it is as though there is an extra section of cable between the VNA and the DUT. Adding this "virtual" cable length corresponds to a rotation of the input impedance on the Smith chart where the amount of rotation corresponds to the *electrical length* of the cable (i.e., its length in wavelengths). However,

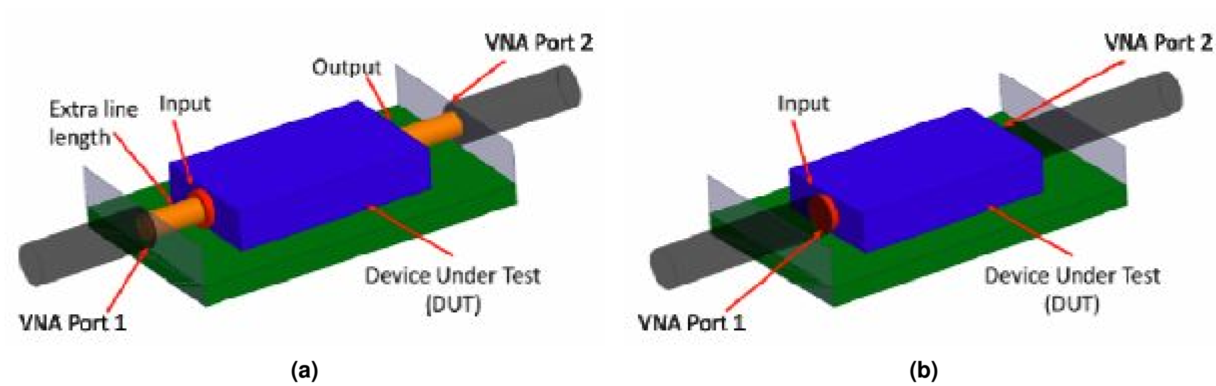


Figure 1 Test fixture: (a) before de-embedding, and (b) after de-embedding.

in this case the direction of rotation is *counter-clockwise* since the measurement port is moving closer to the load (that is, away from the generator, or VNA).

3.2 Introduction to a Double Stub Tuner

A single stub shunt matching network is shown in Fig. 2. The single stub tuner uses two degrees of freedom to achieve a match; an open- or short-circuited stub with a variable length l, and a variable length of transmission line d between the stub and the load. The use of a variable length of transmission line between the load and the first stub is generally seen as a disadvantage in practice. In the case of a reusable tuning network, such as the case in this exercise, adding and removing this section of transmission line is not practical, and can in fact be very difficult to do. In the case of a double stub tuner, the separation between the stubs remains constant, and the addition of a second stub allows for matching almost any load without the need for the addition or removal of sections of transmission line, resulting in an electrically flexible and mechanically simple system.

A double stub shunt matching network is shown in Fig. 3. Stubs l_1 and l_2 separated by distance d_1 transform a specific admittance Y_{L} into Y_0 at the input terminals of the network. Note also that there is an extra section of line of length d_0 separating the first stub from the load, which models the length of the connector between the tuning circuit and the load.

3.3 Basic Smith Chart Operations

The Smith chart is a powerful tool that is commonly used in practice in microwave engineering. In the context of impedance matching, such as the double stub tuner used in this laboratory, there are four basic operations to review.

The first basic operation is transforming an arbitrary load *along* a transmission line. Given a transmission line that is terminated in some load z_{L} (or y_{L}, if using the admittance chart (typ.)), which can take on any complex value, including an open- or short-circuit, the load can be transformed from its physical location to some other arbitrary location on the transmission line by rotating the load along

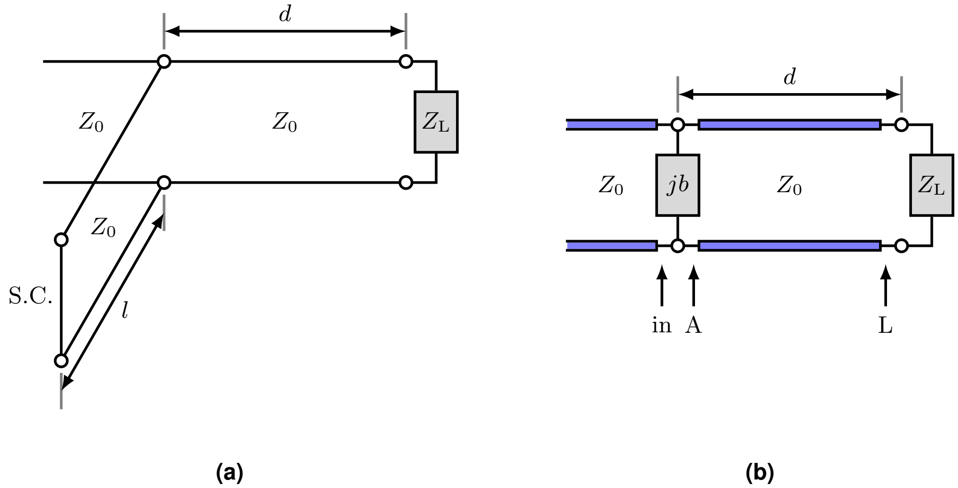


Figure 2 Schematics for a single stub tuner: (a) transmission-line schematic, and (b) equivalent circuit. A match is achieved by varying the stub length l and distance from the load d.

the constant *voltage standing wave ratio* (VSWR) circle on the Smith chart. Example trajectories of this operation can be seen in Fig. 4a.

The second basic operation is the addition of a *purely reactive* component to the load. This component can take the form of a capacitor, an inductor, or a short- or open-circuited stub. In this case, the *real* part of the load impedance (or admittance) will remain constant, whereas the *imaginary* part will be altered. For this reason, the resulting trajectory will be along a *circle of constant resistance (or conductance)*. Example trajectories of this operation can be seen in Fig. 4b.

The third basic operation is the addition of a *purely resistive* component to the load. In this case, the component will clearly be a resistor. As opposed to the previous operation, this time, the *imaginary* part of the load will remain constant, whereas the the *real* part will be altered. This time, the resulting trajectory will be along a *curve of constant reactance (or susceptance)*. Example trajectories of this operation can be seen in Fig. 4c. ***Note: this operation will NOT be used in this laboratory, but is presented for the sake of completeness.***

The fourth basic Smith chart operation that is covered here is the rotation of the $r = 1$ (or $g = 1$) circle, as illustrated in Fig. 4d. This operation is generally specific to multi-stub tuning, as is the case in this laboratory. The purpose of this operation is to create a 'target' for the first stub.

3.4 Double Stub Matching Design Procedure

Before going over the design procedure, there are two important concepts to keep in mind when using a double stub tuner. Firstly, referring to Fig. 3, it is noted that for a perfectly matched load, y_{in} is unity and therefore $y_{\text{C}} = 1 - jb_2$. This means that the y_{C} point is located on the $g = 1$ circle of the Smith chart. Therefore, to find the y_{B} point, the $g = 1$ circle is rotated *counter-clockwise* (toward the load) on the Smith chart by a distance corresponding the stub separation d_1 (in wavelengths). For y_{C}

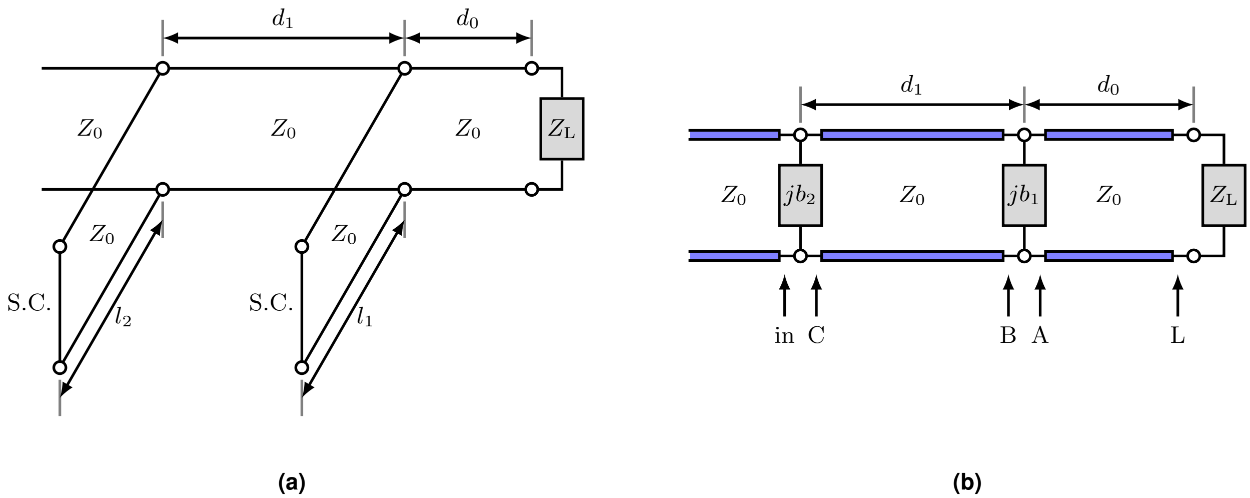


Figure 3 Schematics for a double stub tuner: (a) transmission-line schematic, and (b) equivalent circuit. A match is achieved by varying the stub length l_1 and l_2, whereas the distances between the stubs d_1 and load d_0 are fixed.

to lie on the $g = 1$ circle, y_B point must lie somewhere on the rotated $g = 1$ circle as indicated on the Smith chart.

Secondly, when determining the stub lengths l_1 and l_2, the stubs are treated as transmission lines in their own right. That is, the load at the end of the stubs (either a short- or open-circuit) is plotted on the Smith chart. The length of the stub is determined by rotating *away from the load* (clockwise) until the required reactance (or susceptance) is reached on the outer edge of the Smith chart. Then convert the wavelength reading from the edge of the Smith chart into a physical length. This way, the impedance seen looking in to the stub with that length is the desired reactance (or susceptance).

Finally, the notation for distances associated with Smith chart operations is as follows. *Physical* distances (d_0, d_1, l_1, and l_2, in cm) must be converted to *electrical* distances (normalized to wavelengths λ when using the Smith chart, and are indicated with the 'hat' notation ($\hat{d}_0$, $\hat{d}_1$, $\hat{l}_1$, and $\hat{l}_2$). The double stub tuner used in this laboratory consists of air-filled coaxial lines. As a result, a free-space wavelength λ_0 is assumed.

The design procedure for a double stub shunt tuner with short-circuited stubs is illustrated on the Smith chart in Figs. 6, 7, 8, and 9. In the illustrations, it is assumed that the load impedance is $z_L = 0.5+j0.5$, and the tuner dimensions are $d_0 = d_1 = \lambda/8$ (that is, $\hat{d}_0 = \hat{d}_1 = 1/8$). ***Note that the procedure illustrated in these figures uses an arbitrary load impedance and tuner dimensions. When doing the procedure yourself, ensure that you use the load impedance measured in the lab and the dimensions given for the double stub tuner, NOT that used in the illustrations.*** The procedure is outlined in the following steps (it may help to refer to Fig. 3 while reading the outline below).

1. Plot the normalized (to $50\,\Omega$, unless otherwise specified) load impedance z_L on the Smith chart.

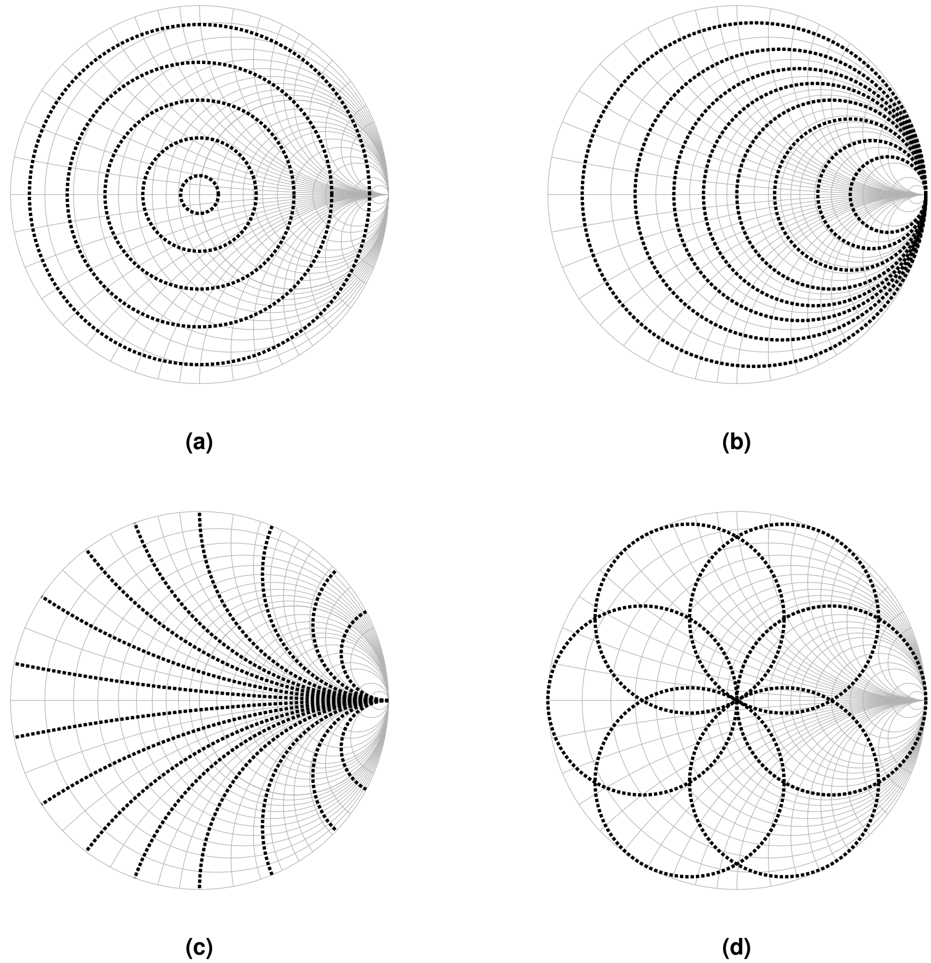


Figure 4 Basic Smith chart operation transformation example trajectories: (a) transforming a load along a transmission line, (b) adding a *purely reactive* component to the load, (c) adding a *purely resistive* component to the load, and (d) the rotation of the $g = 1$ circle.

2. Transform the load impedance z_L to the equivalent admittance y_L by rotating it by 180° about the center of the Smith chart. This can also be interpreted as mirroring the load across the center of the Smith chart.

3. Transform the load admittance y_L by the distance $\hat{d}_0$ to bring the load from the load location to the location of the first stub. Since we're moving along the transmission line, the equivalent Smith chart operation is a rotation about the center of the Smith chart on the VSWR circle associated with the load. The load is being moved *toward the generator* (clockwise) to y_A.

4. The $g = 1$ circle is now transformed *toward the load* (counter-clockwise) by the distance $\hat{d}_1$, to 'move' it to the first stub location. This creates a 'target' for the tune with the first stub, as discussed above.

5. The admittance y_A is now transformed to intersect the rotated $g = 1$ circle. Since $y_\text{B} = y_\text{A} + jb_1$ (y_A and y_B have the same conductance), matching is possible for any load y_A which lies on a

conductance circle intersecting the rotated $g = 1$ circle. That is, the equivalent Smith chart operation is a rotation along the *circle of constant conductance*, because the *real* part of the admittance is constant, whereas the *imaginary* part is being altered.

6. Since there are in general two intersection points of one conductance circle with the rotated $g = 1$ circle, two different designs are possible for a given load y_A, resulting in $y_B = y_A + jb_1$ and $y'_B = y_A + jb'_1$.

7. The susceptances b_1 and b'_1 are given by the required difference in susceptance between y_A and the chosen y_B and y'_B, respectively, on the Smith chart.

8. The admittances y_B and y'_B are now transformed to y_C and y'_C by the distance $\hat{d}_1$ to move them from the first stub location to the second stub location. Since the loads are being moved along the transmission line, they are rotated *toward the generator* (clockwise) about the center of the Smith chart along their respective VSWR circles. Note that because y_B and y'_B lie on the rotated $g = 1$ circle, which has been transformed *toward the load* by the distance $\hat{d}_1$, the transformed loads y_C and y'_C now lie on the original $g = 1$ circle.

9. Since $y_C = 1 + jb_2$ and $y'_C = 1 + jb'_2$, the susceptances b_2 and b'_2 are given by the required differences in susceptance between y_C/y'_C and the matched position at the center of the Smith chart.

10. Determine the two stub lengths for each solution (l_1, l_2, l'_1, and l'_2) using the Smith chart by starting at the short circuit ($y = \infty$) position and rotating *toward the generator* (clockwise) along the VSWR circle ($|\Gamma| = 1$ for a short circuit) until the required susceptance values are encountered. Then convert the wavelength readings from the Smith chart into physical lengths for each stub.

4 Experimental Procedure

It is very important to wear a static bracelet when operating the VNA. The VNA is a very sensitive piece of equipment, and can easily be damaged by electrostatic discharge (ESD). When connecting loads and/or calibration standards, use the provided torque wrench. If no torque wrench is provided, tighten the loads or standards *finger tight*. Do NOT overtighten the loads or standards. The calibration kit (seen in Fig. 5) has protective caps on all standards; please ensure that the caps are replaced after the calibration procedure is completed. If any caps are missing, notify the laboratory teaching assistant (TA) and/or the laboratory manager.

Recall that the double stub tuner used in this laboratory consists of air-filled coaxial lines. As a result, a free-space wavelength λ_0 is assumed when normalizing physical distances to electrical distances. **The design frequency for this exercise is $f = 800$ MHz. The physical dimensions associated with the double stub tuner are $d_0 = 3.4$ cm and $d_1 = 3.8$ cm.**

Notation: In the following instructions, [Command] refers to a softkey on the VNA front panel while *Command* refers to an option appearing on the VNA touchscreen (or monitor if it is hooked up to the

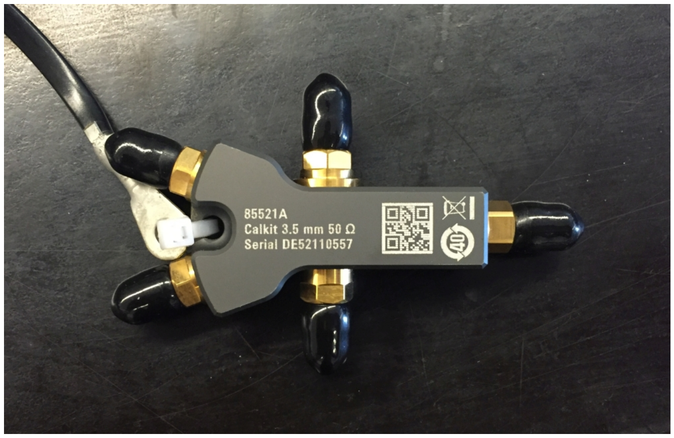


Figure 5 The calibration kit used for ECE320 laboratories. Note the protective caps on all the standards. Please replace the caps after the calibration procedure is completed. If any caps are missing, please notify the teaching assistant and/or laboratory manager.

VNA), and $<$ Command $>$ refers to something that is typed in using the VNA keypad (or keyboard if it is hooked up to the VNA).

1. Calibrate the VNA for a 1-port measurement.

(a) Set the frequency span over which the measurements and calibration will be performed. On the VNA front panel, press [Start] $\rightarrow$ $<$ 300 MHz $>$, and [Stop] $\rightarrow$ $<$ 1.3 GHz $>$.

(b) Set the number of points to be measured. Press [Sweep Setup] $\rightarrow$ *Points* $\rightarrow$ $<$ 1601 $>$ $\rightarrow$ *Return*.

(c) Select the calibration kit. Press [Cal] $\rightarrow$ *Cal Kit* $\rightarrow$ *85521A*.

(d) Initialize the calibration procedure. Press [Cal] $\rightarrow$ *Calibrate* $\rightarrow$ *1-Port Cal*.

(e) Begin by connecting the cable attached to port 1 of the VNA to the 'Open' calibration standard. Then press the corresponding *Open* option on the VNA calibration menu. Repeat for the 'Short' and 'Load' standards.

(f) **Make sure you press *Done* at the bottom of the calibration menu after measuring the three calibration standards. Otherwise the calibration will not be complete, and hence will not be applied.**

(g) When the calibration is complete, reconnect the 'Load' standard, and press [Meas] $\rightarrow$ *S11*, and [Format] $\rightarrow$ *Smith* $\rightarrow$ $R + jX$. A successful calibration will result in a measured locus that appears as a small dot at the center of the Smith chart. If this is not the case, the calibration procedure has not been properly completed. Disconnect the standard and replace all protective caps that have been removed during the calibration process.

2. Measure the unknown load impedance.

(a) Record the color code of your load in your notes.

(b) Connect the load to the SMA cable. Press [Meas] $\rightarrow$ *S11*, and [Format] $\rightarrow$ *Smith* $\rightarrow$ $R+jX$. A data marker can be displayed by pressing [Marker] $\rightarrow$ *Marker 1* $\rightarrow$ $<$ 800 MHz $>$.

(c) Record the measured impedance displayed at the top left of the VNA (or monitor) screen. Export the Smith Chart graph with [System] → *Dump Screen Image* → < Select save location >.

(d) Demonstrate the de-embedding process graphically on a Smith chart. Press [CAL] → *Port Extension* → *Extension ON* → *Extension Port 1* → < 0.2 ns >, and save the resulting image. How many wavelengths (at 800 MHz) does the length increase correspond to? Using your own calculations, show how this increase corresponds to rotating the impedance measured in part 2b) on the chart. *Note: the port extension process corresponds to moving* ***toward the load****. This is because a positive time* (0.2 ns) *was entered, which effectively* ***adds*** *'virtual' cable to the measurement setup. The addition of this virtual cable moves the load away from the generator.*

(e) Turn off the port extension. Press [CAL] → *Port Extension* → *Extension OFF*.

3. Plot the load on a *paper* Smith chart to begin the graphical matching process.

(a) Using a *paper* Smith Chart, determine the normalized impedance of the load z_L transferred to the connection point of the first stub ($d_0 = 3.4\,\text{cm}$). This is the impedance z_A.

(b) Locate the corresponding admittance y_A.

4. Follow the design procedure outlined in Section 3.4 to design a double stub matching network. Record both stub lengths in wavelengths as well as in cm. The distance between the centerlines of the two stubs is $d_1 = 3.8\,\text{cm}$. Note that there are two *independent* solutions for each stub length.

5. Replicate the matching process using the physical double stub tuner and the load. Attach the double stub tuner and load to the SMA cable attached to port 1. Adjust the stub lengths to find a good impedance match for both sets of solutions and compare the final stub lengths with your calculated values.

6. Plot the resulting VSWR vs. frequency of the matched line using [Format] → *SWR*. Find the bandwidth of each solution for a VSWR of less than 2. What does this correspond to in terms of the reflection coefficient and its value in decibels ($20\log|\Gamma|$)? Compare the two measured bandwidths and suggest reasons for this bandwidth limitation.

Make sure you speak to you TA before leaving the lab, so that you can be assessed your oral grade for this experiment!

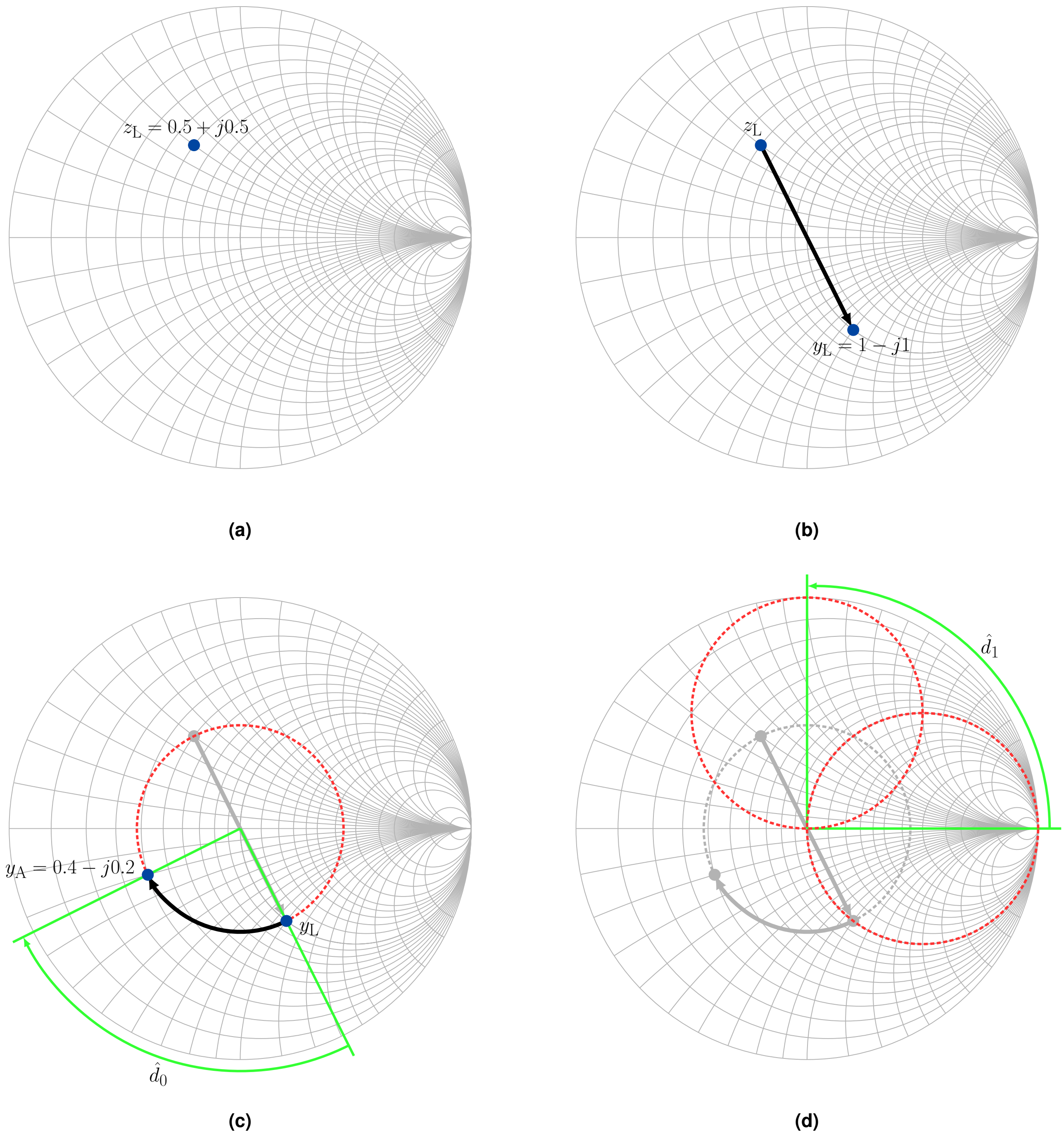


Figure 6 Beginning the double stub matching procedure. Plotting (a) the normalized load impedance z_L on the *impedance* Smith chart and (b) transforming it to the equivalent load admittance on the *admittance* Smith chart. Then, transforming (c) the load y_L from the load location *back towards the generator* (counter-clockwise) by $d_0 = \lambda/8$ ($\hat{d}_0 = 1/8$) to y_A at the first stub location, and (d) the $g = 1$ circle *forward towards the load* (clockwise) by $d_1 = \lambda/8$ ($\hat{d}_1 = 1/8$). Since the movement is *along the transmission line*, the equivalent Smith chart operation is a rotation along the VSWR circle.

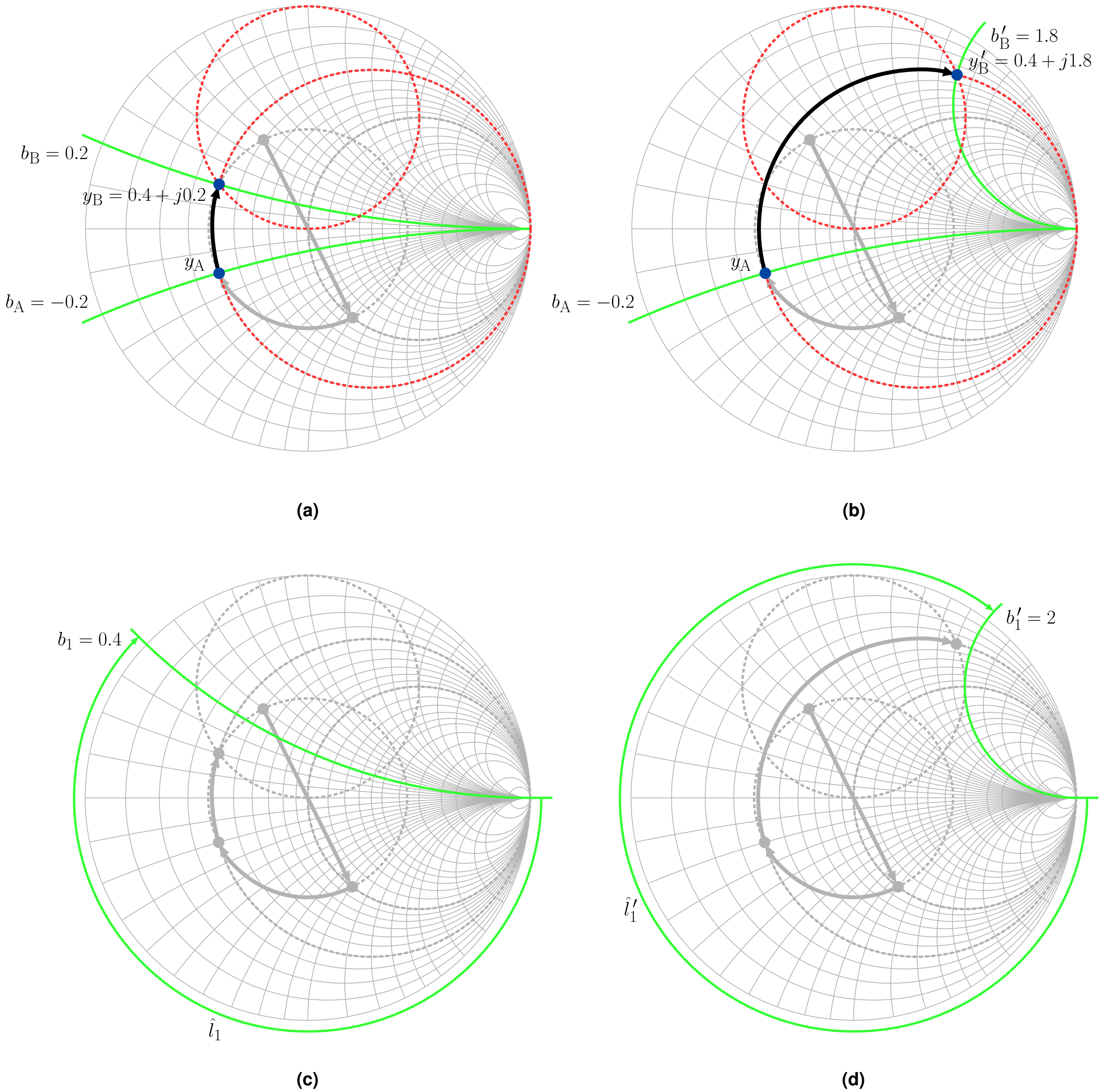
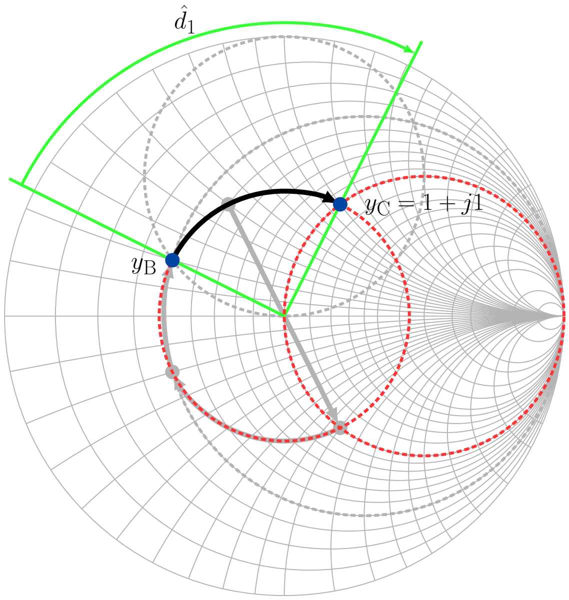
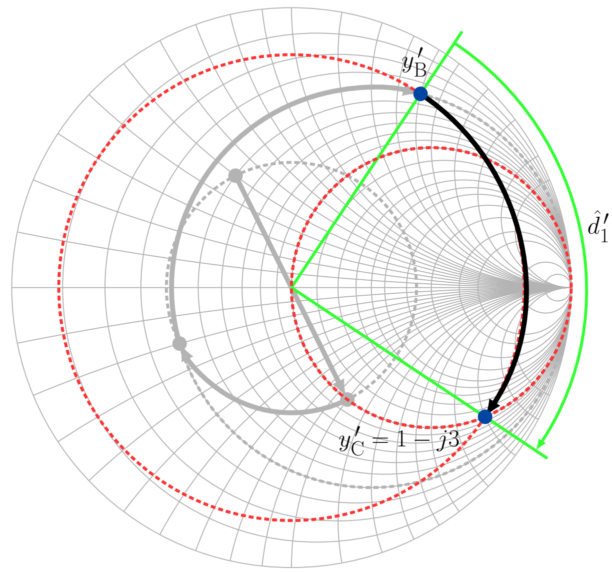


Figure 7 The two independent solutions to transform y_{A} to the rotated $g = 1$ circle; (a) solution 1 (y_{B}) and (b) solution 2 (y'_{B}). The stubs are purely reactive, and hence change only the *imaginary* part of the admittance. As a result, the *real* part of the admittance remains constant, and the load is transformed along the $g = 0.4$ circle. It is noted that: (c) solution 1 requires $b_1 = b_{\mathrm{B}} - b_A = 0.4$, and (d) solution 2 requires $b'_1 = b'_{\mathrm{B}} - b_A = 2$. For both solutions, the stub lengths l_1 and l'_1 are determined by beginning at the short circuit on the right of the Smith chart ($y_{\mathrm{L}} = \infty$), and rotating *toward the generator* (clockwise) until the required susceptance (read on the outer edge of the Smith chart) is reached.

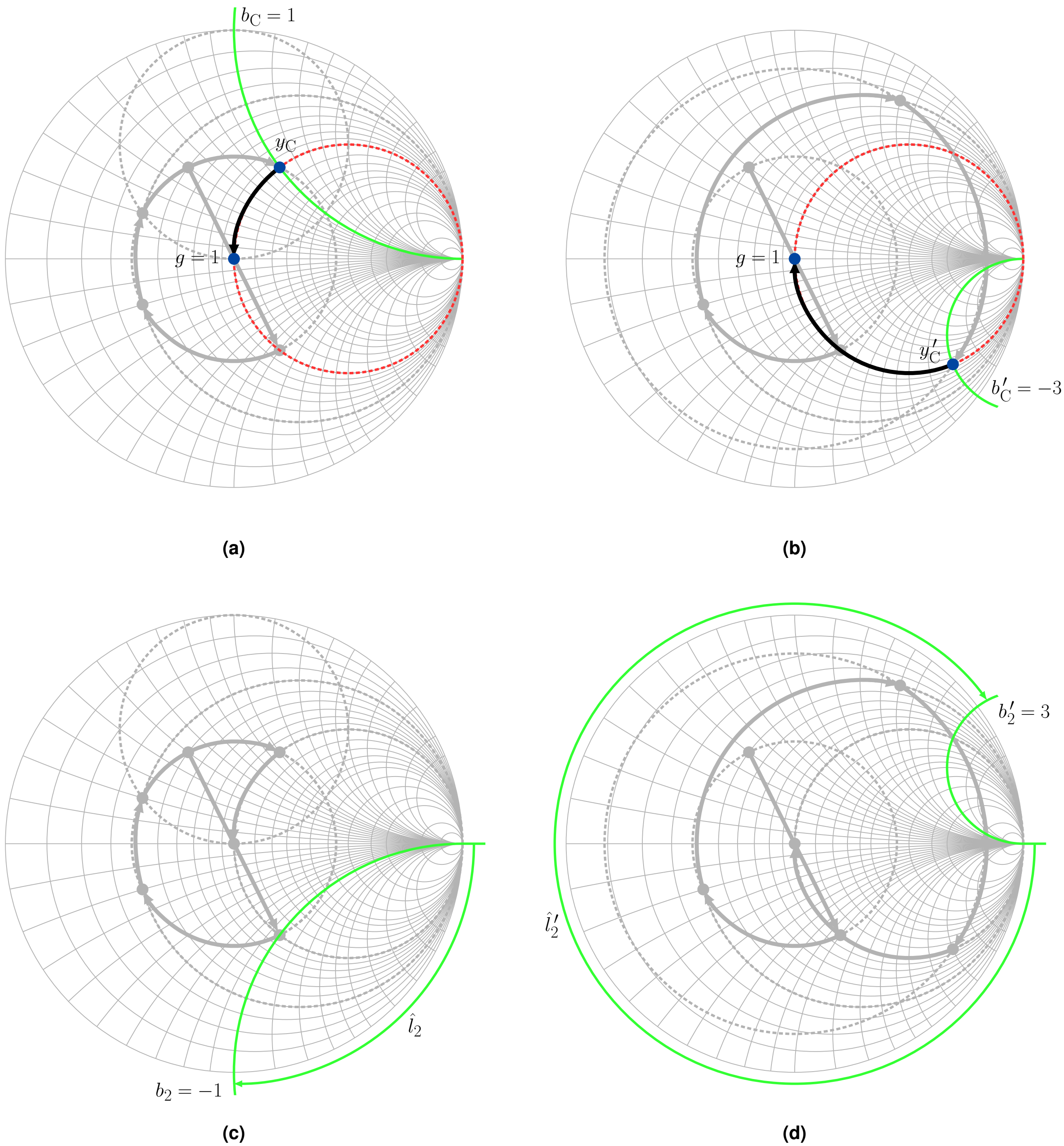


(a)



(b)

Figure 8 Transforming y_B and y'_B from the first stub location *toward the generator* (clockwise) to the second stub location. Since the load y_A has been transformed to $y_\mathrm{B}/y'_\mathrm{B}$ on the *rotated* $g = 1$ circle, $y_\mathrm{B}/y'_\mathrm{B}$ are now rotated back on to the *original* $g = 1$ circle.



(a) **(b)** **(c)** **(d)**

Figure 9 Transforming y_C and y'_C to $g = 1$. As is the case with the first set of stubs, the second set of stubs are purely reactive and change only the *imaginary* part of the admittance. As a result, the *real* part of the load admittance remains constant, and the load is transformed along the $g = 1$ circle. It is noted that: (c) solution 1 requires $b_2 = 0 - b_C = -1$, and (d) solution 2 requires $b'_2 = 0 - b'_C = 3$. For both solutions, the stub lengths l_2 and l'_2 are determined by beginning at the short circuit on the right of the Smith chart ($y_L = \infty$), and rotating *toward the generator* (clockwise) until the required susceptance (read on the outer edge of the Smith chart) is reached.