ECE320: Fields and Waves

Lab 1 Report: Waves on Transmission Lines

PRA106

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1 Introduction

This laboratory focused on investigating the characteristics of transmission lines, studying voltage and current propagation along them, as well as its depedance on the nature of load impedance.

2 Determining the Characteristic Impedance, Z_0

We varied the load on the switch box until we saw little or no traces of reflected waves. This was at $Z_L = 50\Omega$ which is also equal to the charactertic impedance since we know that the reflections nullify when $Z_L = Z_0$. The corresponding waveforms captured at the generator input (channel 1, top) and the transmission line input (channel 2, bottom) are shown in Figure 1.

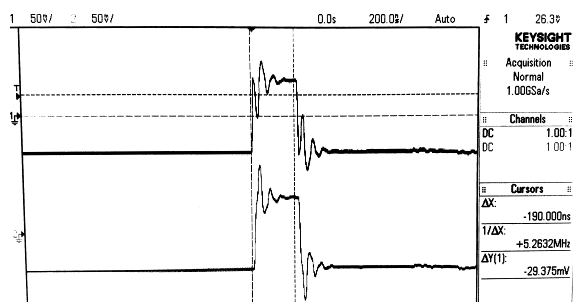


Figure 1: Transmission line terminated with load $Z_L = Z_0$

3 Determining Z_0 using $\frac{\tilde{V}^+(z=-L)}{\tilde{I}^+(z=-L)}$

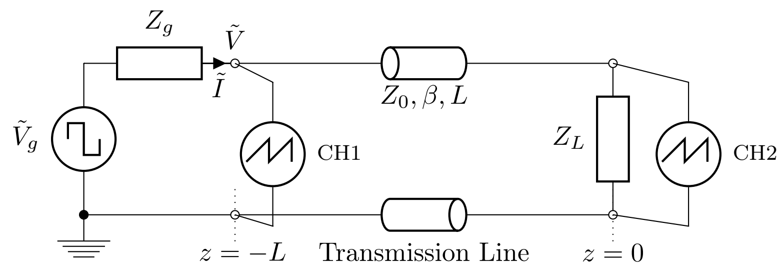
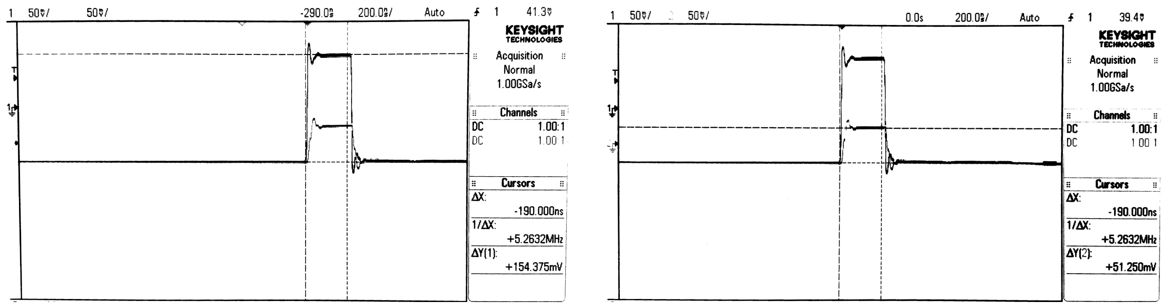


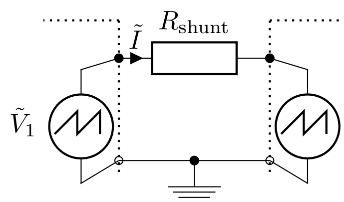
Figure 2: Laborartory setup for studying characteristcs of tranmission lines



(a) $\tilde{V}_1 \approx 154\text{mV}$ (b) $\tilde{V}_2 \approx 51\text{mV}$

Figure 3: $\tilde{V}$ measured across $R_{\text{shunt}} = 100\Omega$

As seen in Figure 3, the voltage at $\tilde{V}_1 = 154\text{mV}$ and $\tilde{V}_2 = 51\text{mV}$. Given the value of $R_{\text{shunt}} = 100\Omega$, we can calculate $\tilde{I}^+ = \frac{0.154-0.051}{100} = 1.03\text{mA}$. From Figure 4, we can see that $\tilde{V}_2 = \tilde{V}^+$, and therefore we can confirm the value of Z_0 through the relation, $Z_0 := \frac{\tilde{V}^+(z=-L)}{\tilde{I}^+(z=-L)} = \frac{51\text{mV}}{1.03\text{mA}} = 49.51\Omega \approx 50\Omega$.



$$\tilde{I} = \frac{\tilde{V}_1 - \tilde{V}_2}{R_{\text{shunt}}}$$

Figure 4: Estimating input current through a shunt resistance

4 Observation of Travelling Waves

Observed waveforms at different points on the transmission line can be found in Figure 5.

The recorded time delays, Δt, relative to the input signal are listed in Table 1.

Port	**Δt (ns)**
D	130
E	244
F	368

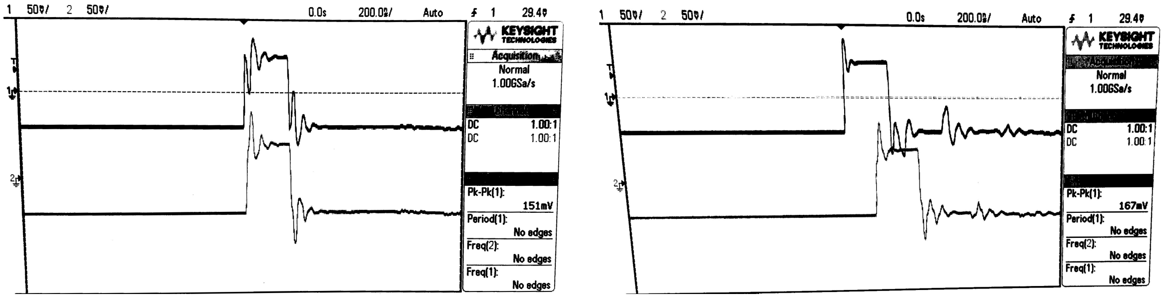
Table 1: Recorded time delay at different locations along the transmission line

5 Determining Velocity of Propagation

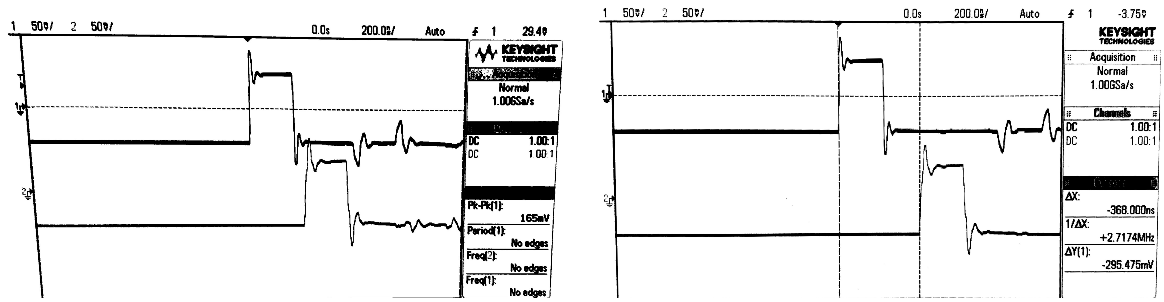
The velocity of propagation of the signal can be calculated by the relation $v_p = \frac{\Delta L}{\Delta t}$, given that we are able to track the same point on the waveform. Using the data from Table 1, we get that the average velocity of propagation is $v_{p,\text{avg}} = 2.44 \cdot 10^8\text{m/s}$.

Now, to find the relative permittivity, we also know that the phase velocity of an electromagentic wave in an electrical transmission line with magnetic permeability, $\mu \approx \mu_0$, and electric permittivity, $\epsilon = \epsilon_r \epsilon_0$, is given by:

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} \implies \frac{v_p}{c} \approx \sqrt{\frac{\mu_0\epsilon_0}{\mu_0\epsilon_0\epsilon_r}} \implies \epsilon_r \approx \frac{c^2}{v_p^2} \approx 1.51$$



(a) Port C (0m) (b) Port D (30m)



(c) Port E (60m) (d) Port F (90m)

Figure 5: Measured $V(t)$ at different loactions along the transmission line with $Z_L = Z_0$

The theoretical $V(t)$ plots in Figure 6 closely match the observations in Figure 5.

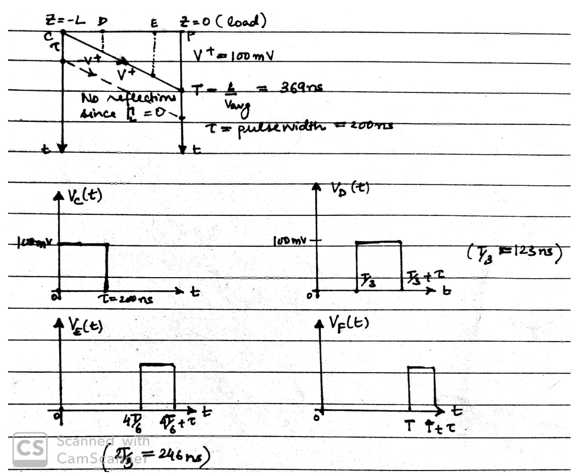


Figure 6: Theoretical bounce diagram and $V(t)$ plots along the transmission line

6 Simple Reflection

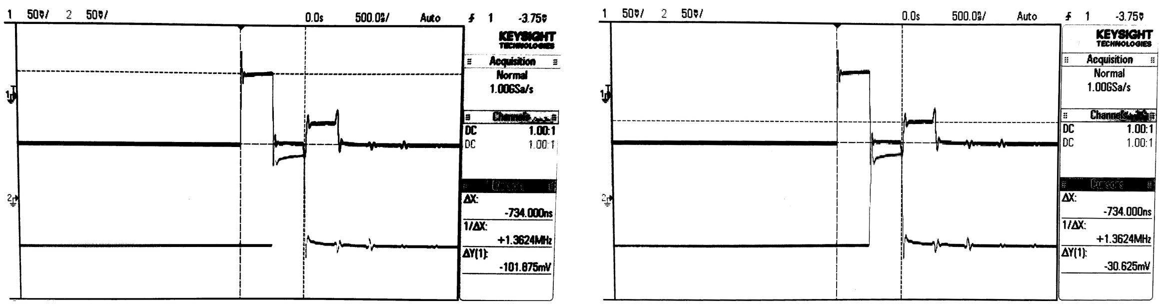
For an input pulse signal, the magnitude of the first voltage pulse across the transmission line input will always be $\tilde{V}_1^+ = \tilde{V}_g \frac{Z_0}{Z_0+Z_g}$, regardless of Z_L. There will be a load mismatch if $Z_L \neq Z_0$ and we know that, in this case, reflection of current and voltage occurs at the load, i.e. $\tilde{V}^- \neq 0$ and $\tilde{I}^- \neq 0$. The load reflection coefficient, Γ_L, if defined as:

$$\Gamma_L := \frac{\tilde{V}^-}{\tilde{V}^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Thus, in cases where there is a load mismatch, $\Gamma_L \neq 0$, there will be a relection of intensity $\tilde{V}^- = \Gamma_L \tilde{V^+}$

along the transmission line towards the generator and the steady state for a step input will be achieved after the voltage and current wave has travelled back and forth once, and this can also be verified using a bounce diagram. The Theoretically for $Z_L = 100\Omega$, $\Gamma_L = \frac{50\Omega}{150\Omega} = \frac{1}{3}$ and through the measurements, we observe the reflection coefficient $\Gamma_L = \frac{30\text{mV}}{100\text{mV}} = \frac{3}{10} \approx \frac{1}{3}$.

The measurements for reflected voltage waves can be found in Figure 7. Channel 1 (top) waveform has been recorded at port C (0m) and channel 2 (bottom) waveform has been recorded at port F (90m). The pulsewidth τ of the signal was set such that it was equal to the time delay T for a pulse to reach port F from port C, which made the signal recordings at port C and F exactly out of phase. In terms of magnitude, the signal at port F was a superposition of the incident and reflected wave $\tilde{V}_F = \tilde{V}^+ + \Gamma_L\tilde{V}^+$. Similarly at port C, the signal at $t \in (2T, 3T)$, $\tilde{V}_C = \Gamma_L\tilde{V}^+ = \tilde{V}^-$, which can all again be verified using a bounce diagram. The theoretical $V(d)$ graphs for different time points can be found in Figure 8.



(a) $\tilde{V}_1 = \tilde{V}^+ \approx 100\text{mV}$

(b) $\tilde{V}_2 = \Gamma_L\tilde{V}^+ = \tilde{V}^- \approx 30\text{mV}$

Figure 7: $\tilde{V}$ measured at port C and port F for input signal with pulsewidth $\tau = T$

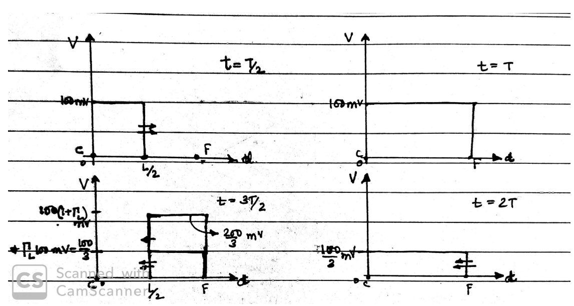


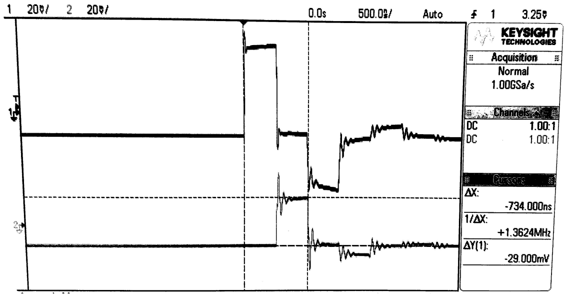
Figure 8: $V(d)$ plots at different time points

7 Multiple Reflections

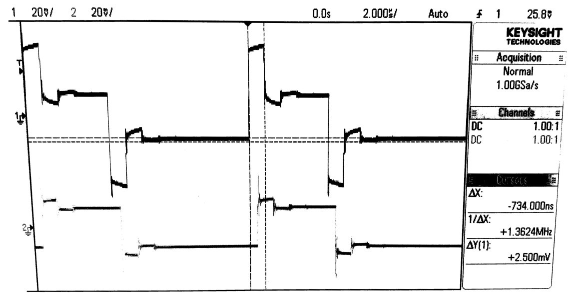
For mismatched generator side, $Z_g = 150\Omega$, as well as the load side, $Z_L = 20\Omega$, there will be reflections on both ends of the transmission line. The observed waveforms can be found in Figure 9, with channel 1 (top) recording signals at port C and channel 2 (bottom) recording signals at port F.

At the generator end of the transmission line, $\Gamma_g = \frac{100\Omega}{200\Omega} = \frac{1}{2}$, and at the load end $\Gamma_L = \frac{-30\Omega}{70\Omega} = \frac{-3}{7}$. In Figure 9 (a), we measure $\tilde{V}^+ = \tilde{V}_1^+ = 51.5\text{mV}$ on channel 1 for $t \in (0, T)$, whereas on channel 2 for $t \in (T, 2T)$, we measure $\tilde{V} = \tilde{V}^+ + \tilde{V}^- = 29\text{mV}$. This implies that $\tilde{V}^- = \Gamma_L\tilde{V}^+ = 29 - 51.5\text{mV} = -22.5\text{mV}$. Therefore, the observed $\Gamma_L = \frac{\tilde{V}^-}{\tilde{V}^+} = \frac{-22.5\text{mV}}{51.5\text{mV}} = -0.437 \approx \frac{-3}{7}$. Similarly, in Figure 9 (a), we measure $\tilde{V} = \Gamma_L\tilde{V}_1^+ + \Gamma_g\Gamma_L\tilde{V}_1^+ = -32\text{mV}$ at port C (channel 1) for $t \in (2T, 3T)$. Here we use the previous measurement of $\Gamma_L = -0.437$, we measure $\Gamma_g = 0.42 \approx \frac{1}{2}$. Thus, both of our measurements closely match the theoretical values for the reflection coefficients.

Theoretical bounce diagrams and $V(t)$ plots for signals observed at ports C and F can be found in Figure 10. In Figures 9 and 10, we observe such behavior of pulses because the input pulse creates a reflection at the



(a) $\tau = T$



(b) $\tau = 10T$

Figure 9: Multiply reflected signals for input signal with different pulsewidths

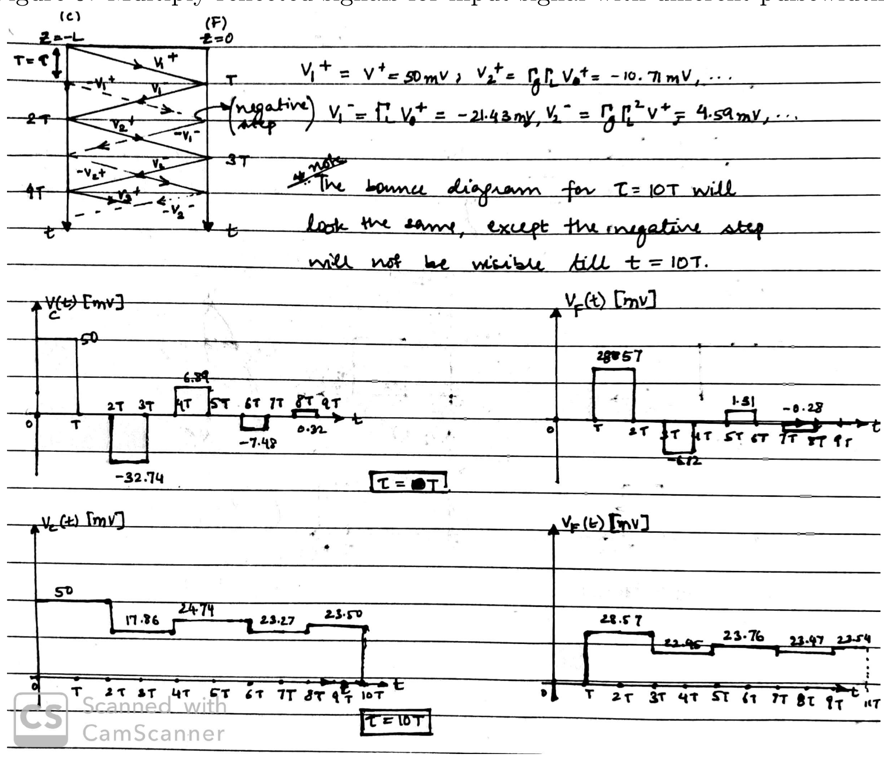


Figure 10: Theoretical bounce diagrams and $V(t)$ plots for $\tau = T$ and $\tau = 10T$

load which comes back to the source; however, when the reflection reaches the source, it is also being reflected and, thus, the source and load in this setup keep the pulse bouncing back and forth until it completely dies out beacuse of resistive losses along the transmission line.

8 Input Impedance and Transmission Line Length

While sweeping the input sinusoid frequency from 1MHz to 4MHz, we recorded the frequencies at which $||\tilde{V}_{\text{in}}||$ achieved its minimum. These frequencies are listed in Table 2.

To explain this effect for the short circuited (zero) load, we know that $\Gamma_L = -1 \implies \phi_Gamma = \pi$ and the voltage along the transmission line is $\tilde{V}(z) = \tilde{V}_0^+ e^{-j\beta z}(1 + ||\Gamma_L|| e^{j2\beta z + j\phi_\Gamma})$. Thus, the voltage minimum occurs when $\phi_\Gamma + 2\beta z = (2k+1)\pi, k \in \mathbb{Z}$. Since $\beta \propto \omega$, as we increase frequencies, there is a change in $||\tilde{V}(z)||$. Also, since current $\tilde{I} = \frac{\tilde{V}_0}{Z_0} e^{-j\beta z}(1 - ||\Gamma|| e^{j2\beta z + j\phi_\Gamma})$, there is a current maximum at voltage minimum, as when $||(1 + e^{j2\beta z + j\phi_\Gamma})||$ is minimum $||(1 - e^{j2\beta z + j\phi_\Gamma})||$ will be at its maximum.

$f_{\min}$ **Shorted**	$f_{\min}$ **Capacitive**
1.4	1.6
2.8	3.0
4.0	4.2

Table 2: Corresponding $f_{\min}$ for minimum $\tilde{V}_{\text{in}}$ for zero and capacitive loads in MHz

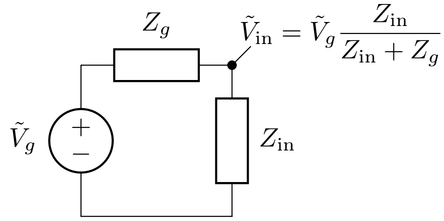


Figure 11: Voltage division over net input impedance, Z_{in}

To explain voltage minimum for the capacitive load, we know that net input impedance of the load and transmission line at the source, Z_in, varies with frequency:

$$Z_{\text{in}} = Z(z)\big|_{z=-L} = Z_0 \frac{1 + \Gamma e^{j2\beta z}}{1 - \Gamma e^{j2\beta z}}\bigg|_{z=-L} = Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L}$$

Thus, at certain frequencies, when Z_{in} is minimum, $||\tilde{V}_{\text{in}}||$ will also be minimum. To find frequencies at which this minimum will occur, we know that now $\Gamma = \frac{jX_c(\omega) - Z_0}{jX_c(\omega) + Z_0} = ||\Gamma|| e^{j\phi_\Gamma}$. Here, we note that for a pure capacitive load $||\Gamma|| = 1$ and phase is a function of frequency, $\phi_\Gamma(\omega) = 2\arctan(\frac{10^8 j}{50\omega})$ for $C = 0.01\mu F$, as opposed to the short circuited load where $\phi_\Gamma = \pi$. Thus, the voltage minimums will occur at $2\beta z + \phi_\Gamma(\omega) = (2k+1)\pi, k \in \mathbb{Z}$, where $f_{\min}$ will be skewed beacuse of the frequency-depedance of reflection coefficient's phase

9 Notes

All images taken during the lab were post-processed in a batch using a custom script that bit-wise inverts the pixels and binarizes the resulting image based on a custom threshold. No adjustments or modifications were made to the readings, for which the oscilloscope's measurements are also shown alongside the waveforms. All scripts and related work can be found at `github.com/pranshumalik14/ece320-labs`.