ECE320: Fields and Waves

Lab 3 Report: Design of a Double Stub Matching Network

PRA106

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1 Introduction

This laboratory session was focused on investigating the input impedance and reflection coefficient at the load, and making transformations to the load impedance using a double stub network such that we are able to match the input impedance to the characteristic impedance of the transmission line. We also measured the voltage standing wave ratio (VSWR) for the same matching network to characterize its bandwidth. Figure 1 shows the schematic for a double-stub tuner and its equivalent circuit. Our theoretical work and measurements are based on the unknown "orange" labelled load given to us.

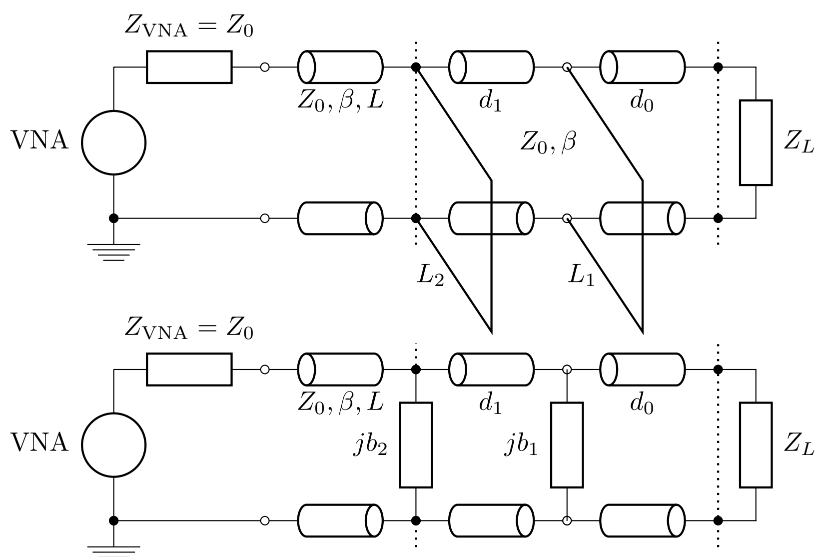


Figure 1: A double-stub matching network with $d_0 = 3.4$ cm and $d_1 = 3.8$ cm

2 Measurement of the Unknown Load Impedance

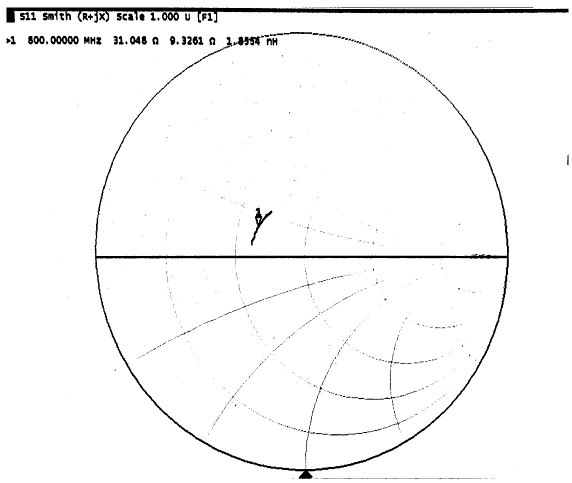
The unknown load impedance was measured over a frequency range of [300MHz, 1.3GHz], with and without de-embedding, using the VNA on a Smith Chart shown in Figure 2 (a) and (b). The values of the load impedance and admittance with its normalized form taking $Z_0 = 50\Omega$ are given in Table 1.

Parameter	**Value**
Z_L	$31.08 + 9.32j \quad [\Omega]$
Y_L	$0.029 + 0.089j \quad [\Omega^{-1}]$
$Z_{\text{L,N}}$	$0.622 + 0.186j$
$Y_{\text{L,N}}$	$1.476 - 0.441j$

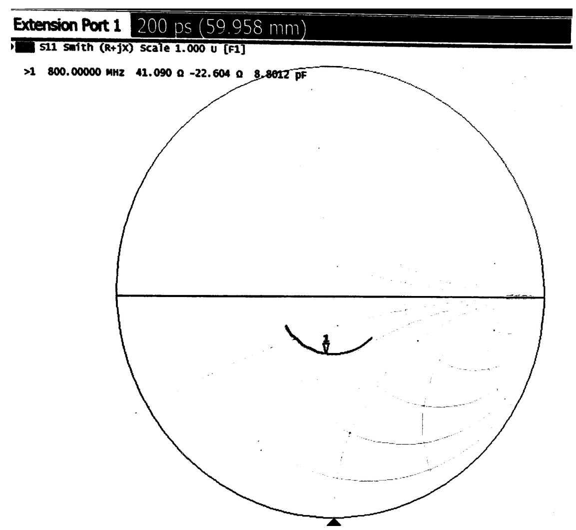
Table 1: Measured impedance and admittance of the unknown load

Since there is an adapter and extension ports between the ends of the BNC cable and the load, they act as additional sections of transmission lines which cause a rotation of the impedance on the Smith chart, and we can undo this rotation by rotating (de-embedding) the measurement anti-clockwise (towards the load) by the same extension length. The lab manual asks to de-embed our load by $T = 0.2$ ns at $f = 800$Mhz, which corresponds to a distance of $\frac{c \cdot T}{c/f} = (0.2 \cdot 10^{-9} \cdot 800 \cdot 10^{6})\lambda = 0.16\lambda \approx 6$ cm, where λ is the wavelength of a wave propagating at the speed of light, c, with frequency f.

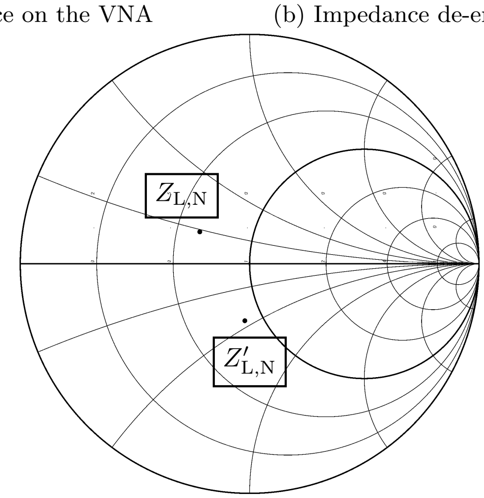
To de-embed the load we rotate $Z_{\mathrm{L,N}}$ anti-clockwise (towards the load) by 0.16λ on the SWR circle, shown in Figure 2 (c), and we get $Z'_{\mathrm{L,N}} = 0.85 - 0.45j$, which is coincidently very near $Y_{\mathrm{A,N}}$ (see section 3), and corresponds to $Z'_L = 42.5 - 22.5j[\Omega]$, closely matching the result shown by the VNA in Figure 2 (b).



(a) Measured impedance on the VNA



(b) Impedance de-embedded by 0.2 ns on the VNA



(c) Theoretically de-embedded impedance

Figure 2: Measured and de-embedded imepedance of the unknown load

3 Designing a Double-stub Matching Network

To start the matching process, we need to transform Z_L by moving it towoards the generator by d_0 to reach the first stub. The impedance at that loaction corresponds to rotating $Z_{\mathrm{L,N}}$ by 0.09067λ clockwise on the SWR circle leading to $Z_{\mathrm{A,N}} = 0.92 + 0.49j$, and $Y_{\mathrm{A,N}} = 0.85 - 0.45j$ which was obtained by rotating $Z_{\mathrm{A,N}}$ by $\frac{\lambda}{4}$ (or π radians) on the same (SWR) circle. From here, we followed the double-stub matching procedure provided in the laboratory manual and the Smith charts and our calculations for the lengths of both stubs are attached with this report.

Fundamental Solution	**Length in wavelengths**, λ
$\hat{L}_1$	0.324λ
$\hat{L}_2$	0.210λ
$\hat{L}'_1$	0.449λ
$\hat{L}'_2$	0.445λ

(a) Calculated matching stub length pairs in wavelengths

Fundamental Solution	**Length in** cm
L_1	12.15
L_2	7.88
L'_1	16.84
L'_2	16.69

(b) Calculated matching stub length pairs

The general solution in wavelengths for $n \in \mathbb{N} + \{0\}$ is,

$$\hat{L}_1 = 0.324\lambda + \frac{n\lambda}{2} \quad , \quad \hat{L}_2 = 0.210\lambda + \frac{n\lambda}{2}$$
$$\hat{L}'_1 = 0.449\lambda + \frac{n\lambda}{2} \quad , \quad \hat{L}'_2 = 0.445\lambda + \frac{n\lambda}{2}$$

and the general solution in centimeters at $f = 800$ MHz can be written as,

$$L_1 = 12.15 + 18.75 \cdot n \quad , \quad L_2 = 7.88 + 18.75 \cdot n$$
$$L'_1 = 16.84 + 18.75 \cdot n \quad , \quad L'_2 = 16.69 + 18.75 \cdot n$$

4 Experimental Measurement and Verification

Our measurements were in close agreement with our theoretical calculations and can be found in Table 3. The measured values were within an error of ± 2 cm which is mainly due to the accumulation of error over a large number of operations done by hand on the Smith chart, and also due to the error in recording lengths of the stubs in the matching network. The Smith chart plots measured on the VNA can be found in Figure 3.

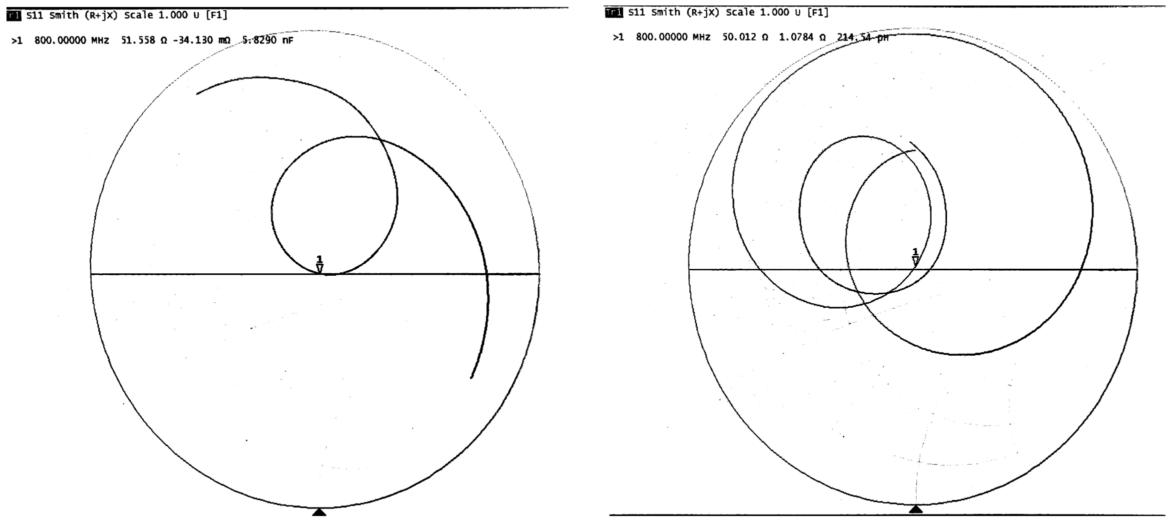
Fundamental Solution	**Length in** cm
L_1	10.5
L_2	5.5
L'_1	17.1
L'_2	16.8

Table 3: Experimentally measured stub lengths

5 The Standing Wave Ratio and Bandwidth Calculations

Since the input impedenace of the matching network changes with frequency, it is important to see the signal transfer characteristics of the matching network to determine the bandwidth of frequencies that "pass" over to the load without much distortion and reduction in amplitude. We use the voltage standing wave ratio (VSWR) to define bandwidth, BW, as,

$$\text{VSWR}(\omega) = \frac{1 + ||\Gamma_L(\omega)||}{1 - ||\Gamma_L(\omega)||} \quad \text{and} \quad \text{BW} = \left\{\omega \in \mathbb{R}^+ \middle| \text{VSWR}(\omega) \leq 2\right\}$$



(a) Matched load using the shorter length stubs

(b) Matched load using the longer length stubs

Figure 3: Measuring input impedance over our frequency range for short and long stub length solutions

Here we note that the time-averaged power transmitted to load is given by $\langle P\rangle = \frac{||V_0^+||^2}{2Z_0}(1 - ||\Gamma_L||^2) = \langle P_{\text{max}}\rangle \cdot (1 - ||\Gamma_L||^2)$, where $\langle P_{\text{max}}\rangle$ is the maximum time-averaged power that can be delivered to the load which happens for a matched load when $\Gamma_L = 0$. At $\text{VSWR} = 2, \Gamma_L = \frac{1}{3} \implies \langle P\rangle \approx 0.89 \cdot \langle P_{\text{max}}\rangle$, and thus our definition considers bandwith to be the range of frequencies where the power is delivered to the load at approximately 90% efficiency. Also note that at bandwidth limiting frequencies, $20\log(\Gamma_L) = \Gamma_{\text{L, dB}} \approx -10$ dB, which suggests that $\Gamma_L \geq -10$ dB are acceptable values for the refelection coefficient such that the delivered signal's power is not degraded as much. We have also recorded the VSWR versus frequency plots on the VNA for both fundamental solutions, shown in Figure 4, and the bandwidths can be found in Table 4.

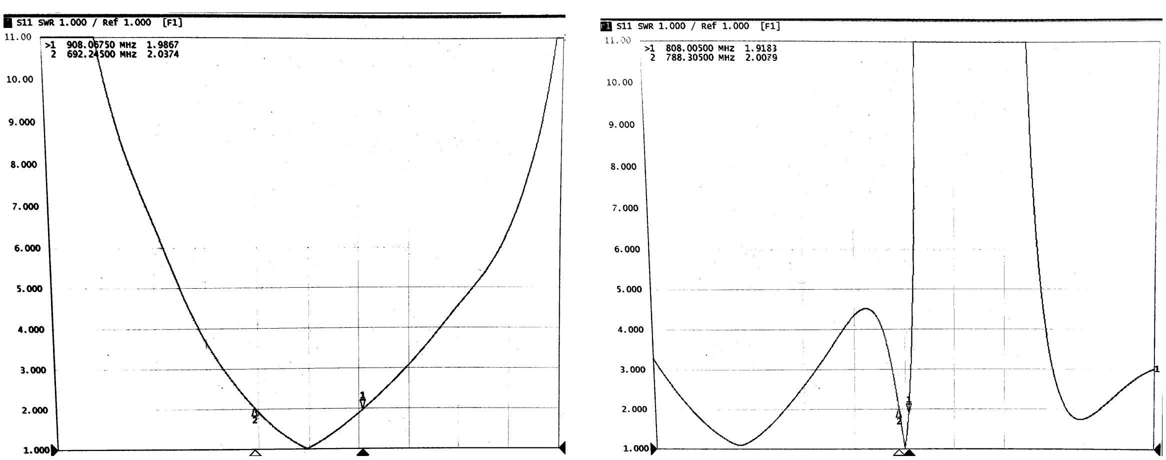
Stub Pair	**Bandwidth in** MHz
Short	$[692, 908]$
Long	$[788, 808]$

Table 4: Bandwidth of the short and long matching networks

We note that shorter stubs give better performance in terms of bandwidth since they have less variation of electrical parameters, specifcially input impedance, with frequency compared to the longer stubs. This is beacuse the normalized input admittance of short circuited stubs is $Y_{\text{in,N}} = -j\cot(\beta L)$, where $\beta = \omega\sqrt{L'C'}$, where L is the length of the stub and L', C' are distributed inductance and capacitance of the stub, respectively. Thus, for a fixed frequency, ω, if L is increased, $Y_{\text{in,N}}$ and Γ_L will vary faster, which can also be seen in Figure 3. As a result, for longer stubs we get a smaller window of frequencies we can use to fit the bandwidth constraint, whereas for shorter stubs the bandwidth is larger.

6 Notes

All images taken during the lab were post-processed in a batch using a custom script that bit-wise inverts the pixels and binarizes the resulting image based on a custom threshold. No adjustments or modifications were made to the readings, for which the measurements on the VNA are also shown alongside the waveforms. All scripts and related work can be found at `github.com/pranshumalik14/ece320-labs`.



(a) VSWR vs frequency for the shorter length stubs (b) VSWR vs frequency for the longer length stubs

Figure 4: Variation of VSWR over our frequency range