

ECE421 - Winter 2022

Homework Problems - Tutorial #8

Theme: Gaussian Mixture Model

Due: March 20, 2022 11:59 PM

Question 1 (Problem 5 - Final 2018)

Consider an already-trained Gaussian mixture model (GMM) that is trained to fit data on student performance in a class. The GMM uses two components ($K = 2$) as the class consists of two categories of students: undergraduate students (category 1) and graduate students (category 2). The learned parameters of the GMM are as follows.

- The weights of the two categories are $w_1 = 2/3$ (undergraduate) and $w_2 = 1/3$ (graduate).
 - The distribution that fits scores in category 1 is $\mathcal{N}(x; 70, 10^2)$.
 - The distribution that fits scores in category 2 is $\mathcal{N}(x; 80, 5^2)$.
- (a) According to the GMM, what is the probability that an arbitrarily selected student scores greater than 80%? That is, compute $\Pr[X \geq 80]$, where X denotes the score of the student. (In your computation, use the approximation that for zero-mean σ^2 -variance random variable X , i.e., $X \sim \mathcal{N}(x; 0, \sigma^2)$, $\Pr[|X| \leq \sigma] = 2/3$).
- (b) If a particular student has a score greater than 80, what is the probability that the student is from category 1? That is, compute $\Pr[\text{class} = 1 | X \geq 80]$. (Use the same approximation as in the previous part.)