Lab4

Saturday, March 12, 2022 8:15 PM

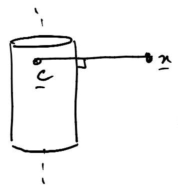
Prelab

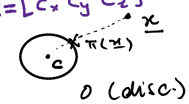
1. Cylindrical $\mathcal{O}$: We first project $\underline{x}$ onto $z = x_z$ plane.

Then, we a problem treat this as in $\mathbb{R}^2$.

$$c = [\underbrace{c_x \; c_y}_{\text{constant}} \; \underbrace{c_z}_{x_z}]^T$$

$\underline{x} \; \hookrightarrow = \begin{bmatrix} x_x \\ x_y \\ x_z \end{bmatrix}$.





Thus, C

Now, define $c = [c_x, c_y]$., $\underline{x}' = [x_x, x_y]$

$$\pi(x) = \begin{cases} \begin{bmatrix} \underline{c} + \frac{R}{\|\underline{x}' - \underline{c}\|}(\underline{x}' - \underline{c}) \\ x_z \end{bmatrix}, & \|\underline{x}' - \underline{c}\| \geq R. \\ \begin{bmatrix} \underline{x}' \\ x_z \end{bmatrix} & \|x - c\| \leq R \end{cases}$$

$$O_i(q) - b = O_i(q) - \pi(x) = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} \left[\begin{matrix} c_x \\ c_y \end{matrix}\right] - \frac{R}{\left\|\left[\begin{matrix} x_i - c_x \\ y_i - c_y \end{matrix}\right]\right\|}\left(\left[\begin{matrix} x_i - c_x \\ y_i - c_y \end{matrix}\right]\right) \\ z_i \end{bmatrix} \quad \text{if } \|O_i(q) - c\| \geq R$$

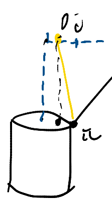
$$= \begin{bmatrix} (x_i - c_x)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ (y_i - c_y)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ 0 \end{bmatrix}$$

$$O_i(q) - b = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = 0 \quad \text{if } \|O_i(q) - c\| \leq R$$

$$\boxed{\begin{aligned} &\text{if } z_i < h \\ &O_i(q) - b = \begin{cases} \begin{bmatrix} (x_i - c_x)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ (y_i - c_y)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ 0 \end{bmatrix} & \text{if } \|O_i(q) - c\| \geq R \\ 0 & \text{if } \|O_i(q) - c\| \leq R \end{cases} \\ &\|O_i(q) - b\| = \begin{cases} \left\|\begin{bmatrix} (x_i - c_x)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ (y_i - c_y)\left(1 - \frac{R}{\sqrt{(x_i - c_x)^2 + (y_i - c_y)^2}}\right) \\ 0 \end{bmatrix}\right\| & \text{if } \|O_i(q) - c\| \geq R \\ 0 & \text{if } \|O_i(q) - c\| \leq R \end{cases} \end{aligned}}$$

if $z_i > h$

then



$$\pi(x) = \begin{cases} \begin{bmatrix} \underline{c} + \frac{R}{\|\underline{x}'-\underline{c}\|}(\underline{x}'-\underline{c}) \\ h \end{bmatrix}, & \|\underline{x}'-\underline{c}\| > R. \\ \begin{bmatrix} x' \\ h \end{bmatrix}, & \|x'-c\| \le R \quad (\text{planar case}) \end{cases}$$

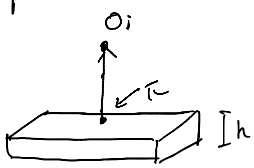
if $z_i > h$ & $\|x'-c\| > R$

$$O_i(q) - b = O_i(q) - \pi(x) = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} \begin{bmatrix} c_x \\ c_y \end{bmatrix} - \frac{R}{\left\| \begin{bmatrix} x_i \\ y_i \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \end{bmatrix} \right\|} \left(\begin{bmatrix} x_i \\ y_i \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \end{bmatrix} \right) \\ h \end{bmatrix}$$

if $z_i > h$ & $\|x'-c\| < R$

$$O_i(q) - b = \begin{bmatrix} 0 \\ 0 \\ z_i - h \end{bmatrix}, \quad \|O_i(q) - b\| = z_i - h$$

2. planar



$$\pi(x) = \begin{bmatrix} x_i \\ y_i \\ h \end{bmatrix}$$

$$O_i(q) - b = \begin{cases} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \text{If } z_i \le h \\ \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ z_i - h \end{bmatrix} & \text{If } z_i > h \end{cases}$$