Lab 2

Friday, 25 February 2022 9:43 AM

Lab Prep

DH parameters of link i : a_i : length of common normal measured from ℓ_{i-1} to ℓ_i with sign determined by the orientation of x_i

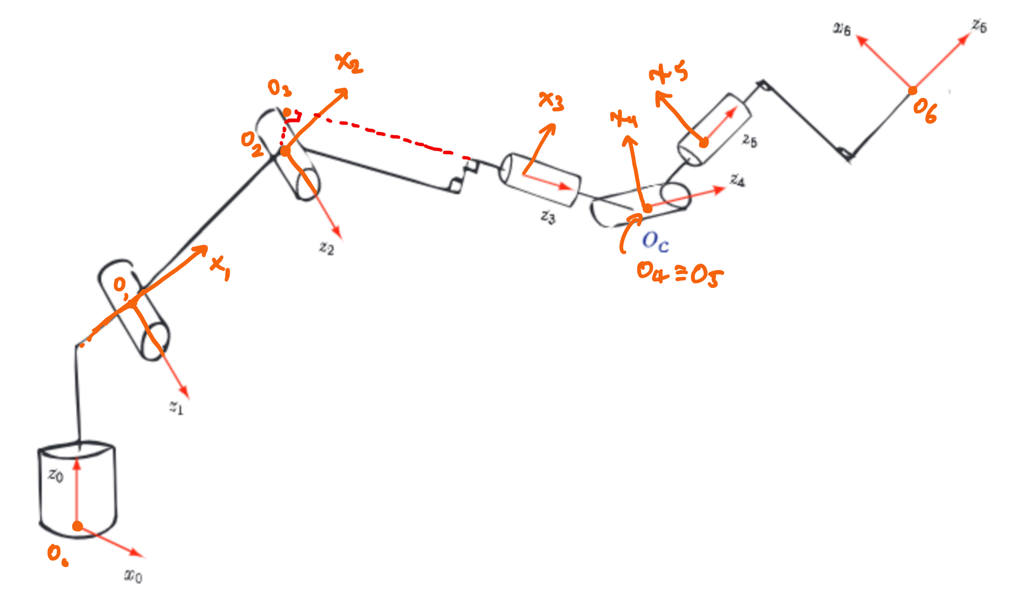
$i = 1, \dots, n$

d_i : distance from O_{i-1} to O_i along the z_{i-1} axis, with sign determined by the orientation of z_{i-1}.

θ_i : angle from x_{i-1} to x_i measured as a right-handed rotation about axis z_{i-1}

α_i : angle from z_{i-1} to z_i measured as a right-handed rotation about axis x_i

① DH



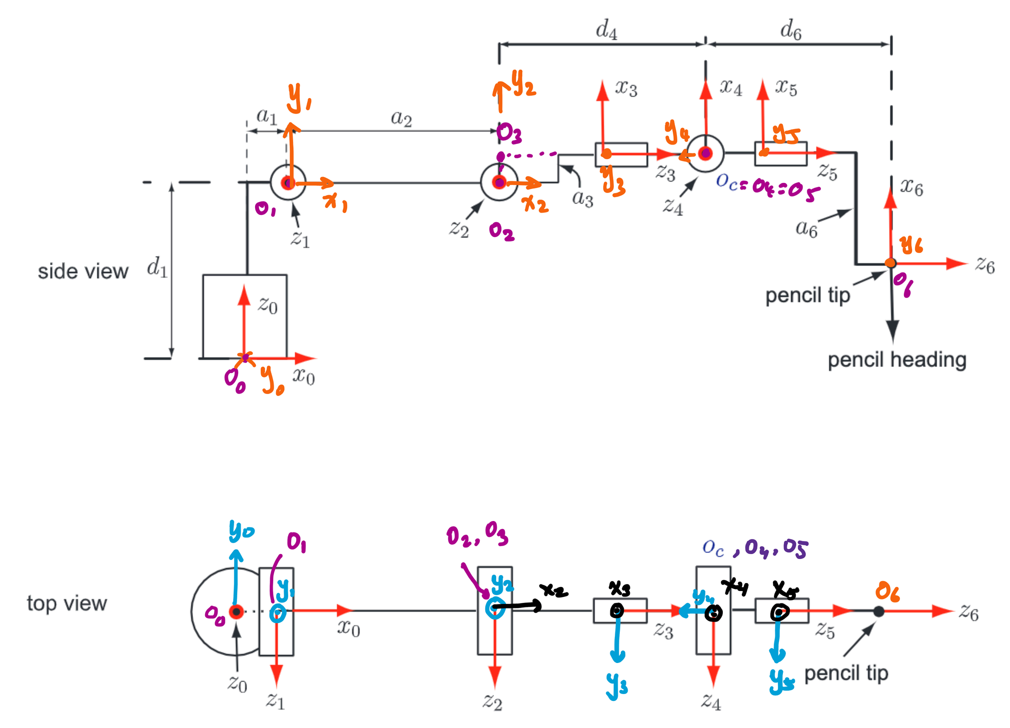


Figure 2: Schematic diagrams illustrating the KUKA kinematic parameters

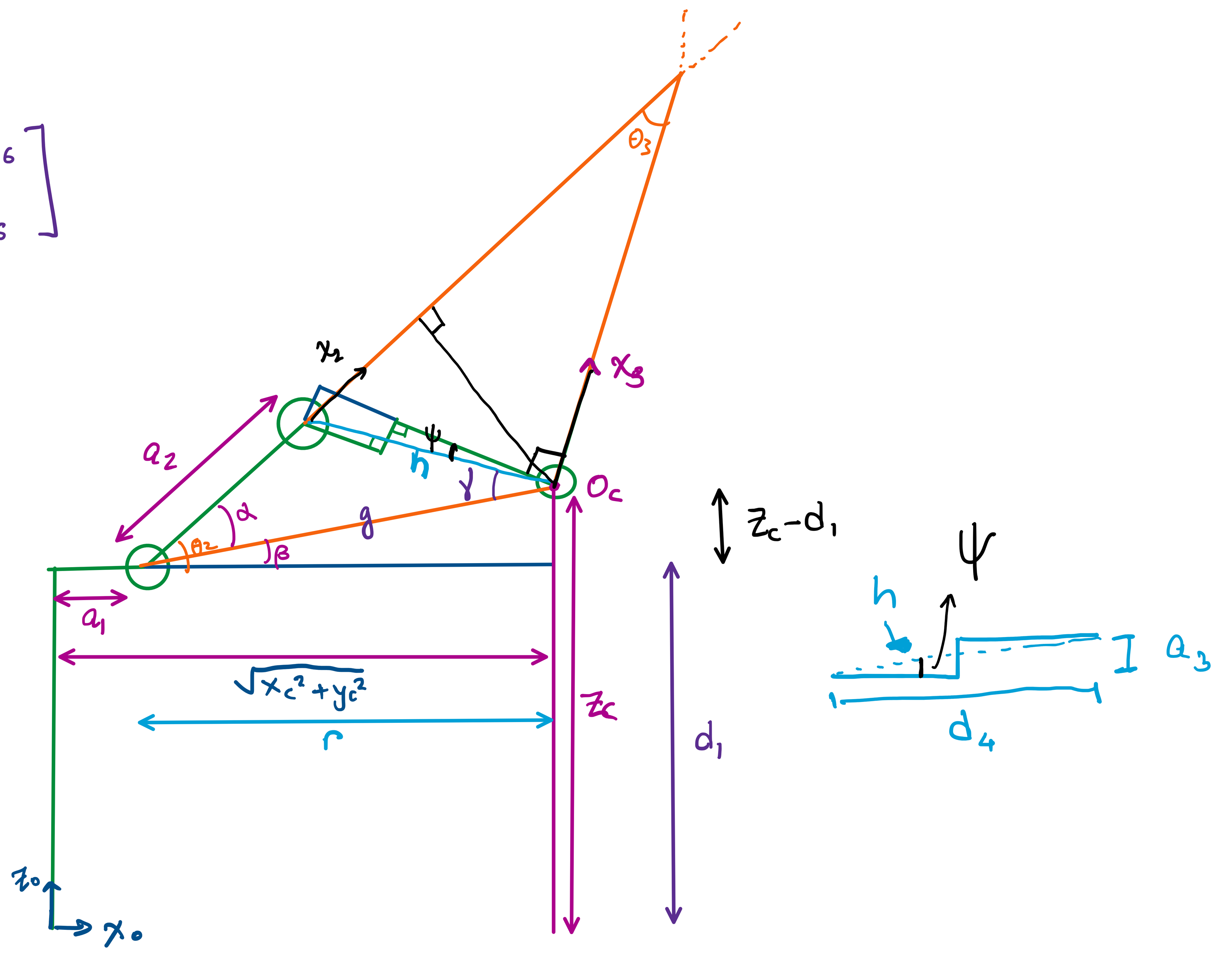
D.H Table : [mm]

Link #	a_i	α_i	d_i	θ_i
1	25	$\pi/2$	400	θ_1
2	315	0	0	θ_2
3	35	$\pi/2$	0	θ_3
4	0	$-\pi/2$	365	θ_4
5	0	$\pi/2$	0	θ_5
6	-296.33	0	161.44	θ_6

② Inverse Kinematics

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = O_d^0 - R_d \begin{bmatrix} -a_6 \\ 0 \\ d_6 \end{bmatrix}$$

$$\boxed{\theta_1 = \text{arctan2}(y_c, x_c)}$$



$r = \sqrt{x_c^2 + y_c^2} - a_1$

$h = \sqrt{d_4^2 + a_3^2}$; $g = \sqrt{r^2 + (z_c - d_1)^2}$

$D := \dfrac{h^2 - a_2^2 - g^2}{-2 \cdot a_2 \cdot g} = \cos(\alpha) \Rightarrow \boxed{\alpha = \text{atan2}\left(\pm\sqrt{1-D^2},\ D\right)}$ $\quad$ [atan(Y, X)]

$\beta = \text{atan2}(z_c - d_1,\ r)$

$$\boxed{\theta_2 = \alpha + \beta}$$

Using cosine laws for θ_3:

$a_2^2 = g^2 + h^2 - 2gh\cos(\gamma)$

$\cos(\gamma) = \dfrac{a_2^2 - g^2 - h^2}{-2 \cdot g \cdot h} =: D' \Rightarrow \gamma = \text{atan2}\left(\pm\sqrt{1-D'^2},\ D'\right)$

$\psi = \text{atan2}(a_3,\ d_4)$

$$\boxed{\theta_3 = \frac{\pi}{2} - \alpha - \gamma - \psi}$$

Now to solve the eqn $R_6^3(q_4, q_5, q_6) = M$, we can use the formulas for the ZYZ Euler angles

: if $m_{13}^2 + m_{23}^2 \neq 0$:

$$\boxed{\begin{array}{lll} \theta_4^* = \text{atan2}(m_{23}, m_{13}) & \text{or} & \theta_4^* = \text{atan2}(-m_{23}, -m_{13}) \\ \theta_5^* = \text{atan2}\left(\sqrt{1-m_{33}^2}, m_{33}\right) & \text{or} & \theta_5^* = \text{atan2}\left(-\sqrt{1-m_{33}^2}, m_{33}\right) \\ \theta_6^* = \text{atan2}(m_{32}, -m_{31}) & \text{or} & \theta_6^* = \text{atan2}(-m_{32}, m_{31}) \end{array}}$$

If $m_{13}^2 + m_{23}^2 = 0$:

$$\boxed{\theta_5 = 0 \text{ or } 2\pi n}$$