ECE537 Random Processes

Programming Assignment 3

Question 1 - A random walk process $Z(t)$ is constructed from

$$Z(n) = \sum_{i=1}^{n} X_i$$

where X_i are independent random variables with the probability mass function $P(X_i = +1) = p$ and $P(X_i = -1) = 1 - p$.

(a) Assume $n = 1, \ldots, 500$, and $p = 0.5$. Generate 50 independent traces of the random process $Z(n)$ and plot them all together in the same figure as a function of n.

(b) Estimate the expected value of $Z(n)$ for all $n = 1, \ldots, 500$ and plot as a function of n. Explain your observation.

(c) Estimate the variance of $Z(n)$ for all $n = 1, \ldots, 500$ and plot as a function of n. Is $Z(n)$ a stationary process? Explain.

(d) Repeat part (a) with $p = 0.6$. Explain what you see in the plot.

(e) Repeat part (a) with $p = 0.4$. Explain what you see in the plot.

Question 2 - Let $N(t)$ be defined as

$$N(t) = \sum_{i=1}^{\infty} I(X_1 + \ldots + X_i < t)$$

where X_i's are independent exponentially distributed random variables with parameter λ, the identifier function $I(\zeta)$ is defined as

$$I(\zeta) = \begin{cases} 1 & \text{if the predicate } \zeta \text{ is true} \\ 0 & \text{if the predicate } \zeta \text{ is false} \end{cases}$$

(a) Let $\lambda = 2$, and $t = 5$. Generate 5 independent traces of $N(t)$ and plot them all together in the same figure.

(b) Generate 1000 independent traces of $N(t)$ for $t = 5$ and $\lambda = 2$. Plot the histogram of $N(t)$. In the same plot, draw the PDF of a Poisson distribution with mean λt, as

$$P[\widehat{N}(t) = k] = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \qquad k = 0, 1, \ldots$$

Compare the histogram with the Poisson distribution and explain your observations.

(c) Estimate the expected value of $N(t)$ from the generated data and compare with the theoretical value.

(d) Estimate the variance of $N(t)$ from the generated data and compare with the theoretical value.