ECE537 Random Processes

Programming Assignment 5

Question 1 - Let N_n be a zero-mean Gaussian i.i.d. random process with unit variance.

(a) generate 600 samples of N_n, and use it as the input to the AR filter

$$X_n = 1.5X_{n-1} - 0.8X_{n-2} + N_n.$$

Throw away the first 88 samples of X_n and only use the last 512 samples. Plot N_n and X_n using the **subplot** command of MATLAB and comment.

(b) Find the autocorrelation function of N_n and X_n for ± 50 lags and plot them using subplot. Find the power of each signal. Comment on the plots.

(c) Create 10 independent traces of N_n and X_n. Find the periodogram at the output of the filter. Plot the periodogram. On the same figure, use the **freqz** command of MATLAB to also plot the actual power spectral density of X_n. Compare the results.

(d) Find the power spectral density using the FFT of the autocorrelation function. Compare the result to that obtained by the periodogram.

(e) Use the Yule-Walker equations to estimate the best 3-element, 4-element, and 5-element predictor for X_n and compare them to the AR filter that generated X_n. Comment on your findings.

Question 2 - Consider a desired signal generated by the AR filter

$$S_n = 0.2S_{n-1} - 0.8S_{n-2} + N_n$$

where N_n is the unit-variance zero-mean additive white Gaussian noise. Such as Question 1, generate 600 samples of S_n and throw away the first 88 samples and use the remaining 512 samples for the rest of this question.

(a) Generate the observation vector $X_n = S_n + W_n$, where W_n is a unit-variance zero-mean additive white Gaussian noise uncorrelated from N_n.

(b) Find the sample autocorrelation function $R_X(k)$ for ± 256 samples and plot it. Is it an even function? Why?

(c) Find the cross-correlation function R_{SX} and plot it. Examine whether or not the cross-correlation function is even. Discuss your findings.

(d) Use the Wiener-Hopf equations to find the 7-element optimal filter

$$Y_n = \sum_{k=0}^{6} h_k X_{n-k}$$

as a linear estimator of the desired signal S_n.

(e) Compute the estimation error using the variance of $(S_n - Y_n)$ and compare to the value obtained from

$$E[e_n^2] = R_S(0) - \sum_{k=0}^{6} h_k R_{SX}(k)$$