## ECE537 Random Processes

## Programming Assignment 2

Question 1 - Let  $X = (X_1, X_2)$  consist of two Gaussian distributed random variables  $X_1$  and  $X_2$ .

- (a) Assume  $E(X_1) = E(X_2) = 0$  and  $\sigma^2(X_1) = \sigma^2(X_2) = 1$ . Let  $\rho = 0$ . Take N = 100 samples from the distribution of X and plot the scattergram (samples shown in the  $X_1$ - $X_2$  plane). Explain your observation.
- (b) Let  $E(X_1) = 1$  and  $E(X_2) = 2$  and  $\sigma^2(X_1) = 2$  and  $\sigma^2(X_2) = 1$ . For  $\rho = -1, -0.5, 0, 0.5, 1$  take N = 100 samples from the distribution of X and plot the scattergram (five different plots). Explain your observation.
- (c) Use the samples of X in part (b) with  $\rho = 0.5$  to find the marginal PDF of  $X_1$  and plot as a histogram. Estimate the expected value and the variance of the marginal PDF. Explain your observation.

Question 2 - Let  $X_i$  be a uniform random variable distributed in the interval [0,1]. Define

$$S_n = \sum_{i=1}^n X_i,$$

where  $X_i$ 's are independent.

- (a) For n = 1, ..., 1000, plot  $\frac{1}{n}S_n$  as a function of n. Does  $\frac{1}{n}S_n$  converge? What is the limit?
- (b) Let n = 100. Define

$$Z_{100} = \frac{S_{100} - 50}{\sqrt{100/12}}$$

Note that  $Z_{100}$  is a random variable. Generate 1000 samples of  $Z_{100}$  and plot the histogram of  $Z_n$ .

- (c) Overlap a Gaussian pdf with the histogram in part (b). Explain your observation.
- (d) Use your simulated data to estimate the expected value and the variance of  $Z_n$  and compare with the theoretical values.