ECE367: Matrix Algebra and Optimization
Reference Notes

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Contents

Note: This document is not meant to be a comprehensive treatment of the material or contain the complete course notes. The primary purpose of this document is to summarise key concepts along with some analysis and proofs, that can aid in revising the course or serve as a quick reference to a particular topic. Any suggestions or corrections are appreciated and can be sent directly to `pranshu.malik@mail.utoronto.ca`

1 Optimization Problems in Standard Form

A typical optimization problem formulation is as follows:

Definition 1.1

For decision variables $x_1, x_2, \ldots, x_n$

2 Mathematical Preliminaries

2.1 Sets

2.2 Functions

2.3 Fields

3 Vector Spaces

3.1 Subspaces

3.2 Affine Sets

4 Norms and Inner Products

4.1 Norms

4.2 Inner Products

5 Projection on Subspaces

5.1 Orthogonal Projection on One-dimensional Subspaces

5.2 Induced Norm

5.3 Metrics and Induced Metrics

5.4 Projection on Subspaces

5.5 Computing Projections with General Basis

5.6 Computing Projections with Orthonormal Basis

5.7 The Gram-Schmidt Procedure

5.8 Projection on the Orthogonal Complement

5.9 Projection on Affine Sets, including Lines and Hyperplanes

6 Linear Transformations

6.1 The Matrix of a Linear Transformation

6.2 The Fundamental Theorem of Linear Algebra

7 Operators

7.1 Invariant Subspaces

7.2 Upper Triangular Matrices for Operators on Complex Vector Spaces

7.2.1 Theorems for Operators

7.2.2 Change of Basis

7.2.3 Theorems for Matrices

7.3 Diagonal Matrices for Operators

8 Operators on Inner-Product Spaces

8.1 Upper Triangular Matrices

8.2 Linear Functionals and Adjoints

8.3 Operators on Inner Product Spaces