

Solving Generalized Grouping Problems in Cellular Manufacturing Systems Using a Network Flow Model

Md. Kutub Uddin^a, Md. Saiful Islam^b, Md Abrar Jahin^c, Md. Saiful Islam Seam^b, and M. F. Mridha^{d,*}

^aDepartment of Mechanical Engineering, Khulna University of Engineering & Technology, Khulna 9203, Bangladesh

^bDepartment of Industrial Engineering and Management, Khulna University of Engineering & Technology, Khulna 9203, Bangladesh

^cPhysics and Biology Unit, Okinawa Institute of Science and Technology Graduate University, Okinawa 904-0412, Japan

^dDepartment of Computer Science, American International University–Bangladesh, Dhaka 1229, Bangladesh

*Corresponding Author

Email addresses: kutubuddin@me.kuet.ac.bd (Md. Kutub Uddin), saifuliem@iem.kuet.ac.bd (Md. Saiful Islam), abrar.jahin.2652@gmail.com (Md Abrar Jahin), seam1911017@stud.kuet.ac.bd (Md. Saiful Islam Siam), firoz.mridha@aiub.edu (M. F. Mridha)

Solving Generalized Grouping Problems in Cellular Manufacturing Systems Using a Network Flow Model

Md. Kutub Uddin^a, Md. Saiful Islam^b, Md Abrar Jahin^c, Md. Saiful Islam Seam^b, and M. F. Mridha^{d,†}

^a*Department of Mechanical Engineering, Khulna University of Engineering & Technology, Khulna 9203, Bangladesh*

^b*Department of Industrial Engineering and Management, Khulna University of Engineering & Technology, Khulna 9203, Bangladesh*

^c*Physics and Biology Unit, Okinawa Institute of Science and Technology Graduate University, Okinawa 904-0412, Japan*

^d*Department of Computer Science, American International University–Bangladesh, Dhaka 1229, Bangladesh*

Abstract

This paper focuses on the generalized grouping problem in the context of cellular manufacturing systems (CMS), where parts may have more than one process route. A process route lists the machines corresponding to each operation of the part. Inspired by the extensive and widespread use of network flow algorithms, this research formulates the process route family formation for generalized grouping as a unit capacity minimum cost network flow model. The objective is to minimize dissimilarity (based on the machines required) among the process routes within a family. The proposed model optimally solves the process route family formation problem without pre-specifying the number of part families to be formed. The process route of family formation is the first stage in a hierarchical procedure. For the second stage (machine cell formation), two procedures, a quadratic assignment programming (QAP) formulation, and a heuristic procedure, are proposed. The QAP simultaneously assigns process route families and machines to a pre-specified number of cells in such a way that total machine utilization is maximized. The heuristic procedure for machine cell formation is hierarchical in nature. Computational results for some test problems show that the QAP and the heuristic procedure yield the same results.

Keywords: cellular manufacturing systems; group technology; generalized grouping problems; process route; network flow model

1. Introduction

The world of manufacturing is undergoing rapid growth due to increasing demand for all kinds of products. This growth is accompanied by radical changes, transforming the face of manufacturing altogether. Cellular manufacturing systems (CMS), based on the philosophy of group technology (GT), have been recognized as a technological innovation in job shop or batch-type production systems to gain economic advantages similar to those of mass production systems. CMS is capable of producing small to medium-sized batches of a large variety of parts in a flow-line manner. The concept of CMS involves dividing the entire production system into smaller autonomous subsystems

[†]Corresponding Author

Email addresses: kutubuddin@me.kuet.ac.bd (Md. Kutub Uddin), saifuliem@iem.kuet.ac.bd (Md. Saiful Islam), abrar.jahin.2652@gmail.com (Md Abrar Jahin), seam1911017@stud.kuet.ac.bd (Md. Saiful Islam Seam), firoz.mridha@aiub.edu (M. F. Mridha)

to improve shop floor control, material handling, tooling, and scheduling. This approach leads to decreased setup times, in-process inventories, and throughput times. To achieve this decomposition, it is necessary to identify subsets of parts with similar processing or design requirements so that each subset of parts can be processed by a single subsystem. In the context of GT, each such subsystem is termed a machine cell, and each subset of parts is referred to as a part family. The identification of machine cells and part families is known as machine-component grouping and is the first step in the design of a CMS. This partitioning of machines and parts in a factory is achieved through the application of GT. The application of GT does not depend on the degree of automation in a factory, and hence, GT can be applied at any level of automation, from manual production to fully automated systems.

The grouping problem varies depending on factors like the availability of information and the level of decision-making. In a simple grouping problem, each part has only one process plan, and each operation of the process plan can be performed on only one machine. As a result, each part has only one process route. Here, the terms ‘process plan’ and ‘process route’ have different meanings. A process plan lists the operations required to complete a part, while a process route lists the machines to which the various operations are assigned. On the other hand, in a generalized grouping problem, each part has more than one process route. The objective of the grouping problem, in this case, is to identify a single process route for each part and determine the process route families and machine cells. The goal is to ensure that each machine cell is capable of handling at least one process route family.

The generalized grouping problem is a significant area of research within the field of network flow models, where the primary objective is to optimally assign groups under certain constraints. Starting from the initial work on grouping based on graph theory by Rajagopalan and Batra (1975), numerous graph-theoretic methodologies have been developed by researchers over the past few decades. These approaches have been thoroughly studied in the most recent literature, including applications in social networks and computer science (Majeed and Rauf, 2020). This problem is particularly important in contexts such as telecommunications, transportation, and logistics, where efficient resource allocation is critical. The graph-theory-based methodologies can be classified into two main categories: graph decomposition (or partition) and network flow.

The graph decomposition method constructs a graph from the given parts' process routes. The nodes in these graphs correspond to either machines, parts, or both. The arcs represent the associations between nodes and carry a weight representing some kind of dissimilarity or interaction between the nodes. Depending on the node type, three types of graphs can be constructed: machine graph, part graph, and machine-and-part graph (Werner, 2020). These are described in Table 1.

Table 1: Types of graphs for grouping.

Type of graph	Nodes	Arcs and weightage on arcs
Machine graph	Machines	Arcs represent the relationships between machines, indicating that there is movement of one or more parts between the machines, constituting the pair of nodes. The weight on the arcs represents one of the following: dissimilarity between the two machines, the total number of part movements between the two machines, or similar metrics.
Part graph	Parts	Arcs representing the relationship between parts indicate that one or more machines are common to the pair of parts constituting the nodes. The weight on the arcs represents some kind of dissimilarity or similarity between the two parts.

Machine-part graph	Machines and parts	Arcs indicate the relationship between a machine and a part, showing that a certain part utilizes a specific machine. The weight on an arc may indicate how long the part takes to process on the machine, or it could simply be assigned a value of 1 to show that the part uses the machine. No arcs exist between two parts or between two machines.
--------------------	--------------------	---

To obtain machine cells and part families, graphs are successively decomposed into sub-graphs, ensuring minimal interaction among them, or resulting in disconnected sub-graphs. Numerous studies have shown that there is no single strategy that works most effectively for obtaining part families and machine cells. For example, some research (Uddin and Shanker, 2002; Kulkarni, 2021) discussed the use of genetic algorithm (GA) to solve generalized grouping problems in CMS, demonstrating improvements in optimizing these groupings without pre-specifying the number of groups. In the network flow method, a network is constructed by creating two nodes for each machine or part. The nodes are connected by directed arcs, and the weight on the arcs represents some kind of cost, indicating dissimilarity between the parts or machines. These formulations solve the grouping problem optimally without pre-specifying the number of groups. A pioneering work by Lee and Garcia-Diaz (1993) formulated the machine grouping problem as a capacitated circulation network for a simple grouping case.

Network flow algorithms have been successfully applied to various production planning problems, including the simple grouping problem. However, no work has been reported on solving the generalized grouping problem using network flow algorithms. In this paper, a methodology is presented that employs a hierarchical approach to solve the generalized grouping problem when process routes are given for each part. In the first stage, process route families are formed using a unit capacity minimum cost network flow model. In the second stage, a heuristic procedure and a quadratic assignment model are presented for machine cell formation and the assignment of process route families to these cells.

The remainder of the paper is structured as follows. Section 2 reviews the relevant research on grouping problems and network flow models. The problem environment considered for the model creation is explained in Section 3. The solution strategy is described in Section 4. The proposed network model for the development of process route (or part) families is presented in Section 5. Section 6 describes the development of machine cells and the assignment of part families to these cells. The application of the proposed model to a few numerical issues is demonstrated in Section 7. Finally, Section 8 presents the conclusions and recommendations for future research.

2. Literature review

Extensive studies have been conducted based on the framework of network flow models, particularly in routing (Yan and Wong, 2009; Cova and Johnson, 2003), planning and scheduling (Rietz *et al.*, 2016), production optimization (Lerlertpakdee *et al.*, 2014), supply chain management and intelligent transportation (Hsu and Wallace, 2007; Rudi *et al.*, 2016), inventory management and distribution (Hovav and Tsadikovich, 2015), social network analysis (Gomez *et al.*, 2013), resilience assessment (Goldbeck *et al.*, 2019; Yin *et al.*, 2022), infrastructural interdependencies (Holden *et al.*, 2013), water supply planning (Hsu and Cheng, 2002), strategic mine planning (Topal and Ramazan, 2012), and energy systems (Quelhas *et al.*, 2007; Quelhas and McCalley, 2007). A number of graph theory-based studies on grouping problems have been published in the literature, where machines or parts are represented by the vertices of graphs, and appropriately determined similarity coefficients are defined by the weights of the arcs. The grouping problem was first solved using graph theory by Rajagopalan and Batra (1975). Using the route cards of the parts, they developed a machine graph in which the vertices represented the machines and the edges represented Jaccard's similarity coefficients. Cells were identified as groups of vertices that were highly connected to one another.

For studying the optimization of cell formation, various algorithmic techniques have been used. A hybrid GA is one example used in CMS for machine-part grouping. The formation of machine groups and part families is a complex operation in cellular manufacturing, combining the fast production rate of flow lines with the flexibility of job shops. The integration of local search heuristics with GA, as proposed by Tariq *et al.* (2009), has demonstrated encouraging results in quickly obtaining optimal solutions, indicating the effectiveness of hybrid techniques in addressing real-world manufacturing challenges. Mak *et al.* (2000) proposed a GA to optimize cell development in manufacturing systems. Their approach provided practical insights into algorithmic techniques and strategies for optimization by dynamically adjusting parameters to maximize efficiency and reduce handling costs. Similarly, Salehi *et al.* (2010) offered complex techniques and frameworks for performance evaluation, analyzing the implementation of GA to address cell formation problems. These methods have been further developed by hybrid GA proposed by James *et al.* (2007), demonstrating efficient optimization in practical situations.

The grouping problem was first solved by Lee and Garcia-Diaz (1993) as a circulation network flow problem. The idea is to group machines into cells so that each family of parts can be processed in a single machine cell. To achieve this, a directed graph for the machines is created, and its network flow problem is solved. The solution to the network flow problem consists of one complete loop and several sub-loops, each loop corresponding to a machine cell. Compared to the p -median model, this approach is reported to have excellent potential for providing computationally efficient and optimal solutions for simple grouping problems. Cheng *et al.* (2019) developed generalized grouping strategies in coded caching, and Garg and Arora (2018) developed fuzzy soft-set decision-making frameworks, demonstrating the flexibility of grouping techniques. These methods integrate expert preferences and efficiently manage uncertainty, offering powerful tools for decision-making. The use of grouping concepts in decision theory and computational problems is further illustrated by the grouping functions proposed by Bustince *et al.* (2011) and the generalized group testing procedures proposed by Malinovsky (2019). These approaches enhance efficiency and scalability across various fields. This thorough analysis highlights the usefulness and potential of network flow models in addressing a range of grouping problems.

It is evident that network flow models have been successfully applied across various domains, including social networks, decision-making, industry, and infrastructure, to ensure computational efficiency and practical application. These models provide robust frameworks for enhancing resilience, improving decision-making, and optimizing complex systems. Consequently, they offer valuable insights for the development of a network flow model for the generalized grouping problem.

3. Problem environment

For the proposed network flow model, a generalized grouping problem is considered where each part has more than one process route. The input to the problem is a two-dimensional (0-1) matrix showing the requirement of machines for various operations of each process route. The entry value in the matrix is 1 if an operation of a process route requires a particular machine and 0 otherwise. The grouping problem involves selecting one process route for each part from the given alternatives and grouping them into process route families, each of which can be processed by a single machine cell. Several process route families can be processed by a machine cell. The objective is to minimize the distance (dissimilarity) among the process routes within a family. The proposed model is expected to result in machine cells with the minimum possible inter-cell movements.

We consider a problem situation where there are K parts, and each part has a set of distinct process routes. Different process routes for a part generally require different sets of machines to complete the operations. In total, combining all the parts, there are N process routes. There are M machines of different types, and a maximum of C cells are to be created. The objective is to minimize the inter-cell movements of parts and to maximize machine utilization.

4. The solution approach

The proposed model solves the problem in a hierarchical manner. First, process route families are formed based on minimum dissimilarity among the members of each family using a network flow model. In the second stage, machine cells are formed, and process route families are assigned to these cells simultaneously, with the objective of maximizing machine utilization. For machine cell formation and process route family assignment, a heuristic and a QAP formulation are proposed.

The mechanism of process route family formation is as follows. First of all, for each part, there is a source or supply node (k_s) and a sink or demand node (k_d). For each process route, there are two nodes, a and b , and a directed arc (a, b). From each part source node, arcs go to the corresponding process route node a . All arcs coming to a part sink node come from the corresponding process route node b . It is obvious that only one process route must be selected for each part. Let part k have $TPR(k)$ process routes. So, $TPR(k)-1$ routes are eliminated from consideration by including the corresponding arcs in directed paths from source to sink ($k_s \rightarrow i_a \rightarrow i_b \rightarrow k_d$). The flow value on all process route arcs is constrained to 1. The supply at the source and demand at the sink are fixed at $TPR(k)-1$ for all parts. This results in exactly one process route, with no supply from the source for each part. However, since all process routes are constrained to have flow value 1, even the process route arcs getting no supply from the source node have to have the flow conditions satisfied. This is possible only if such unsatisfied arcs form cycles within themselves, which are nothing but process route families. Further, to encourage better formation of process route cycles, the arc costs from one process route to another are taken as the corresponding dissimilarity values between them, the total of which is to be minimized over the whole network. In this way, the network flow model tries to group similar process routes together. The calculations of dissimilarity values are described in Sub-section 5.1.

At the stage of machine cell formation and route family assignment, the machine requirements for the various process route families are evaluated. Machine cells are created based on these requirements. Some process route families may be assigned to a cell if their machine requirements are fully met by that cell. If the requirements do not match completely, the family will be assigned to a cell where most of its requirements are satisfied, which may result in inter-cell movements.

The following notations are used for the development of the network flow model for process route family formation and machine cell formation model.

Indices

i, j = process route

k = part

m = machine

c = machine cell

r = process route family

Parameters

K = total number of parts

$PR(k)$ = set of process routes for part k

$TPR(k)$ = total number of process routes for part k

$= |PR(k)|$

A = set of process routes over all parts

N = total number of process routes over all parts

$$= |A|$$

$$= \sum_{k=1}^K TPR(k)$$

M = total number of machines

C = maximum number of machine cells that can be formed

R = total number of process route families

max_c = maximum number of machines that can be assigned to a cell c

$\omega(i, j)$ = cost of unit flow on arc (i, j)

$$a_{im} = \begin{cases} 1 & \text{if process route } i \text{ requires machine } m \text{ for} \\ & \text{processing (irrespective of the number of} \\ & \text{times it is required)} \\ 0 & \text{otherwise} \end{cases}$$

u_{mr} = usage factor for machine m in process route family r indicating the number of processes routes in a process route family r using machine m

$$= \sum_{i \in r} a_{im}$$

Decision variables

$$Z_{mc} = \begin{cases} 1 & \text{if machine } m \text{ is assigned to cell } c \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{rc} = \begin{cases} 1 & \text{if process route family } r \text{ is assigned to machine cell } c \\ 0 & \text{otherwise} \end{cases}$$

$f(x, y)$ = flow from node x to node y

5. Proposed network flow model for route family formation

The proposed network flow model consists of three steps. The first step involves computing the pairwise dissimilarity between process routes, as described in subsection 5.1. The second step constructs the unit capacity minimum cost flow network, detailed in subsection 5.2. The third step identifies the process route families by solving the unit capacity minimum cost network flow problem, as described in subsection 5.3.

5.1 Computation of dissimilarities (or distances)

The pairwise dissimilarities between process routes are computed as follows:

The dissimilarity value between a pair of process routes is measured as the number of machines that are not common to them. Let d_{ij} be the dissimilarity between process routes i and j . The value of d_{ij} (dissimilarity) is an indicator

showing the degree of dissimilarity between process routes i and j . The value gets smaller as the two process routes require more and more common machines for processing. The process route-machine matrix $A = [a_{im}]$ ($i = 1, \dots, N$; $m = 1, \dots, M$) is used to calculate the elements d_{ij} of the dissimilarity matrix $D = [d_{ij}]$. The Hamming metric, to calculate the distance between a pair of binary row vectors, is used to calculate the dissimilarity between two process routes of different parts as shown below:

$$d_{ij} = \sum_{m=1}^M \delta(a_{im}, a_{jm}) \quad \forall i \in PR(k_1), j \in PR(k_2), k_1, k_2 = 1, \dots, K, k_1 \neq k_2$$

$$\text{where } \delta(a_{im}, a_{jm}) = \begin{cases} 1 & \text{if } a_{im} \neq a_{jm} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

For example, consider process routes 1 and 3, where the number of machines in route 1 is 7 and the number of machines for route 3 is 5, out of which 3 machines are common. Then the dissimilarity between the pair (1, 3) is taken as (total number of machines – total number of common machines) = $(5+7) - (2 \times 3) = 6$. To illustrate the calculation of the total dissimilarity value for a process route family, let us consider a process route family having process routes 1, 2, 3, and 4. Now, one cyclic order for this family could be (1, 2), (2, 3), (3, 4), and (4, 1). The dissimilarity value for this cyclic order will be $d_{12} + d_{23} + d_{34} + d_{41}$, where d_{ij} is the dissimilarity value between processes routes i and j . There will be $4!$ such cyclic orders ($n!$ cyclic orders for n process routes), and some of them will have the same dissimilarity value because of the symmetric nature of the dissimilarity coefficients, $d_{ij} = d_{ji}$. The dissimilarity value for a process route family is taken as the minimum of total dissimilarity values amongst all cyclic orders.

5.2 Construction of the network

The construction of the unit capacity minimum cost flow network is based on the work of Lee and Garcia-Diaz (1993), who proposed the network flow formulation for a simple grouping problem. The network is constructed using the $N \times N$ dissimilarity matrix D obtained in subsection 5.1. The creation of nodes and arcs connecting the nodes and the assignment of weightage on arcs are carried out as follows:

5.2.1 Creation of nodes

Creation of supply nodes

Create K supply nodes designated as $I_s, 2_s, \dots, K_s$, and one node corresponding to each part (the subscript s denotes that it is a supply node). The supply capacity of the node k_s is taken as $TPR(k)-1$ units, i.e., one less than the total number of process routes of part k ; $k = 1, \dots, K$.

Creation of demand nodes

Create K demand nodes designated as $I_d, 2_d, \dots, K_d$, again one node corresponding to each part (the subscript d denotes that it is a demand node). The demand requirement of node k_d is again taken as $TPR(k)-1$ units; $k = 1, \dots, K$.

Creation of intermediate nodes

Corresponding to each process route i , create a pair of nodes i_a and i_b . The two sets of nodes are designated as $I_a, 2_a, \dots, N_a$ and $I_b, 2_b, \dots, N_b$; $i = 1, \dots, N$. These $2N$ nodes are used as transshipment nodes, i.e., these nodes do not possess any supply capacity, nor do they have any demand.

5.2.2 Creation of arcs

Create arcs and assign a capacity-cost triplet $[U, L, C]$ to each arc to indicate an upper bound on its flow (U), a lower bound on its flow (L), and the per unit cost of flow (C) respectively, according to the following rules:

Rule I:

For each supply node k_s , $k = 1, \dots, K$ and the corresponding transshipment node i_a , $i \in PR(k)$; create supply arcs directed from k_s to i_a , signifying that part k uses process route i . For example, arc $(1_s, 1_a)$ indicates that part 1 uses process route 1 for its processing. Assign capacity-cost triplet $[U, L, C] = [1, 0, 0]$ to all arcs (k_s, i_a) .

Rule II:

For each node pair i_a and i_b , $i = 1, \dots, N$; create transshipment arcs directed from i_a to i_b . The two nodes i_a and i_b , connected by a directed arc, represent a process route. For example, arc $(1_a, 1_b)$ represents process route 1, arc $(2_a, 2_b)$ represents process route 2, and so on. Assign capacity-cost triplet $[U, L, C] = [1, 1, 0]$ to all arcs (i_a, i_b) .

Rule III:

For each node i_b , $i \in PR(k)$, $k = 1, \dots, K$ and the corresponding node k_d , create demand arcs directed from i_b to k_d . The directed arc (i_b, k_d) signifies that part k uses process route i ; for example, the directed arc $(1_b, 1_d)$ indicates that part 1 uses process route 1 for its processing. Assign capacity-cost triplet $[U, L, C] = [1, 0, 0]$ to all arcs (i_b, k_d) .

Rule IV:

For each node i_b and j_a , $i \in PR(k)$, $j \notin PR(k)$, $k = 1, \dots, K$; create relational arcs directed from i_b to j_a and assign capacity-cost triplet $[U, L, C] = [1, 0, d_{ij}]$. The cost of shipping one unit of flow from node i_b to node j_a is taken as the dissimilarity value between process routes i and j .

The network constructed for the problem environment described in section 4, according to the steps described above, is shown in Figure 1.

Observations

1. The upper and the lower bound on flow on transshipment arcs (i_a, i_b) , $i = 1, \dots, N$ is 1 unit. Thus, in accordance with the flow conservation, a node i_b can supply 1 unit of flow to either destination node j_a or to demand node k_d , $i \in PR(k)$, $j \notin PR(k)$, $k = 1, \dots, K$.
2. If a part k has $TPR(k) = n$ process routes, n units of flow are needed to satisfy the flow condition on the n transshipment arcs, say, (i_a, i_b) , $((i+1)_a, (i+1)_b)$, ... $((i+n-1)_a, (i+n-1)_b)$. However, according to the network construction, the supply node k_s can supply only $TPR(k) - 1 = n - 1$ units of flow, that is, to only $n - 1$ of the transshipment arcs. The remaining 1 unit of flow required on the unsatisfied arc can be satisfied by including it either in a path with satisfied process route arcs of some other parts or in a cycle with unsatisfied process route arcs of some other parts. In general, such a cycle or path may involve more than one part and its process routes. The relational arcs (i_b, j_a) (i and j must not belong to the same part) are used to create these cycles or paths, and the weightage assigned to each such arc (i_b, j_a) is the dissimilarity value between process routes i and j , namely, d_{ij} . In the framework of the proposed minimum cost network flow problem, the relational arc that has the minimum dissimilarity (i.e., cost of flow) will be chosen for creating the cycle or path, and this is true for all other parts. Therefore, any optimal (or feasible) solution of the proposed minimum cost network flow problem will contain paths (from supply nodes to demand nodes) with flow value 1 or both paths and cycles with flow value 1. Two types of paths could be there. The first kind, direct path $(k_s \rightarrow i_a \rightarrow i_b \rightarrow k_d)$, includes only one process route, whereas the second kind, indirect path, includes more than one process route. The direct path does not contain any relational arcs, but the indirect path does.

3. The cost of flow on all arcs is zero except on relational arcs (i_b, j_a) , $i \in PR(k)$, $j \notin PR(k)$, $k = 1, \dots, K$; whose cost of flow is the dissimilarity value between process routes i and j , i.e., d_{ij} . Therefore, minimizing the total cost of flow in the network is equivalent to minimizing the total cost of flow on the relational arcs (i_b, j_a) with flow value 1, which, in turn, is equivalent to minimizing the total dissimilarity values among the process routes involved in creating the paths and cycles.

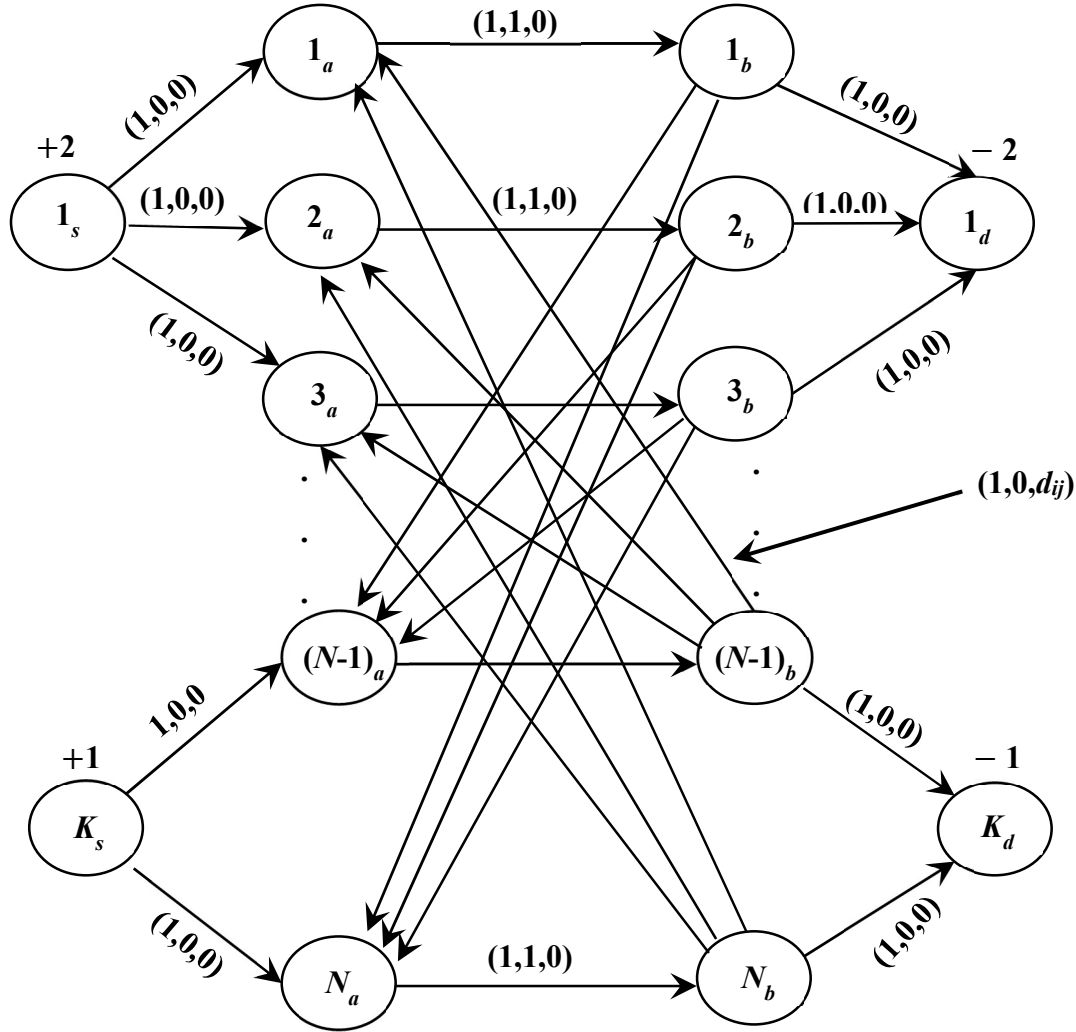


Figure 1: Network flow representation of process route family formation.

5.2.3 Mathematical formulation

Objective function

The objective function is to minimize the total cost of flow in the network and consists of four components: cost of flow on supply arcs, cost of flow on transshipment arcs, cost of flow on relational arcs, and cost of flow on demand arcs. Assuming the linearity, these costs are computed as the product of flow rate $f(.,.)$ and cost of unit flow $\omega(.,.)$. The four cost components can be written as follows:

$$\text{Cost of flow on supply arcs} = \sum_{k=1}^K \sum_{i \in PR(k)} f(k_s, i_a) \cdot \omega(k_s, i_a)$$

$$\text{Cost of flow on transshipment arcs} = \sum_{i=1}^N f(i_a, i_b) \cdot \omega(i_a, i_b)$$

$$\text{Cost of flow on relational arcs} = \sum_{k=1}^K \sum_{i \in PR(k)} \sum_{j \notin PR(k)} f(i_b, j_a) \cdot \omega(i_b, j_a)$$

$$\text{Cost of flow on demand arcs} = \sum_{k=1}^K \sum_{i \in PR(k)} f(i_b, k_d) \cdot \omega(i_b, k_d)$$

The cost of flow on supply arcs, transshipment arcs, and demand arcs will always be zero because their cost of flow is zero according to *Rule I*, *Rule II*, and *Rule IV*, respectively. The cost of flow for the relational arcs, $\omega(i_b, j_a)$, is taken as d_{ij} , the dissimilarity value between the two process routes i and j according to *Rule III*. Therefore, the objective function is reduced to:

$$\min \sum_{k=1}^K \sum_{i \in PR(k)} \sum_{j \notin PR(k)} d_{ij} \cdot f(i_b, j_a) \quad (2)$$

Constraints

1. Flow conservation at source nodes k_s

Flow emanating from these nodes must be equal to their supply capacities:

$$\sum_{i \in PR(k)} f(k_s, i_a) = TPR(k) - 1 \quad k = 1, \dots, K \quad (3)$$

2. Flow conservation at nodes i_a

Since these are transshipment nodes, therefore, the incoming flow must be equal to the outgoing flow:

$$f(k_s, i_a) + \sum_{j=1}^N f(j_b, i_a) = 1 \quad k=1, \dots, K; i \in PR(k); j \notin PR(k) \quad (4)$$

3. Flow conservation at nodes i_b

These nodes are also transshipment nodes. Therefore, the incoming flow must be equal to the outgoing flow:

$$\sum_{j=1}^N f(i_b, j_a) + f(i_b, k_d) = 1 \quad k=1, \dots, K; i \in PR(k); j \notin PR(k) \quad (5)$$

4. Flow conservation at sink nodes k_d

These are the sink nodes. Therefore, the flow incoming to these nodes must be equal to their requirement:

$$\sum_{i \in PR(k)} f(i_b, k_d) = - (TPR(k) - 1) \quad k=1, \dots, K \quad (6)$$

5. Side constraints - only one process route arc on any path from supply to demand node

This constraint is used to ensure that the path from a supply node to a demand node with flow value 1 must include only one process route and thus will eliminate from the network any indirect paths, i.e., the paths from supply nodes to demand nodes with flow value 1 which include more than one process routes. It can be written as:

$$f(k_s, i_a) = f(i_b, k_d) \quad \forall i \in PR(k); k = 1, \dots, K \quad (7)$$

6. The integrality constraints for flow variables

$$f(k_s, i_a) = 0 \text{ or } 1 \quad \forall i \in PR(k); k = 1, \dots, K$$

$$f(i_b, k_d) = 0 \text{ or } 1 \quad \forall i \in PR(k); k = 1, \dots, K$$

$$f(i_b, j_a) = 0 \text{ or } 1 \quad \forall i \in PR(k); j \notin PR(k); k = 1, \dots, K \quad (8)$$

The model formulated in equations 2 to 8 is a 0-1 integer linear programming problem, which can be solved using any procedure designed for 0-1 integer linear programming problems. Except for constraint 7, all other constraints and the objective function conform to the standard network structure and can be solved using any minimum-cost network flow algorithm. However, due to constraint 7, a specialized network flow algorithm is required to solve this problem. In this work, CPLEX, a general mixed integer optimization package (version 22.1.0.0), is used to solve the problem. The output consists of binary (0-1) flow values for all arcs.

5.3 Process route (part) family identification

As stated in subsection 5.2 (observation 2), the network solution will contain cycles and paths (indirect paths) to satisfy the flow conditions on unsatisfied arcs. However, due to side constraint 7, the solution cannot contain any indirect paths, i.e., paths that include more than one process route and contain relational arcs. Therefore, the only way to satisfy the flow conditions on unsatisfied arcs is by creating cycles. The cost of flow on these arcs is the dissimilarity value between the two process routes connected by a relational arc. Since the objective function minimizes the cost of flow on relational arcs and only cycles can contain relational arcs, minimizing the total cost of flow on relational arcs is equivalent to minimizing the cost of flow on cycles. In other words, this means minimizing the dissimilarity values (which are the costs of flow for relational arcs) between the process routes included in the cycles. Generally, a cycle contains at least two process routes (each from a different part). These cycles will represent process route families in the context of GT and can be identified from the solution of the mathematical model described in subsection 5.2 by isolating the relational arcs with a flow value of 1. This identification of cycles is illustrated in Figure 2. The network shown in this figure represents a grouping problem with 4 machines and 4 parts (with a total of 8 process routes). To illustrate the identification of cycles, let the solution of the model have flow value 1 on relational arcs $(2_b, 3_a)$, $(3_b, 2_a)$, $(6_b, 7_a)$, and $(7_b, 6_a)$. From these, two cycles can be identified. The first cycle consists of the following four arcs:

$$\text{Cycle 1: } f(2_b, 3_a) = f(3_a, 3_b) = f(3_b, 2_a) = f(2_a, 2_b) = 1$$

The second cycle also consists of four arcs:

$$\text{Cycle 2: } f(6_b, 7_a) = f(7_a, 7_b) = f(7_b, 6_a) = f(6_a, 6_b) = 1$$

From these two cycles, two process route families can be identified:

1. Process route family 1 consists of process routes 1 and 4, corresponding to parts 1 and 2, respectively.
2. Process route family 2 consists of process routes 6 and 7, corresponding to parts 3 and 4, respectively.

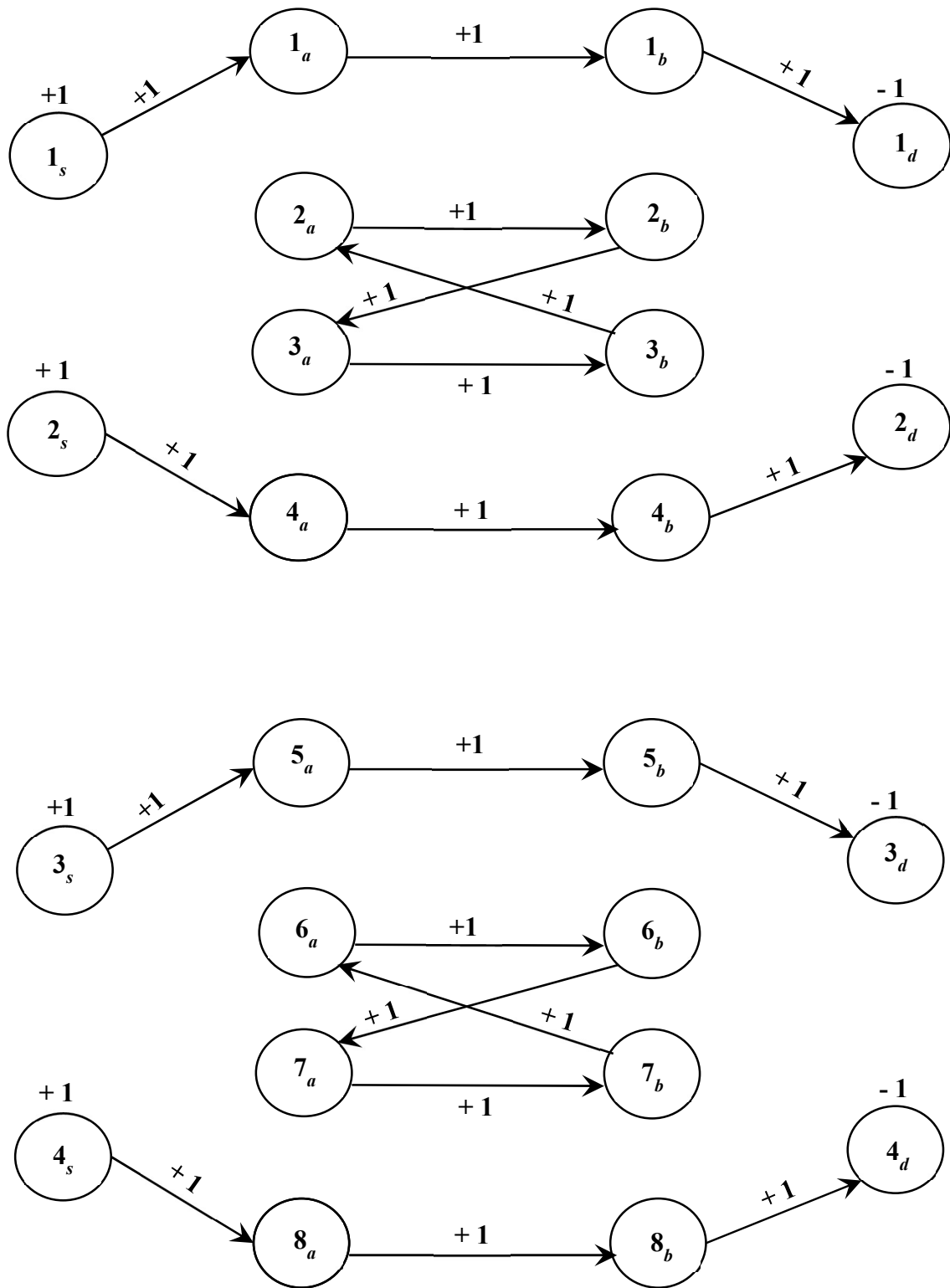


Figure 2: Network showing the paths and cycles for process route family formation.

6. Machine cell formation

For machine cell formation, we propose a QAP formulation and a heuristic procedure. The descriptions are given in the following subsections.

6.1 The QAP formulation

The QAP formulation assigns part families and machines to cells simultaneously. The inputs to the model are the maximum number of cells that can be formed, the maximum number of machines that can be assigned to a cell, and the machine usage factors (i.e., the number of process routes in a particular family using a particular machine) for all process route (part) families. The objective of the formulation is to maximize machine utilization. The output of the model is the assignment of part families and machines to cells.

Objective function

The objective of the model is to maximize machine utilization and can be written as:

$$\max \sum_{c=1}^C \sum_{r=1}^R \sum_{m=1}^M u_{mr} Z_{mc} Y_{rc} \quad (9)$$

Constraints

1. Each machine is assigned to only one cell

$$\sum_{c=1}^C Z_{mc} = 1 \quad m = 1, \dots, M \quad (10)$$

2. Each process route family is assigned to only one cell

$$\sum_{c=1}^C Y_{rc} = 1 \quad r = 1, \dots, R \quad (11)$$

3. Maximum number of machines that can be assigned to a cell

$$\sum_{m=1}^M Z_{mc} \leq \max_c \quad c = 1, \dots, C \quad (12)$$

6.2 The heuristic procedure

The objective of the heuristic procedure is also to maximize machine utilization. It follows a hierarchical approach and consists of three steps. The first step combines process route families wherever possible. The second step assigns machines to the process route families. The third step merges process route families and machine cells wherever feasible within the system constraints. The inputs to the procedure are the maximum number of machines that can be assigned to a cell, the process route families, and the machines they require. The details of the heuristic are as follows:

Step-1. Combining the process route families

1. Take a process route family r
2. Check if the machines used by this process route family r are a subset or superset of the machines used by any other process route family. If the answer is yes, merge the two process route families; otherwise, go to 3.
3. Repeat 1 and 2 for all route families till no merging is possible.

Step-2. Assigning the machines to process route families

1. Compute for each machine m the usage factor u_{mr} for each process route family r
2. Assign machine m to a process route family r where u_{mr} is maximum, the ties being broken arbitrarily.
3. Repeat 2 for all machines.

Step-3. Merging the process route families and machine cells

Merge two process route families and the corresponding machine cells if there is some inter-cell movement between them, but only if the merging does not violate the cell size limit. Stop when no more merging is possible.

7. Numerical illustrations

To test the effectiveness of the present formulation, two data sets are chosen from the literature on generalized grouping problems: one without exceptional elements (i.e., where disjoint groups exist) and the other with exceptional elements (i.e., where disjoint groups do not exist). The number of exceptional elements is used as a critical criterion for measuring the effectiveness of cell formation, as it is widely used by researchers in generalized grouping theory.

7.1 Example problem I (without exceptional elements)

The first problem shown in Table 2 is the incidence matrix. There are 5 parts with a total of 11 process routes and 4 machines. In the process route family formation stage, two cycles are identified.

$$\text{Cycle 1: } f(2_a, 2_b) = f(2_b, 7_a) = f(7_a, 7_b) = f(7_b, 2_a) = 1$$

$$\text{Cycle 2: } f(5_b, 11_a) = f(11_a, 11_b) = f(11_b, 9_a) = f(9_a, 9_b) = f(9_b, 5_a) = f(5_a, 5_b) = 1$$

The two cycles are shown in Figures 3 and 4, respectively. From the two cycles, two process route families can be identified.

Table 2: Example problem I.

Part (k)	Process route for part k ($i \in PR(k)$)	Process route SI. No. ($i \in A$)	Machine No. (m)			
			1	2	3	4
1	1	1	0	0	1	1
	2	2	0	1	0	1
	3	3	1	1	0	0
2	1	4	0	1	1	0
	2	5	1	0	1	0
3	1	6	1	0	0	1
	2	7	0	1	0	1
4	1	8	1	0	0	1
	2	9	1	0	1	0
5	1	10	0	0	1	1
	2	11	1	0	0	0

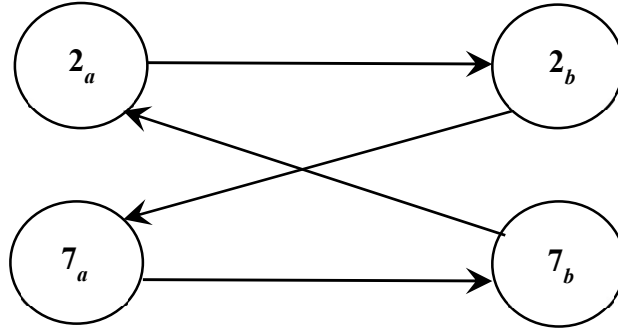


Figure 3: Cycle 1 for part family 1 of example problem I.

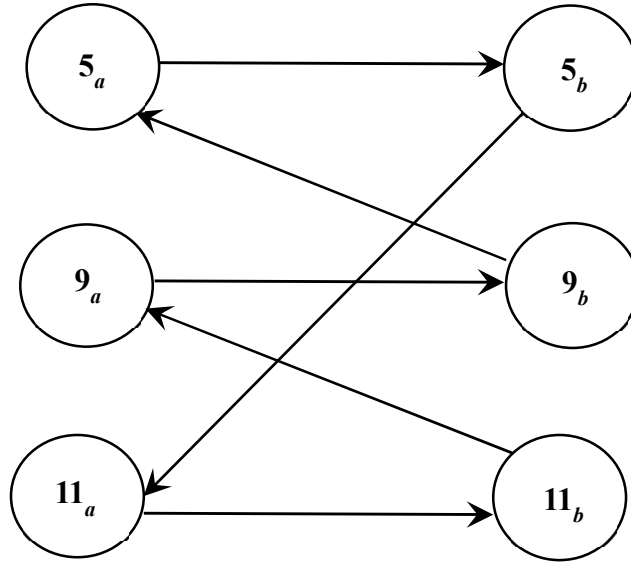


Figure 4: Cycle 2 for part family 2 of example problem I.

1. Process route family 1 consists of process routes 2 and 7, corresponding to parts 1 and 3, respectively.
2. Process route family 2 consists of process routes 5, 9, and 11, corresponding to parts 2, 4, and 5, respectively.

In the machine cell formation stage, the machine utilizations u_{mr} for each process route family r are calculated and shown in Table 3.

Table 3: Machine utilization u_{mr} for example problem I.

Process route family (r) → Machine (m) ↓	1	2
1	0	3
2	2	0
3	0	2
4	2	0

From Table 3, it is obvious that none of the machines are utilized by more than one process route family, and hence, two machine cells are easily identified as:

1. Machine cell 1 consisting of machines 2 and 4
2. Machine cell 2 consisting of machines 1 and 3

After rearranging the matrix according to the results obtained from the process route family formation and the machine cell formation stage, the resultant matrix is shown in Table 4.

Table 4: Solution to example problem I in block diagonal form.

Part (k)	Process route for part k ($i \in PR(k)$)	Process route SI. No. ($i \in A$)	Machine No. (m)			
			1	3	2	4
2	2	5	1	1	0	0
4	2	9	1	1	0	0
5	2	11	1	0	0	0
1	2	2	0	0	1	1
3	2	7	0	0	1	1

7.2 Example problem II (with exceptional elements)

The second problem is given in Table 5. This problem has 20 parts having a total of 51 process routes and 20 machines. In the process route family formation stage, seven cycles are identified as:

Table 5: Example problem II.[illegible]

Table 5: Example problem II (continued).

Part (k)	Process route set ($PR(k)$)	Process route number (i)	Machine No. (m)																			
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
14	1	39	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	1	0	0
	2	40	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0
	3	41	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1
15	1	42	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0
	2	43	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0
	3	44	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
16	1	45	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0
	2	46	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0
	3	47	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1
17	1	48	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0
18	1	49	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
19	1	50	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
20	1	51	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

Cycle 1: $f(2_a, 2_b) = f(2_b, 11_a) = f(11_a, 11_b) = f(11_b, 6_a) = f(6_a, 6_b) = f(6_b, 2_a) = 1$

Cycle 2: $f(30_a, 30_b) = f(30_b, 33_a) = f(33_a, 33_b) = f(33_b, 36_a) = f(36_a, 36_b) = f(36_b, 30_a) = 1$

Cycle 3: $f(49_a, 49_b) = f(49_b, 51_a) = f(51_a, 51_b) = f(51_b, 50_a) = f(50_a, 50_b) = f(50_b, 49_a) = 1$

Cycle 4: $f(4_a, 4_b) = f(4_b, 8_a) = f(8_a, 8_b) = f(8_b, 4_a) = 1$

Cycle 5: $f(41_a, 41_b) = f(41_b, 47_a) = f(47_a, 47_b) = f(47_b, 41_a) = 1$

Cycle 6: $f(44_a, 44_b) = f(44_b, 48_a) = f(48_a, 48_b) = f(48_b, 44_a) = 1$

Cycle 7: $f(12_a, 12_b) = f(12_b, 17_a) = f(17_a, 17_b) = f(17_b, 23_a) = f(23_a, 23_b) = f(23_b, 21_a) =$
 $f(21_a, 21_b) = f(21_b, 28_a) = f(28_a, 28_b) = f(28_b, 12_a) = 1$

The seven cycles are shown in Figures 5 to 11.

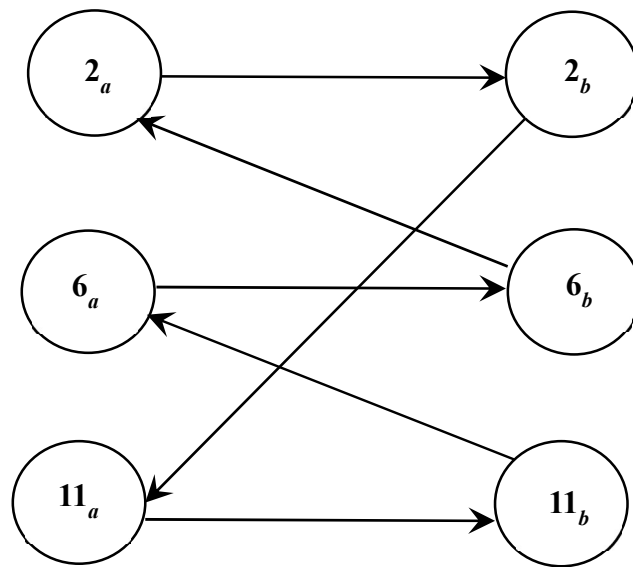


Figure 5: Cycle 1 for part family 2 of example problem II.

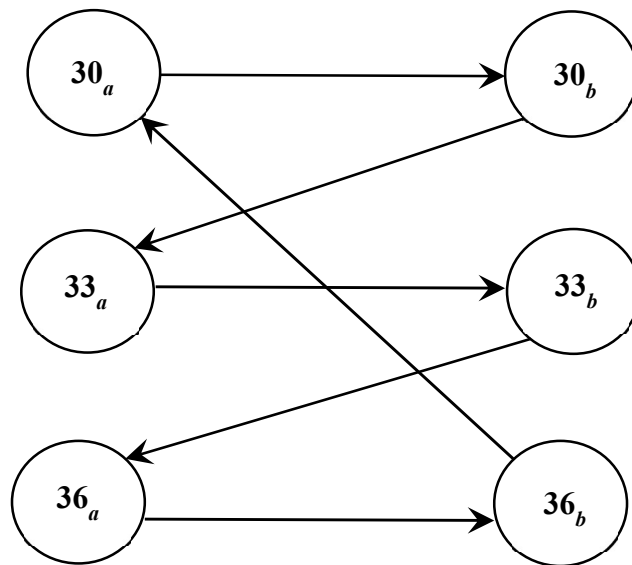


Figure 6: Cycle 2 for part family 2 of example problem II.

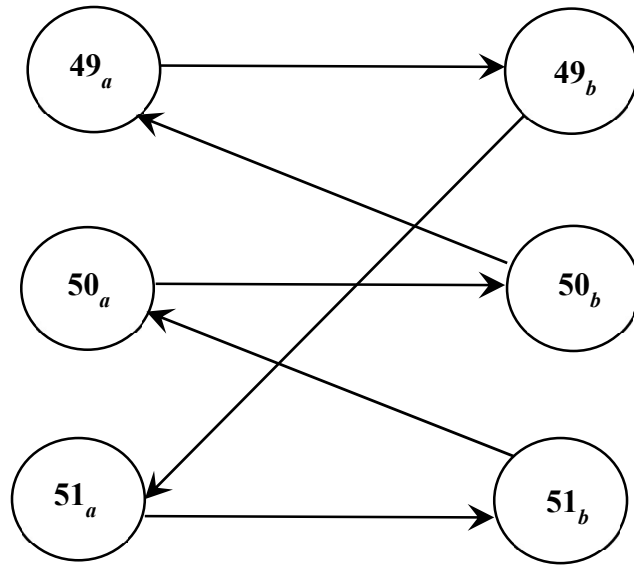


Figure 7: Cycle 3 for part family 2 of example problem II.

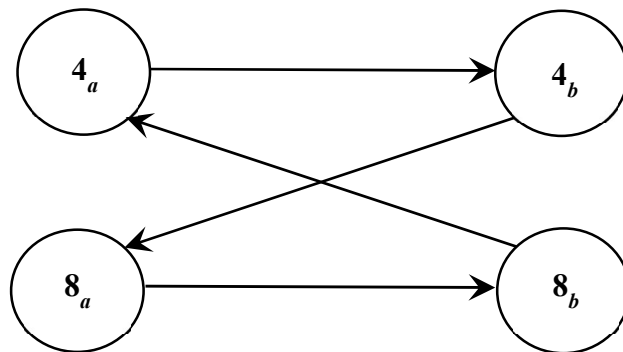


Figure 8: Cycle 4 for part family 2 of example problem II.

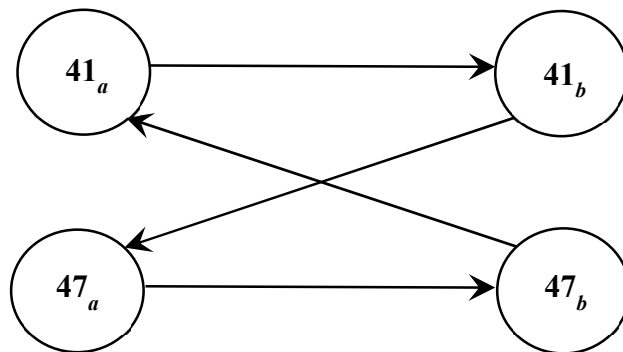


Figure 9: Cycle 5 for part family 2 of example problem II.

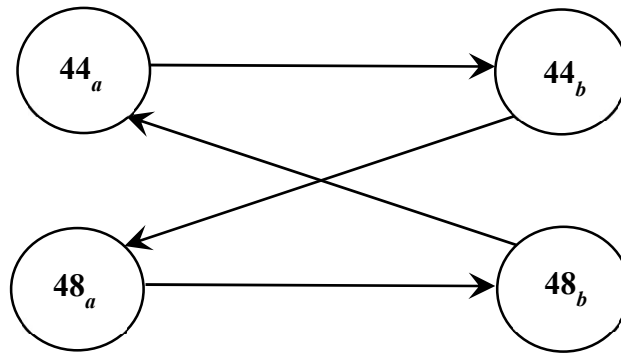


Figure 10: Cycle 6 for part family 2 of example problem II.

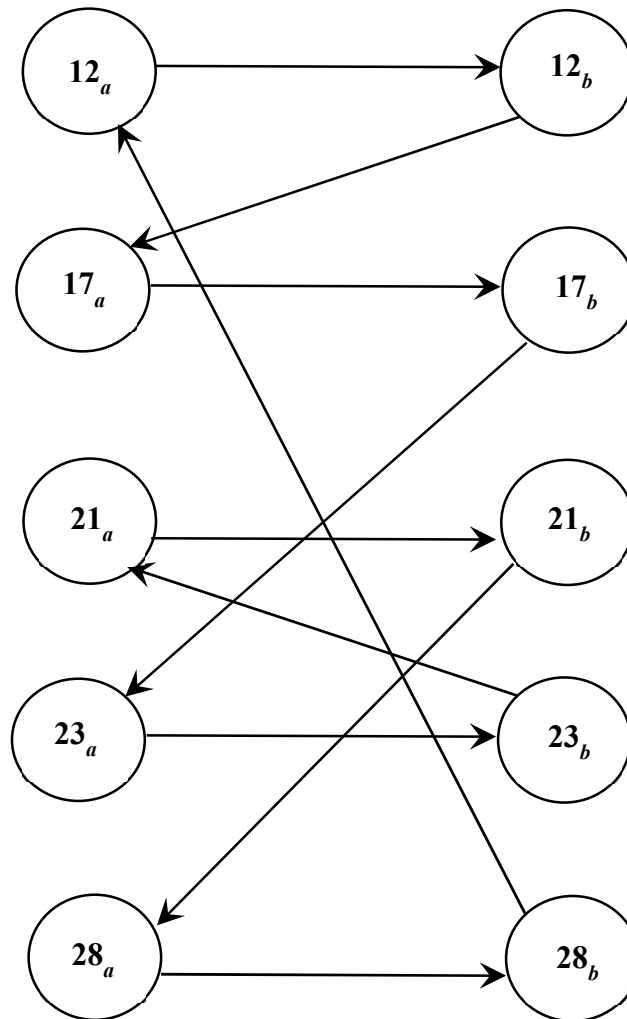


Figure 11: Cycle 7 for part family 2 of example problem II.

From the seven cycles shown in Figures 5 to 11, seven process route families can be identified as:

- (i). Process route family 1 consisting of process routes 2, 6 and 11 corresponding to parts 1, 3 and 5 respectively.
- (ii). Process route family 2 consisting of process routes 30, 33 and 36 corresponding to parts 11, 12 and 13 respectively.
- (iii). Process route family 3 consisting of process routes 49, 50 and 51 corresponding to parts 18, 19 and 20 respectively.
- (iv). Process route family 4 consisting of process routes 4 and 8 corresponding to parts 2 and 4 respectively.
- (v). Process route family 5 consisting of process routes 41 and 47 corresponding to parts 14 and 16 respectively.
- (vi). Process route family 6 consisting of process routes 44 and 48 corresponding to parts 15 and 17 respectively.
- (vii). Process route family 7 consisting of process routes 12, 17, 21, 23, and 28 corresponding to parts 6, 7, 8, 9, and 10 respectively.

In the machine cell formation stage, applying machine cell formation heuristic, five machine cells are obtained with only one exceptional element. These results are same as those obtained by solving the QAP formulation for machine cell formation:

- (i). Machine cell 1 consisting of machines [1, 7, 9, 12]. Process route families assigned to it are 1 and 4 which contain process routes [2, 4, 6, 8, 11].
- (ii). Machine cell 2 consisting of machines [2, 5, 6, 16, 19]. Process route family assigned to it is 7 which contains process routes [12, 17, 21, 23, 28].
- (iii). Machine cell 3 consisting of machines [3, 8, 11, 18]. Process route family assigned to it is 2 which contains process routes [30, 33, 36].
- (iv). Machine cell 4 consisting of machines [10, 14, 17, 20]. Process route families assigned to it are 5 and 6 which contain process routes [41, 44, 47, 48].
- (v). Machine cell 5 consisting of machines [4, 13, 15]. Process route family assigned to it is 3 which contains process routes [49, 50, 51].

After rearranging the process route-machine incidence matrix, the resultant matrix is shown in Table 6. The present model yields one exceptional element, whereas the p -median formulation yields three exceptional elements. For comparison, the results obtained from the p -median model are shown in matrix form in Table 7.

Table 6: Solution to example problem II in block diagonal form by present model.

Part (k)	Process route set ($PR(k)$)	Process route number (i)	Machine No. (m)																			
			1	9	12	7	2	5	6	16	19	3	8	11	18	10	14	17	20	4	13	15
1	2	2	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	6	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	3	11	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
4	2	4	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	2	8	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	12	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
7	4	17	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
8	2	21	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
9	2	23	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
10	1	28	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
11	1	30	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
12	1	33	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0

13	1	36	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0
14	3	41	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0
15	3	47	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0
16	3	44	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0
17	1	48	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
18	1	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
19	1	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
20	1	51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1

Table 7: Solution to example problem II in block diagonal form by p -median model.

Part (k)	Process route set ($PR(k)$)	Process route number (i)	Machine No. (m)																			
			7	9	12	1	20	2	5	6	16	19	3	8	11	18	10	14	17	4	13	15
1	2	2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	4	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	2	6	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	2	8	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	3	11	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	12	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
7	4	17	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0
8	2	21	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0
9	2	23	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0
10	1	28	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
11	1	30	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
12	1	33	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0
13	1	36	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
14	2	40	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0
15	2	43	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0
16	2	46	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0
17	1	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
18	1	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
19	1	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
20	1	51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1

8. Conclusions and future research scopes

In this paper, we propose a procedure for forming part families and machine cells in a generalized grouping environment, where each part has more than one process route. The grouping problem involves selecting a process route for each part, grouping them into part families, and forming machine cells such that each machine cell can process at least one process route (part) family. The procedure organizes the formation of part families and machine cells hierarchically. For the process route family formation stage, a unit capacity minimum cost network flow model is developed. The proposed model solves the part family formation problem optimally without pre-specifying the

number of part families to be formed and is not an iterative process. The method is flexible in that it can be used in both situations where disjoint groups exist and where they do not. Our computational results show that it provides better solutions than the p -median model in terms of exceptional elements.

For the machine cell formation stage, a QAP formulation and a heuristic procedure are proposed. The QAP formulation simultaneously assigns process route families and machines to the pre-specified number of cells to maximize machine utilization. One advantage of this formulation is that, even if a large number of cells is specified, the model will form only the optimal number of cells within the specified limit, ensuring maximized machine utilization while satisfying system constraints. The heuristic procedure is hierarchical in nature. In the first stage, it aims to reduce the number of process route families wherever possible. In the second stage, machines are assigned to process route families based on maximum utilization among competing families. In the final stage, the procedure attempts to merge cells (and corresponding process route families) wherever feasible within system constraints. Computational results show that the QAP and heuristic procedure yield the same results.

In the presence of exceptional elements in the final solution, an improvement scheme can be employed. In reality, particularly in flexible manufacturing systems environments where machines have inherent flexibility, it may be possible to reassign exceptional operations that create inter-cell movements within the cell itself, provided that the reassignment does not violate machine capacity constraints. For solving the network model, we used a mixed-integer optimization package (CPLEX 22.1.0.0). Except for side constraint 7, all other constraints and the objective function conform to the standard unit capacity minimum cost network flow problem, which can be solved using any minimum cost network flow algorithm.

Due to constraint 7, a specialized network flow algorithm is required to solve the problem, and the development of such an algorithm could be a future avenue of research. The network flow model developed in this work involves certain side constraints that prevent it from being solved using standard network flow algorithms. It would be worthwhile to develop a specialized network flow algorithm for this model.

References

- Bustince, H., Pagola, M., Mesiar, R., Hullermeier, E., & Herrera, F. (2012). Grouping, Overlap, and Generalized Bientropic Functions for Fuzzy Modeling of Pairwise Comparisons. *IEEE Transactions on Fuzzy Systems*, 20(3), 405–415. <https://doi.org/10.1109/TFUZZ.2011.2173581>
- Cheng, M., Jiang, J., Wang, Q., & Yao, Y. (2019). A Generalized Grouping Scheme in Coded Caching. *IEEE Transactions on Communications*, 67(5), 3422–3430. <https://doi.org/10.1109/TCOMM.2019.2896960>
- Cova, T. J., & Johnson, J. P. (2003). A network flow model for lane-based evacuation routing. *Transportation Research Part A: Policy and Practice*, 37(7), 579–604. [https://doi.org/10.1016/S0965-8564\(03\)00007-7](https://doi.org/10.1016/S0965-8564(03)00007-7)
- Garg, H., & Arora, R. (2018). Generalized and group-based generalized intuitionistic fuzzy soft sets with applications in decision-making. *Applied Intelligence*, 48(2), 343–356. <https://doi.org/10.1007/s10489-017-0981-5>
- Goldbeck, N., Angeloudis, P., & Ochieng, W. Y. (2019). Resilience assessment for interdependent urban infrastructure systems using dynamic network flow models. *Reliability Engineering & System Safety*, 188, 62–79. <https://doi.org/10.1016/j.ress.2019.03.007>
- Gómez, D., Figueira, J. R., & Eusébio, A. (2013). Modeling centrality measures in social network analysis using bicriteria network flow optimization problems. *European Journal of Operational Research*, 226(2), 354–365. <https://doi.org/10.1016/j.ejor.2012.11.027>

- Holden, R., Val, D. V., Burkhard, R., & Nodwell, S. (2013). A network flow model for interdependent infrastructures at the local scale. *Safety Science*, 53, 51–60. <https://doi.org/10.1016/j.ssci.2012.08.013>
- Hovav, S., & Tsadikovich, D. (2015). A network flow model for inventory management and distribution of influenza vaccines through a healthcare supply chain. *Operations Research for Health Care*, 5, 49–62. <https://doi.org/10.1016/j.orhc.2015.05.003>
- Hsu, C., & Wallace, W. A. (2007). An industrial network flow information integration model for supply chain management and intelligent transportation. *Enterprise Information Systems*, 1(3), 327–351. <https://doi.org/10.1080/17517570701504633>
- Hsu, N.-S., & Cheng, K.-W. (2002). Network Flow Optimization Model for Basin-Scale Water Supply Planning. *Journal of Water Resources Planning and Management*, 128(2), 102–112. [https://doi.org/10.1061/\(ASCE\)0733-9496\(2002\)128:2\(102\)](https://doi.org/10.1061/(ASCE)0733-9496(2002)128:2(102))
- James, T. L., Brown, E. C., & Keeling, K. B. (2007). A hybrid grouping genetic algorithm for the cell formation problem. *Computers & Operations Research*, 34(7), 2059–2079. <https://doi.org/10.1016/j.cor.2005.08.010>
- LEE, H., & GARCIA-DIAZ, A. (1993). A network flow approach to solve clustering problems in group technology. *International Journal of Production Research*, 31(3), 603–612. <https://doi.org/10.1080/00207549308956746>
- Lerlertpakdee, P., Jafarpour, B., & Gildin, E. (2014). Efficient Production Optimization With Flow-Network Models. *SPE Journal*, 19(06), 1083–1095. <https://doi.org/10.2118/170241-PA>
- Majeed, A., & Rauf, I. (2020). Graph Theory: A Comprehensive Survey about Graph Theory Applications in Computer Science and Social Networks. *Inventions*, 5(1), 10. <https://doi.org/10.3390/inventions5010010>
- Mak, K. L., Wong, Y. S., & Wang, X. X. (2000). An Adaptive Genetic Algorithm for Manufacturing Cell Formation. *The International Journal of Advanced Manufacturing Technology*, 16(7), 491–497. <https://doi.org/10.1007/s001700070057>
- Malinovsky, Y. (2019). Sterrett Procedure for the Generalized Group Testing Problem. *Methodology and Computing in Applied Probability*, 21(3), 829–840. <https://doi.org/10.1007/s11009-017-9601-4>
- Prafulla, D., & Kulkarni, C. (2021). Solving Generalized groupings problems in Cellular manufacturing systems by genetic algorithms. *Journal of Science & Technology (JST)*, 6(5), 82–88. <https://doi.org/10.46243/jst.2021.v6.i05.pp82-88>
- Quelhas, A., Gil, E., McCalley, J. D., & Ryan, S. M. (2007). A Multiperiod Generalized Network Flow Model of the U.S. Integrated Energy System: Part I—Model Description. *IEEE Transactions on Power Systems*, 22(2), 829–836. <https://doi.org/10.1109/TPWRS.2007.894844>
- Quelhas, A., & McCalley, J. D. (2007). A Multiperiod Generalized Network Flow Model of the U.S. Integrated Energy System: Part II—Simulation Results. *IEEE Transactions on Power Systems*, 22(2), 837–844. <https://doi.org/10.1109/TPWRS.2007.894845>
- RAJAGOPALAN, R., & BATRA, J. L. (1975). Design of cellular production systems A graph-theoretic approach. *International Journal of Production Research*, 13(6), 567–579. <https://doi.org/10.1080/00207547508943029>
- Rietz, J., Alves, C., Braga, N., & Valério de Carvalho, J. (2016). An exact approach based on a new pseudo-polynomial network flow model for integrated planning and scheduling. *Computers & Operations Research*, 76, 183–194. <https://doi.org/10.1016/j.cor.2016.07.008>

- Rudi, A., Fröhling, M., Zimmer, K., & Schultmann, F. (2016). Freight transportation planning considering carbon emissions and in-transit holding costs: A capacitated multi-commodity network flow model. *EURO Journal on Transportation and Logistics*, 5(2), 123–160. <https://doi.org/10.1007/s13676-014-0062-4>
- Salehi, M., & Moghaddam, R. T. (2010). A genetic algorithm-based grouping method for a cell formation problem with the efficacy measure. *International Journal of Industrial and Systems Engineering*, 6(3), 340. <https://doi.org/10.1504/IJISE.2010.035016>
- Tariq, A., Hussain, I., & Ghafoor, A. (2009). A hybrid genetic algorithm for machine-part grouping. *Computers & Industrial Engineering*, 56(1), 347–356. <https://doi.org/10.1016/j.cie.2008.06.007>
- Topal, E., & Ramazan, S. (2012). Strategic mine planning model using network flow model and real case application. *International Journal of Mining, Reclamation and Environment*, 26(1), 29–37. <https://doi.org/10.1080/17480930.2011.600827>
- Uddin, M. K., & Shanker, K. (2002). Grouping of parts and machines in presence of alternative process routes by genetic algorithm. *International Journal of Production Economics*, 76(3), 219–228. [https://doi.org/10.1016/S0925-5273\(01\)00164-5](https://doi.org/10.1016/S0925-5273(01)00164-5)
- Werner, F. (2020). Graph-Theoretic Problems and Their New Applications. *Mathematics*, 8(3), 445. <https://doi.org/10.3390/math8030445>
- Yan, T., & Wong, M. D. F. (2009). A correct network flow model for escape routing. *Proceedings of the 46th Annual Design Automation Conference*, 332–335. <https://doi.org/10.1145/1629911.1630001>
- Yin, Y., Val, D. V., Zou, Q., & Yurchenko, D. (2022). Resilience of Critical Infrastructure Systems to Floods: A Coupled Probabilistic Network Flow and LISFLOOD-FP Model. *Water*, 14(5), 683. <https://doi.org/10.3390/w14050683>

Statements and Declarations

Funding

The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

Competing Interests

The authors have no relevant financial or non-financial interests to disclose.

Data availability statement

Data will be provided anonymously upon request.