Physical Hamiltonians, the Zeros of L-Functions, and a Hybrid Quantum–Classical Search Pipeline

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Abstract

Short answer: Yes—in principle. If one finds (or reliably engineers) a self-adjoint physical Hamiltonian whose spectrum equals the nontrivial zeros of an L-function, then measuring that system's spectrum provides direct physical access to the zeros. Because the zeros determine prime-counting via explicit formulas, the primes or their distribution can be recovered from the measured spectrum instead of by evaluating the L-function numerically.

This document explains the potential, the obstacles, and a concrete hybrid pipeline (KAN $\rightarrow$ VQE $\rightarrow$ Krylov/SBQD $\rightarrow$ FFT $\rightarrow$ QPE) to search for and validate candidate Hamiltonians.

1 Why this would be transformative

1. **Direct physical measurement of zeros.** Quantum phase estimation (QPE) or high-resolution spectroscopy on a system whose eigenvalues coincide with the zeros will output those zeros (or highly accurate approximations) without evaluating $\zeta(s)$ numerically.

2. **Primes follow from zeros.** The explicit trace/Weil formulas relate prime-counting functions to sums over zeros. Substituting measured zeros into those formulas reconstructs prime-distribution data (e.g. $\pi(x)$, Chebyshev functions) — effectively a "physical prime oracle."

3. **New viewpoint / potential proofs.** A concrete self-adjoint operator whose spectrum equals the zeros would amount to a Hilbert–Pólya style demonstration and provide deep conceptual payoff beyond computation.

2 Why it is not automatic or practical yet

- **No known explicit Hamiltonian.** Several proposals exist (Berry–Keating, scattering/trace-formula models, quantum graphs) but no accepted self-adjoint Hamiltonian reproducing the nontrivial zeros exactly is known.

- **Statistics vs. identity.** Many physical systems reproduce local statistics of zeros (GUE spacings), but statistical similarity does not imply the exact spectral identity required for recovering primes.

- **Precision and resolution.** High-accuracy prime estimates require zeros to many digits. Physical measurement to that precision demands either many ancilla qubits in QPE or extremely fine spectroscopic control.

- **Encoding arithmetic structure.** The Hamiltonian must encode arithmetic features (e.g. Euler product structure) — how to embed that into realistic physical models remains open.

3 The Berry–Keating Model and Hermiticity

3.1 The bare operator

The original Berry–Keating proposal considers the classical Hamiltonian $H = xp$. Upon quantization in position representation, this becomes

$$\hat{H} = -i\hbar x \frac{d}{dx}.$$

However, this operator is not Hermitian (self-adjoint) on $L^2(\mathbb{R})$. Integration by parts shows an extra term appears in the inner product, violating $\langle \phi | \hat{H}\psi \rangle = \langle \hat{H}\phi | \psi \rangle$. Thus, $\hat{H} = xp$ is not a valid physical Hamiltonian.

3.2 Symmetrized version

To fix Hermiticity, one defines the symmetrized operator

$$\hat{H}_{BK} = \frac{1}{2}(\hat{x}\hat{p} + \hat{p}\hat{x}) = -i\hbar \left(x\frac{d}{dx} + \frac{1}{2} \right),$$

which is formally Hermitian for wavefunctions vanishing at infinity. The spectrum of this operator is continuous (real-valued), so it is physically consistent but not yet discrete.

3.3 Self-adjoint extension and boundary regularization

The operator can be made self-adjoint by restricting its domain or imposing boundary conditions:

$$\psi(x_{\min}) = e^{i\theta}\psi(x_{\max}),$$

with finite cutoffs $x_{\min}, x_{\max}$. This compactifies the domain, giving a discrete spectrum and turning the operator into a legitimate quantum Hamiltonian. Such regularized or discretized Berry–Keating models are self-adjoint and yield real, discrete eigenvalues.

3.4 Discretized matrix realization

For numerical work, the discretized version uses a finite grid $x_j \in [x_{\min}, x_{\max}]$ and represents $\hat{p}$ by finite differences. The Hamiltonian matrix is then

$$H_{jk} = \frac{1}{2}\left(x_j p_{jk} + p_{jk} x_k \right),$$

where p_{jk} is anti-Hermitian (imaginary and skew-symmetric), making H_{jk} Hermitian. This ensures real eigenvalues and enables numerical diagonalization. In the hybrid pipeline, this discretized Hermitian form serves as the base physical model for parameter optimization.

4 How the hybrid pipeline helps discover such a Hamiltonian

We propose a search-and-validate workflow that uses a hybrid quantum–classical pipeline to (i) propose plausible parametric Hamiltonians, (ii) measure/estimate their spectra efficiently, and (iii) optimize parameters to match known zeros.

4.1 Parametric Hamiltonian family

Choose a flexible, physically plausible family $H(\alpha)$ with tunable parameters α. Examples:

- Regularized/discretized Berry–Keating model with boundary or regularization parameters.
- One-dimensional scattering or quantum-graph families with tunable potentials or edge lengths.
- Small many-body lattice Hamiltonians with adjustable couplings (e.g. BEC-inspired lattices).

4.2 Fast spectral probe (pipeline)

For a given α:

(a) Use **KAN** to select ansatz parameters θ that prepare seed states with good overlap cheaply.

(b) Run short **VQE** refinements to improve eigenstate overlaps where useful.

(c) Build a **Krylov subspace / SBQD** around seed states to extract multiple approximate eigenvalues in blocks.

(d) Use **FFT** on time-series $C(t) = \langle\psi|e^{-iHt}|\psi\rangle$ or short-time evolutions to identify spectral peaks (coarse localization).

(e) Run **QPE** on promising spectral windows to refine eigenvalues to target precision.

This yields a list of extracted eigenvalues $\{\tilde{\lambda}_i(\alpha)\}$.

4.3 Objective and metrics

Compare $\{\tilde{\lambda}_i(\alpha)\}$ to true nontrivial zeros $\{\gamma_i\}$ (computed classically for the range targeted). Suggested metrics:

- Absolute error: $\max_i |\tilde{\lambda}_i - \gamma_i|$ for the first N zeros.
- Pair-correlation / spacing statistics: nearest-neighbor spacing distribution, two-point correlation.
- Trace-formula residual: mismatch between semiclassical spectral density $\rho(E)$ and the zero density.
- Primes-derived residual: insert $\tilde{\lambda}$ into an explicit formula for the Chebyshev or prime-counting functions and compare to known values.

Combine into a weighted loss

$$L(\alpha) = w_1\,\mathrm{abs\_err} + w_2\,\mathrm{stat\_err} + w_3\,\mathrm{trace\_res}.$$

Use KAN for surrogate modeling of $L(\alpha)$ to reduce expensive evaluations.

4.4 Optimize over α

Use classical optimization (derivative-free, Bayesian, or surrogate-based) to minimize $L(\alpha)$. Iterate until a candidate $\alpha^\star$ yields an acceptably small loss.

4.5 Validate and test generalization

- Test $\tilde{\lambda}(\alpha^\star)$ against zeros outside the fitted range (out-of-sample verification).
- Test robustness to noise and parameter drift.

5 Practical outcomes and limitations

- If a Hamiltonian matches zeros to arbitrary high accuracy and generalizes, measuring its spectrum provides a direct route to primes — a revolutionary outcome.
- More realistically, one may find Hamiltonians that match zeros approximately over some window; these models are still valuable for exploring physics–number-theory connections.
- Resource demands for optimization and high-precision QPE will be significant; careful error mitigation is required.

6 Concrete next steps (implementable)

Possible tasks (user-selectable):

1. **Prototype search code:** Implement the full loop for a chosen parametric family (KAN sampling, VQE, Krylov+FFT spectral extraction, objective evaluation vs. first N classical zeros, and parameter optimization). Example target: first 20 nontrivial zeros.
2. **FFT-enhanced spectral pipeline:** Build the FFT-based spectral estimation from short-time evolutions plus Krylov/SBQD and hooks to QPE.
3. **Validation scripts:** Functions to compute explicit-formula-based prime estimates from measured zeros and compare to $\pi(x)$ and Chebyshev functions.
4. **Scoring and visualization:** Produce plots tracking spectral match during optimization and prime-distribution residuals.

7 Acknowledgements and references

This document summarizes an idea and an actionable hybrid quantum–classical plan. Relevant literature includes Berry–Keating models, quantum graph spectral theory, semiclassical trace-formula methods, and modern variational/Krylov quantum eigensolvers. The reader is encouraged to consult standard references in analytic number theory and quantum computation for background.