The Refined Hybrid Quantum-Classical Search Framework

Framework Description (Integrated with GMC and Harper-Xu-Wang 2025)

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Abstract

This document outlines the coherent, step-by-step process of the hybrid quantum-classical search pipeline. The framework's goal is to reliably engineer a self-adjoint physical Hamiltonian whose spectrum equals the nontrivial zeros of an L-function. The key refinement is the integration of **Gaussian Multiplicative Chaos (GMC)** for statistical rigor and the **Harper-Xu-Wang (HXW) 2025** results for advanced validation, leading to a **"physically grounded, fractal-chaos-driven discovery process"**.

1 Framework Overview: Why This is Transformative

The primary motivation is to achieve a **physical prime oracle**.

- **Direct Zeros Measurement (Why):** Finding such a Hamiltonian means its eigenvalues are the zeros. Quantum Phase Estimation (QPE) or high-resolution spectroscopy on this system provides direct, physical access to the zeros without requiring numerical evaluation of the L-function.
- **Prime-Counting (How):** Once the zeros are measured, they are substituted into explicit trace/Weil formulas to reconstruct prime-counting data, such as $\pi(x)$.
- **Hilbert-Pólya (Why):** A successful demonstration would constitute a Hilbert-Pólya style demonstration, yielding deep conceptual payoff beyond mere computation.

2 The Refined Hybrid Pipeline Steps (KAN $\to \cdots \to$ Validation)

The pipeline, summarized as KAN $\to$ VQE $\to$ Krylov/SBQD $\to$ FFT $\to$ QPE/Lindbladian $\to$ Validation, is detailed below with the integrated concepts.

2.1 Step 1: KAN (Koopman-Active Network) and GMC Pretraining

- **How:** The KAN acts as an intelligent classical surrogate model to explore the vast space of parametric Hamiltonians. It is specifically suggested to **Pretrain with synthetic GMC data**.
- **Why:** This step encodes the required **fractal correlations** into the initial Hamiltonian parameters. Since the zeros are known to follow Gaussian Multiplicative Chaos (GMC) statistics, starting the search with a model already exhibiting these log-correlated statistics (covariance $\mathbb{E}[(V(x) - V(y))^2] \sim -\log|x - y|$) dramatically constrains and accelerates the search toward physically meaningful candidates.

2.2 Step 2: VQE (Variational Quantum Eigensolver) with GMC Regularization

- **How:** The VQE optimizes the Hamiltonian parameters (sampled from the KAN). The critical addition is the cost function: it must **Optimize both eigenvalue alignment and a GMC statistical regularizer**.

- **Why:** Optimizing only for individual eigenvalues is insufficient, as many unrelated systems share single eigenvalues. The **GMC-regularized cost term** rigorously constrains the search to Hamiltonians whose spectral density statistics (log-covariance) match the $-log|x-y|$ form. This ensures the resulting model is statistically equivalent to the Riemann system.

2.3 Step 3 & 4: Coarse & Fine Spectral Extraction

- **Krylov/FFT (Coarse):**
  - **How:** The **Krylov subspace construction** (or techniques like Spectral-Bundle-Diffusion (SBQD)) and **Fast Fourier Transform (FFT)** are used to estimate coarse spectra and the log-correlation structure.
  - **Why:** These classical/near-term quantum methods provide an initial, fast spectral estimate, allowing the optimizer to compute the GMC regularizer gradient and rapidly refine the parameters before moving to resource-intensive quantum steps.
- **QPE/Lindbladian (Fine):**
  - **How:** **Quantum Phase Estimation (QPE)** or the alternative **Lindbladian Spectral Filtering** technique is used to perform fine, high-precision spectral extraction.
  - **Why:** While QPE offers high resolution, the Lindbladian approach ($\mathcal{L}_H(\rho) = H\rho H - \frac{1}{2}\{H^2, \rho\}$) offers a spectral decomposition with potential resource advantages when coherence time is limited, serving as a robust, open-system alternative for extracting the eigenvalues (zeros).

2.4 Step 5: Refined Prime-Counting Validation (Harper-Xu-Wang 2025)

- **How:** The final validation step **compares the measured zeros with Harper-Xu-Wang prime-counting predictions**. This involves replacing the classical explicit formula with the HXW short-interval model for evaluating residuals.
- **Why:** The HXW 2025 results provide a more precise and sensitive metric for prime-counting statistics over short intervals. Since a quantum computer can only measure a small number of zeros (e.g., the first 20-50) with high precision, the HXW model serves as a **superior, high-sensitivity validation benchmark** compared to the classical explicit formula, which converges slowly and requires a massive number of zeros for accurate results.