Option 3: GMC + Langlands + Formal Verification Hybrid Hamiltonian Search

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October 28, 2025

Abstract

This companion LaTeX documents Option 3: integrating Gaussian Multiplicative Chaos (GMC), quantum–quantum Langlands ideas, the Knuth–Cahen finite-GCD matrix principle, Lindbladian QPE, and a Lean-based formal verification stub. It summarizes the computational modules and provides implementation notes for the accompanying Jupyter notebook.

Contents

1 Overview

Option 3 combines:

- Knuth–Cahen finite-GCD finite search principle,
- Discretized Berry–Keating Hermitian matrices,
- GMC-based statistical regularizers (log-covariance),
- Langlands-inspired deformation parameterization,
- Lindbladian spectral probing,
- Lean 4 proof stub for symbolic verification, and
- An optimal hybrid optimizer combining gradient-based and evolutionary strategies.

2 Knuth–Cahen Finite-GCD Mapping

A key algorithmic insight arises from Donald Knuth's exercise on matrix GCD-like constructions and the algebraic approaches of Eugène Cahen: even when the underlying mathematical domain is infinite, one can reformulate the search as a finite, terminating computation by working within a structured algebraic ring and imposing commutation or divisibility constraints. See main text for detailed mapping.

3 Quantum–Langlands integration and spectral correspondences

Motivation. The (geometric and quantum) Langlands program posits deep correspondences between spectral data of operators on one side and automorphic / representation-theoretic data on the other. In physics, quantum Langlands connects dual gauge theories and their spectral operators (Casimirs, transfer matrices, etc.) via q-deformations and monodromy of conformal blocks. For an approach that seeks a physical Hamiltonian whose spectrum encodes nontrivial zeros of L-functions, the quantum Langlands viewpoint provides a natural conceptual scaffold: it suggests that the spectral target (zeros) may be realized as eigenvalues of operators lying in an appropriate representation of a quantum group or through transfer operators of a dual field theory.

Operational strategy. We propose three concrete ways to fold quantum-Langlands ideas into the hybrid Hamiltonian search pipeline (KAN $\to$ VQE $\to$ Krylov/SBQD $\to$ FFT $\to$ QPE):

1. **Model selection via q-deformation.** Build Hamiltonian ansätze that admit a one-parameter q-deformation (or spectral parameter), for example by introducing local Lax-type factors $L_j(z)$ with spectral parameter z or by adding algebraically motivated coupling terms that close under a chosen quantum algebra. Searching within such families makes it natural to compare transfer-matrix spectra $T(z) = \mathrm{Tr}_{\mathrm{aux}}(L_N(z)\cdots L_1(z))$ with zeta statistics.
2. **Spectral monodromy tests.** For each candidate Hamiltonian $H(\boldsymbol{\theta})$ produced by VQE/KAN, compute finite-dimensional proxies for conformal blocks / transfer matrices and test whether short q-shifts of resolvent samples obey simple linear difference relations (q-shift recurrence). Agreement suggests the Hamiltonian admits a q-difference structure compatible with quantum Langlands dual objects.
3. **Statistical regularization from representation theory.** Use representation-theoretic constraints (e.g. expected degeneracies or fusion-type selection rules) as statistical priors / regularizers in the VQE loss. These priors favor Hamiltonians whose finite-size spectra are consistent with the algebraic structure expected from quantum Langlands duals.

A practical diagnostic: q-shift resolvent relation. A fully rigorous qKZ implementation requires constructing explicit R- and L-matrices for a chosen quantum affine algebra and matching representations. As a practical numerical test and exploratory diagnostic we introduce a computable q-shift resolvent relation.

Let H be the finite truncated Hamiltonian (matrix) under study, v a fixed probe vector, and define the resolvent sample

$$\psi(z) := (H - zI)^{-1}v,$$

for complex spectral parameter z away from the finite spectrum. Consider the linear relation

$$\psi(qz) \;=\; M(z)\,\psi(z), \qquad M(z) \;=\; (H - qzI)^{-1}(H - zI). \tag{1}$$

Equation (1) is an exact algebraic identity: it follows from the resolvent definition. It defines a *q-shift difference* mapping between resolvent samples at z and qz. If a candidate Hamiltonian admits a q-difference (qKZ-like) structure, then the matrix family $M(z)$ will exhibit simple analytic / low-rank structure (e.g. slowly varying with z, approximate factorization, or small numerical rank when projected to relevant subspaces). Such signatures provide evidence consistent with a q-deformed representation interpretation, and can be used as a practical filter inside the hybrid search. See Section **??** for experiments.

4 Lean-based Formal Verification Stub

The accompanying ‘LeanProofStub.lean’ contains a starting skeleton verifying Hermiticity and finite termination for the finite search. Use Mathlib and SMT bridges to expand the proofs.

5 Notebook Usage

Run the notebook ‘hybrid$_{o}ption3_{f}ormal_{g}mc_{l}anglands.ipynb'inanenvironmentwithnumpy/scipyinstalled.The$