Physical Hamiltonians, the Zeros of L-Functions, and a Hybrid Quantum–Classical Search Pipeline

Prepared by ChatGPT based on user text and latest integrations

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Abstract

Short answer: Yes—in principle. If one finds (or reliably engineers) a self-adjoint physical Hamiltonian whose spectrum equals the nontrivial zeros of an L-function, then measuring that system's spectrum provides direct physical access to the zeros. Because the zeros determine prime-counting via explicit formulas, the primes or their distribution can be recovered from the measured spectrum instead of by evaluating the L-function numerically.

This document explains the potential, the obstacles, and a concrete hybrid pipeline (KAN $\rightarrow$ VQE $\rightarrow$ Krylov/SBQD $\rightarrow$ FFT $\rightarrow$ QPE) to search for and validate candidate Hamiltonians. It integrates new sections on Gaussian Multiplicative Chaos (GMC), Lindbladian spectral methods, and Harper–Xu–Wang 2025 results for a physically grounded, fractal-chaos-aware approach.

Contents

1 Why this would be transformative

1. **Direct physical measurement of zeros.** Quantum phase estimation (QPE) or high-resolution spectroscopy on a system whose eigenvalues coincide with the zeros will output those zeros (or highly accurate approximations) without evaluating $\zeta(s)$ numerically.

2. **Primes follow from zeros.** The explicit trace/Weil formulas relate prime-counting functions to sums over zeros. Substituting measured zeros into those formulas reconstructs prime-distribution data (e.g. $\pi(x)$, Chebyshev functions) — effectively a "physical prime oracle."

3. **New viewpoint / potential proofs.** A concrete self-adjoint operator whose spectrum equals the zeros would amount to a Hilbert–Pólya style demonstration and provide deep conceptual payoff beyond computation.

2 Why it is not automatic or practical yet

- **No known explicit Hamiltonian.** Several proposals exist (Berry–Keating, scattering/trace-formula models, quantum graphs) but no accepted self-adjoint Hamiltonian reproducing the nontrivial zeros exactly is known.

- **Statistics vs. identity.** Many physical systems reproduce local statistics of zeros (GUE spacings), but statistical similarity does not imply the exact spectral identity required for recovering primes.

- **Precision and resolution.** High-accuracy prime estimates require zeros to many digits. Physical measurement to that precision demands either many ancilla qubits in QPE or extremely fine spectroscopic control.

- **Encoding arithmetic structure.** The Hamiltonian must encode arithmetic features (e.g. Euler product structure) — how to embed that into realistic physical models remains open.

3 The Berry–Keating Model and Hermiticity

3.1 The bare operator

The original Berry–Keating proposal considers the classical Hamiltonian $H = xp$. Upon quantization in position representation, this becomes

$$\hat{H} = -i\hbar x \frac{d}{dx}.$$

However, this operator is not Hermitian (self-adjoint) on $L^2(\mathbb{R})$. Integration by parts shows an extra term appears in the inner product, violating $\langle \phi | \hat{H} \psi \rangle = \langle \hat{H} \phi | \psi \rangle$. Thus, $\hat{H} = xp$ is not a valid physical Hamiltonian.

3.2 Symmetrized version

To fix Hermiticity, one defines the symmetrized operator

$$\hat{H}_{BK} = \frac{1}{2}(\hat{x}\hat{p} + \hat{p}\hat{x}) = -i\hbar \left(x \frac{d}{dx} + \frac{1}{2} \right),$$

which is formally Hermitian for wavefunctions vanishing at infinity. The spectrum of this operator is continuous (real-valued), so it is physically consistent but not yet discrete.

3.3 Self-adjoint extension and boundary regularization

The operator can be made self-adjoint by restricting its domain or imposing boundary conditions:

$$\psi(x_{\min}) = e^{i\theta}\psi(x_{\max}),$$

with finite cutoffs $x_{\min}, x_{\max}$. This compactifies the domain, giving a discrete spectrum and turning the operator into a legitimate quantum Hamiltonian. Such regularized or discretized Berry–Keating models are self-adjoint and yield real, discrete eigenvalues.

3.4 Discretized matrix realization

For numerical work, the discretized version uses a finite grid $x_j \in [x_{\min}, x_{\max}]$ and represents $\hat{p}$ by finite differences. The Hamiltonian matrix is then

$$H_{jk} = \frac{1}{2}\left(x_j p_{jk} + p_{jk} x_k\right),$$

where p_{jk} is anti-Hermitian (imaginary and skew-symmetric), making H_{jk} Hermitian. This ensures real eigenvalues and enables numerical diagonalization. In the hybrid pipeline, this discretized Hermitian form serves as the base physical model for parameter optimization.

4 Lindbladian QPE and Open-System Extensions

4.1 Lindbladian generator

Define a Lindbladian superoperator associated with H:

$$\mathcal{L}_H(\rho) = H\rho H - \frac{1}{2}\{H^2, \rho\}.$$

This dissipative evolution decoheres off-diagonal elements in the energy basis while preserving eigenvalues. Hence, $\mathcal{L}_H$ can serve as a spectral filter to isolate eigenvalues, a useful alternative to QPE when coherence time is limited.

4.2 Fast-forwarding and spectral filtering

Recent works show that certain Lindbladians can be simulated faster than Hamiltonians (quadratic speedups). Using $e^{t\mathcal{L}_H}$ on an initial mixed state effectively performs a spectral decomposition with resource advantages over direct QPE.

5 Gaussian Multiplicative Chaos (GMC) and Harper–Xu–Wang 2025

5.1 Background

Recent research (Harper, Xu, Soundararajan, Wang, 2025) established that fine-scale Riemann zero statistics can be described using **Gaussian Multiplicative Chaos** (GMC). The energy landscape of $\log|\zeta(\frac{1}{2} + it)|$ behaves like a fractal random field with covariance

$$\mathbb{E}[(V(x) - V(y))^2] \sim -\log|x - y| + O(1).$$

This provides a rigorous connection between primes, zeros, and fractal chaos.

5.2 Interpretation for our framework

GMC gives a new statistical structure for the spectrum: instead of only matching eigenvalues, the Hamiltonian search should also match *log-correlated statistics*. The onset of chaotic fluctuations in prime-counting corresponds to GMC variance thresholds. This defines new metrics for spectral validation beyond pair correlations.

5.3 Integration into the hybrid pipeline

We enhance the hybrid search:

- Add a **GMC-regularized cost term** in VQE: penalize deviation between empirical spectral log-covariance and the GMC-predicted $-\log|x-y|$ form.
- Use FFT-based spectral density to compute multifractal exponents and feed them into the optimizer.
- In validation, replace classical Riemann explicit formula with **Harper–Xu–Wang short-interval prime-counting model** for evaluating residuals.

6 Refined Hybrid Pipeline Integration

1. **KAN:** Pretrain with synthetic GMC data to encode fractal correlations.
2. **VQE:** Optimize both eigenvalue alignment and GMC statistical regularizer.
3. **Krylov/FFT:** Estimate coarse spectra and log-correlation structure.
4. **QPE/Lindbladian:** Perform fine spectral extraction.
5. **Validation:** Compare with Harper–Xu–Wang prime-counting predictions.

7 Next Steps and Implementation

- Integrate the GMC-based regularizer into the VQE cost.
- Extend FFT post-processing to estimate log-correlated variance.
- Add Lindbladian simulation cells for spectral filtering in the Jupyter pipeline.
- Benchmark recovered spectra vs. true zeta zeros and fractal dimension predictions.

8 Conclusion

Integrating Gaussian Multiplicative Chaos, Lindbladian QPE, and Harper–Xu–Wang short-interval results transforms the hybrid Hamiltonian search from purely spectral fitting into a physically grounded, fractal-chaos-driven discovery process. The resulting framework unifies quantum simulation, spectral statistics, and modern number-theoretic insights.