Physical Hamiltonians, the Zeros of L-Functions, and a Hybrid Quantum–Classical Search Pipeline

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Abstract

Short answer: Yes—in principle. If one finds (or reliably engineers) a self-adjoint physical Hamiltonian whose spectrum equals the nontrivial zeros of an L-function, then measuring that system's spectrum provides direct physical access to the zeros. Because the zeros determine prime-counting via explicit formulas, the primes or their distribution can be recovered from the measured spectrum instead of by evaluating the L-function numerically.

This document explains the potential, the obstacles, and a concrete hybrid pipeline (KAN $\rightarrow$ VQE $\rightarrow$ Krylov/SBQD $\rightarrow$ FFT $\rightarrow$ QPE) to search for and validate candidate Hamiltonians.

1 Why this would be transformative

1. **Direct physical measurement of zeros.** Quantum phase estimation (QPE) or high-resolution spectroscopy on a system whose eigenvalues coincide with the zeros will output those zeros (or highly accurate approximations) without evaluating $\zeta(s)$ numerically.
2. **Primes follow from zeros.** The explicit trace/Weil formulas relate prime-counting functions to sums over zeros. Substituting measured zeros into those formulas reconstructs prime-distribution data (e.g. $\pi(x)$, Chebyshev functions) — effectively a "physical prime oracle."
3. **New viewpoint / potential proofs.** A concrete self-adjoint operator whose spectrum equals the zeros would amount to a Hilbert–Pólya style demonstration and provide deep conceptual payoff beyond computation.

2 Why it is not automatic or practical yet

- **No known explicit Hamiltonian.** Several proposals exist (Berry–Keating, scattering/trace-formula models, quantum graphs) but no accepted self-adjoint Hamiltonian reproducing the nontrivial zeros exactly is known.
- **Statistics vs. identity.** Many physical systems reproduce local statistics of zeros (GUE spacings), but statistical similarity does not imply the exact spectral identity required for recovering primes.
- **Precision and resolution.** High-accuracy prime estimates require zeros to many digits. Physical measurement to that precision demands either many ancilla qubits in QPE or extremely fine spectroscopic control.
- **Encoding arithmetic structure.** The Hamiltonian must encode arithmetic features (e.g. Euler product structure) — how to embed that into realistic physical models remains open.

3 How the hybrid pipeline helps discover such a Hamiltonian

We propose a search-and-validate workflow that uses a hybrid quantum–classical pipeline to (i) propose plausible parametric Hamiltonians, (ii) measure/estimate their spectra efficiently, and (iii) optimize parameters to match known zeros.

3.1 Parametric Hamiltonian family

Choose a flexible, physically plausible family $H(\alpha)$ with tunable parameters α. Examples:

- Regularized/discretized Berry–Keating model with boundary or regularization parameters.
- One-dimensional scattering or quantum-graph families with tunable potentials or edge lengths.
- Small many-body lattice Hamiltonians with adjustable couplings (e.g. BEC-inspired lattices).

3.2 Fast spectral probe (pipeline)

For a given α:

(a) Use **KAN** to select ansatz parameters θ that prepare seed states with good overlap cheaply.

(b) Run short **VQE** refinements to improve eigenstate overlaps where useful.

(c) Build a **Krylov subspace** / **SBQD** around seed states to extract multiple approximate eigenvalues in blocks.

(d) Use **FFT** on time-series $C(t) = \langle \psi | e^{-iHt} | \psi \rangle$ or short-time evolutions to identify spectral peaks (coarse localization).

(e) Run **QPE** on promising spectral windows to refine eigenvalues to target precision.

This yields a list of extracted eigenvalues $\{\tilde{\lambda}_i(\alpha)\}$.

3.3 Objective and metrics

Compare $\{\tilde{\lambda}_i(\alpha)\}$ to true nontrivial zeros $\{\gamma_i\}$ (computed classically for the range targeted). Suggested metrics:

- Absolute error: $\max_i |\tilde{\lambda}_i - \gamma_i|$ for the first N zeros.
- Pair-correlation / spacing statistics: nearest-neighbor spacing distribution, two-point correlation.
- Trace-formula residual: mismatch between semiclassical spectral density $\rho(E)$ and the zero density.
- Primes-derived residual: insert $\tilde{\lambda}$ into an explicit formula for the Chebyshev or prime-counting functions and compare to known values.

Combine into a weighted loss

$$L(\alpha) = w_1 \, \text{abs\_err} + w_2 \, \text{stat\_err} + w_3 \, \text{trace\_res}.$$

Use KAN for surrogate modeling of $L(\alpha)$ to reduce expensive evaluations.

3.4 Optimize over α

Use classical optimization (derivative-free, Bayesian, or surrogate-based) to minimize $L(\alpha)$. Iterate until a candidate $\alpha^\star$ yields an acceptably small loss.

3.5 Validate and test generalization

- Test $\tilde{\lambda}(\alpha^\star)$ against zeros outside the fitted range (out-of-sample verification).
- Test robustness to noise and parameter drift.

4 Practical outcomes and limitations

- If a Hamiltonian matches zeros to arbitrary high accuracy and generalizes, measuring its spectrum provides a direct route to primes — a revolutionary outcome.
- More realistically, one may find Hamiltonians that match zeros approximately over some window; these models are still valuable for exploring physics–number-theory connections.
- Resource demands for optimization and high-precision QPE will be significant; careful error mitigation is required.

5 Concrete next steps (implementable)

Possible tasks (user-selectable):

1. **Prototype search code:** Implement the full loop for a chosen parametric family (KAN sampling, VQE, Krylov+FFT spectral extraction, objective evaluation vs. first N classical zeros, and parameter optimization). Example target: first 20 nontrivial zeros.
2. **FFT-enhanced spectral pipeline:** Build the FFT-based spectral estimation from short-time evolutions plus Krylov/SBQD and hooks to QPE.
3. **Validation scripts:** Functions to compute explicit-formula-based prime estimates from measured zeros and compare to $\pi(x)$ and Chebyshev functions.
4. **Scoring and visualization:** Produce plots tracking spectral match during optimization and prime-distribution residuals.

6 Acknowledgements and references

This document summarizes an idea and an actionable hybrid quantum–classical plan. Relevant literature includes Berry–Keating models, quantum graph spectral theory, semiclassical trace-formula methods, and modern variational/Krylov quantum eigensolvers. The reader is encouraged to consult standard references in analytic number theory and quantum computation for background.