CLEVELAND STATE UNIVERSITY

CIS 606 – ANALYSIS OF ALGORITHMS

FINAL EXAM

Name: Prudhvi Reddy Araga

Login id: praraga

Question – 1

Question – 1 (a)

Answer: **True**

Given $f(n) = O(g(n))$

$O(g(n)) = \{ f(n)$: there exists positive constants c and n_0 such that $0 \leq f(n) \leq c.g(n)$ for $n_0 \leq n$

Let us consider the following:

$f(n) = 2n^2+n$

$g(n) = n^2$

➔ $2n^2+n = O(n^2)$

$2n^2+n \leq c.\ n^2$

Let us assume the constant value c = 3

$2n^2+n \leq 3.\ n^2$

$n \leq n^2$

$1 \leq n$ (where 1 is the value of n_0)

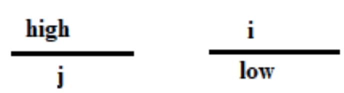
Hence, $f(n) = O(g(n))$ is true.

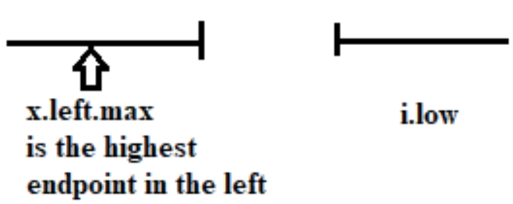
Question – 1 (b)

Answer: True

If the search goes right => x.left.max < i.low

i, j no overlapping condition => i.low > j.high
(or)
j.low > i.high



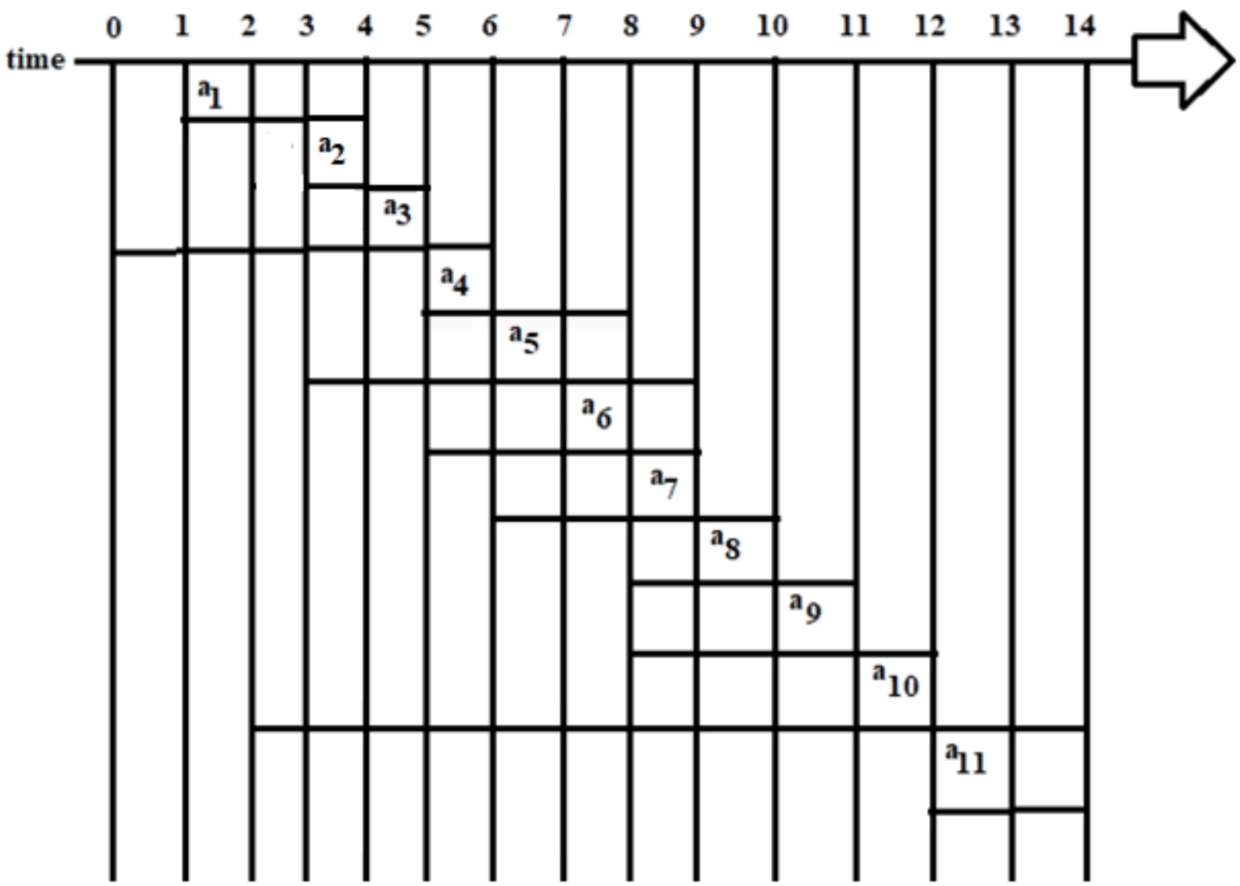


Search going right means i.low > x.left.max (highest endpoint in the left subtree).

If there is no overlap in right => no overlap in left.

Question 1 (c)

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	14



Solution 1 = $\{a_1, a_4, a_8, a_{11}\}$
Solution 2 = $\{a_2, a_4, a_9, a_{11}\}$

As per the given question, we need to select the activity with least duration then the solution with maximum subset of compatible activities are $\{a_2, a_8, a_{11}\}$.

The $\{a_2, a_8, a_{11}\}$ solution doesn't produce maximum subset of all activities, because the $\{a_2, a_4, a_9, a_{11}\}$ and $\{a_2, a_4, a_9, a_{11}\}$ are the optimal solutions.

Question 1 (d)

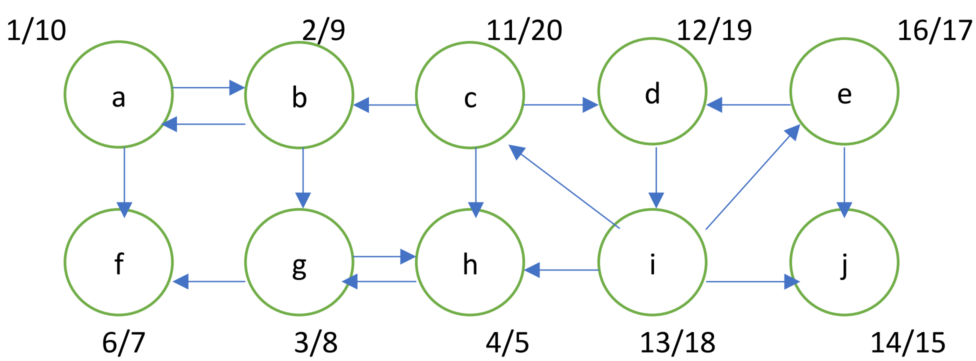
Answer: False

If any instance of X reduces to Y in polynomial time and Y is NP-Complete, it implies that X is almost as hard as Y.

$X \leq_p Y$ ➔ Y is harder than X i.e., if we can solve Y in polynomial time, then we can solve X in polynomial time. If we can't solve X in polynomial time X is incomplete, then Y is NP Complete and not vice versa.

Question – 2

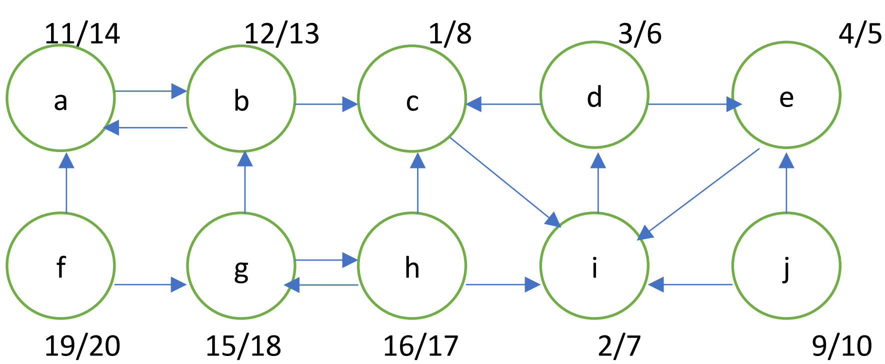
G =



Transpose of the Graph G is as follows

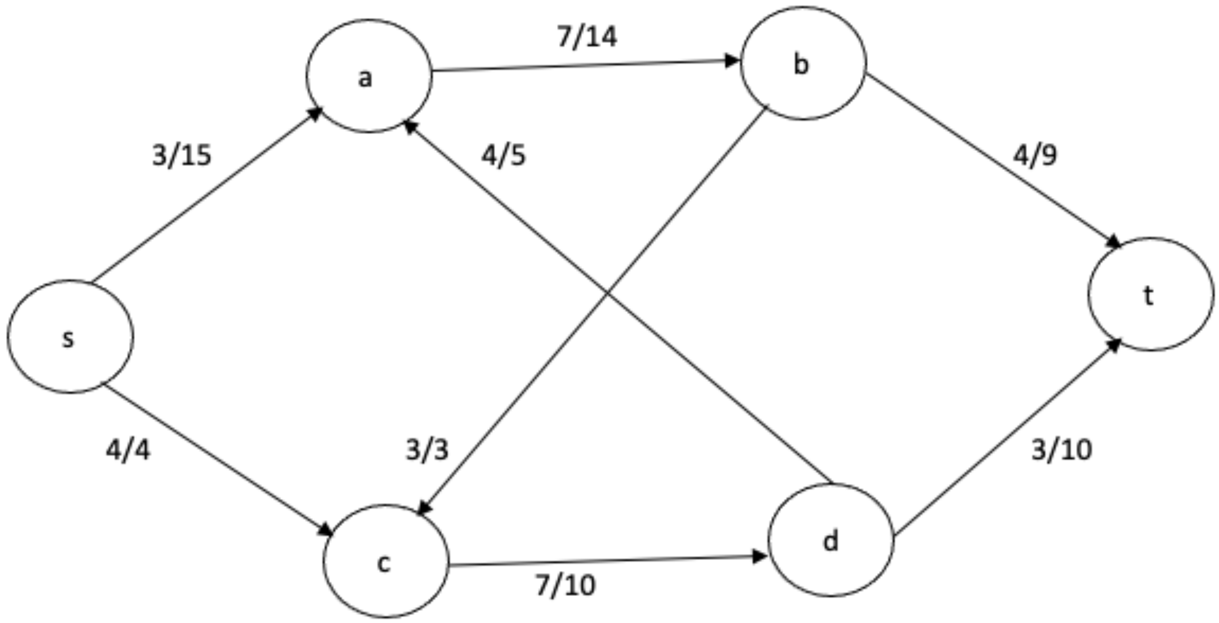
Start considering the nodes with highest finished time from Graph G.

G^T ➔



Simple Connected Components are
SCC1 = {c,d,e,i}
SCC2 = {j}
SCC3 = {a,b}
SCC4 = {g,h}
SCC5 = {f}

Question – 3



Start with s, there are two nodes a,c. But for edge(s,c), f(u,v) = c(u,v).

So, we are left with one edge (s,a).

Now, "a" is our current node and we have only one outgoing edge (a,b).

From "b", we have only one outgoing edge (b,t)

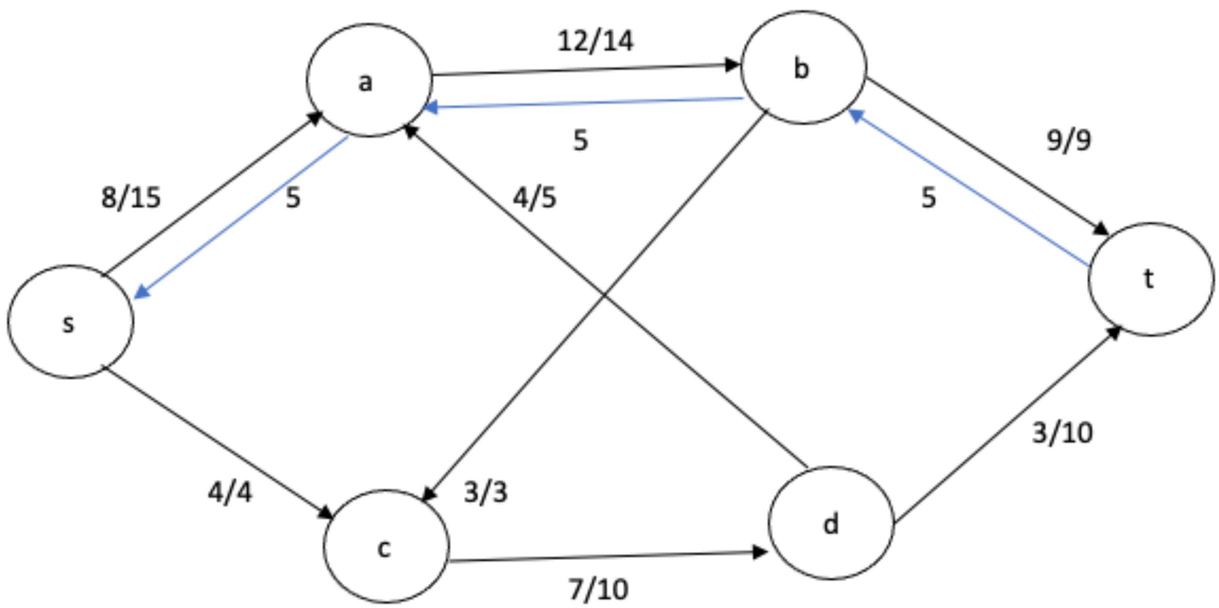
Now, we have reached from s → t.

There is only one path which we can consider:
s → a → b → t

Max residual capacity is the min of all

- ⇨ c(u,v) – t(u,v) i.e..,
- ⇨ min(15-3, 14-7, 9-4)
- ⇨ min(12, 7, 5)

Hence, we could increase the flow in this path by "5".



Other Paths,

- s → c → d → t

  We have residual capacity from s → c as 0.

  So, this could not be considered.

- s → a → b → c → d → t

  We cannot consider this residual capacity from b → c which is 0.

So, we are left with only one s → a → b → t.

Maxflow is sum of f(u, v) is from b → t and d → t, i.e..,

9 + 3 = 12

Question - 4

Given X = 0011001
Y = 1010010

From Equation 15.9 (in the textbook),

$C[i, j] = 0 \quad \text{if } i = 0 \text{ or } j = 0$

$C[i, j] = c[i-1, j-1] + 1 \quad \text{if } i, j > 0 \text{ and } x_i = y_j$

$C[i, j] = \max(c[i, j-1], c[i-1, j]) \quad \text{if } i, j > 0 \text{ and } x_i \neq y$

	Y	1	0	1	0	0	1	0
X	0	0	0	0	0	0	0	0
0	0	↑ 0	↖ 1	← 1	↖ 1	↖ 1	← 1	↖ 1
0	0	↑ 0	↖ 1	↑ 1	↖ 2	↖ 2	← 2	↖ 2
1	0	↖ 1	↑ 1	↖ 2	↑ 2	↑ 2	↖ 3	← 3
1	0	↖ 1	↑ 1	↖ 2	↑ 2	↑ 2	↖ 3	↑ 3
0	0	↑ 1	↖ 2	↑ 2	↖ 3	↖ 3	↑ 3	↖ 4
0	0	↑ 1	↖ 2	↑ 2	↖ 3	↖ 4	← 4	↖ 4
1	0	↖ 1	↑ 2	↖ 3	↑ 3	↑ 4	↖ 5	← 5

LCS(X,Y) = 5

The sequence of LCS is found by encountering the symbol “↖“ in entry b[i,j] which implies $x_i = y_j$ is an element of LCS.

LCS = 0 1 0 0 1

As per the above diagram, the LCS length for given sequence = 5

Question 5

Given array of devices d[1….n], for each d[i], 1 ≤ i ≤ n, d[i].low and d[i].high represent its left and right signal reachable endpoints.

Using the Activity selection problem, we need to consider the high and low endpoints.

The shaded networking devices are a group of 5.