CIS 606 **Homework 2** **Fall 2020**

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1. $F(n) = \Omega(g(n))$ implies $g(n)$ =0$(f(n))$

   Answer: True

   By definition
   $f(n) = \Omega(g(n))$ which implies $O <= c.g(n) <= f(n)$
   $g(n)$ =0$(f(n))$ which implies $O <= g(n) <= c.f(n)$

   Let us assume that $f(n) = 100n^2$, $g(n) = n^2$

   $f(n) >= c.g(n)$
   $100.n^2 >= c.n^2$
   Consider the constant c = 50

   $100n^2 >= 50n^2$
   $2 >= 1$

   $g(n)$ =O$(f(n))$ which is equal to

   $c.f(n) >= g(n)$
   $c.100.n^2 >= n^2$
   $50.100.n^2 >= n^2$
   $5000 >= 1$

   Based on the above equations $F(n) = \Omega(g(n))$ implies $g(n)$ =O$(f(n))$ is true.

2. $T(n) = 2T(n/2) + n^4$

   $T(n) = 2T(n/2) + n^4$ – step 1

   Let us assume that $n = n/2$

   $T(n/2) = 2T(n/4) + (n/2)^4$
   $T(n/2) = 2T(n/4) + n^4/16$ – step 2

   Substitute 2 in 1

   $T(n) = 2[2T(n/4) + n^4/2^4] + n^4$
   $T(n) = 2^2T(n/2^2) + 1/2^3.n^4 + n^4$ – step 3
   Substitute $n = n/2$ in step 2
   $T(n/4) = 2T(n/8) + (n/2)^4.1/16$

$= 2T(n/8) + n^4/2^8$ – step 4

Substitute 4 in 3

$T(n) = 2^2[2T(n/8) + n^4/2^8] + (n^4/2^3) + n^4$
$=2^3.T(n/2)^3 + n^4/2^6 + n^4/2^3 + n^4$
$=2^3(T(n/2^3)) + n^4(1/8^2 + 1/8 + 1)$

After K substitutions

$= 2^kT(n/2^k) + ((2^k)^4).\sum_{k=1}^{k-1}(1/8)$

Let $n = 2^k$, k=logn

$T(n) = nT(1) + n^4, (1/8)^k$
$= n + n^4.(1^logn)/(8^logn)$
$= n + n^4.(0.125)^logn$

As n increases $(0.125)^logn$ decreases and it can be ignored.

So, $T(n) = \theta(n^4)$