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1. How quickly can you multiply a  $kn \times n$  matrix by an  $n \times kn$  matrix, using Strassen's algorithm as a subroutine? Answer the same question with the order of the input matrices reversed.

Let us assume that  $A = kn \times n$  ;  $B = n \times kn$

Based on the above rule, the matrix multiplication  $A \times B = kn \times kn$  which implies that  $k^2 (n \times n)$ .

As per the definition of Strassen's matrix multiplication algorithm, the given matrices are splitted into smaller matrices until the elements are computable.

If the value of  $n=1$ , then the time complexity is  $\theta(1)$ .

If the value of  $n > 1$ , then the time complexity is  $\theta(n^{\log 7})$ . (This is based on section 4.5 and equation 4.18 from textbook using the master's theorem)

Therefore, the value of  $AB$  matrix is  $k^2 n \times n$  matrices and the complexity of each  $n \times n$  matrix is  $\theta(n^{\log 7})$ .

So, the resultant time complexity is  $\theta(K^2 n^{\log 7})$ .

In case if we reverse the order of matrices, then  $BA$  will have only  $k-1$  additions to add those products.

Hence, the time complexity is  $\theta(K n^{\log 7})$ .

2. As per the definition of Karatsuba algorithm,  $(a+bi)(c+di)$  can be computed as  $m_1, m_2, m_3$  where:

$$m_1 = a * c$$

$$m_2 = b * d$$

$$m_3 = (a + b)(c + d)$$

$$(a+bi)(c+di) = m_1 - m_2 + i(m_3 - m_1 - m_2)$$

$$= ac - bd + i((a+b)(c+d) - ac - bd)$$

$$= ac - bd + i(ac + ad + bc + bd - ac - bd)$$

$$= ac - bd + i(ad + bc)$$

Hence,  $ac - bd$  is the real component and  $ad + bc$  is the imaginary component.

3. b)

In best case, the first element can be the search element and it's time complexity is constant which is  $O(1)$ .

In average case, we compute the average of all possible positions and if the algorithm is sufficiently random then total no. of comparisons can be  $n+x$  where  $x$  is a constant which can be ignored.

$$\begin{aligned} E(x) &= \sigma P(x_i) x_i \\ &= 1 \cdot \frac{1}{n} + 2 \cdot \frac{n-1}{n} \cdot \frac{1}{n-1} + 3 \cdot \frac{n-1}{n} \cdot \frac{n-2}{n-1} \cdot \frac{1}{n-2} + \dots + n \cdot \frac{n-1}{n} \cdot \frac{n-2}{n-1} \dots \\ &= \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n}{n} \\ &= \frac{1}{n} (1 + 2 + 3 + \dots + n) - \text{step 1} \end{aligned}$$

where numerator is the no. of comparisons needed.

As per the above equation, the formula for sum on  $n$  positive integers is  $\frac{n(n+1)}{2}$  - Equation A.1 Arithmetic series (page-1146)

Replacing the formula in step 1

$$= \frac{n(n+1)}{2}$$

$$= \frac{n+1}{2}$$

The time complexity of the equation can be derived as

$$= O(n)$$

In Worst case, we can expect to find the search element in last which is  $O(n + x)$  which is equivalent to  $O(n)$  as x is a constant.

c)

Time Complexity to get K occurrences =  $O(n)$

Time complexity to get 1 occurrence = ?

$$k - - > O(n)$$

$$1 - - > ?$$

$$= O(n/k)$$

If there is only element, its time complexity is  $O(n)$ . If there are 'k' occurrences of an element and we terminate it once we find any one, it will be reduced by k times which is  $O(n/k)$ .