CIS 606 **Homework 2** **Fall 2020**

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1. Question 2:
$F(n) = \Omega(g(n))$ implies $g(n)$ =0$(f(n))$
Answer: True
By definition
$f(n) = \Omega(g(n))$ which implies $O <= c.g(n) <= f(n)$
$g(n)$ =0$(f(n))$ which implies $O <= g(n) <= c.f(n)$

Let us assume that $f(n) = 100n^2$, $g(n) = n^2$

$f(n) >= c.g(n)$
$100.n^2 >= c.n^2$
Consider the constant c = 50

$100n^2 >= 50n^2$
$2 >= 1$

$g(n)$ =O$(f(n))$ which is equal to

$c.f(n) >= g(n)$
$c.100.n^2 >= n^2$
$50.100.n^2 >= n^2$
$5000 >= 1$
Based on the above equations $F(n) = \Omega(g(n))$ implies $g(n)$ =O$(f(n))$ is true.

2. Question 3:
$T(n) = 2T(n/2) + n^4$

$T(n) = 2T(n/2) + n^4$ – step 1

Let us assume that $n = n/2$
$T(n/2) = 2T(n/4) + (n/2)^4$
$T(n/2) = 2T(n/4) + n^4/16$ – step 2
Substitute 2 in 1
$T(n) = 2[2T(n/4) + n^4/2^4] + n^4$
$T(n) = 2^2T(n/2^2) + 1/2^3.n^4 + n^4$ – step 3
Substitute $n = n/2$ in step 2
$T(n/4) = 2T(n/8) + (n/2)^4.1/16$
$= 2T(n/8) + n^4/2^8$ – step 4
Substitute 4 in 3
$T(n) = 2^2[2T(n/8) + n^4/2^8] + (n^4/2^3) + n^4$
$=2^3.T(n/2)^3 + n^4/2^6 + n^4/2^3 + n^4$
$=2^3(T(n/2^3)) + n^4(1/8^2 + 1/8 + 1)$
After K substitutions
$= 2^kT(n/2^k) + ((2^k)^4).\sum_{k=1}^{k-1}(1/8)$
Let $n = 2^k$, k=logn

$T(n) = nT(1) + n^4, (1/8)^k$
$= n + n^4.1^{logn}/8^{logn}$
$= n + n^4.(0.125)^{logn})$
As n increases $(0.125)^{logn}$ decreases and it can be ignored.

So, $T(n) = \theta(n^4)$

3. Question 4:
$T(n) = 16T(n/4) + n^2$
$T(n) = 16T(n/4) + n^2$ – step 1
Substitute $n = n/4$ in step 1
$T(n) = 16T(n/16) + (n^2)/(4^2)$ – step 2
Substitute step 2 in step 1
$T(n) = 16(16.T(n/16) + n^2/16) + n^2$
$= (16^2)T(n/16) + n^2 + n^2$ – step 3
substitue $n = n/4$ in step 2

$T(n/16) = 16T(n/64) + n^2/256$ – step 4
Substiture step 4 in step 3
$T(n) = 16^2(16.T(n/64) + n^2/16^2) + n^2 + n^2$
$=16^3T(n/64) + 3n^2$
$=4^6.T(n/4^3) + 3n^2$
After k substitutions
$=4^2kT(n/4^k) + kn^2$
$= n^2.T(1) + n^2log_4n$
4^k = n
k = log_4n
By ignoring the lower order terms and constants
$T(n) = \theta(n^2Logn)$