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1. $F(n) = \Omega(g(n))$ implies $g(n) = O(f(n))$

Answer: True

By definition

$$f(n) = \Omega(g(n)) \text{ which implies } O \leq c.g(n) \leq f(n)$$

$$g(n) = O(f(n)) \text{ which implies } O \leq g(n) \leq c.f(n)$$

Let us assume that $f(n) = 100n^2$, $g(n) = n^2$

$$f(n) \geq c.g(n)$$

$$100.n^2 \geq c.n^2$$

Consider the constant $c = 50$

$$100n^2 \geq 50n^2$$

$$2 \geq 1$$

$g(n) = O(f(n))$ which is equal to

$$c.f(n) \geq g(n)$$

$$c.100.n^2 \geq n^2$$

$$50.100.n^2 \geq n^2$$

$$5000 \geq 1$$

Based on the above equations $F(n) = \Omega(g(n))$ implies $g(n) = O(f(n))$ is true.

2. $T(n) = 2T(n/2) + n^4$

$$T(n) = 2T(n/2) + n^4 - \text{step 1}$$

Let us assume that $n = n/2$

$$T(n/2) = 2T(n/4) + (n/2)^4$$

$$T(n/2) = 2T(n/4) + n^4/16 - \text{step 2}$$

Substitute 2 in 1

$$T(n) = 2[2T(n/4) + n^4/2^4] + n^4$$

$$T(n) = 2^2T(n/2^2) + 1/2^3.n^4 + n^4 - \text{step 3}$$

Substitute $n = n/2$ in step 2

$$T(n/4) = 2T(n/8) + (n/2)^4.1/16$$

$$= 2T(n/8) + n^4/2^8 - \text{step 4}$$

Substitute 4 in 3

$$\begin{aligned} T(n) &= 2^2[2T(n/8) + n^4/2^8] + (n^4/2^3) + n^4 \\ &= 2^3 \cdot T(n/2^3) + n^4/2^6 + n^4/2^3 + n^4 \\ &= 2^3(T(n/2^3)) + n^4(1/8^2 + 1/8 + 1) \end{aligned}$$

After K substitutions

$$= 2^k T(n/2^k) + ((2^k)^4) \cdot \sum_{k=1}^{k-1} (1/8)$$

Let $n = 2^k$, $k = \log n$

$$\begin{aligned} T(n) &= nT(1) + n^4 \cdot (1/8)^k \\ &= n + n^4 \cdot 1^{\log n} / 8^{\log n} \\ &= n + n^4 \cdot (0.125)^{\log n} \end{aligned}$$

As n increases $(0.125)^{\log n}$ decreases and it can be ignored.

So, $T(n) = \theta(n^4)$

$$\begin{aligned} 3. \quad 4 \quad T(n) &= 16T(n/4) + n^2 \\ T(n) &= 16T(n/4) + n^2 - \text{step 1} \end{aligned}$$

Substitute $n = n/4$ in step 1

$$T(n) = 16T(n/16) + (n^2)/(4^2) - \text{step 2}$$

Substitute step 2 in step 1

$$T(n) = 16(16T(n/16) + n^2/16) + n^2$$

$$= (16^2)T(n/16) + n^2 + n^2 - \text{step 3}$$

substitutue $n = n/4$ in step 2

$$T(n/16) = 16T(n/64) + n^2/256 - \text{step 4}$$

Substitute step 4 in step 3

$$T(n) = 16^2(16.T(n/64) + n^2/16^2) + n^2 + n^2$$

$$= 16^3 T(n/64) + 3n^2$$

$$= 4^6 . T(n/4^3) + 3n^2$$

After k substitutions

$$= 4^{2k} T(n/4^k) + kn^2$$

$$= n^2 . T(1) + n^2 \log_4 n$$

$$4^k = n \quad k = \log_4 n$$

By ignoring the lower order terms constants

$$T(n) = \theta(n^2 \log n)$$