CIS 606 **Homework 3** **Fall 2020**

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1. How quickly can you multiply a kn xn matrix by an n x kn matrix, using Strassens algorithm as a subroutine? Answer the same question with the order of the input matrices reversed.

   Let us assume that A = kn x n ; B =n x kn
   Based on the above rule, the matrix multiplication A x B = kn x kn which implies that k^2 (n x n).
   As per the definition of Strassen's matrix multiplication algorithm, the given matrices are splitted into smaller matrices until the elements are computable.
   If the value of n=1, then the time complexity is $\theta(1)$.
   If the value of $n > 1$, then the time complexity is $\theta(n^{log7})$. (This is based on section 4.5 and equation 4.18 from textbook using the master's theorem)
   Therefore, the value of AB matrix is k^2 n x n matrices and the complexity of each n x n matrix is $\theta(n^{log7})$.
   So, the resultant time complexity is $\theta(K^2 n^{log7})$.
   In case if we reverse the order of matrices, then BA will have only k-1 additions to add those products.
   Hence, the time complexity is $\theta(Kn^{log7})$.

2. As per the definition of Karatsuba algorithm, (a+bi)(c+di) can be computed as m_1, m_2, m_3 where:
   $m_1 = a * c$
   $m_2 = b * d$
   $m_3 = (a + b)(c + d)$
   (a+bi)(c+di) = $m_1 - m_2 + i(m_3 - m_1 - m_2)$
   = ac - bd + i((a+b)(c+d) - ac - bd)
   = ac-bd + i(ac+ad+bc+bd-ac-bd)
   = ac - bd + i(ad+bc)
   Hence, ac-bd is the real component and ad+bc is the imaginary component.

3. b)
   In best case, the first element can be the search element and it's time complexity is constant which is O(1).
   In average case, we compute the average of all possible positions and if the algorithm is sufficiently random then total no.of comparisons can be n+x where x is a constant which can be ignored.
   $E(x) = \sigma P(x_i)x_i$
   $= 1.\frac{1}{n} + 2.\frac{n-1}{n}.\frac{1}{n-1} + 3.\frac{n-1}{n}.\frac{n-2}{n-1}.\frac{1}{n-2} + ... + n.\frac{n-1}{n}.\frac{n-2}{n-1}..$
   $= \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + ... + \frac{n}{n}$
   $=\frac{1}{n}(1 + 2 + 3 + ...n)$ – step 1
   where numerator is the no. of comparisons needed.
   As per the above equation, the formula for sum on n positive integers is $\frac{n(n+1)}{2}$ – Equation A.1 Arithmetic series (page-1146)

Replacing the formula in step 1
$=\frac{n(n+1)}{2*n}$
$=\frac{n+1}{2}$
The time complexity of the equation can be derived as
$=O(n)$
In Worst case, we can expect to find the search element in last which is $O(n+x)$ which is equivalent to $O(n)$ as x is a constant. c)
Time Complexity to get K occurences = O(n) Time complexity to get 1 occurence = ? k −¿ O(n) 1 −¿ ? $=O(n/k)$ If there is only element, its time complexity is $O(n)$. If there are 'k' occurences of an element and we terminate it once we find any one, it will be reduced by k times which is $O(n/k)$.