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Question-1:

a) Code for inner product of vectors X and Y

```
import numpy as np
x = np.array([4, 5, 6,8,9])
y = np.array([7, 10, 1, 2, 3])
print("Original vectors:")
print(x)
print(y)
print("Inner product of said vectors:")
print(np.dot(x, y))
```

Output:

```
Original vectors:
[4 5 6 8 9]
[ 7 10  1  2  3]
Inner product of said vectors:
127
```

The time complexity of inner product of two vectors is θ(n).

b) Recursive function to calculate inner product of X and Y using divide and conquer approach.

```
int innerproduct(int vector_x[], int vector_y[])
{
        int product = 0;
        for (int i = 0; i < size; i++)
        product = product + vector_x[i] * vector_y[i];
        return product;
}

int main()
{
      int vector_a[] = { 4, 2, -1 };
      int vector_b[] = { 5, 7, 1 };
      int temp[size];
      cout << "Inner product:";
     cout << innerproduct(vector_a, vector_b) << endl;
     for (int i = 0; i < size; i++)
          cout << temp[i] << " ";
     return 0;
}
```

The recurrence relation for the above program is as follows:

$T(n) = 2T(n/2) + 1$

Using the masters theorem, the values are $a = 2$, $b = 2$, $c = 1$

The time complexity is $\theta(n \log n)$ [This is based on equation 4.7 from textbook].

Question – 2:

The algorithm that is used for finding the median of 2n elements is as follows:

The middle element of X and Y arrays:

if(X==Y) then this is the median

if (X[midsize] < Y[midsize]), then the median is median of elements from left side of X and elements from right side of Y.

if (X[midsize] > Y[midsize]), then the median is median of elements from left side of Y and elements from right side of X.

Algorithm findMedian(X, Y, m):

```
if m == 1,
   return (X[0]+Y[0])/2
midsize = [n/2]
         if X[midsize] == Y[midsize]
                   return X[mid]
         if X[midsize] < Y[midsize]
                  return findMedian(X[0..midsize], Y[midsize+1..n])
return findMedian(Y[0..midsize], X[midsize+1..n])
```

Therefore, time complexity:

$$T(n) = T(n/2) + O(1)$$

As per the master theorem

$a = 1$, $b = 2$, and $f(n) = O(1)$

Therefore, $T(n) = O(n \text{ Log } n) = O(\text{ Log } n)$

Question – 4:

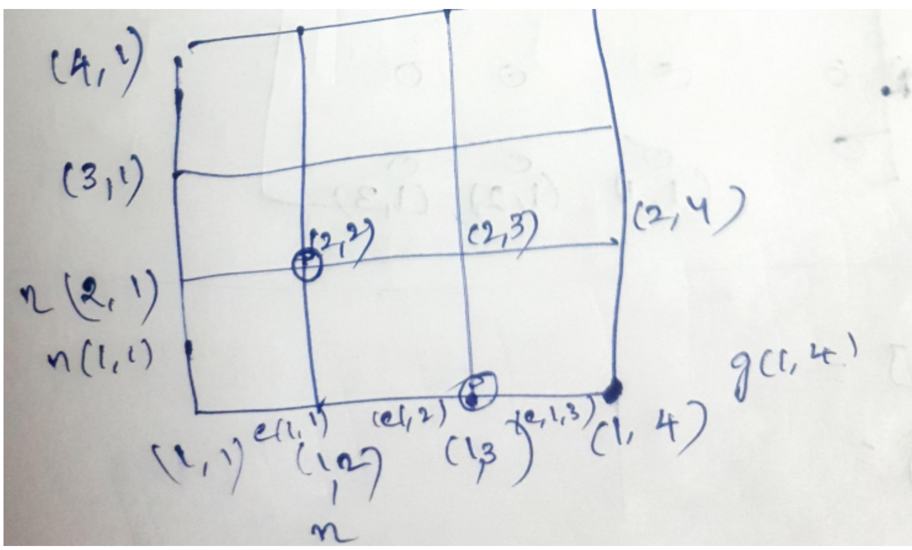
$$\text{Matrix n} = \begin{array}{c} \begin{bmatrix} 7 & 4 & 8 & 17 \\ 16 & 17 & 11 & 23 \\ 15 & 12 & 18 & 11 \end{bmatrix} \\ \begin{matrix} 11 & 12 & 13 & 14 \end{matrix} \end{array}$$

$$\text{Matrix e} = \begin{array}{c} \begin{bmatrix} 5 & 10 & 5 \\ 10 & 15 & 12 \\ 20 & 23 & 17 \\ 10 & 12 & 15 \end{bmatrix} \\ \begin{matrix} 11 & 12 & 13 \end{matrix} \end{array}$$

From g[1][1], there are two paths. We can move to either g[1][2] or g[2][1]. Similarly, there are two ways to move from any one place to the next place and so on.

So, to reach g[m][m], it should reach from g[m-1][m] or g[m][m-1].

Cost to move all intermediate are accumulated and at each step based on minimum cost to reach from start point to current position, path is chosen.



$$g[i][j] = \sum_{i=2,j=2}^{m,n} \min(\, g[i][j-1] + e[i][j-1], g[i-1][j] + n[i-1][j] \,)$$

g[i][j] = g[i-1][j] + n[i-1][j]   j = 1, j > 1

```
g[i][j] = g[i][j-1] + e[i][j-1]      i = 1, j > 1

g[i][j] = 0                          i = 1, j = 1
```

b. Shortest Path (n,e):

```
initialize g[m][m] to 0
initialize path[m][n] to (0,0)
i = 1
for j in range of (m)
        g[i][j] = g[i][j-1] + e[i][j-1]
        path[i][j] = (i,j-1)
j =1
for i in range(0,m)
        g[i][j] = g[i=1][j] + n[i-1][j]

        path[i][j] = (i-1,j)
for i in range (1,m) :   → m times
for j in range (1,m):    → m times

if( g[i][j-1] + e[i][j-1] > g[i-1][j] + n[i-1][j] )
        g[i][j] = g[i-1][j] + n[i-1][j]
        path[i][j] = (i-1,j)
else
        g[i][j] = g[i][j-1] + e[i][j-1]
        path[i][j] = (i,J-1)
```

g[i][j] contains mincost and path[i][j] contains previous node and by traversing from path[i][j] to path[o][o], we can get shortest path.

Initialize shortest [n] to (0,0)

c. Time Complexity is **O(m2)** where, m is the size or length of the matrix.

There will be m2 ways or paths in which we can travel from source to destination i.e..,
rows*columns.