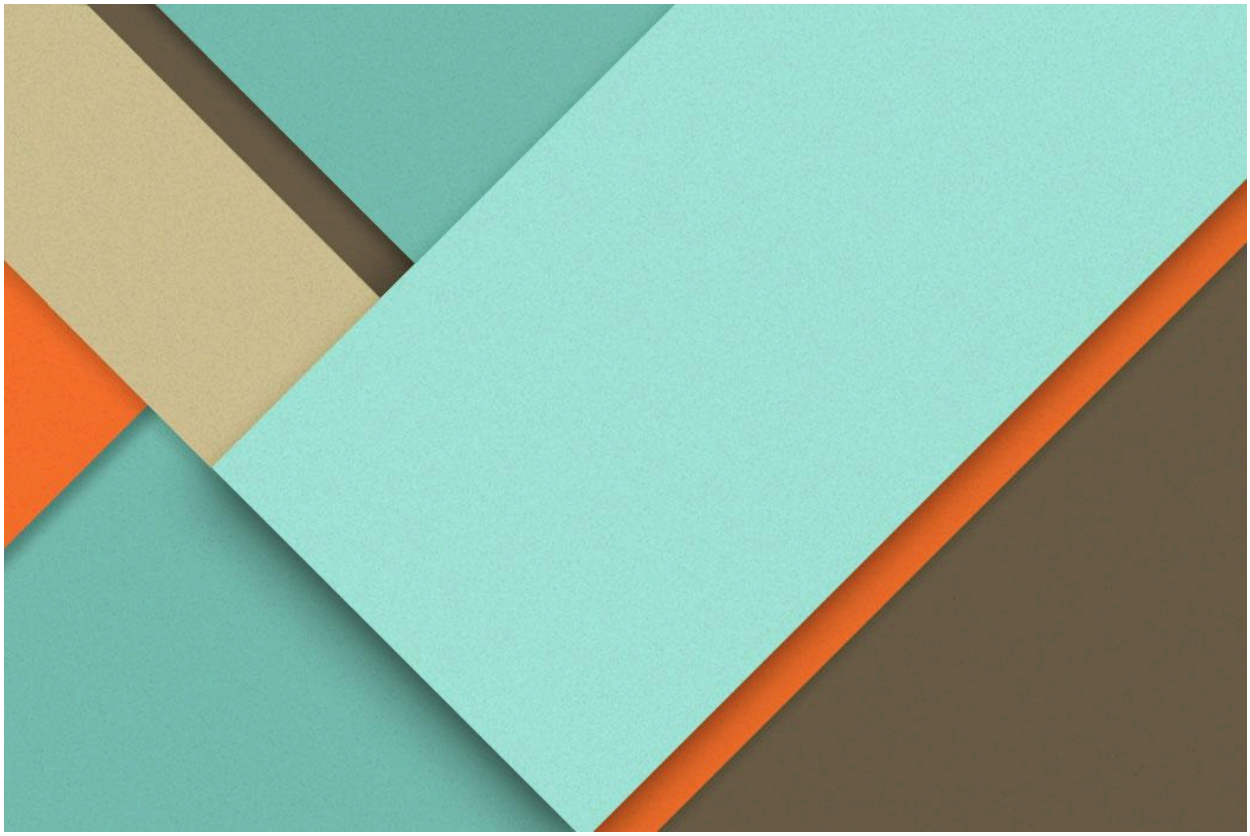

Credit Risk Management

29/08/2024

Team Members:

Sachin Bansal

Nitin Kumar

Shivprasad Nirmale

Introduction

The objective of this model is to estimate the Expected loss associated with lending to borrowers in the event of a default. Risk Parameters like Exposure at Default(EAD) , Probability of Default(PD), Loss Given Default(LGD) are used to calculate the Expected Loss.

Expected Loss is the anticipated loss on a portfolio of loans. It represents the average loss a lender can expect to incur in case borrower defaults.

Goals

1. Estimate the Expected loss of a financial institution in lending a borrower in case of a default.
2. Calculate Exposure at default(EAD) , Probability of Default(PD), Loss Given Default(LGD), to calculate Expected Loss(EL)

Context

Financial institutions like banks and corporations must be aware of the credit risk associated with lending to borrowers in the event of a default. So it is important to have a robust model to estimate the expected loss.

Key components

Overview of Risk Parameters(Modelling Components) :

1. **Exposure at Default(EAD)** : EAD is the total value that a lender is exposed to when a borrower defaults. It is viewed as an important tool in the calculation of Expected Losses(EL) when a borrower fully defaults the loan.
2. **Probability of Default(PD)** : It is the likelihood that a borrower will default on their obligations within a specific time frame.

3. **Loss Given Default(LGD)** : It is the portion of the Exposure that the lender will lose when the borrower defaults. It accounts for recoveries.

Data Collection

We have used the Historical data of the stock prices and last 5 years annual balance sheet of the **Reliance Infra (RELIANCEINFRA.NS)** . We have taken the data from Yahoo finance.

Methodology

$$EL = EAD \times PD \times LGD$$

Expected Loss(EL) is calculated using the Risk parameters PD, LGD and EAD. It is the product of EAD and PD and LGD.

1. Exposure at Default

Definition

EAD is an estimated value to what extent the lender can suffer loss in case of borrower's default. It is equal to the current outstanding loan along with any further amounts that may be defaulted

Factors Contributing to EAD:

★ **Outstanding Debt :**

It is the amount of debt that the borrower already owes to the lender. Thus, it includes the total amount the borrower has withdrawn along with interest.

★ **Unused Commitment :**

It is the available credit that is unutilised by the borrower. Although it is not currently borrowed, it can still be drawn up to the the time of default, hence increasing EAD.

★ **UGD (Usage Given Default) :**

It is the percentage of the unused commitment that is likely to be drawn before the time of default. It refers to the extent to which a borrower may utilize available credit upto default. It refers to what extent the borrower may utilize available credit before defaulting.

It is worth emphasizing that companies with higher probability to default(e.g. CCC rated companies) are expected to have lower UGD than that of companies with lower probability(AAA rated companies). This is mainly due to following reasons:

➢ **Higher Risk of Default :**

Given significant risk of default for lower rated companies, they are highly vulnerable to not being able to clear dues in case it uses more of unused commitment.

➢ **Lender Restrictions and Monitoring :**

Lenders are more likely to closely monitor lower rated companies by intervening or imposing additional constraints. Moreover, sometimes loan agreements include provisions that allow lenders to limit further borrowing if the financial situation worsens.

➢ **Liquidity Management Concerns :**

Lower rated companies often have weaker cash flows so they have to manage liquidity carefully. Drawing down from unused commitments further increases debt and interests which can afterwards strain their cash flows.

In a nutshell, lower rated companies are highly vulnerable to non-clearance of dues and pose significant risk of default. Hence, they are wary of increasing debts than they already are.

UGD Table for S&P rated companies :

Rating	UGD
AAA	69%
AA	73%
A	71%
BBB	65%
BB	52%
B	48%
CCC	44%

Micheal Ong : Internal Credit Risk Model :-

The value of EAD is given as :

EAD = Outstanding Debt + (Unused Commitment * UGD)

Calculation :

Outstanding Debt = 99.42 billion Rs.

Unused Commitment = 30 billion Rs.

UGD(in %age) = 65%

*EAD = 99.42 + (30 * 65%)*

= 118.92

EAD = 118.92 billion Rs

2. Probability of Default(PD)

Introduction

It is the likelihood that a borrower will default on the debt obligations within a specific time frame.

We will use the Merton Model to calculate PD.

Data Input

We will use the historical data of stocks and annual balance sheet of Reliance infra of the past 5 years.

Merton Model

- The Merton model calculates the probability of default based on the structure of a firm's balance sheet.
- The model assumes that a firm will default if the value of its assets falls below the value of its debt obligations.
- Key input variable is the **Value of Assets** at any given point of time.
- If the value of firm's assets at time T exceeds the value of its debt ($A_t > D_t$) , the debt is fully repaid, and remaining value is attributed to the equity holders

The value of a firm's assets is modeled as a stochastic process, following the following geometric Brownian Motion.

$$dA_t = \mu_A A_t \, dt + \sigma_A A_t \, dW_t$$

where μ_A is the drift rate, σ_A is the volatility of assets, and dW_t represents a Wiener process (standard Brownian motion).

- In this framework-

- ➢ **The shareholders hold a call option** to buy the firm's assets by repaying the debt. The option will be exercised if the value of the assets (A_t) exceeds the debt(D_t) at the maturity.
- ➢ In case $A_t > D_t$, the shareholders exercise the option and repay the debt, and the remaining value will be retained as equity.
- ➢ If At is less than Dt the option exercising is worthless, meaning the shareholders will not repay the debt and the equity value will be null.

❖ For Debt Holders-

- ★ **The debt holders own a put option** on the firm's assets. If the firm's asset value falls below the debt value, the shareholders default and the debt holders exercise their put option and take ownership of the firm's assets.

The solution to the stochastic differential equation of the firm's asset will be as follows:

$$A_t = A_0 \exp\left(\left(\mu_A - \frac{1}{2}\sigma_A^2\right) t + \sigma_A W_t\right)$$

At maturity T, the asset value will be -

$$A_T = A_0 \exp\left(\left(\mu_A - \frac{1}{2}\sigma_A^2\right) T + \sigma_A W_T\right)$$

The value of debt will be assumed as constant at maturity.

The Probability of Default (PD) is the likelihood that A_t will be less than or equal to D_t

$$PD = P(A_t <= D_t)$$

Taking the natural logarithm in the solution of the stochastic differential equation of the firm's asset we get-

$$\ln(A_T) = \ln(A_0) + \left(\mu_A - \frac{1}{2}\sigma_A^2\right) T + \sigma_A W_T$$

Therefore,

$$\frac{\ln\left(\frac{A_T}{D_T}\right) + \frac{1}{2}\sigma_A^2 T - \mu_A T}{\sigma_A\sqrt{T}} \sim \mathcal{N}(0,1)$$

Thus, Probability of default is equal to

$$\text{PD} = N(-d_2)$$

where d_2 is given by:

$$d_2 = \frac{\ln\left(\frac{A_T}{D_T}\right) + \left(r - \frac{\sigma_A^2}{2}\right) T}{\sigma_A\sqrt{T}}$$

Result : The formulae for the probability of default is the normal CDF of d2.

Interpretation from the Input Data

From the last year's Balance sheet data we can get the data like Asset value , Debt value.

Asset Value(A_t) = 591,808,900 (0.591 billion Rs.)

Debt Value(Asset-Equity) = 453,227,700(0.453 billion Rs.)

*Annualized Volatility (std.dev * sqrt(252)) = 64.45%*

Time to Maturity = 1 year

Risk Free rate = 0.05(5%)

Know we know the formulae of PD so we have toh find the value of d_2 first,

The value of d_2 after using the formulae(derived above) will be - 0.6586

And we know that,

$$PD = N(-d_2)$$

So the value of PD we get from here is :- 25.51%

Probability of default = 25.51%

3. Loss given default(LGD)

Definition:

- ★ Loss Given Default (LGD) is the percentage of the exposure that a lender expects to lose if a borrower defaults. It represents the severity of the loss when a default occurs.
- ★ LGD is a measure of the potential loss severity in the event of default, taking into account any recoveries that the lender might receive from the collateral, guarantees, or other sources.
- ★ LGD is influenced by several factors, it includes the quality and liquidity of the collateral, the seniority of the debt, and the legal and regulatory environment.
- ★ For example, if a loan has an LGD of 70%, it means that if the borrower defaults, the lender expects to recover 30% of the exposure, while the remaining 70% would be lost.
- ★ Loans secured by high-quality collateral generally have lower LGD, while unsecured loans or loans in distressed sectors may have higher LGD.

Introduction to Beta Distribution:

The Beta distribution is a continuous probability distribution defined on the interval [0, 1]. It is parameterized by two shape parameters, α and β, which control the distribution's shape. The probability density function (PDF) is given by:

$$f(x;\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, \quad 0 \le x \le 1,$$

Where B(α,β) is the Beta function, defined as:

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt.$$

Properties of Beta Distribution

The Beta distribution is flexible and can model a variety of shapes depending on α and β:

- Symmetric if α = β.
- Left-skewed if α < β.
- Right-skewed if α > β.

The mean and variance are given by:

$$\text{Mean} = \frac{\alpha}{\alpha+\beta},$$

$$\text{Variance} = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}.$$

Where,

α= center parameter of beta distribution

β= shape parameter of beta distribution

Parameter Estimation

To fit a Beta distribution to LGD data, we estimate the shape parameters α and β using methods such as Maximum Likelihood Estimation (MLE). Given a dataset {LGD_1, LGD_2, . . . , LGD_n}, we maximize the likelihood function:

$$L(\alpha, \beta) = \prod_{i=1}^{n} f(\mathrm{LGD}_i; \alpha, \beta).$$

The estimated parameters αˆ and βˆ are those that maximize L(α, β).

$$\alpha = \frac{\mu}{\max}\left[\frac{\mu\,(Max - \mu)}{Max\ \sigma^2} - 1\right] \quad and \quad \beta = \alpha\left[\frac{Max}{\mu} - 1\right]$$

Where,

α = implied alpha

β= Implied beta

max= set 1.1 for bonds , otherwise is 1.0

LGD = 1- mean recovery

Where,

Mean recovery = Implied alpha/(Implied alpha + implied beta)

Calculation

LGD data for the past years given below:

TABLE 3

LGD BY YEAR OF DEFAULT

	OBLIGOR LEVEL		OBLIGATION LEVEL	
YEAR OF DEFAULT	NUMBER OF DEFAULTS	LGD	NUMBER OF FACILITIES	LGD
2000	456	35%	742	35%
2001	837	33%	1,412	32%
2002	875	29%	1,363	27%
2003	654	23%	1,039	22%
2004	291	20%	491	17%
2005	344	19%	565	19%
2006	346	19%	535	20%
2007	412	29%	756	29%
2008	1,151	31%	2,000	30%
2009	1,926	20%	3,544	19%
2010	1,019	19%	1,833	19%
2011	727	22%	1,311	22%
2012	765	19%	1,418	22%
2013	549	20%	911	20%
2014	350	24%	602	23%
2015	351	28%	544	27%
2016	474	13%	739	15%
TOTAL	11,527	24%	19,805	23%

Here we are using the LGD of Number of defaults of the past 16 years.

By calculating Maximum likelihood estimation of this LGD data,we get

mean=23.51%

Standard deviation=6.07%

Alpha (center)=11.39

Beta (shape)=36.69

Implied alpha=9.40

Implied beta=34.22

Mean recovery=21.55%

LGD=78.45%

Expected loss calculation

EL = EAD*PD*LGD

EL = 118.92 * 25.51*78.45

EL = 23.798 bn Rs.

Hence the Expected loss that lender will incur in event that Reliance infra will default is 23.798 bn Rs.