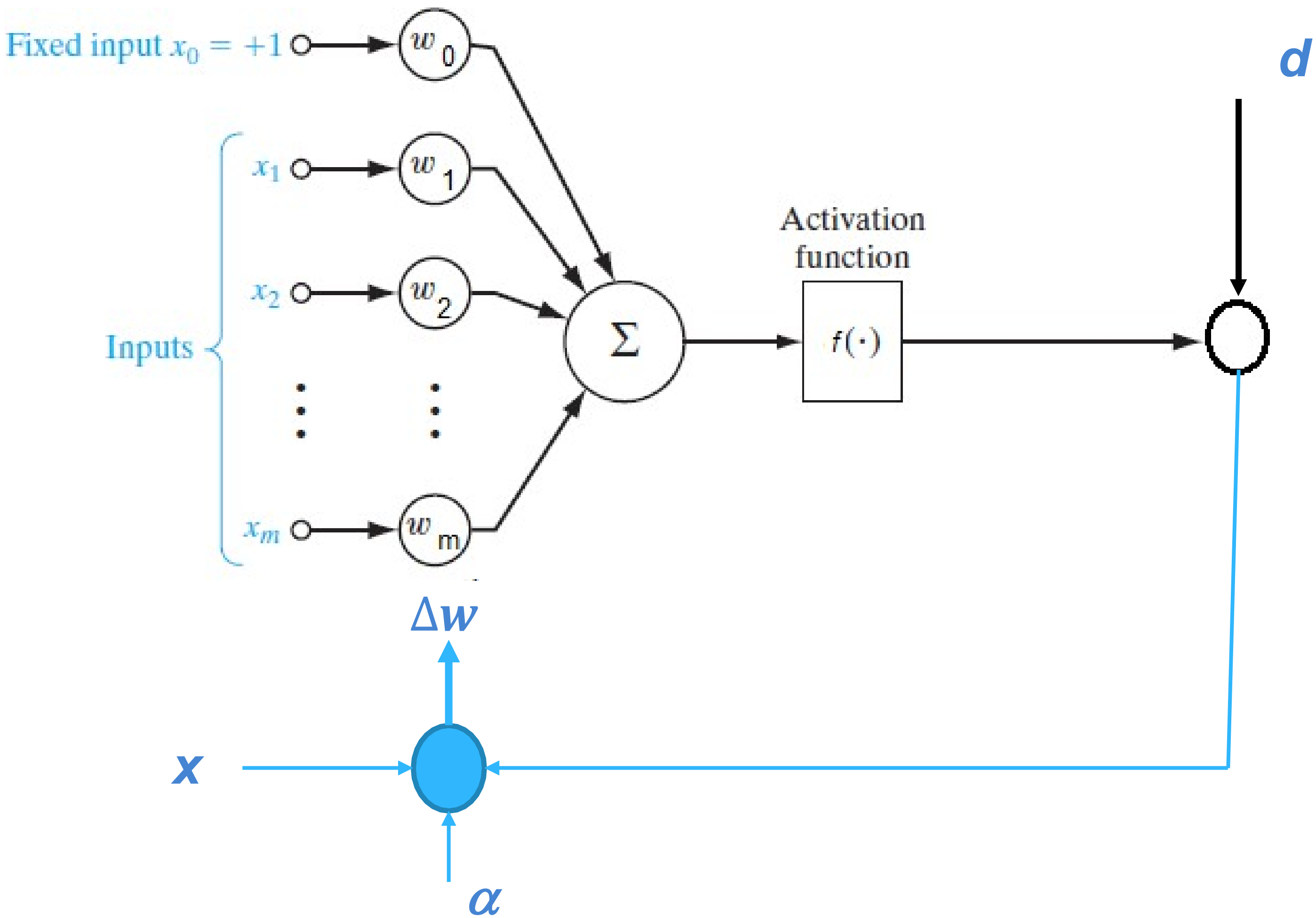
DELTA RULE LEARNING



DELTA RULE LEARNING: SINGLE NEURON

$$\boldsymbol{net} = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}$$

Using a linear activation function $\boldsymbol{y} = \boldsymbol{f}(\mathbf{net})$

The error between the network output and the desired output is

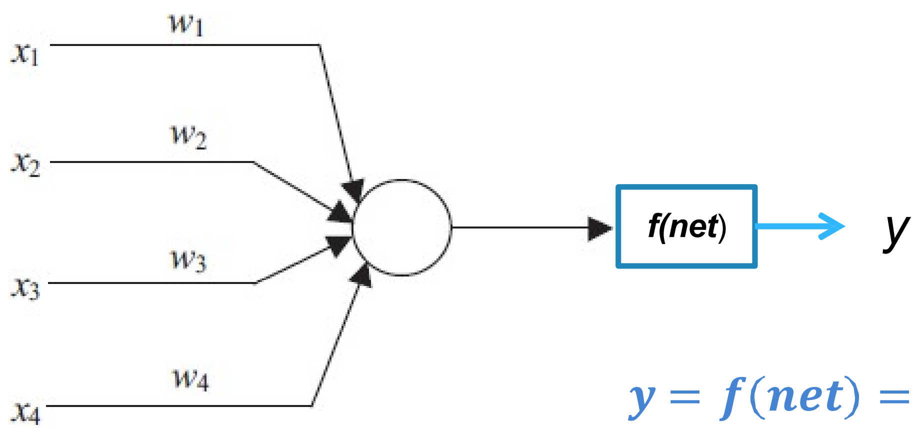
$$\boldsymbol{E} = \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{d} - \boldsymbol{y})^{\mathbf{2}}$$

The derivative w.r.t. weights is $$\frac{\boldsymbol{dE}}{\boldsymbol{dw_i}} = \frac{\boldsymbol{dE}}{\boldsymbol{dy}}\frac{\boldsymbol{dy}}{\boldsymbol{dw_i}} = -(\boldsymbol{d} - \boldsymbol{y})\boldsymbol{f}'(\boldsymbol{net})\,\boldsymbol{x_i}$$

Update the weighs using delta rule $$\boldsymbol{w_i} = \boldsymbol{w_i} - \boldsymbol{\alpha}\frac{\boldsymbol{dE}}{\boldsymbol{dw_i}}$$

In vector format: $\boldsymbol{W} = \boldsymbol{W} - \boldsymbol{\alpha}\,\nabla\boldsymbol{E}$

EXAMPLE



$$y = f(net) = \frac{1 - e^{-net}}{1 + e^{-net}}$$

$$Note: y' = 0.5(1 - y^2)$$

$$w^1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}, x^1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix}, x^2 = \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} \text{ and } x^3 = \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$$

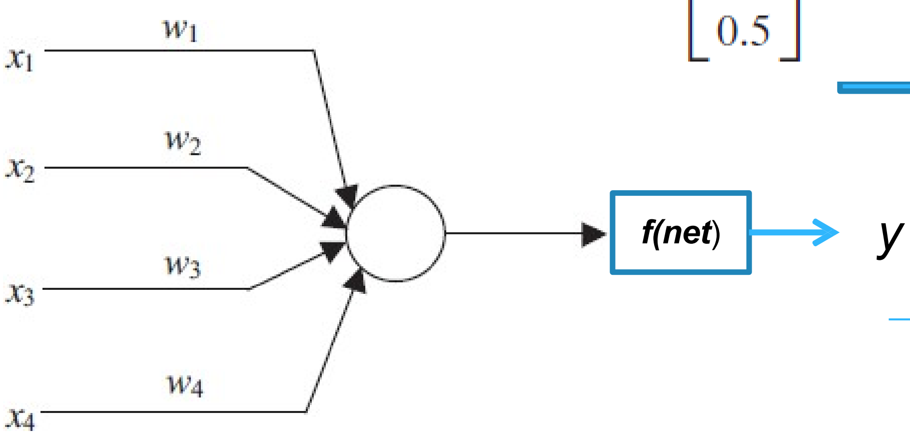
Desired output, d = [-1 -1 1]

Use **Delta Rule** learning to update the weights

with $\alpha = 0.1$

EXAMPLE

$$w^1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix},\ x^1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix},\ x^2 = \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} \text{ and } x^3 = \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$$



Desired output, d = [-1 -1 1]

Iteration One
Pattern One

$$net = w^T x = [1 \quad -1 \quad 0 \quad 0.5]\begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix} = 2.5$$

$$y = \frac{1 - e^{-net}}{1 + e^{-net}} \Rightarrow y = 0.848$$

$$y' = 0.5\,(1 - y^2) = 0.140$$

$$\nabla E = -\alpha\,(d - y)\,y' x$$

$$= -0.1\,(-1 - 0.848)(0.14)\begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix}$$

$$= 0.0259\begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.0259 \\ -0.0518 \\ 0 \\ -0.0259 \end{bmatrix}$$

$$w = w - \nabla E = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} - \begin{bmatrix} 0.0259 \\ -0.0518 \\ 0 \\ -0.0259 \end{bmatrix} = \begin{bmatrix} 0.9741 \\ -0.9482 \\ 0 \\ 0.5259 \end{bmatrix}$$

EXAMPLE

$$w^1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}, x^1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix}, x^2 = \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} \text{ and } x^3 = \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$$

Desired output, d = [-1 -1 1]

Pattern Two

$$net = w^T x$$

$$= [\,0.974 \quad -0.948 \quad 0 \quad 0.5259]\begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix}$$

$$= -1.948$$

$$y = \frac{1 - e^{-net}}{1 + e^{-net}} \Rightarrow y = -0.7505$$

$$y' = 0.5\,(1 - y^2) = 0.2184$$

$$\nabla E = -\alpha\,(d - y)\,y' x$$

$$= -0.1\,(-1 + 0.7505)(0.2184)\begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.0082 \\ -0.0027 \\ -0.0054 \end{bmatrix}$$

$$w = w - \nabla E = \begin{bmatrix} 0.974 \\ -0.948 \\ 0 \\ 0.5259 \end{bmatrix} - \begin{bmatrix} 0 \\ 0.0082 \\ -0.0027 \\ -0.0054 \end{bmatrix} = \begin{bmatrix} 0.9741 \\ -0.9563 \\ 0.0027 \\ 0.5314 \end{bmatrix}$$