LOGARITHMS

IMPORTANT FORMULAS

1. **Logarithm:**
   If a is a positive real number, other than 1 and $a^m = x$, then we write:
   m* = log$_a$*x and we say that the value of log x to the base a is m.
   Examples:
   (i). 10^3 1000 $\Rightarrow$ $\log_{10} 1000 = 3$.
   (ii). $3^4 = 81$ $\Rightarrow$ $\log_3 81 = 4$.
   (iii). $2^{-3} = \frac{1}{8}$ $\Rightarrow$ $\log_2 \frac{1}{8} = -3$.
   (iv). $(.1)^2 = .01$ $\Rightarrow$ $\log_{(.1)} .01 = 2$.

2. **Properties of Logarithms:**
   1. $\log_a (xy) = \log_a x + \log_a y$
   2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
   3. $\log_x x = 1$
   4. $\log_a 1 = 0$
   5. $\log_a (x^n) = n(\log_a x)$
   6. $\log_a x = \frac{1}{\log_x a}$
   7. $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a}$.

3. **Common Logarithms:**

   Logarithms to the base 10 are known as common logarithms.

4. The logarithm of a number contains two parts, namely 'characteristic' and 'mantissa'.
   Characteristic: The internal part of the logarithm of a number is called its **characteristic.**

   Case I: When the number is greater than 1.

   In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

   Case II: When the number is less than 1.

   In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.
   Instead of -1, -2 etc. we write 1 (one bar), 2 (two bar), etc.

QUESTIONS –

1. Which of the following statements is not correct?

   A. $\log_{10} 10 = 1$

   B. $\log (2 + 3) = \log (2 \times 3)$

   C. $\log_{10} 1 = 0$

   D. $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

   Answer: Option **B**

   Explanation:

   (a) Since $\log_a a = 1$, so $\log_{10} 10 = 1$.

   (b) $\log (2 + 3) = \log 5$ and $\log (2 \times 3) = \log 6 = \log 2 + \log 3$

   $\therefore \log (2 + 3) \neq \log (2 \times 3)$

   (c) Since $\log_a 1 = 0$, so $\log_{10} 1 = 0$.

   (d) $\log (1 + 2 + 3) = \log 6 = \log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$.

   So, (b) is incorrect.

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2. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, the value of $\log_5 512$ is:

   A. 2.870

   B. 2.967

   C. 3.876

   D. 3.912

   Answer: Option **C**

   Explanation:

$$\log_5 512 = \frac{\log 512}{\log 5}$$

$$= \frac{\log 2^9}{\log (10/2)}$$

$$= \frac{9 \log 2}{\log 10 - \log 2}$$

$$= \frac{(9 \times 0.3010)}{1 - 0.3010}$$

$$= \frac{2.709}{0.699}$$

$$= \frac{2709}{699}$$

$= 3.876$

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3. $\dfrac{\log \sqrt{8}}{\log 8}$ is equal to:

A. $\dfrac{1}{\sqrt{8}}$

B. $\dfrac{1}{4}$

C. $\dfrac{1}{2}$

D. $\dfrac{1}{8}$

Answer: Option **C**
Explanation:

$\dfrac{\log \sqrt{8}}{\log 8} = \dfrac{\log (8)^{1/2}}{\log 8} = \dfrac{\frac{1}{2}\log 8}{\log 8} = \dfrac{1}{2}.$

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4. If $\log 27 = 1.431$, then the value of $\log 9$ is:

A. 0.934

B. 0.945

C. 0.954

D. 0.958

Answer: Option **C**
Explanation:

$\log 27 = 1.431$

$\Rightarrow \log (3^3) = 1.431$

$\Rightarrow 3 \log 3 = 1.431$

$\Rightarrow \log 3 = 0.477$

$\therefore \log 9 = \log(3^2) = 2 \log 3 = (2 \times 0.477) = 0.954.$

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5. If $\log \dfrac{a}{b} + \log \dfrac{b}{a} = \log (a + b)$, then:

A. $a + b = 1$

B. $a - b = 1$

C. $a = b$

D. $a^2 - b^2 = 1$

Answer: Option **A**

Explanation:

$\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b)$

$\Rightarrow \log (a + b) = \log\left(\frac{a}{b} \times \frac{b}{a}\right) = \log 1.$

So, $a + b = 1$.

6. If $\log_{10} 7 = a$, then $\log_{10}\left(\frac{1}{70}\right)$ is equal to:

A. $- (1 + a)$

B. $(1 + a)^{-1}$

C. $\frac{a}{10}$

D. $\frac{1}{10a}$

Answer: Option **A**

Explanation:

$\log_{10}\left(\frac{1}{70}\right) = \log_{10} 1 - \log_{10} 70$

$= - \log_{10} (7 \times 10)$

$= - (\log_{10} 7 + \log_{10} 10)$

$= - (a + 1).$

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7. If $\log_{10} 2 = 0.3010$, then $\log_2 10$ is equal to:

A. $\frac{699}{301}$

B. $\frac{1000}{301}$

C. 0.3010

D. 0.6990

Answer: Option **B**

Explanation:

$\log_2 10 = \dfrac{1}{\log_{10} 2} = \dfrac{1}{0.3010} = \dfrac{10000}{3010} = \dfrac{1000}{301}$.

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8. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 80$ is:

A. 1.6020

B. 1.9030

C. 3.9030

D. None of these

Answer: Option **B**

Explanation:

$\log_{10} 80 = \log_{10} (8 \times 10)$

$= \log_{10} 8 + \log_{10} 10$

$= \log_{10} (2^3) + 1$

$= 3 \log_{10} 2 + 1$

$= (3 \times 0.3010) + 1$

$= 1.9030.$

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9. If $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$, then x is equal to:

A. 1

B. 3

C. 5

D. 10

Answer: Option **B**

Explanation:

$\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$

$\Rightarrow \log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + \log_{10} 10$

$\Rightarrow \log_{10} [5 (5x + 1)] = \log_{10} [10(x + 5)]$

$\Rightarrow 5(5x + 1) = 10(x + 5)$

$\Rightarrow 5x + 1 = 2x + 10$

$\Rightarrow 3x = 9$

$\Rightarrow x = 3.$

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10. The value of $\left(\dfrac{1}{\log_3 60} + \dfrac{1}{\log_4 60} + \dfrac{1}{\log_5 60}\right)$ is:

A. 0

B. 1

C. 5

D. 60

Answer: Option **B**

Explanation:

Given expression $= \log_{60} 3 + \log_{60} 4 + \log_{60} 5$

$= \log_{60} (3 \times 4 \times 5)$

$= \log_{60} 60$

$= 1.$

11. If log 2 = 0.30103, the number of digits in 2^{64} is:

A. 18

B. 19

C. 20

D. 21

Answer: Option **C**

Explanation:

$\log (2^{64}) = 64 \times \log 2$

$= (64 \times 0.30103)$

$= 19.26592$

Its characteristic is 19.

Hence, then number of digits in 2^{64} is 20.

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12. If $\log_x \left(\dfrac{9}{16}\right) = -\dfrac{1}{2}$, then x is equal to:

A. $-\dfrac{3}{4}$

B. $\dfrac{3}{}$

4

C. $\frac{81}{256}$

D. $\frac{256}{81}$

Answer: Option **D**

Explanation:

$\log_x\left(\frac{9}{16}\right) = -\frac{1}{2}$

$\Rightarrow x^{-1/2} = \frac{9}{16}$

$\Rightarrow \frac{1}{\sqrt{x}} = \frac{9}{16}$

$\Rightarrow \sqrt{x} = \frac{16}{9}$

$\Rightarrow x = \left(\frac{16}{9}\right)^2$

$\Rightarrow x = \frac{256}{81}$

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13. If $a^x = b^y$, then:

A. $\log \frac{a}{b} = \frac{x}{y}$

B. $\frac{\log a}{\log b} = \frac{x}{y}$

C. $\frac{\log a}{\log b} = \frac{y}{x}$

D. None of these

Answer: Option **C**

Explanation:

$a^x = b^y$

$\Rightarrow \log a^x = \log b^y$

$\Rightarrow x \log a = y \log b$

$\Rightarrow \frac{\log a}{\log b} = \frac{y}{x}$.

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14. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is:

A. 2^{10}

B. 2^{100}

C. 2^{1000}

D. 2^{10000}

Answer: Option **C**

Explanation:

$\log_2 x = 10 \quad \Rightarrow \quad x = 2^{10}$.

$\therefore \log_x y = 100$

$\Rightarrow y = x^{100}$

$\Rightarrow y = (2^{10})^{100}$ [put value of x]

$\Rightarrow y = 2^{1000}$.

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15. The value of $\log_2 16$ is:

A. $\frac{1}{8}$

B. 4

C. 8

D. 16

Answer: Option **B**

Explanation:

Let $\log_2 16 = n$.

Then, $2^n = 16 = 2^4 \quad \Rightarrow \quad n = 4$.

$\therefore \log_2 16 = 4$.