MIXTURES AND ALLIGATIONS

IMPORTANT FORMULAS

1. **Alligation:**

   It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of desired price.
2. **Mean Price:**

   The cost of a unit quantity of the mixture is called the mean price.
3. **Rule of Alligation:**

   If two ingredients are mixed, then

   $$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}}\right) = \left(\frac{\text{C.P. of dearer - Mean Price}}{\text{Mean price - C.P. of cheaper}}\right)$$

   We present as under:

   C.P. of a unit quantity of cheaper | Mean Price | C.P. of a unit quantity of dearer

   (c) | | (d)

   $(d - m)$ | (m) | $(m - c)$

   $\therefore$ (Cheaper quantity) : (Dearer quantity) = $(d - m) : (m - c)$.
4. Suppose a container contains x of liquid from which y units are taken out and replaced by water.

   After n operations, the quantity of pure liquid = $\left[x\left(1 - \frac{y}{x}\right)^n\right]$ units.

1. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

   A. $\frac{1}{3}$

   B. $\frac{1}{4}$

   C. $\frac{1}{5}$

   D. $\frac{1}{7}$

   Answer: Option **C**

   Explanation:

Suppose the vessel initially contains 8 litres of liquid.

Let x litres of this liquid be replaced with water.

Quantity of water in new mixture = $\left(3 - \frac{3x}{8} + x\right)$ litres

Quantity of syrup in new mixture = $\left(5 - \frac{5x}{8}\right)$ litres

$\therefore \left(3 - \frac{3x}{8} + x\right) = \left(5 - \frac{5x}{8}\right)$

$\Rightarrow 5x + 24 = 40 - 5x$

$\Rightarrow 10x = 16$

$\Rightarrow x = \frac{8}{5}$.

So, part of the mixture replaced = $\left(\frac{8}{5} \times \frac{1}{8}\right) = \frac{1}{5}$.

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2. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be:

A. Rs. 169.50

B. Rs. 170

C. Rs. 175.50

D. Rs. 180

Answer: Option **C**

Explanation:

Since first and second varieties are mixed in equal proportions.

So, their average price = Rs. $\left(\frac{126 + 135}{2}\right)$ = Rs. 130.50

So, the mixture is formed by mixing two varieties, one at Rs. 130.50 per kg and the other at say, Rs. x per kg in the ratio 2 : 2, *i.e.,* 1 : 1. We have to find x.

By the rule of alligation, we have:

Cost of 1 kg of 1st kind		Cost of 1 kg tea of 2nd kind
Rs. 130.50	Mean Price Rs. 153	Rs. x
(x - 153)		22.50

$\therefore \frac{x - 153}{22.50} = 1$

$\Rightarrow x - 153 = 22.50$

$\Rightarrow x = 175.50$

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3. A can contains a mixture of two liquids A and B is the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially?

A. 10

B. 20

C. 21

D. 25

Answer: Option **C**

Explanation:

Suppose the can initially contains $7x$ and $5x$ of mixtures A and B respectively.

Quantity of A in mixture left = $\left(7x - \frac{7}{12} \text{ x } 9\right)$ litres = $\left(7x - \frac{21}{4}\right)$ litres.

Quantity of B in mixture left = $\left(5x - \frac{5}{12} \text{ x } 9\right)$ litres = $\left(5x - \frac{15}{4}\right)$ litres.

$\therefore \quad \frac{\left(7x - \frac{21}{4}\right)}{\left(5x - \frac{15}{4}\right) + 9} = \frac{7}{9}$

$\Rightarrow \frac{28x - 21}{20x + 21} = \frac{7}{9}$

$\Rightarrow 252x - 189 = 140x + 147$

$\Rightarrow 112x = 336$

$\Rightarrow x = 3.$

So, the can contained 21 litres of A.

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4. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5?

A. 4 litres, 8 litres

B. 6 litres, 6 litres

C. 5 litres, 7 litres

D. 7 litres, 5 litres

Answer: Option **B**

Explanation:

Let the cost of 1 litre milk be Re. 1

Milk in 1 litre mix. in 1st can = $\frac{3}{4}$ litre, C.P. of 1 litre mix. in 1st can Re. $\frac{3}{4}$

Milk in 1 litre mix. in 2nd can = $\frac{1}{2}$ litre, C.P. of 1 litre mix. in 2nd can Re. $\frac{1}{2}$

Milk in 1 litre of final mix. =$\frac{5}{8}$ litre, Mean price = Re. $\frac{5}{8}$

By the rule of alligation, we have:

C.P. of 1 litre mixture in 1st can		C.P. of 1 litre mixture in 2nd can
$\frac{3}{4}$	Mean Price $\frac{5}{8}$	$\frac{1}{2}$
$\frac{1}{8}$		$\frac{1}{8}$

$\therefore$ Ratio of two mixtures = $\frac{1}{8} : \frac{1}{8}$ = 1 : 1.

So, quantity of mixture taken from each can = $\left(\frac{1}{2} \text{x } 12\right)$ = 6 litres.

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5. In what ratio must a grocer mix two varieties of pulses costing Rs. 15 and Rs. 20 per kg respectively so as to get a mixture worth Rs. 16.50 kg?

A. 3 : 7

B. 5 : 7

C. 7 : 3

D. 7 : 5

Answer: Option **C**
Explanation:

By the rule of alligation:

Cost of 1 kg pulses of 1st kind		Cost of 1 kg pulses of 2nd kind
Rs. 15	Mean Price Rs. 16.50	Rs. 20
3.50		1.50

$\therefore$ Required rate = 3.50 : 1.50 = 7 : 3.

6. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is:

A. 4%

B. $6\frac{1}{4}\%$

C. 20%

D. 25%

Answer: Option **C**
Explanation:

Let C.P. of 1 litre milk be Re. 1

Then, S.P. of 1 litre of mixture = Re. 1, Gain = 25%.

C.P. of 1 litre mixture = Re. $\left(\frac{100}{125} \times 1\right) = \frac{4}{5}$

By the rule of alligation, we have:

C.P. of 1 litre of milk		C.P. of 1 litre of water
Re. 1	Mean Price	0
$\frac{4}{5}$	Re. $\frac{4}{5}$	$\frac{1}{5}$

$\therefore$ Ratio of milk to water = $\frac{4}{5} : \frac{1}{5} = 4 : 1$.

Hence, percentage of water in the mixture = $\left(\frac{1}{5} \times 100\right)\%$ = 20%.

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7. How many kilogram of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs. 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg?

A. 36 kg

B. 42 kg

C. 54 kg

D. 63 kg

Answer: Option **D**
Explanation:

S.P. of 1 kg of mixture = Rs. 9.24, Gain 10%.

$\therefore$ C.P. of 1 kg of mixture = Rs. $\left(\frac{100}{110} \times 9.24\right)$ = Rs. 8.40

By the rule of allilation, we have:

C.P. of 1 kg sugar of 1st kind		Cost of 1 kg sugar of 2nd kind
Rs. 9	Mean Price	Rs. 7
1.40	Rs. 8.40	0.60

$\therefore$ Ratio of quantities of 1st and 2nd kind = 14 : 6 = 7 : 3.
Let *x* kg of sugar of 1st be mixed with 27 kg of 2nd kind.
Then, 7 : 3 = *x* : 27
$\Rightarrow x = \left(\frac{7 \times 27}{3}\right)$ = 63 kg.

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8. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?

A. 26.34 litres

B. 27.36 litres

C. 28 litres

D. 29.16 litres

Answer: Option **D**
Explanation:

Amount of milk left after 3 operations = $\left[40\left(1 - \frac{4}{40}\right)^3\right]$ litres

= $\left(40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}\right)$ = 29.16 litres.

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9. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

Answer: Option **B**
Explanation:

By the rule of alligation, we have:

Strength of first jar		Strength of 2nd jar
40%	Mean Strength	19%
7	26%	14

So, ratio of 1st and 2nd quantities = 7 : 14 = 1 : 2

$\therefore$ Required quantity replaced = $\frac{2}{3}$

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10. In what ratio must water be mixed with milk to gain $16\frac{2}{3}$% on selling the mixture at cost price?

A. 1 : 6

B. 6 : 1

C. 2 : 3

D. 4 : 3

Answer: Option **A**
Explanation:

Let C.P. of 1 litre milk be Re. 1.

S.P. of 1 litre of mixture = Re.1, Gain = $\frac{50}{3}$%.

$\therefore$ C.P. of 1 litre of mixture = $\left(100 \times \frac{3}{350} \times 1\right) = \frac{6}{7}$

By the rule of alligation, we have:

C.P. of 1 litre of water		C.P. of 1 litre of milk
0	Mean Price	Re. 1
$\frac{1}{7}$	Re. $\frac{6}{7}$	$\frac{6}{7}$

$\therefore$ Ratio of water and milk = $\frac{1}{7} : \frac{6}{7} = 1 : 6$.

11. Find the ratio in which rice at Rs. 7.20 a kg be mixed with rice at Rs. 5.70 a kg to produce a mixture worth Rs. 6.30 a kg.

A. 1 : 3

B. 2 : 3

C. 3 : 4

D. 4 : 5

Answer: Option **B**
Explanation:

By the rule of alligation:

Cost of 1 kg of 1st kind		Cost of 1 kg of 2nd kind
720 p	Mean Price	570 p
60	630 p	90

$\therefore$ Required ratio = 60 : 90 = 2 : 3.

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12. In what ratio must a grocer mix two varieties of tea worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg he may gain 10%?

A. 3 : 2

B. 3 : 4

C. 3 : 5

D. 4 : 5

Answer: Option **A**
Explanation:

S.P. of 1 kg of the mixture = Rs. 68.20, Gain = 10%.

C.P. of 1 kg of the mixture = Rs. $\left(\frac{100}{110} \times 68.20\right)$ = Rs. 62.

By the rule of alligation, we have:

Cost of 1 kg tea of 1st kind.		Cost of 1 kg tea of 2nd kind.
Rs. 60	Mean Price Rs. 62	Rs. 65
3		2

∴ Required ratio = 3 : 2.

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13. The cost of Type 1 rice is Rs. 15 per kg and Type 2 rice is Rs. 20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is:

A. Rs. 18

B. Rs. 18.50

C. Rs. 19

D. Rs. 19.50

Answer: Option **A**
Explanation:
Let the price of the mixed variety be Rs. x per kg.

By rule of alligation, we have:

Cost of 1 kg of Type 1 rice		Cost of 1 kg of Type 2 rice
Rs. 15	Mean Price Rs. x	Rs. 20
$(20 - x)$		$(x - 15)$

$\therefore \frac{(20 - x)}{(x - 15)} = \frac{2}{3}$
$\Rightarrow 60 - 3x = 2x - 30$
$\Rightarrow 5x = 90$
$\Rightarrow x = 18.$

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14. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of water is 16 : 65. How much wine did the cask hold originally?

A. 18 litres

B. 24 litres

C. 32 litres

D. 42 litres

Answer: Option **B**

Explanation:

Let the quantity of the wine in the cask originally be x litres.

Then, quantity of wine left in cask after 4 operations = $\left[x\left(1 - \frac{8}{x}\right)^4\right]$ litres.

$\therefore \left(\frac{x(1 - (8/x))^4}{x}\right) = \frac{16}{81}$

$\Rightarrow \left(1 - \frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^4$

$\Rightarrow \left(\frac{x - 8}{x}\right) = \frac{2}{3}$

$\Rightarrow 3x - 24 = 2x$

$\Rightarrow x = 24.$

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15. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is:

A. 400 kg

B. 560 kg

C. 600 kg

D. 640 kg

Answer: Option **C**

Explanation:

By the rule of alligation, we have:

Profit on 1st part		Profit on 2nd part
8%	Mean Profit	18%
4	14%	6

Ration of 1st and 2nd parts = 4 : 6 = 2 : 3

$\therefore$ Quantity of 2nd kind = $\left(\frac{3}{5} \text{ x } 1000\right)$ kg = 600 kg.