# RATIO AND PROPORTION

# IMPORTANT FORMULAS

1. **Ratio:**

   The ratio of two quantities $a$ and $b$ in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.
   In the ratio $a : b$, we call $a$ as the first term or **antecedent** and b, the second term or **consequent**.

   Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

   **Rule:** The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

   Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

2. **Proportion:**

   The equality of two ratios is called proportion.
   If $a : b = c : d$, we write $a : b :: c : d$ and we say that $a, b, c, d$ are in proportion.
   Here $a$ and $d$ are called **extremes**, while $b$ and $c$ are called **mean terms**.

   Product of means = Product of extremes.
   Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. **Fourth Proportional:**
   If $a : b = c : d$, then $d$ is called the fourth proportional to $a, b, c$.
   **Third Proportional:**
   $a : b = c : d$, then $c$ is called the third proportion to $a$ and $b$.

   **Mean Proportional:**
   Mean proportional between $a$ and $b$ is $ab$.

4. **Comparison of Ratios:**

   We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

5. **Compounded Ratio:**

6. The compounded ratio of the ratios: $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

7. **Duplicate Ratios:**
   Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.
   Sub-duplicate ratio of $(a : b)$ is $(a : b)$.
   Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.
   Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.
   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

8. **Variations:**
We say that $x$ is directly proportional to $y$, if $x = ky$ for some constant $k$ and we write, $x \propto y$.
We say that $x$ is inversely proportional to $y$, if $xy = k$ for some constant $k$ *and*
we write, $x \propto \frac{1}{y}$.

# QUESTIONS –

1. A and B together have Rs. 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much amount does B have?

   **A.** Rs. 460

   **B.** Rs. 484

   **C.** Rs. 550

   **D.** Rs. 664

   **Answer:** Option **B**
   **Explanation:**

   $\frac{4}{15}A = \frac{2}{5}B$

   $\Rightarrow A = \left(\frac{2}{5} \times \frac{15}{4}\right)B$

   $\Rightarrow A = \frac{3}{2}B$

   $\Rightarrow \frac{A}{B} = \frac{3}{2}$

   $\Rightarrow A : B = 3 : 2.$

   $\therefore$ B's share = Rs. $\left(1210 \times \frac{2}{5}\right)$ = Rs. 484.

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2. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:

   **A.** 2 : 5

   **B.** 3 : 5

   **C.** 4 : 5

   **D.** 6 : 7

   **Answer:** Option **C**
   **Explanation:**
   Let the third number be $x$.
   Then, first number = 120% of $x$ = $\frac{120x}{} = \frac{6x}{}$

Second number = 150% of $x$ = $\frac{150x}{100}$ = $\frac{3x}{2}$ (100, 5)

$\therefore$ Ratio of first two numbers = $\left(\frac{6x}{5} : \frac{3x}{2}\right)$ = $12x : 15x = 4 : 5$.

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3. A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1000 more than D, what is B's share?

**A.** Rs. 500

**B.** Rs. 1500

**C.** Rs. 2000

**D.** None of these

**Answer:** Option **C**
**Explanation:**
Let the shares of A, B, C and D be Rs. 5$x$, Rs. 2$x$, Rs. 4$x$ and Rs. 3$x$ respectively.
Then, $4x - 3x = 1000$
$\Rightarrow x = 1000$.
$\therefore$ B's share = Rs. $2x$ = Rs. (2 x 1000) = Rs. 2000.

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4. Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

**A.** 2 : 3 : 4

**B.** 6 : 7 : 8

**C.** 6 : 8 : 9

**D.** None of these

**Answer:** Option **A**
**Explanation:**
Originally, let the number of seats for Mathematics, Physics and Biology be 5$x$, 7$x$ and 8$x$ respectively.
Number of increased seats are (140% of 5$x$), (150% of 7$x$) and (175% of 8$x$).
$\Rightarrow \left(\frac{140}{100} \text{ x } 5x\right)$ , $\left(\frac{150}{100} \text{ x } 7x\right)$ and $\left(\frac{175}{100} \text{ x } 8x\right)$
$\Rightarrow 7x, \frac{21x}{2}$ and $14x$.
$\therefore$ The required ratio = $7x : \frac{21x}{2} : 14x$
$\Rightarrow 14x : 21x : 28x$
$\Rightarrow 2 : 3 : 4$.

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5. In a mixture 60 litres, the ratio of milk and water 2 : 1. If this ratio is to be 1 : 2, then the quantity of water to be further added is:

**A.** 20 litres

**B.** 30 litres

**C.** 40 litres

**D.** 60 litres

**Answer:** Option **D**

**Explanation:**

Quantity of milk = $\left(60 \times \frac{2}{3}\right)$ litres = 40 litres.

Quantity of water in it = (60- 40) litres = 20 litres.

New ratio = 1 : 2

Let quantity of water to be added further be $x$ litres.

Then, milk : water = $\left(\frac{40}{20+x}\right)$ .

Now, $\left(\frac{40}{20+x}\right) = \frac{1}{2}$

$\Rightarrow 20 + x = 80$

$\Rightarrow x = 60.$

$\therefore$ Quantity of water to be added = 60 litres.

6. The ratio of the number of boys and girls in a college is 7 : 8. If the percentage increase in the number of boys and girls be 20% and 10% respectively, what will be the new ratio?

**A.** 8 : 9

**B.** 17 : 18

**C.** 21 : 22

**D.** Cannot be determined

**Answer:** Option **C**

**Explanation:**

Originally, let the number of boys and girls in the college be $7x$ and $8x$ respectively.

Their increased number is (120% of $7x$) and (110% of $8x$).

$\Rightarrow \left(\frac{120}{100} \times 7x\right)$ and $\left(\frac{110}{100} \times 8x\right)$

$\Rightarrow \frac{42x}{5}$ and $\frac{44x}{5}$

$\therefore$ The required ratio = $\left(\frac{42x}{5} : \frac{44x}{5}\right)$ = 21 : 22.

7. Salaries of Ravi and Sumit are in the ratio 2 : 3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40 : 57. What is Sumit's salary?

**A.** Rs. 17,000

**B.** Rs. 20,000

**C.** Rs. 25,500

**D.** Rs. 38,000

**Answer:** Option **D**
**Explanation:**
Let the original salaries of Ravi and Sumit be Rs. $2x$ and Rs. $3x$ respectively.
Then, $\frac{2x + 4000}{3x + 4000} = \frac{40}{57}$
$\Rightarrow 57(2x + 4000) = 40(3x + 4000)$
$\Rightarrow 6x = 68,000$
$\Rightarrow 3x = 34,000$
Sumit's present salary = $(3x + 4000)$ = Rs.(34000 + 4000) = Rs. 38,000.

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8. If 0.75 : $x$ :: 5 : 8, then $x$ is equal to:

**A.** 1.12

**B.** 1.2

**C.** 1.25

**D.** 1.30

**Answer:** Option **B**
**Explanation:**
$(x \times 5) = (0.75 \times 8) \Rightarrow x = \left(\frac{6}{5}\right) = 1.20$

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9. The sum of three numbers is 98. If the ratio of the first to second is 2 :3 and that of the second to the third is 5 : 8, then the second number is:

**A.** 20

**B.** 30

**C.** 48

**D.** 58

**Answer:** Option **B**
**Explanation:**

Let the three parts be A, B, C. Then,

A : B = 2 : 3 and B : C = 5 : 8 = $\left(5 \times \frac{3}{5}\right) : \left(8 \times \frac{3}{5}\right) = 3 : \frac{24}{5}$
$\Rightarrow$ A : B : C = 2 : 3 : $\frac{24}{}$ = 10 : 15 : 24

$\Rightarrow$ B = $\left(98 \times \frac{15}{49}\right)^{5}$ = 30.

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10. If Rs. 782 be divided into three parts, proportional to $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$, then the first part is:

**A.** Rs. 182

**B.** Rs. 190

**C.** Rs. 196

**D.** Rs. 204

**Answer:** Option **D**
**Explanation:**

Given ratio = $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$ = 6 : 8 : 9.

$\therefore$ 1st part = Rs. $\left(782 \times \frac{6}{23}\right)$ = Rs. 204

11. The salaries A, B, C are in the ratio 2 : 3 : 5. If the increments of 15%, 10% and 20% are allowed respectively in their salaries, then what will be new ratio of their salaries?

**A.** 3 : 3 : 10

**B.** 10 : 11 : 20

**C.** 23 : 33 : 60

**D.** Cannot be determined

**Answer:** Option **C**
**Explanation:**

Let A = 2*k*, B = 3*k* and C = 5*k*.

A's new salary = $\frac{115}{100}$ of 2*k* = $\left(\frac{115}{100} \times 2k\right) = \frac{23k}{10}$

B's new salary = $\frac{110}{100}$ of 3*k* = $\left(\frac{110}{100} \times 3k\right) = \frac{33k}{10}$

C's new salary = $\frac{120}{100}$ of 5*k* = $\left(\frac{120}{100} \times 5k\right)$ = 6*k*

$\therefore$ New ratio $\left(\frac{23k}{10} : \frac{33k}{10} : 6k\right)$ = 23 : 33 : 60

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12. If 40% of a number is equal to two-third of another number, what is the ratio of first number to the second number?

**A.** 2 : 5

**B.** 3 : 7

**C.** 5 : 3

**D.** 7 : 3

**Answer:** Option **C**
**Explanation:**

Let 40% of A = $\frac{2}{3}$ B

Then, $\frac{40A}{100} = \frac{2B}{3}$

$\Rightarrow \frac{2A}{5} = \frac{2B}{3}$

$\Rightarrow \frac{A}{B} = \left(\frac{2}{3} \times \frac{5}{2}\right) = \frac{5}{3}$

$\therefore$ A : B = 5 : 3.

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13. The fourth proportional to 5, 8, 15 is:

**A.** 18

**B.** 24

**C.** 19

**D.** 20

**Answer:** Option **B**
**Explanation:**

Let the fourth proportional to 5, 8, 15 be *x*.
Then, 5 : 8 : 15 : *x*
$\Rightarrow 5x = (8 \times 15)$
$x = \frac{(8 \times 15)}{5} = 24.$

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14. Two number are in the ratio 3 : 5. If 9 is subtracted from each, the new numbers are in the ratio 12 : 23. The smaller number is:

**A.** 27

**B.** 33

**C.** 49

**D.** 55

**Answer:** Option **B**
**Explanation:**

Let the numbers be $3x$ and $5x$.

Then, $\frac{3x - 9}{5x - 9} = \frac{12}{23}$

$\Rightarrow 23(3x - 9) = 12(5x - 9)$

$\Rightarrow 9x = 99$

$\Rightarrow x = 11.$

$\therefore$ The smaller number = (3 x 11) = 33.

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15. In a bag, there are coins of 25 p, 10 p and 5 p in the ratio of 1 : 2 : 3. If there is Rs. 30 in all, how many 5 p coins are there?

**A.** 50

**B.** 100

**C.** 150

**D.** 200

**Answer:** Option **C**

**Explanation:**

Let the number of 25 p, 10 p and 5 p coins be $x$, $2x$, $3x$ respectively.

Then, sum of their values = Rs. $\left(\frac{25x}{100} + \frac{10 \text{ x } 2x}{100} + \frac{5 \text{ x } 3x}{100}\right)$ = Rs. $\frac{60x}{100}$

$\therefore \frac{60x}{100} = 30 \Leftrightarrow x = \frac{30 \text{ x } 100}{60} = 50.$

Hence, the number of 5 p coins = (3 x 50) = 150.