VECTORS

1 10^{th} Maths - EXERCISE-7.3

1. That a median of a triangle divides it into two triangles of equal areas.
   verify this result for $\triangle ABC$ whose vertices are $\mathbf{A}(4,-6), \mathbf{B}(3,-2)$ and $\mathbf{C}(5,2)$.

2 SOLUTION

Given points are

$$\mathbf{A} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \tag{1}$$

$$\mathbf{D} = \frac{\mathbf{B}+\mathbf{C}}{2} \tag{2}$$

$$= \frac{\begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}}{2} \tag{3}$$

$$= \frac{\begin{pmatrix} 8 \\ 0 \end{pmatrix}}{2} \tag{4}$$

$$\mathbf{D} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5}$$

The ar(ABD) can be expressed as

$$\frac{1}{2} \left\|(\mathbf{A}-\mathbf{B}) \times (\mathbf{A}-\mathbf{D})\right\| \tag{6}$$

$$\mathbf{A}-\mathbf{B}=\begin{pmatrix}4\\-6\end{pmatrix}-\begin{pmatrix}3\\-2\end{pmatrix}=\begin{pmatrix}1\\-4\end{pmatrix} \tag{7}$$

$$\mathbf{A}-\mathbf{D}=\begin{pmatrix}4\\-6\end{pmatrix}-\begin{pmatrix}4\\0\end{pmatrix}=\begin{pmatrix}0\\-6\end{pmatrix} \tag{8}$$

Substituting the values of (7) and (8) in (6),

$$ar(ABD)=\frac{1}{2}\begin{vmatrix}1 & 0\\-4 & -6\end{vmatrix} \tag{9}$$

$$=\frac{6}{2} \tag{10}$$

$$ar(ABD)=3 \tag{11}$$

Also, the ar(ACD) can be expressed as

$$\frac{1}{2}\left\|(\mathbf{A}-\mathbf{C})\times(\mathbf{A}-\mathbf{D})\right\| \tag{12}$$

$$\mathbf{A}-\mathbf{C}=\begin{pmatrix}4\\-6\end{pmatrix}-\begin{pmatrix}5\\2\end{pmatrix}=\begin{pmatrix}-1\\-8\end{pmatrix} \tag{13}$$

$$\mathbf{A}-\mathbf{D}=\begin{pmatrix}4\\-6\end{pmatrix}-\begin{pmatrix}4\\0\end{pmatrix}=\begin{pmatrix}0\\-6\end{pmatrix} \tag{14}$$

Substituting the values of (13) and (14) in (12),

$$ar(ACD)=\frac{1}{2}\begin{vmatrix}-1 & 0\\-8 & -6\end{vmatrix} \tag{15}$$

$$=\frac{6}{2} \tag{16}$$

$$ar(ACD)=3 \tag{17}$$

The median of the triangle is both side areas are equal $\triangle ABD=\triangle ACD$

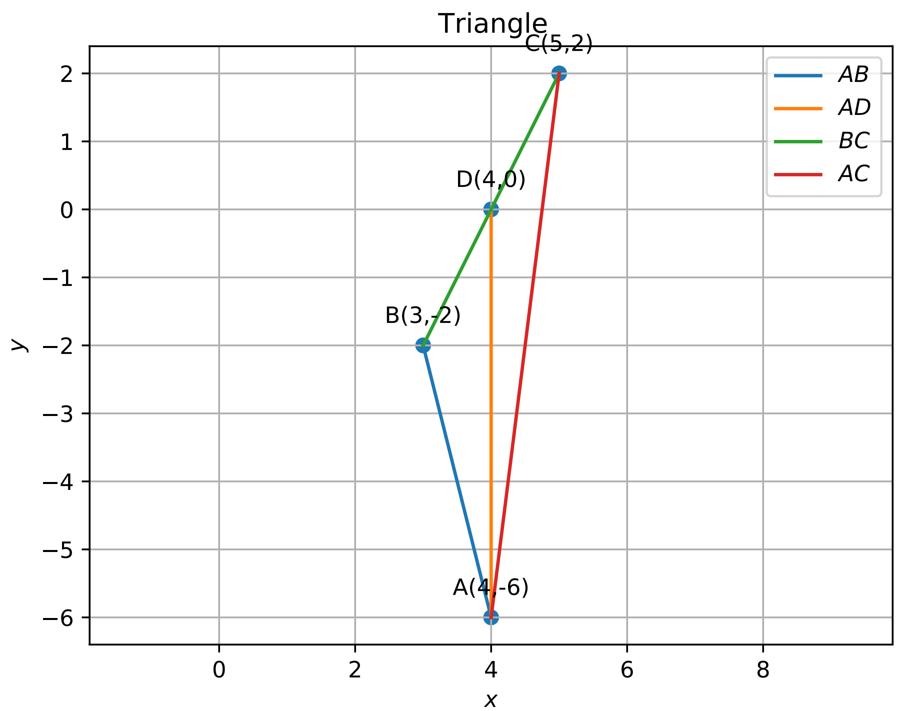

Figure 1: Triangle