

CHAPTER-9
CIRCLES

1 EXERCISE-10.5

1. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

2 SOLUTION

The input parameters are the length

Symbol	Value	Description
r	1	Radius
\mathbf{O}	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	circle point
\mathbf{P}	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\mathbf{P} = \mathbf{e}_1$
θ	60°	—
α	130°	—
β	-40°	—

Table 1: chords are intersecting in a circle

Take three points Q,R and P on a unit circle at angles θ, α , and β .Then

$$\mathbf{Q} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}, \mathbf{S} = \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \quad (1)$$

$$\cos \angle QRP = \frac{(Q - R)(P - R)}{|Q - R| |P - R|} \quad (2)$$

Where

$$(Q - R, P - R) = (\cos \theta - \cos \alpha, \sin \theta - \sin \alpha), (1 - \cos \alpha, 0 - \sin \alpha) \quad (3)$$

$$= (\cos \theta - \cos \alpha) \cos \alpha + (\sin \theta - \sin \alpha) \quad (4)$$

$$= 2 \sin \frac{\theta - \alpha}{2} \sin \frac{\theta + \alpha}{2} \cos \alpha + 2 \cos \frac{\theta + \alpha}{2} \sin \frac{\theta - \alpha}{2} \quad (5)$$

$$= (\cos \alpha - \cos \theta) \cos \alpha + (\sin \theta - \sin \alpha) \quad (6)$$

$$\begin{aligned}
|Q - R|^2 |P - R|^2 &= ((\cos \theta - \cos \alpha)^2 + (\sin \theta - \sin \alpha)^2)((1 - \cos \alpha)^2 + (0 - \sin \alpha)^2) \\
&= (2 - 2 \cos \theta \cos \alpha - 2 \sin \theta \sin \alpha)(2 - \cos \alpha)
\end{aligned}
\tag{7}$$

substituting the (6) and (8) in (2)

$$\cos \angle QRP = \frac{2.079}{4.323} \tag{9}$$

$$\angle QRP = \cos^{-1} 0.480 \tag{10}$$

$$\angle QRP = 66^\circ \tag{11}$$

$$\cos \angle QSP = \frac{(Q - S)(P - S)}{|Q - S| |P - S|} \tag{12}$$

$$(Q - S, P - S) = (\cos \theta - \cos \beta, \sin \theta - \sin \beta), (1 - \cos \beta, 0 - \sin \beta) \tag{13}$$

$$= (\cos \theta - \cos \beta) \cos \beta + (\sin \theta - \sin \beta) \tag{14}$$

$$= 2 \sin \frac{\theta - \beta}{2} \sin \frac{\theta + \beta}{2} \cos \beta + 2 \cos \frac{\theta + \beta}{2} \sin \frac{\theta - \beta}{2} \tag{15}$$

$$= (\cos \beta - \cos \theta) \cos \beta + (\sin \theta - \sin \beta) \tag{16}$$

$$|Q - S|^2 |P - S|^2 = ((\cos \theta - \cos \beta)^2 + (\sin \theta - \sin \beta)^2)((1 - \cos \beta)^2 + (0 - \sin \beta)^2) \tag{17}$$

$$= (2 - 2 \cos \theta \cos \beta - 2 \sin \theta \sin \beta)(2 - \cos \beta) \tag{18}$$

substituting the (16) and (18) in (12)

$$\cos \angle QSP = \frac{1.048}{1.098} \tag{19}$$

$$\angle QSP = \cos^{-1} 0.954 \tag{20}$$

$$\angle QSP = 17^\circ \tag{21}$$

3 FIGURE

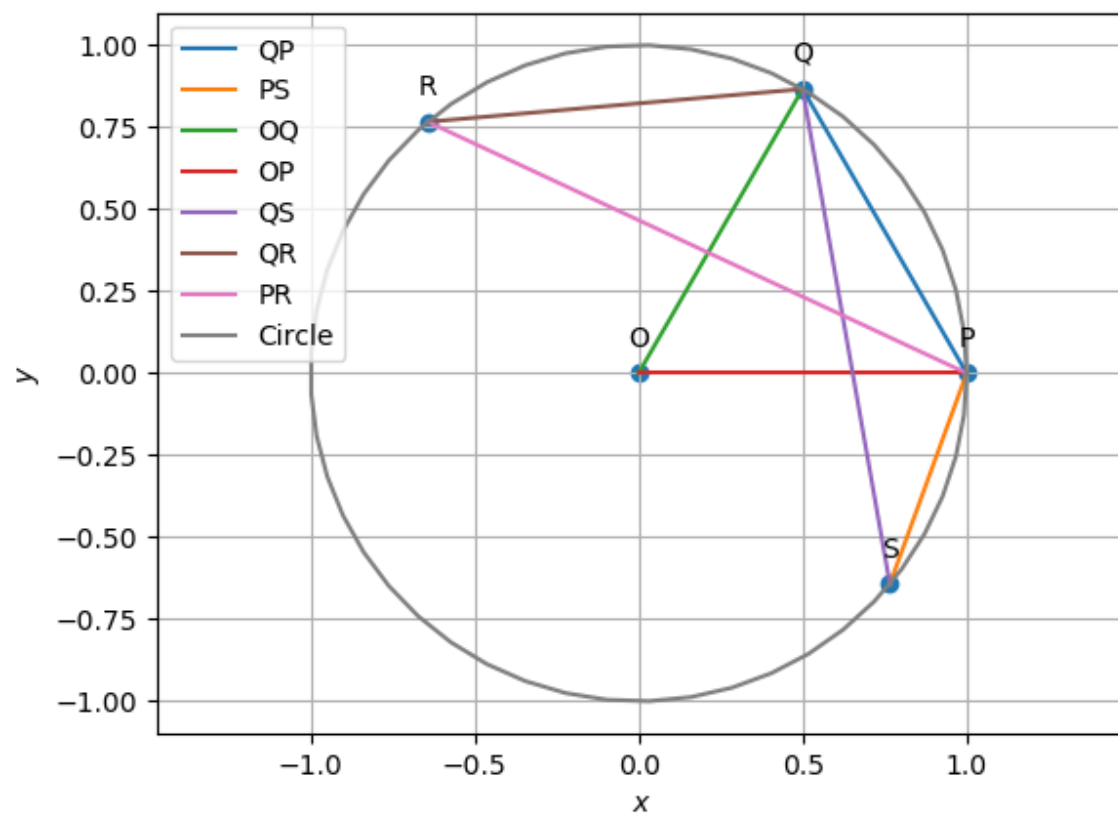


Figure 1: circle