CHAPTER-9
CIRCLES

1 EXERCISE-10.5

1. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

2 SOLUTION

The input parameters are the length

Symbol	Value	Description
r	1	Radius
O	$\begin{pmatrix}0\\0\end{pmatrix}$	circle point
P	$\begin{pmatrix}1\\0\end{pmatrix}$	$\mathbf{P}=\mathbf{e_1}$
θ	60°	–
α	130°	–
β	-40°	–

Table 1: chords are intersecting in a circle

Take three points Q,R and P on a unit circle at angles $\theta, \alpha,$ and β.Then

$$\mathbf{Q}=\begin{pmatrix}cos\theta\\sin\theta\end{pmatrix},\mathbf{R}=\begin{pmatrix}cos\alpha\\sin\alpha\end{pmatrix},\mathbf{S}=\begin{pmatrix}cos\beta\\sin\beta\end{pmatrix} \tag{1}$$

$$\cos\angle QRP=\frac{(Q-R)(P-R)}{\left|Q-R\right|\left|P-R\right|} \tag{2}$$

Where

$$(Q-R,P-R)=(\cos\theta-\cos\alpha,\sin\theta-\sin\alpha),\big(1-\cos\alpha,\quad o-\sin\alpha\big) \tag{3}$$

$$=(\cos\theta-\cos\alpha)\cos\alpha+(\sin\theta-\sin\alpha) \tag{4}$$

$$=2\sin\frac{\theta-\alpha}{2}\sin\frac{\theta+\alpha}{2}\cos\alpha+2\cos\frac{\theta+\alpha}{2}\sin\frac{\theta-\alpha}{2} \tag{5}$$

$$=(\cos\alpha-\cos\theta)\cos\alpha+(\sin\theta-\sin\alpha) \tag{6}$$

$$\left|Q-R\right|^2\left|P-R\right|^2=((\cos\theta-\cos\alpha)^2+(\sin\theta-\sin\alpha)^2)((1-\cos\alpha)^2+(0-\sin\alpha)^2) \tag{7}$$

$$=(2-2\cos\theta\cos\alpha-2\sin\theta\sin\alpha)(2-\cos\alpha) \tag{8}$$

substituing the (6) and (8) in (2)

$$\cos\angle QRP=\frac{2.079}{4.323} \tag{9}$$

$$\angle QRP=\cos^{-1}0.480 \tag{10}$$

$$\angle QRP=66^\circ \tag{11}$$

$$\cos\angle QSP=\frac{(Q-S)(P-S)}{\left|Q-S\right|\left|P-S\right|} \tag{12}$$

$$(Q-S,P-S)=(\cos\theta-\cos\beta,\sin\theta-\sin\beta),\left(1-\cos\beta,\quad o-\sin\beta\right) \tag{13}$$

$$=(\cos\theta-\cos\beta)\cos\beta+(\sin\theta-\sin\beta) \tag{14}$$

$$=2\sin\frac{\theta-\beta}{2}\sin\frac{\theta+\beta}{2}\cos\beta+2\cos\frac{\theta+\beta}{2}\sin\frac{\theta-\beta}{2} \tag{15}$$

$$=(\cos\beta-\cos\theta)\cos\beta+(\sin\theta-\sin\beta) \tag{16}$$

$$\left|Q-S\right|^2\left|P-S\right|^2=((\cos\theta-\cos\beta)^2+(\sin\theta-\sin\beta)^2)((1-\cos\beta)^2+(0-\sin\beta)^2) \tag{17}$$

$$=(2-2\cos\theta\cos\beta-2\sin\theta\sin\beta)(2-\cos\beta) \tag{18}$$

substituing the (16) and (18) in (12)

$$\cos\angle QSP=\frac{1.048}{1.098} \tag{19}$$

$$\angle QSP=\cos^{-1}0.954 \tag{20}$$

$$\angle QSP=17^\circ \tag{21}$$

3 FIGURE

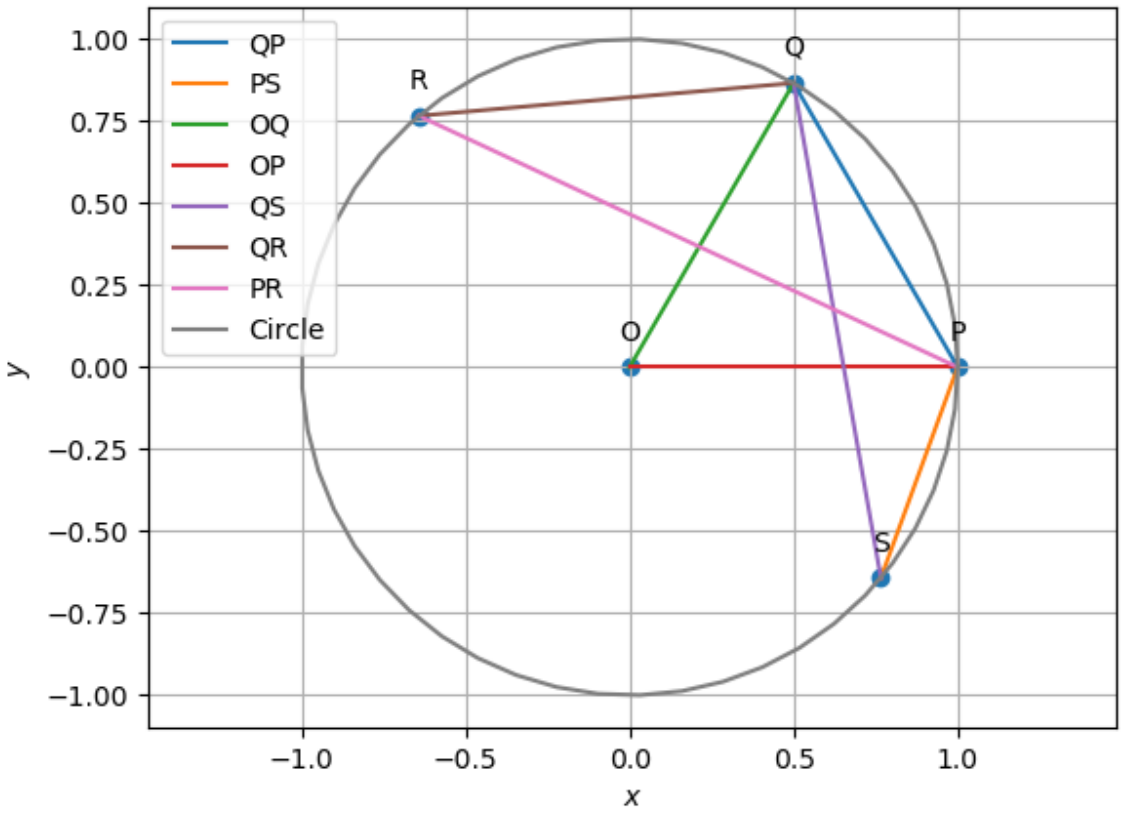

Figure 1: circle