CHAPTER-9
CIRCLES

1 EXERCISE-10.5

1. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

2 SOLUTION

The input parameters are

Symbol	**Value**	**Description**
r	1	Radius
$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	circle point
$\mathbf{P}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	standard basis vector$\mathbf{e_1}$
θ	$60°$	$\angle QOP$
α	$130°$	$\angle QRP$
β	$-40°$	$\angle QSP$

Table 1:

Take three points Q,R and P on a unit circle at angles $\theta, \alpha,$ and β.Then

$$\mathbf{Q} = \begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix}, \mathbf{R} = \begin{pmatrix} cos\alpha \\ sin\alpha \end{pmatrix}, \mathbf{S} = \begin{pmatrix} cos\beta \\ sin\beta \end{pmatrix} \tag{1}$$

$$\cos \angle QRP = \frac{(\mathbf{Q} - \mathbf{R})(\mathbf{P} - \mathbf{R})}{\|\mathbf{Q} - \mathbf{R}\| \|\mathbf{P} - \mathbf{R}\|} \tag{2}$$

Where

$$(\mathbf{Q} - \mathbf{R})(\mathbf{P} - \mathbf{R}) = (\cos\theta - \cos\alpha \sin\theta - \sin\alpha) \begin{pmatrix} 1 - \cos\alpha & o - \sin\alpha \end{pmatrix} \tag{3}$$

$$= (\cos\theta - \cos\alpha)\cos\alpha + (\sin\theta - \sin\alpha) \tag{4}$$

$$= 2\sin\frac{\theta - \alpha}{2}\sin\frac{\theta + \alpha}{2}\cos\alpha + 2\cos\frac{\theta + \alpha}{2}\sin\frac{\theta - \alpha}{2} \tag{5}$$

$$= (\cos\alpha - \cos\theta)\cos\alpha + (\sin\theta - \sin\alpha) \tag{6}$$

$$\|\mathbf{Q}-\mathbf{R}\|^2 \|\mathbf{P}-\mathbf{R}\|^2 = (\cos\theta-\cos\alpha)^2 + (\sin\theta-\sin\alpha)^2 (1-\cos\alpha)^2 + (0-\sin\alpha)^2 \tag{7}$$

$$= (2-2\cos\theta\cos\alpha-2\sin\theta\sin\alpha)(2-\cos\alpha) \tag{8}$$

substituing the (6) and (8) in (2)

$$\cos\angle QRP = \frac{2.079}{4.323} \tag{9}$$

$$\angle QRP = \cos^{-1} 0.480 \tag{10}$$

$$\angle QRP = 66^\circ \tag{11}$$

$$\cos\angle QSP = \frac{(\mathbf{Q}-\mathbf{S})(\mathbf{P}-\mathbf{S})}{\|\mathbf{Q}-\mathbf{S}\| \|\mathbf{P}-\mathbf{S}\|} \tag{12}$$

$$(\mathbf{Q}-\mathbf{S})(\mathbf{P}-\mathbf{S}) = (\cos\theta-\cos\beta\sin\theta-\sin\beta)\left(1-\cos\beta \quad o-\sin\beta\right) \tag{13}$$

$$= (\cos\theta-\cos\beta)\cos\beta + (\sin\theta-\sin\beta) \tag{14}$$

$$= 2\sin\frac{\theta-\beta}{2}\sin\frac{\theta+\beta}{2}\cos\beta + 2\cos\frac{\theta+\beta}{2}\sin\frac{\theta-\beta}{2} \tag{15}$$

$$= (\cos\beta-\cos\theta)\cos\beta + (\sin\theta-\sin\beta) \tag{16}$$

$$\|\mathbf{Q}-\mathbf{S}\|^2 \|\mathbf{P}-\mathbf{S}\|^2 = (\cos\theta-\cos\beta)^2 + (\sin\theta-\sin\beta)^2 (1-\cos\beta)^2 + (0-\sin\beta)^2) \tag{17}$$

$$= (2-2\cos\theta\cos\beta-2\sin\theta\sin\beta)(2-\cos\beta) \tag{18}$$

substituing the (16) and (18) in (12)

$$\cos\angle QSP = \frac{1.048}{1.098} \tag{19}$$

$$\angle QSP = \cos^{-1} 0.954 \tag{20}$$

$$\angle QSP = 17^\circ \tag{21}$$