CHAPTER-9
CIRCLES

1 EXERCISE-10.5

1. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

2 SOLUTION

The input parameters are the length

Symbol	Value	Description
R	1	Radius
O	$\begin{pmatrix}0\\0\end{pmatrix}$	circle point
P	$\begin{pmatrix}1\\0\end{pmatrix}$	$\mathbf{P}=\mathbf{e_1}$
θ	60°	$\mathbf{Q}=\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$
α	130°	$\mathbf{R}=\begin{pmatrix}\cos\alpha\\\sin\alpha\end{pmatrix}$
β	-40°	$\mathbf{S}=\begin{pmatrix}\cos\beta\\\sin\beta\end{pmatrix}$

Table 1: chords are intersecting in a circle

Take three points Q,R and P on a unit circle at angles $\theta, \alpha,$ and β.Then

$$\mathbf{Q}=\begin{pmatrix}cos\theta\\sin\theta\end{pmatrix},\mathbf{R}=\begin{pmatrix}cos\alpha\\sin\alpha\end{pmatrix},\mathbf{S}=\begin{pmatrix}cos\beta\\sin\beta\end{pmatrix} \tag{1}$$

$$\cos\angle QRP=\frac{(Q-R)(P-R)}{\left|Q-R\right|\left|P-R\right|} \tag{2}$$

Where

$$(Q-R, P-R) = (\cos\theta - \cos\alpha, \sin\theta - \sin\alpha), \big(1-\cos\alpha, \quad o - \sin\alpha\big) \tag{3}$$
$$= (\cos\theta - \cos\alpha)\cos\alpha + (\sin\theta - \sin\alpha) \tag{4}$$
$$= 2\sin\frac{\theta-\alpha}{2}\sin\frac{\theta+\alpha}{2}\cos\alpha + 2\cos\frac{\theta+\alpha}{2}\sin\frac{\theta-\alpha}{2} \tag{5}$$
$$= (\cos\alpha - \cos\theta)\cos\alpha + (\sin\theta - \sin\alpha) \tag{6}$$

$$\big|Q-R\big|^2\big|P-R\big|^2 = ((\cos\theta-\cos\alpha)^2 + (\sin\theta-\sin\alpha)^2)((1-\cos\alpha)^2 + (0-\sin\alpha)^2) \tag{7}$$
$$= (2 - 2\cos\theta\cos\alpha - 2\sin\theta\sin\alpha)(2-\cos\alpha) \tag{8}$$

substituing the (6) and (8) in (2)

$$\cos\angle QRP = \frac{2.079}{4.323} \tag{9}$$
$$\angle QRP = \cos^{-1} 0.480 \tag{10}$$
$$\angle QRP = 66^\circ \tag{11}$$

$$\cos\angle QSP = \frac{(Q-S)(P-S)}{\big|Q-S\big|\big|P-S\big|} \tag{12}$$

$$(Q-S, P-S) = (\cos\theta - \cos\beta, \sin\theta - \sin\beta), \big(1-\cos\beta, \quad o - \sin\beta\big) \tag{13}$$
$$= (\cos\theta - \cos\beta)\cos\beta + (\sin\theta - \sin\beta) \tag{14}$$
$$= 2\sin\frac{\theta-\beta}{2}\sin\frac{\theta+\beta}{2}\cos\beta + 2\cos\frac{\theta+\beta}{2}\sin\frac{\theta-\beta}{2} \tag{15}$$
$$= (\cos\beta - \cos\theta)\cos\beta + (\sin\theta - \sin\beta) \tag{16}$$

$$\big|Q-S\big|^2\big|P-S\big|^2 = ((\cos\theta-\cos\beta)^2 + (\sin\theta-\sin\beta)^2)((1-\cos\beta)^2 + (0-\sin\beta)^2) \tag{17}$$
$$= (2 - 2\cos\theta\cos\beta - 2\sin\theta\sin\beta)(2-\cos\beta) \tag{18}$$

substituing the (16) and (18) in (12)

$$\cos\angle QSP = \frac{1.048}{1.098} \tag{19}$$
$$\angle QSP = \cos^{-1} 0.954 \tag{20}$$
$$\angle QSP = 17^\circ \tag{21}$$

3 FIGURE

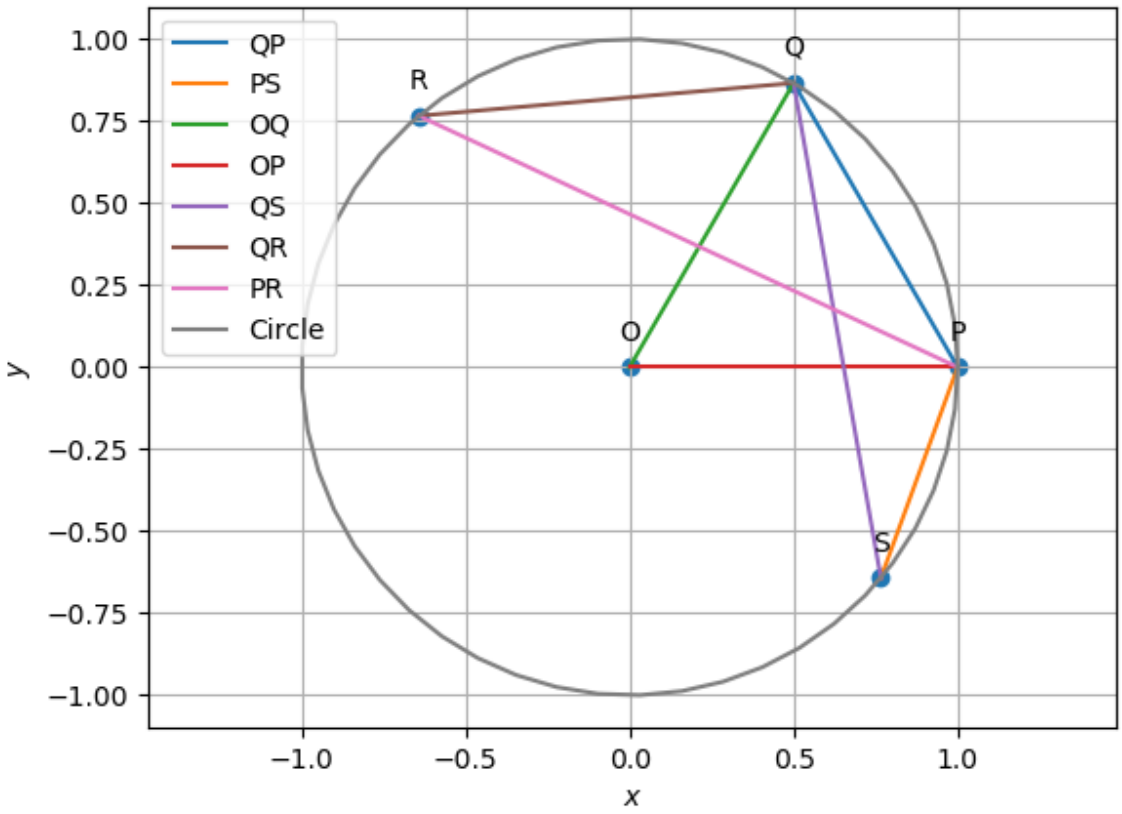

Figure 1: circle