

CHAPTER-9

CIRCLES

1 EXERCISE-10.5

1. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

2 SOLUTION

The input parameters are

Symbol	Value	Description
r	1	Radius
\mathbf{O}	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	circle point
\mathbf{P}	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	standard basis vector \mathbf{e}_1
θ	60°	$\angle QOP$
α	130°	$\angle QRP$
β	-40°	$\angle QSP$

Table 1:

Take three points Q,R and P on a unit circle at angles θ, α , and β .Then

$$\mathbf{Q} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}, \mathbf{S} = \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \quad (1)$$

$$\cos \angle QRP = \frac{(\mathbf{Q} - \mathbf{R}) (\mathbf{P} - \mathbf{R})}{\|\mathbf{Q} - \mathbf{R}\| \|\mathbf{P} - \mathbf{R}\|} \quad (2)$$

Where

$$(\mathbf{Q} - \mathbf{R}) (\mathbf{P} - \mathbf{R}) = (\cos \theta - \cos \alpha \sin \theta - \sin \alpha) (1 - \cos \alpha - \sin \alpha) \quad (3)$$

$$= (\cos \theta - \cos \alpha) \cos \alpha + (\sin \theta - \sin \alpha) \quad (4)$$

$$= 2 \sin \frac{\theta - \alpha}{2} \sin \frac{\theta + \alpha}{2} \cos \alpha + 2 \cos \frac{\theta + \alpha}{2} \sin \frac{\theta - \alpha}{2} \quad (5)$$

$$= (\cos \alpha - \cos \theta) \cos \alpha + (\sin \theta - \sin \alpha) \quad (6)$$

$$\begin{aligned}
\|\mathbf{Q} - \mathbf{R}\|^2 \|\mathbf{P} - \mathbf{R}\|^2 &= (\cos \theta - \cos \alpha)^2 + (\sin \theta - \sin \alpha)^2 (1 - \cos \alpha)^2 + (0 - \sin \alpha)^2 \\
&= (2 - 2 \cos \theta \cos \alpha - 2 \sin \theta \sin \alpha) (2 - \cos \alpha)
\end{aligned}
\tag{7}$$

substituting the (6) and (8) in (2)

$$\cos \angle QRP = \frac{2.079}{4.323} \tag{9}$$

$$\angle QRP = \cos^{-1} 0.480 \tag{10}$$

$$\angle QRP = 66^\circ \tag{11}$$

$$\cos \angle QSP = \frac{(\mathbf{Q} - \mathbf{S})(\mathbf{P} - \mathbf{S})}{\|\mathbf{Q} - \mathbf{S}\| \|\mathbf{P} - \mathbf{S}\|} \tag{12}$$

$$(\mathbf{Q} - \mathbf{S})(\mathbf{P} - \mathbf{S}) = (\cos \theta - \cos \beta \sin \theta - \sin \beta) (1 - \cos \beta - \sin \beta) \tag{13}$$

$$= (\cos \theta - \cos \beta) \cos \beta + (\sin \theta - \sin \beta) \tag{14}$$

$$= 2 \sin \frac{\theta - \beta}{2} \sin \frac{\theta + \beta}{2} \cos \beta + 2 \cos \frac{\theta + \beta}{2} \sin \frac{\theta - \beta}{2} \tag{15}$$

$$= (\cos \beta - \cos \theta) \cos \beta + (\sin \theta - \sin \beta) \tag{16}$$

$$\|\mathbf{Q} - \mathbf{S}\|^2 \|\mathbf{P} - \mathbf{S}\|^2 = (\cos \theta - \cos \beta)^2 + (\sin \theta - \sin \beta)^2 (1 - \cos \beta)^2 + (0 - \sin \beta)^2 \tag{17}$$

$$= (2 - 2 \cos \theta \cos \beta - 2 \sin \theta \sin \beta) (2 - \cos \beta) \tag{18}$$

substituting the (16) and (18) in (12)

$$\cos \angle QSP = \frac{1.048}{1.098} \tag{19}$$

$$\angle QSP = \cos^{-1} 0.954 \tag{20}$$

$$\angle QSP = 17^\circ \tag{21}$$

3 FIGURE

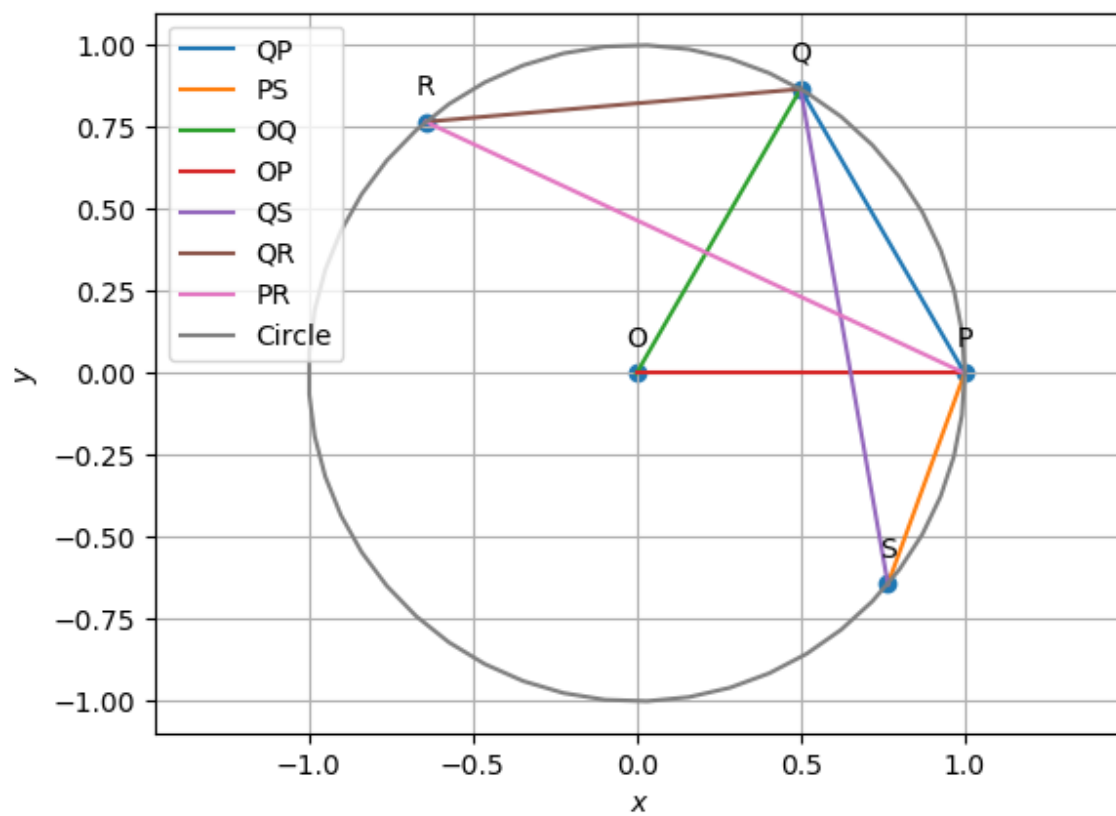


Figure 1: