VECTORS

1 10^{th} Maths - EXERCISE-7.4

Let $A(4,2), B(6,5)\text{and}C(1,4)$ be the vertices of $\triangle ABC$

1. The median from A meets BC at D. Find the coordinates of the point D.
2. Find the coordinates of the point P on AD such that $AP : PD = 2 : 1$
3. Find the coordinates of points Q and R on medians BE and CF respectively such that $BQ : QE = 2 : 1\text{and}CR : RF = 2 : 1$.
4. What do yo observe?
5. If $A(x_1, y_1), B(x_2, y_2)\text{and}C(x_3, y_3)$ are the vertices of $\triangle ABC$, find the coordinates of the centroid of the triangle.

Given points are

$$\mathbf{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{1}$$

1. Solution for problem 1

$$\mathbf{D} = \frac{B+C}{2} \tag{2}$$

$$= \frac{\begin{pmatrix}6\\5\end{pmatrix} + \begin{pmatrix}1\\4\end{pmatrix}}{2} \tag{3}$$

$$= \frac{\begin{pmatrix}7\\9\end{pmatrix}}{2} \tag{4}$$

$$\mathbf{D} = \begin{pmatrix}\frac{7}{2}\\\frac{9}{2}\end{pmatrix} \tag{5}$$

$$\mathbf{E} = \frac{A+C}{2} \tag{6}$$

$$= \frac{\begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}1\\4\end{pmatrix}}{2} \tag{7}$$

$$= \frac{\begin{pmatrix}5\\6\end{pmatrix}}{2} \tag{8}$$

$$\mathbf{E} = \begin{pmatrix}\frac{5}{2}\\3\end{pmatrix} \tag{9}$$

$$\mathbf{F} = \frac{A+B}{2} \tag{10}$$

$$= \frac{\begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}6\\5\end{pmatrix}}{2} \tag{11}$$

$$= \frac{\begin{pmatrix}10\\7\end{pmatrix}}{2} \tag{12}$$

$$\mathbf{F} = \begin{pmatrix}5\\\frac{7}{2}\end{pmatrix} \tag{13}$$

2. Solution for problem 2

$$n = \frac{2}{1} \tag{14}$$

$$\mathbf{P} = \frac{1}{1+n}\left((A + nD)\right) \tag{15}$$

$$= \frac{1}{1+\frac{2}{1}}\left(\begin{pmatrix}4\\2\end{pmatrix} + \frac{2}{1}\begin{pmatrix}\frac{7}{2}\\\frac{9}{2}\end{pmatrix}\right) \tag{16}$$

$$= \frac{1}{3}\left(\begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}7\\9\end{pmatrix}\right) \tag{17}$$

$$= \frac{1}{3}\left(\begin{pmatrix}11\\11\end{pmatrix}\right) \tag{18}$$

$$\mathbf{P} = \begin{pmatrix}\frac{11}{3}\\\frac{11}{3}\end{pmatrix} \tag{19}$$

3. solution for problem 3

$$
\begin{aligned}
n &= \frac{2}{1} && (20)\\
\mathbf{Q} &= \frac{1}{1+n}\left((B+nE)\right) && (21)\\
&= \frac{1}{1+\frac{2}{1}}\left(\begin{pmatrix}6\\5\end{pmatrix}+\frac{2}{1}\begin{pmatrix}\frac{5}{2}\\3\end{pmatrix}\right) && (22)\\
&= \frac{1}{3}\left(\begin{pmatrix}6\\5\end{pmatrix}+\begin{pmatrix}5\\6\end{pmatrix}\right) && (23)\\
&= \frac{1}{3}\left(\begin{pmatrix}11\\11\end{pmatrix}\right) && (24)\\
\mathbf{Q} &= \begin{pmatrix}\frac{11}{3}\\\frac{11}{3}\end{pmatrix} && (25)\\
\mathbf{R} &= \frac{1}{1+n}\left((C+nF)\right) && (26)\\
&= \frac{1}{1+\frac{2}{1}}\left(\begin{pmatrix}1\\4\end{pmatrix}+\frac{2}{1}\begin{pmatrix}5\\\frac{7}{2}\end{pmatrix}\right) && (27)\\
&= \frac{1}{3}\left(\begin{pmatrix}1\\4\end{pmatrix}+\begin{pmatrix}10\\7\end{pmatrix}\right) && (28)\\
&= \frac{1}{3}\left(\begin{pmatrix}11\\11\end{pmatrix}\right) && (29)\\
\mathbf{R} &= \begin{pmatrix}\frac{11}{3}\\\frac{11}{3}\end{pmatrix} && (30)
\end{aligned}
$$

4. solution for problem 4
   I have observed the $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ are the same values and coincidence where median intersect is known as centrid of triangle.

5. solution for problem 5

$$
\begin{aligned}
\mathbf{G} &= \frac{D+E+F}{3} && (31)\\
&= \begin{pmatrix}\frac{7}{2}\\\frac{9}{2}\end{pmatrix}+\begin{pmatrix}\frac{5}{2}\\3\end{pmatrix}+\begin{pmatrix}5\\\frac{7}{2}\end{pmatrix} && (32)\\
\mathbf{G} &= \begin{pmatrix}\frac{11}{3}\\\frac{11}{3}\end{pmatrix} && (33)
\end{aligned}
$$

2 FIGURE

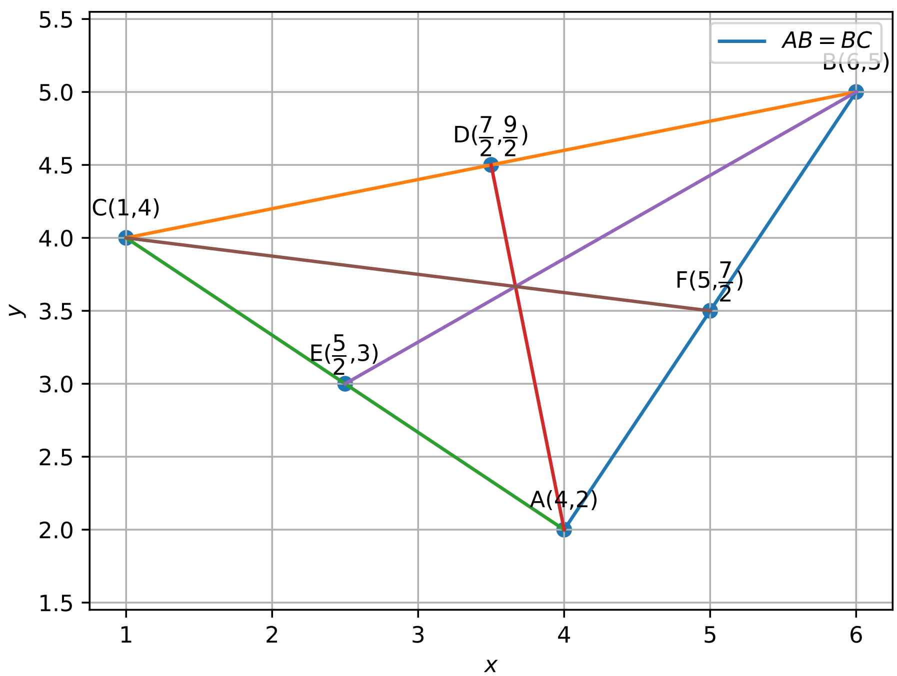

Figure 1: median of triangle