# Circle Assignment

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CONTENTS



## I. PROBLEM

The abscissa of the two points A and B are the roots of the equation x2+2ax-b2=0 and their ordinates are the roots of the equation x2+2px-q=0. Find the equation and the radius of the circle with AB as diameter.

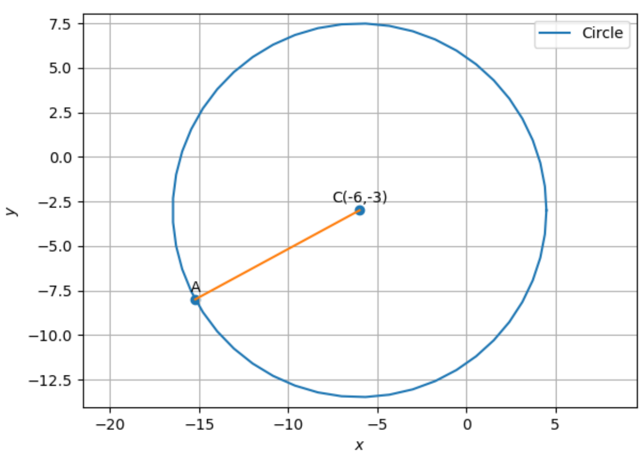


Figure of Construction

## II. CONSTRUCTION

| Symbol | Value | Description |
|---|---|---|
| **C** | $\begin{pmatrix}-6\\-3\end{pmatrix}$ | Center of the circle $C_1$ |
| **r** | 10.4 | Radius of the Circle |

## III. SOLUTION

The roots of the equation $\mathbf{x^2+2ax-b^2=0}$ are

$$\begin{pmatrix}\mathbf{x1}\\\mathbf{y1}\end{pmatrix}=\begin{pmatrix}\mathbf{-a+\sqrt{a^2+b^2}}\\\mathbf{-a-\sqrt{a^2+b^2}}\end{pmatrix}\tag{1}$$

The roots of the equation $\mathbf{x^2+2px-q^2=0}$ are

$$\begin{pmatrix}\mathbf{x2}\\\mathbf{y2}\end{pmatrix}=\begin{pmatrix}\mathbf{-p+\sqrt{p^2+q^2}}\\\mathbf{-p-\sqrt{p^2+q^2}}\end{pmatrix}\tag{2}$$

From question point A and B becomes

$$\mathbf{A}=\begin{pmatrix}\mathbf{-a+\sqrt{a^2+b^2}}\\\mathbf{-p+\sqrt{p^2+q^2}}\end{pmatrix}\tag{3}$$

$$\mathbf{B}=\begin{pmatrix}\mathbf{-a-\sqrt{a^2+b^2}}\\\mathbf{-p-\sqrt{p^2+q^2}}\end{pmatrix}\tag{4}$$

Given AB is diameter so, Center C will be midpoint of A and B

$$\therefore \mathbf{C}=\frac{\mathbf{A+B}}{\mathbf{2}}\tag{5}$$

We get,

$$\mathbf{C}=\begin{pmatrix}\mathbf{-a}\\\mathbf{-p}\end{pmatrix}\tag{6}$$

Radius of circle is given by,

$$\mathbf{r}=\|\mathbf{A-C}\|\tag{7}$$

We get,

$$\mathbf{r}=\sqrt{\mathbf{a^2+b^2+p^2+q^2}}\tag{8}$$

From

$$\mathbf{r}=\sqrt{\|\mathbf{c}\|^2-\mathbf{f}}\tag{9}$$

We get,

$$\mathbf{f=-b^2-q^2}\tag{10}$$

From

$$\mathbf{C=V^{-1}u}\tag{11}$$

We get,

$$\mathbf{u}=\begin{pmatrix}\mathbf{a}\\\mathbf{p}\end{pmatrix}\tag{12}$$

$$\mathbf{x}^\top\mathbf{Vx}+\mathbf{2u}^\top\mathbf{x}+f=0\tag{13}$$

Substituiting u and f in standard equation of conics, we get equation of circle.