Matrix Assignment - Line

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Get Python code for the figure from

https://github.com/SurabhiSeetha/Fwciith2022/tree/main/Assignment%201/codes/src

Get LaTex code from

https://github.com/SurabhiSeetha/Fwciith2022/tree/main/avr%20gcc

1 Question-Class 9, Exercise 9.3, Q(1)

In the Figure, E is any point on median AD of a Δ ABC. Show that ar(ABE) = ar(ACE).

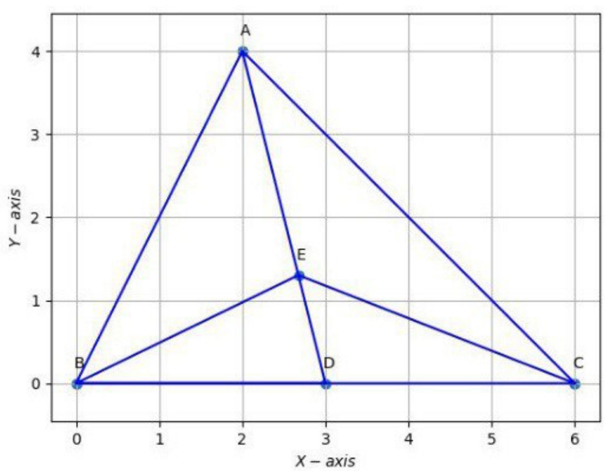

Figure 1 - Triangle ABC

2 Construction

Symbol	Value	Description
r	$\sqrt{20}$	radius
θ	63.45	angle
B	$\begin{pmatrix}0\\0\end{pmatrix}$	Vertex B
C	$\begin{pmatrix}6\\0\end{pmatrix}$	Vertex C
D	$\frac{B+C}{2}$	Mid-point of BC
A	$\begin{pmatrix}rcos\theta\\rsin\theta\end{pmatrix}$	Vertex A
E	$\begin{pmatrix}Ex\\Ey\end{pmatrix}$	point on AD
N	$\begin{pmatrix}Ax\\0\end{pmatrix}$	AN $\perp$ BC
M	$\begin{pmatrix}Ex\\0\end{pmatrix}$	EM $\perp$ BC

3 Solution

3.1 Part-1:

We wish to show that $Ar(\Delta ABE) = Ar(\Delta ACE)$

But to do so, firstly, we need to prove that $Ar(\Delta ABD) = Ar(\Delta ACD)$

Since the formula for area of a triangle is $\frac{1}{2} \times B \times H$, Let us draw an altitude $AN \perp BC$.

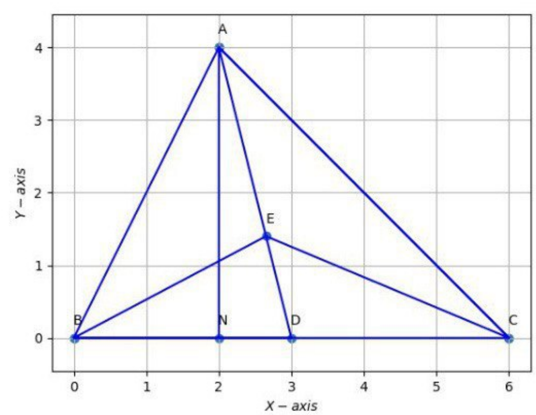

Figure 2 - Δ ABC with altitude $AN \perp BC$

Now, $Ar(\Delta ABD) = \frac{1}{2} \times$ base $\times$ altitude of ΔABD

$$= \frac{1}{2} \times \|\mathbf{D} - \mathbf{B}\| \times \|\mathbf{N} - \mathbf{A}\|$$

$$= \frac{1}{2} \times \|\mathbf{C} - \mathbf{D}\| \times \|\mathbf{N} - \mathbf{A}\|$$

$$[\because \|\mathbf{D} - \mathbf{B}\| = \|\mathbf{C} - \mathbf{D}\|]$$

$= \frac{1}{2} \times$ base $\times$ altitude of ΔACD

$$= Ar(\Delta ACD)$$

$$\therefore Ar(\Delta ABD) = Ar(\Delta ACD) \quad (3.1.1)$$

3.2 Part-2:

Next step is to show that $Ar(\Delta EDB) = Ar(\Delta EDC)$ by using the formula of area of a triangle.

To do so, now we need to draw another perpendicular from the point E onto the base $\overrightarrow{BC}$

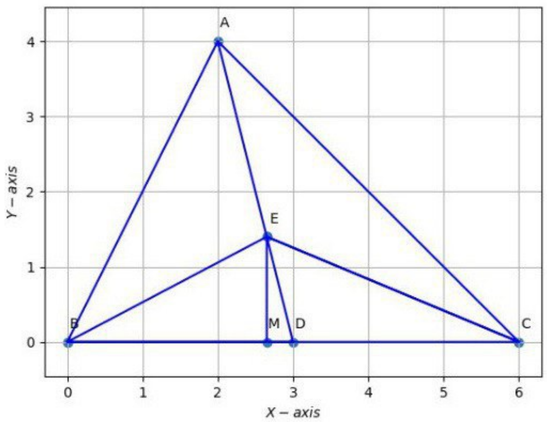

Figure 3 - Δ ABC with altitude $EM \perp CB$

Now, $Ar(\Delta EDB) = \frac{1}{2} \times \text{base} \times \text{altitude of } \Delta EDB$

$$= \frac{1}{2} \times \|\mathbf{D} - \mathbf{B}\| \times \|\mathbf{M} - \mathbf{E}\|$$
$$= \frac{1}{2} \times \|\mathbf{C} - \mathbf{D}\| \times \|\mathbf{M} - \mathbf{E}\|$$
$$[\because \|\mathbf{D} - \mathbf{B}\| = \|\mathbf{C} - \mathbf{D}\|]$$
$$= \frac{1}{2} \times \text{base} \times \text{altitude of } \Delta EDC$$
$$= Ar(\Delta ECD)$$

$$\therefore Ar(\Delta EBD) = Ar(\Delta ECD) \quad (3.2.1)$$

From Fig.1, we can write that

$$Ar(\Delta ABD) = Ar(\Delta ABE) + Ar(\Delta EBD) \quad (3.2.2)$$

And,

$$Ar(\Delta ACD) = Ar(\Delta ACE) + Ar(\Delta ECD) \quad (3.2.3)$$

from the equation 3.1.1, we can say that,

$$Ar(\Delta ABD) = Ar(\Delta ACD)$$

. Hence, from Eq.3.2.2 and eq.3.2.3 we get,

$$Ar(\Delta ABE) + Ar(\Delta EBD) = Ar(\Delta ACE) + Ar(\Delta ECD) \quad (3.2.4)$$

From eq. 3.2.1, we proved that

$$Ar(\Delta EBD) = Ar(\Delta ECD)$$

Hence, from the above equations 3.2.1 and 3.1.1 we can conclude that,

$$\boxed{\therefore \text{Ar}(\Delta\ \text{ABE}) = \text{Ar}(\Delta\ \text{ACE})}$$

Hence Proved