Circle Assignment

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The centre of the circle passing through (0,0)and (1,0) and touching the circle $x^2 + y^2 = 9$

2 Construction

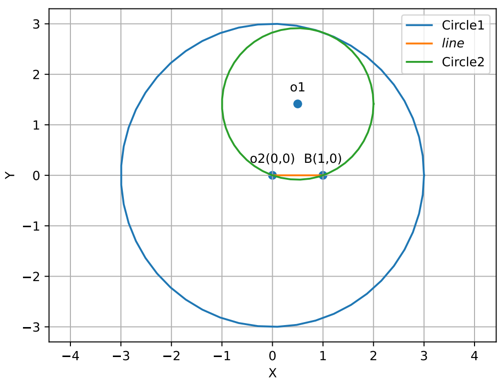

Figure of construction

3 Solution

The standard circle equation

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

Given Circle equation : $x^2 + y^2 = 9$
The given circle can be expressed as conics with parameters

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & 0 \end{pmatrix} \mathbf{x} + f_2 = 0 \tag{2}$$

$$\mathbf{V_1} = \mathbf{I}, \mathbf{u_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f_2 = -9 \tag{3}$$

Radius and Centre are

$$r_2 = \sqrt{\mathbf{u_2}^\top \mathbf{u_2} - f_2} \tag{4}$$

$$r_2 = 3 \tag{5}$$

The given circle can be expressed as conics with parameters For finding center $\mathbf{u_1}$

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f_1 = 0 \tag{6}$$

$$\mathbf{A}^\top \mathbf{A} + 2\mathbf{u_1}^\top \mathbf{A} + f_1 = 0 \tag{7}$$

$$\mathbf{B}^\top \mathbf{B} + 2\mathbf{u_1}^\top \mathbf{B} + f_1 = 0 \tag{8}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{9}$$

when substitute $\mathbf{A}, \mathbf{B}$ in eq 6and 7
$f_1 = 0,$
by eq 7

$$1 + 2\mathbf{u_1}^\top \begin{pmatrix} 1 \\ 0 \end{pmatrix} + f_1 = 0 \tag{10}$$

$$1 + 2\mathbf{u_1}^\top \mathbf{e_1} = 0 \tag{11}$$

$$\mathbf{u_1}^\top \mathbf{e_1} = -1/2 \tag{12}$$

$$\mathbf{u_1} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{13}$$

substitute in eq-11

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -1/2 \tag{14}$$

$$x = -1/2 \tag{15}$$

substitute x in eq-13

$$\mathbf{u_1} = \begin{pmatrix} -1/2 \\ y \end{pmatrix} \tag{16}$$

$$\|\mathbf{u_1} - \mathbf{u_2}\| = (r_1 - r_2)^2 \tag{17}$$

$$r_1 = \frac{r_2}{2} \tag{18}$$

$$\|\mathbf{u_1}\|^2 = r_1^2 \tag{19}$$

$$\|\mathbf{u_1}\|^2 = (r_2/2)^2 \tag{20}$$

by using eq-15

$$\|\mathbf{u_1}\|^2 = 1/4 + y^2 = \frac{r_2^2}{4} \tag{21}$$

$$y = \pm\sqrt{\frac{r_2^2}{4} - \frac{1}{4}} \tag{22}$$

yielding,

$$y = \pm\sqrt{2} \tag{23}$$

substituting y in eq-15

$$\mathbf{u_1} = \begin{pmatrix} -1/2 \\ -\sqrt{2} \end{pmatrix} \tag{24}$$

$$\mathbf{o_1} = -\mathbf{u_1} \tag{25}$$

$$r_1 = \sqrt{\mathbf{u_1}^\top \mathbf{u_1} - f_1} \tag{26}$$

$$r_1 = 3/2 \tag{27}$$

The input parameters for this construction are

Symbol	**Value**	**Description**
$\mathbf{A}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Centre1,point p1
$\mathbf{B}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	point2
$\mathbf{o_2}$	$\begin{pmatrix} -1/2 \\ -\sqrt{2} \end{pmatrix}$	center2
$\mathbf{R}$	$\begin{pmatrix} 1.5 \end{pmatrix}$	Radius2