CIRCLE

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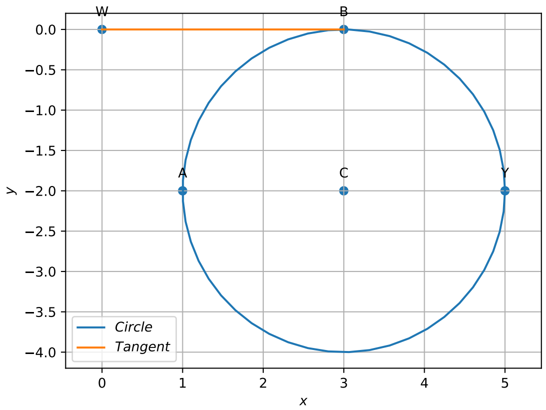
Contents

1 Construction



2 Problem

The circle passing through $\begin{pmatrix}1\\-2\end{pmatrix}$ and touching the axis of x at $\begin{pmatrix}3\\0\end{pmatrix}$ also passes through the point

A : $\begin{pmatrix}-5\\2\end{pmatrix}$

B : $\begin{pmatrix}2\\-5\end{pmatrix}$

C : $\begin{pmatrix}5\\-2\end{pmatrix}$

D : $\begin{pmatrix}-2\\5\end{pmatrix}$

3 Solution

To find the center and Radius The equation of the circle is

$$x^T\mathbf{V}x + 2\mathbf{u}^Tx + f = 0 \tag{1}$$

Circle passes through $\begin{pmatrix}1\\-2\end{pmatrix}$

$$\mathbf{A}\mathbf{A}^T + 2\mathbf{u}^T\mathbf{A} + f = 0 \tag{2}$$

$$\|A\|^2 + 2\mathbf{A}^T\mathbf{u} + f = 0 \tag{3}$$

$$\begin{pmatrix}2\mathbf{A}^T & 1\end{pmatrix}\begin{pmatrix}\mathbf{u}\\f\end{pmatrix} = -\|A\|^2 \tag{4}$$

$$\mathbf{B}\mathbf{B}^T + 2\mathbf{u}^T\mathbf{B} + f = 0 \tag{5}$$

$$\|B\|^2 + 2\mathbf{u}^T(B) + f = 0 \tag{6}$$

$$\begin{pmatrix}2B^T & 1\end{pmatrix}\begin{pmatrix}\mathbf{u}\\f\end{pmatrix} = -\|B\|^2 \tag{7}$$

The equation of the tangent is

$$\mathbf{m}^T(\mathbf{V}q + \mathbf{u}) = 0 \tag{8}$$

$$\mathbf{m}^T\mathbf{B} + \mathbf{m}^T\mathbf{u} = 0 \tag{9}$$

$\mathbf{C} = -\mathbf{u}$

$$\begin{pmatrix}\mathbf{m}^T & 0\\ 2\mathbf{A}^T & 1\\ 2\mathbf{B}^T & 1\end{pmatrix}\begin{pmatrix}\mathbf{u}\\f\end{pmatrix} = \begin{pmatrix}-\mathbf{m}^T\mathbf{B}\\ -\|\mathbf{A}\|^2\\ -\|\mathbf{B}\|^2\end{pmatrix} \tag{10}$$

$$\begin{pmatrix}1 & 0 & 0 & -3\\ 2 & -4 & 1 & -9\\ 6 & 0 & 1 & -5\end{pmatrix} \xrightarrow{R_2 \leftarrow -2R_1 + R_2}$$

$$\begin{pmatrix}1 & 0 & 0 & -3\\ 0 & -4 & 1 & 1\\ 6 & 0 & 1 & -9\end{pmatrix} \xrightarrow{R_3 \leftarrow -6R_1 + R_3}$$

$$\begin{pmatrix}1 & 0 & 0 & -3\\ 0 & -4 & 1 & 1\\ 0 & 0 & 1 & 9\end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/4}$$

$$\begin{pmatrix}1 & 0 & 0 & -3\\ 0 & 1 & \frac{-1}{4} & \frac{-1}{-4}\\ 0 & 0 & 1 & 9\end{pmatrix} \xrightarrow{R_2 \leftarrow \frac{1}{4}R_3 + R_2} \begin{pmatrix}1 & 0 & 0 & -3\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & 9\end{pmatrix}$$

By solving the above equations

The center is $\mathbf{C} = \begin{pmatrix}3\\-2\end{pmatrix}$ and $f = 9$

Radius

$$\mathbf{m} = \begin{pmatrix}1\\-2\end{pmatrix} - \begin{pmatrix}3\\-2\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix} \tag{11}$$

$$\sqrt{\begin{pmatrix}2 & 0\end{pmatrix}\begin{pmatrix}2\\0\end{pmatrix}} = 2 \tag{12}$$

from the given points $\begin{pmatrix}5\\-2\end{pmatrix}$ satisfies the above condition

$$\mathbf{m} = \begin{pmatrix}5\\-2\end{pmatrix} - \begin{pmatrix}3\\-2\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix} \tag{13}$$

$$\sqrt{\begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}} = 2 \tag{14}$$

$\therefore \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ lies on the circle