

Line Assignment

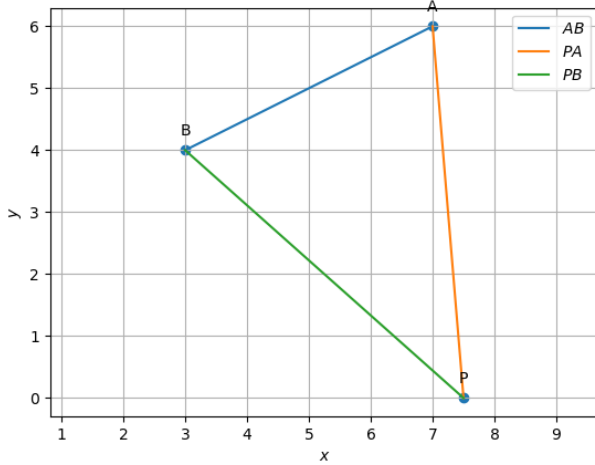
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Problem Statement - Find a point on the x-axis, which is equidistant from the points $\begin{pmatrix} 7 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$$\mathbf{e}_2^\top \mathbf{x} = 0 \quad (2)$$

Symbol	Co-ordinates	Description
A	$\begin{pmatrix} 7 \\ 6 \end{pmatrix}$	co-ordinates of A
B	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$	co-ordinates of B



$$(\mathbf{A} - \mathbf{B})^\top \mathbf{x} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}$$

$$\begin{pmatrix} 0 & 1 \\ 4 & 2 \end{pmatrix} x = \begin{pmatrix} 0 \\ 30 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 4 & 2 & 30 \end{pmatrix}$$

Divide by 2

$$\begin{pmatrix} 0 & 1 & 0 \\ 2 & 1 & 15 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 15 \\ 0 & 1 & 0 \end{pmatrix} \xleftarrow{R_2 \leftarrow R_1}$$

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{15}{2} \\ 0 & 1 & 0 \end{pmatrix} \xleftarrow{R_1 = \frac{R_1}{2}}$$

$$\begin{pmatrix} 1 & 0 & \frac{15}{2} \\ 0 & 1 & 0 \end{pmatrix} \xleftarrow{R_1 = R_1 - \frac{R_2}{2}}$$

$$\begin{pmatrix} 1 & 0 & 7.5 \\ 0 & 1 & 0 \end{pmatrix}$$

on solving we get $x = 7.5$

$$\mathbf{x} = \begin{pmatrix} 7.5 \\ 0 \end{pmatrix}$$

Solution

- Given points $\mathbf{A} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$
- If the point is lying on x-axis then y-axis will be zero i.e..
 $y=0$

From the given information

$$\|(\mathbf{x} - \mathbf{A})\|^2 = \|(\mathbf{x} - \mathbf{B})\|^2 \quad (1)$$

$$\|(\mathbf{x} - \mathbf{A})\|^\top (\mathbf{x} - \mathbf{A}) = \|(\mathbf{x} - \mathbf{B})\|^\top (\mathbf{x} - \mathbf{B})$$

$$\|(\mathbf{x})\|^2 - 2\mathbf{A}^\top \mathbf{x} + \|\mathbf{A}\|^2 = \|(\mathbf{x})\|^2 - 2\mathbf{B}^\top \mathbf{x} + \|\mathbf{B}\|^2$$

$$2(\mathbf{A}^\top - \mathbf{B}^\top) \mathbf{x} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2$$

$$(\mathbf{A}^\top - \mathbf{B}^\top) \mathbf{x} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}$$

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{x} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}$$