FWC22037

ASSIGNMENT-MATRICES

Contents

1 Problem

Find the equation of the tangent and the normal to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at the point (x_0, y_0).

2 Solution

1. The give hyperbola equation can be represented as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$$

$$\mathbf{x}^\top \begin{pmatrix} \frac{1}{a^2} & 1 \\ 0 & \frac{-1}{b^2} \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & 0 \end{pmatrix} \mathbf{x} - 1 = 0$$

2.Equation of tangent is

$$(\mathbf{V}\mathbf{q} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{q} + f = 0$$

$$\left[\begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{-1}{b^2} \end{pmatrix} \begin{pmatrix} x_0 & y_0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right]^\top$$

$$\mathbf{x} + \begin{pmatrix} 0 & 0 \end{pmatrix} q - 1 = 0$$

$$\begin{pmatrix} \frac{x_0}{a^2} & \frac{-y_0}{b^2} \end{pmatrix} \mathbf{x} = 1 \tag{1}$$

Therefore equation of tangent is

$$\left(\frac{x_0}{a^2}\right) x - \left(\frac{y_0}{b^2}\right) y = 1 \tag{2}$$

3. Equation of normal is

$$\mathbf{m}^\top (\mathbf{x} - \mathbf{p}) = 0 \tag{3}$$

$$\mathbf{m} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{n} \tag{4}$$

$$\mathbf{m} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{x_0}{a^2} \\ \frac{-y_0}{b^2} \end{pmatrix}$$

$$\mathbf{m} = \begin{pmatrix} \frac{y_0}{b^2} \\ \frac{x_0}{a^2} \end{pmatrix}$$

$$\mathbf{m}^\top = \begin{pmatrix} \frac{y_0}{b^2} & \frac{x_0}{a^2} \end{pmatrix} \tag{5}$$

Substituting the eq.5 in eq.3

$$\begin{pmatrix} \frac{y_0}{b^2} & \frac{x_0}{a^2} \end{pmatrix} \left[\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \right] = 0 \tag{6}$$

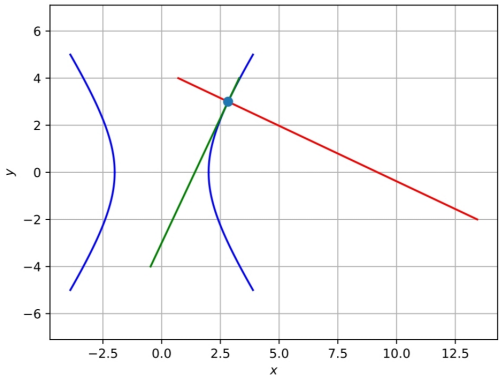
$$\frac{y_0}{b^2}(x - x_0) + \frac{x_0}{a^2}(y - y_0) = 0$$

$$\frac{y_0}{b^2}(x - x_0) = -\frac{x_0}{a^2}(y - y_0)$$

$$\frac{x - x_0}{b^2 x_0} = -\frac{y - y_0}{a^2 y_0}$$

Therefore the equation of normal is

$$\frac{x - x_0}{b^2 x_0} + \frac{y - y_0}{a^2 y_0} = 0$$



Figure

3 Construction

The hyperbol is constructed with

Symbol	Co-ordinates	Description
P	$\begin{pmatrix} 2\sqrt{2} \\ 3 \end{pmatrix}$	point on the hyperbola
a	2	distance from the vertex to the center
b	3	distance perpendicular to the transverse axis from the vertex to the asymptote lines

The figure above is generated using python code provided in the below source code link.

https://github.com/sivagayathri/FWC/blob/main/matrices/conic/conic.py