

circle Assignment

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October 21, 2022

Problem Statement:

Two circles of radii 5cm and 3cm intersect at two points and the distance between their center is 4cm $c1=(0,0)$ $c2=(4,0)$. Find the length of the common chord.

Solution:

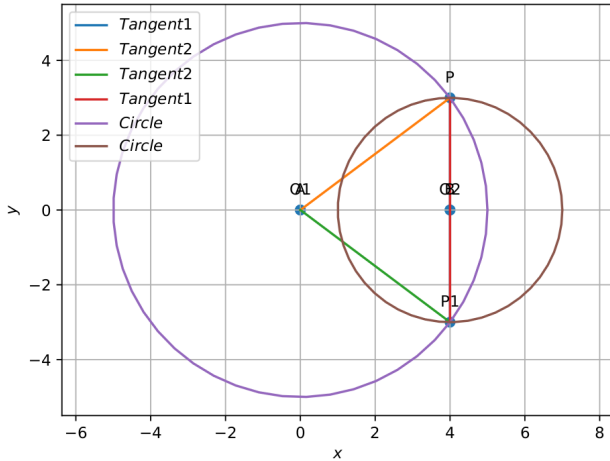


Figure 1: Diagram generated using python

0.1 Theory:

They given two circles radius first circle radii is 5cm(Q1) and second circle radii is 3cm(Q2) distance between circle 1 and circle 2 is 4cm. we have find the length of the chord.

0.2 Mathematical Calculation:

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

0.3 Deriving equation for Circle in matrix form

The equation of circle in matrix form is,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

Where

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -25$$

$$\Rightarrow \mathbf{x}^T \mathbf{I} \mathbf{x} + 2 \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \mathbf{x} - 25 = 0$$

Therefore, the circle equation can be written as

$$\mathbf{x}^T \mathbf{x} + 2 \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \mathbf{x} - 25 = 0 \quad (2)$$

The equation of circle in matrix form is,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (3)$$

Where

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, f = 7$$

$$\Rightarrow \mathbf{x}^T \mathbf{I} \mathbf{x} + 2 \begin{pmatrix} -4 \\ 0 \end{pmatrix}^T \mathbf{x} + 7 = 0$$

Therefore, the circle equation can be written as

$$\mathbf{x}^T \mathbf{x} + 2 \begin{pmatrix} -4 \\ 0 \end{pmatrix}^T \mathbf{x} + 7 = 0 \quad (4)$$

$$= \begin{vmatrix} -25\mu + 7\mu + \mu & -16\mu^2 \\ 0 & -25\mu + 7\mu + \mu \end{vmatrix}$$

$$\mu = -1$$

Here we have to Find the Intersection of Two conics

$$\mathbf{x}^T \mathbf{V}_1 \mathbf{x} + 2 \mathbf{u}_1^T \mathbf{x} + \mathbf{f}_1 = 0 \quad (5)$$

0.4 According to the equation 7

$$\mathbf{x}^T \mathbf{V}_2 \mathbf{x} + 2 \mathbf{u}_2^T \mathbf{x} + \mathbf{f}_2 = 0 \quad (6)$$

The locus of their pair of straight lines

$$\mathbf{x}^T (\mathbf{V}_1 + \mu \mathbf{V}_2) \mathbf{x} + 2(\mathbf{u}_1 + \mu \mathbf{u}_2)^T \mathbf{x} + \mathbf{f}_1 + \mathbf{f}_2 = 0 \quad (9)$$

$$\mathbf{x}^T (\mathbf{V}_1 + \mu \mathbf{V}_2) \mathbf{x} + 2(\mathbf{u}_1 + \mu \mathbf{u}_2)^T \mathbf{x} + \mathbf{f}_1 + \mathbf{f}_2 = 0 \quad (7)$$

$$8x = 32$$

$$x=4$$

$$\begin{vmatrix} \mathbf{V}_1 + \mu \mathbf{V}_2 & \mathbf{u}_1 + \mu \mathbf{u}_2 \\ (\mathbf{u}_1 + \mu \mathbf{u}_2)^T & f_1 + f_2 \end{vmatrix} = 0, \quad (8)$$

$$\text{So, } \mathbf{P} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \mathbf{P}_1 = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

0.5 So The length of the common chord is 6cm

$$= ||\mathbf{P} - \mathbf{P}_1||$$

$$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$= 6$$

Construction

Symbol	Value	Description
r_1	5	Radius
r_2	3	Radius
O	(0, 0)	Center
O ₁	(4, 0)	Center
P	(4, 3)	Point Of intersection
P ₁	(4, -3)	Point Of intersection
P-P ₁	6	Length of the common chord

$$f_1 = -25 \quad f_2 = 7$$

$$= \begin{vmatrix} \mathbf{I} + \mu \mathbf{I} & 0 - \begin{pmatrix} 4 \\ 0 \end{pmatrix} \\ 0 - \mu(4, 0) & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 + \mu & 0 & (-4\mu) \\ 0 & 1 + \mu & 0 \\ -4\mu & 0 & 7\mu - 25 \end{vmatrix}$$