భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
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Matrix Assignment

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1 Problem statement:

Construct a triangle PQR in which QR=6cm, $\angle Q = 60^0$ and PR - PQ = 2cm.

Law of Cosines

The law of Cosines relates the length of the triangle to the cosines of one of its angles. It states that, if the length of two sides and the angle between them is known for a triangle, then we can determine the length of the third side. It is given by:

$$\alpha^2 = \beta^2 + \gamma^2 - 2\beta\gamma\cos\theta \tag{1}$$

SOLUTION:

Steps of Construction:

Step 1:

Let P,Q and R be the vertices of the triangle with coordinates.
Given QR length is a=6cm, So the coordinates of vertices Q,R and P are :

$$Q = \begin{pmatrix}0\\0\end{pmatrix} R = \begin{pmatrix}6\\0\end{pmatrix} P = \alpha\begin{pmatrix}cos\theta\\sin\theta\end{pmatrix}$$

Also given the angle is $Q = 60^0$,so by finding the coordinates of the other sid e we can form a required triangle.

For the input parameters in Table 1.

Symbol	Value	Description
Q	$\begin{pmatrix}0\\0\end{pmatrix}$	Q Point
R	$\begin{pmatrix}6\\0\end{pmatrix}$	R Point
θ	60°	∠PQR
λ	2	PR-PQ
q	α	PR
r	γ	PQ
p	6	QR

Table 1

Given that,

$$\alpha - \gamma = 2 \tag{2}$$

By using the Cosine formula in ΔPQR

$$\alpha^2 = \beta^2 + \gamma^2 - 2\beta\gamma\cos\theta \tag{3}$$

$$\alpha^2 - \gamma^2 = \beta^2 - 2\beta\gamma\cos\theta \tag{4}$$

$$(\alpha+\gamma)(\alpha-\gamma) = \beta^2 - 2\beta\gamma\cos\theta \tag{5}$$

$$(\alpha+\gamma)(\lambda) = \beta^2 - 2\beta\gamma\cos\theta \tag{6}$$

$$\lambda\alpha + \lambda\gamma + 2\beta\gamma\cos\theta = \beta^2 \tag{7}$$

$$\lambda\alpha + \gamma(\lambda + 2\beta\cos\theta) = \beta^2 \tag{8}$$

Step 2:

We know that,

$$\mathbf{AX} = \mathbf{B} \tag{9}$$

Using equation (2) and (8),

$$\begin{pmatrix}1 & -1\\ \lambda & \lambda + 2\beta\cos\theta\end{pmatrix}\begin{pmatrix}\alpha\\\gamma\end{pmatrix} = \begin{pmatrix}\lambda\\\beta^2\end{pmatrix} \tag{10}$$

After substituting values,

$$\begin{pmatrix}1 & -1\\ 1 & 4\end{pmatrix}\begin{pmatrix}\alpha\\\gamma\end{pmatrix} = \begin{pmatrix}2\\18\end{pmatrix} \tag{11}$$

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c}1 & -1 & 2\\ 1 & 4 & 18\end{array}\right) \tag{12}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{cc|c}1 & -1 & 2\\ 0 & 5 & 16\end{array}\right)$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{5}R_2} \left(\begin{array}{cc|c}1 & -1 & 2\\ 0 & 1 & \frac{16}{5}\end{array}\right)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_2} \left(\begin{array}{cc|c}1 & 0 & \frac{26}{5}\\ 0 & 1 & \frac{16}{5}\end{array}\right)$$

$$\implies X = \begin{pmatrix}\frac{26}{5}\\\frac{16}{5}\end{pmatrix} \tag{13}$$

Using equation (7) we get ,

$$\alpha = \frac{26}{5} \tag{14}$$

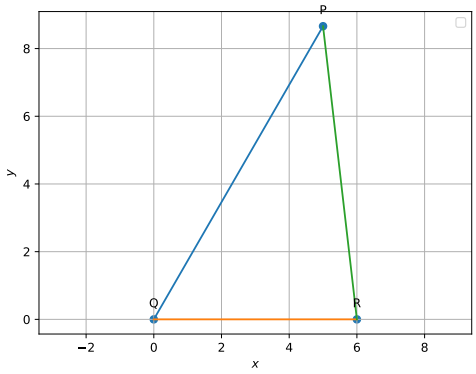
$$\gamma = \frac{16}{5} \tag{15}$$

The vertices of Δ PQR are

$$P = \frac{26}{5}\begin{pmatrix} cos60 \\ sin60 \end{pmatrix}, Q = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, R = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (16)$$

Equation no	Role
1	law of Cosines
7	Matrix form of Linear equation
10	Length of r
11	Length of q

Result



Construction

vertex	coordinates
P	$\begin{pmatrix} 2.6 \\ 4.5 \end{pmatrix}$
Q	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
R	$\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

Implementation

Download the code
https://github.com/Gangagopinath/ASSIGNMENT/tree/main/assignment4