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MATRIX : CONIC ASSIGNMENT

0.1 Problem:

Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

0.2 Solution:

Input Parameters :

Curve Equation : $x^2 = 4y$.
Line Equation : $x = 4y - 2$.

To Find :

1. Comparing the given curve equation with the standard equation of the conics and finding it's parameters.
2. Finding the required parameters for the line equation.
3. Finding the Point of Intersection of the to the curve.
4. Finding the area bounded by the curve and the line.

Step - 1 :

Curve Equation : $x^2 = 4y$.
The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

The given curve can be expressed as conics with parameters

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}, f = 0 \tag{2}$$

Step - 2 :

Line Equation : $x = 4y - 2$.
From the above line equation below vectors are taken

$$\mathbf{q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{3}$$

Step - 3 :

The points of intersection of the line,

$$L : \quad \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \quad \mu \in \mathbb{R} \tag{4}$$

with the conic section,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{5}$$

are given by

$$\mathbf{x}_i = \mathbf{q} + \mu_i \mathbf{m} \tag{6}$$

where,

$$\mu_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}} \Big(-\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)} \Big) \tag{7}$$

On substituting $\mathbf{V}, \mathbf{q}, \mathbf{m}$ in the above equation, we get the values of μ. By substituting the values of μ in eq(6), we get the points of intersection of line with the given curve.

$$i.e., \mathbf{x}_1, \mathbf{x}_2$$

$$\therefore \mathbf{x}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} -1 \\ \frac{1}{4} \end{pmatrix} \tag{8}$$

Step - 4 :

The area bounded by the curve $x^2 = 4y$ and line $x = 4y - 2$ is given by

$$\implies A = \int_{x_2}^{x_1} \left[f(x) - g(x)\right] dx \tag{9}$$

$$\implies A = \int_{-1}^{2} \left(\frac{x+2}{4} - \frac{x^2}{4}\right) dx \tag{10}$$

By solving we get the required area
$\therefore A = \frac{9}{8}$

Code Link :

The below link realises the code of the above construction.

```
https://github.com/19pa1a04e9/FWC-IITH/tree/
main/Assignment-1/MATRICES/Conic/codes/conic.
py
```

0.3 Termux Commands :

bash rncom.sh ..... Using Shell commands.

0.4 Plot :

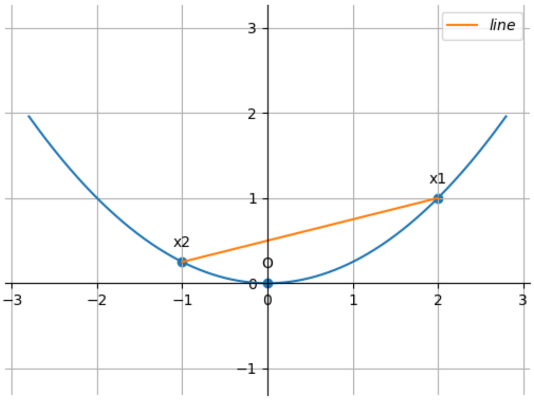

Figure 1