Circle Assignment

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Problem

Show that the tangents of circle drawn at the ends of diameter are parallel.

Solution

Symbol	Value	Description
r	10	circle radius
O	$-\mathbf{u}$	Center
A	$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$	point A
B	$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$	point B

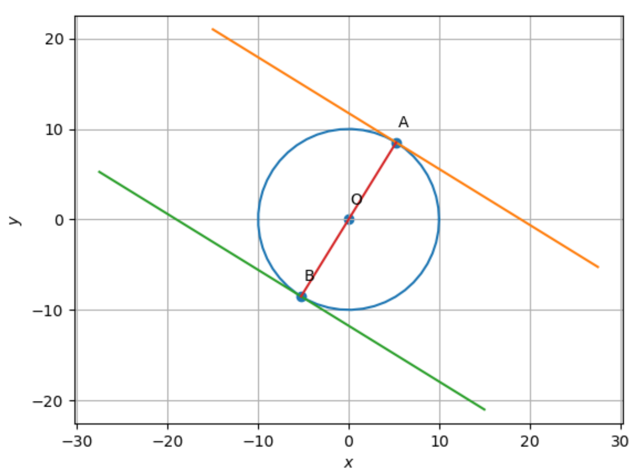

Figure 1: Circle with tangents at ends of it's diameter

Construction

Input taken for the construction of the Circle and the tangents is 'r' radius of the circle.

Let us assume a circle with radius 'r' and center at origin.

$$\boldsymbol{x}^T\boldsymbol{V}\boldsymbol{x} + 2\boldsymbol{u}^T\mathbf{x} + f = 0 \tag{1}$$

but, for a Circle

$$\boldsymbol{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

and the center of the circle is,

$$-\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{3}$$

Let us assume a point $\mathbf{A}$ on the circle and a point $\mathbf{B}$ such that the points form the diameter of the circle. Center of the circle bisect the diameter in equal parts. Then,

$$\frac{\mathbf{A}+\mathbf{B}}{2} = -\mathbf{u} \tag{4}$$

$$\implies \mathbf{A}+\mathbf{B} = -2\mathbf{u} \tag{5}$$

Here, $\mathbf{A}$ and $\mathbf{B}$ are the ends of the diameter and the contact points for the tangents. We know that, for a circle, any line passing through it's center is a normal to the circle at the point of contact.
Tangent intersect the circle at only one point on it's circumference. So, the line intersecting the circle at one point $\mathbf{q}$ is

$$\boldsymbol{m}^T(\boldsymbol{V}\boldsymbol{q} + \boldsymbol{u}) = 0 \tag{6}$$

where $\boldsymbol{m}$ is the directional vector of line at the point of contact.
equation for a tangent at point $\mathbf{A}$ is

$$\boldsymbol{m_1}^T(\boldsymbol{A} + \boldsymbol{u}) = 0 \tag{7}$$

Similarly, Euqation of the tangent at $\boldsymbol{B}$ is given by,

$$\boldsymbol{m_2}^T(\boldsymbol{B} + \boldsymbol{u}) = 0 \tag{8}$$

where $\mathbf{m_1}$&$\mathbf{m_2}$ are the direction vectors of the tangents.
Then, the normal vectors at the point of contact of tangets are

$$\mathbf{A}+\mathbf{u}=k_1\mathbf{n_1} \tag{9}$$

$$\mathbf{B}+\mathbf{u}=k_2\mathbf{n_2} \tag{10}$$

by adding the above equations (9)&(10),

$$\mathbf{A}+\mathbf{B}+2\mathbf{u}=k_1\mathbf{n_1}+k_2\mathbf{n_2} \tag{11}$$

from (5), (11) can be modified as

$$k_1\mathbf{n_1}+k_2\mathbf{n_2}=0 \tag{12}$$

$$k_1\mathbf{n_1}=-k_2\mathbf{n_2} \tag{13}$$

The crossproduct of the normal vectors is zero.

$$\mathbf{n_1}\times\mathbf{n_2}=0 \tag{14}$$

Here, the mormal vectors are parallel n_1 & n_2. So the tangents are parallel to each other.
Hence, we have proved that the tangents at the ends of the diameter of a circle are parallel.