circle Assignment

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CONTENTS

I. PROBLEM

The lines 3x-4y+4=0 and 6x-8y-7=0 are tangents to the same circle.The radius of this circle is .....

II. SOLUTION

The distance between the two parallel lines
$\mathbf{n}^\top \mathbf{x} = \mathbf{C_1}$
$\mathbf{n}^\top \mathbf{x} = \mathbf{C_2}$

$$\mathbf{D} = \frac{|\mathbf{C_2} - \mathbf{C_1}|}{\|\mathbf{n}\|} \tag{1}$$

Termux commands :

```
python3 circle.py
```

Symbol	Value	Description
P	$\begin{pmatrix} 1 \\ \frac{7}{4} \end{pmatrix}$	Point P

The distance between the two parallel lines

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -\mathbf{4} \tag{2}$$

$$\begin{pmatrix} 6 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{7} \tag{3}$$

$$\mathbf{D} = \frac{|\mathbf{C_2} - \mathbf{C_1}|}{\|\mathbf{n}\|} \tag{4}$$

where $\mathbf{C_2} = 3.5, \mathbf{C_1} = -4, \|\mathbf{n}\| = 5$
By using the above values we get $\mathbf{D}$
D=1.5
therefore the radius of the circle is
R=0.5D
Let $\mathbf{P}$ be a point on a line (2)
The equation of a line passing through point $\mathbf{P}$ and perpendicular to the line (2) is
$\mathbf{m}^\top(\mathbf{x} - \mathbf{P}) = 0$
where $\mathbf{m}^\top = \begin{pmatrix} 1 & \frac{3}{4} \end{pmatrix}$
Therefore the equation of a line passing through $\mathbf{P}$ and perpendicular to the line (2) is

$$\begin{pmatrix} 16 & 12 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{37} \tag{5}$$

STEP-1

$$16x + 12y = 37 \tag{6}$$
$$6x - 8y = 7 \tag{7}$$
$$\implies \begin{pmatrix} 16 & 12 \\ 6 & -8 \end{pmatrix} \mathbf{X} = \begin{pmatrix} 37 \\ 7 \end{pmatrix} \tag{8}$$

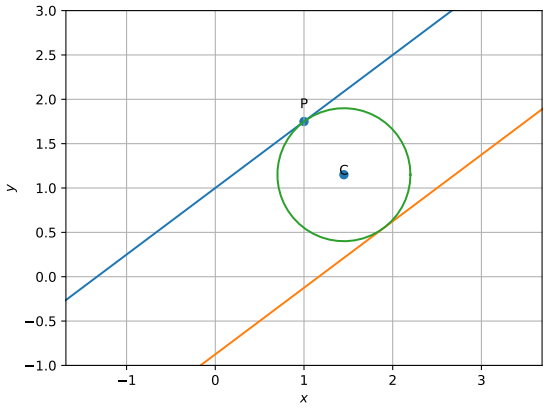
The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c} 16 & 12 & 37 \\ 6 & -8 & 7 \end{array}\right) \tag{9}$$
$$\xleftrightarrow{R_1 \leftarrow R_1/16} \left(\begin{array}{cc|c} 1 & 0.75 & 2.3125 \\ 6 & -8 & 7 \end{array}\right) \tag{10}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 - 6R_1} \left(\begin{array}{cc|c} 1 & 0.75 & 2.3125 \\ 0 & -12.5 & -6.875 \end{array}\right) \tag{11}$$
$$\xleftrightarrow{R_2 \leftarrow R_2/((-12.5)} \left(\begin{array}{cc|c} 1 & 0.75 & 2.3125 \\ 0 & 1 & 0.55 \end{array}\right) \tag{12}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 - (0.75)R_2} \left(\begin{array}{cc|c} 1 & 0 & 1.9 \\ 0 & 1 & 0.55 \end{array}\right) \tag{13}$$
$$\implies \mathbf{X} = \begin{pmatrix} 1.9 \\ 0.55 \end{pmatrix} \tag{14}$$

So point $\mathbf{X}$ is a point on a line (3)
Now the midpoint of $\mathbf{P}$ and $\mathbf{X}$ gives us the centre
C= $\begin{pmatrix} 1.45 \\ 1.15 \end{pmatrix}$
Now $\|\mathbf{P} - \mathbf{C}\|$ gives us the radius of the circle
$\|\mathbf{P} - \mathbf{C}\|$=0.75

III. Construction

Figure of Construction



The below python code realizes the above construction:
https://github.com/ballepu1994/matricescircle