

Contents

1 Problem	1
2 Construction	1
3 Solution	1

1 Problem

In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ. Show that:

- $\triangle APD \cong \triangle CQB$
- $AP = CQ$
- $\triangle AQB \cong \triangle CPD$
- $AQ = CP$
- APCQ is a parallelogram

$$\mathbf{A} = \begin{pmatrix} r \cos \theta \\ r \sin \theta \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} b \\ 0 \end{pmatrix}$$

$$\mathbf{D} = \mathbf{A} + \mathbf{C} - \mathbf{B}$$

$$\mathbf{P} = \frac{\mathbf{B} + K \times \mathbf{D}}{1 + K}$$

$$\mathbf{Q} = \frac{K \times \mathbf{B} + \mathbf{D}}{1 + K}$$

Theorem

A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

"If two directional vectors are equal, implies their magnitude as well as direction are equal to each other."

Two vectors are parallel if they have the same direction (or) are in exactly opposite directions.

Given ABCD is a parallelogram. ,the two points P and Q are taken on diagonal BD such that DP = BQ.

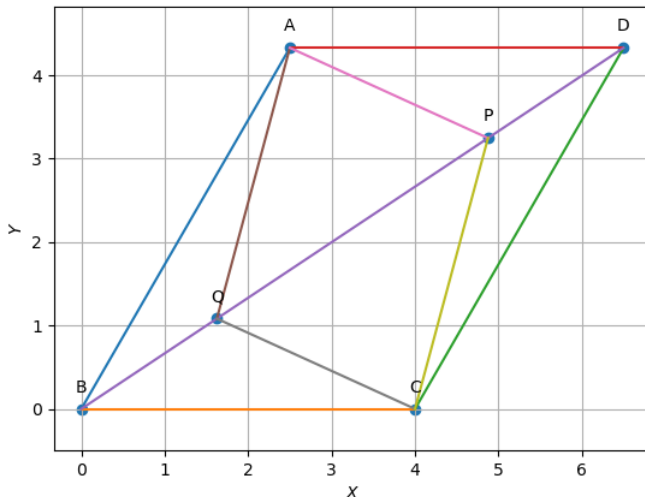


Figure of construction

$$\Rightarrow \mathbf{A} - \mathbf{B} = \mathbf{D} - \mathbf{C} \quad (1)$$

$$\mathbf{A} - \mathbf{D} = \mathbf{B} - \mathbf{C} \quad (2)$$

$$\mathbf{B} - \mathbf{Q} = \mathbf{P} - \mathbf{D} \quad (3)$$

From (1) and (3):

$$\mathbf{A} + \mathbf{C} = \mathbf{B} + \mathbf{D} \quad (4)$$

$$\mathbf{P} + \mathbf{Q} = \mathbf{B} + \mathbf{D}$$

From (4)

$$\mathbf{A} + \mathbf{C} = \mathbf{P} + \mathbf{Q} \quad (5)$$

From (5)

$$\Rightarrow \mathbf{A} - \mathbf{Q} = \mathbf{P} - \mathbf{C} \quad (6)$$

$$\mathbf{A} - \mathbf{P} = \mathbf{Q} - \mathbf{C} \quad (7)$$

1. From (2),(3) and (7)

$$\triangle APD \cong \triangle CQB \quad (8)$$

The input parameters for this construction are

Symbol	Value
r	5
k	3
b	4
θ	$\frac{\pi}{3}$

2.

$$\text{Equation}(7) \implies AP = CQ \quad (9)$$

3. From (1),(3) and (6)

$$\triangle AQB \cong \triangle CPD \quad (10)$$

4.

$$\text{Equation}(6) \implies AQ = CP \quad (11)$$

5. Equation (6) and (7) \implies Quadrilateral APCQ is a parallelogram.

termux commands :

```
bash lines.sh.....using shell command
```

Below python code realizes the above construction :

```
https://github.com/pavan170850/Fwciith2022/  
blob/main/matrices/line/code/Line.py
```