Deriving the equation of Circle with given Area and Center which is crossing point of two diameter lines Using Matrices

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1 Problem statement

The lines 2x-3y=5 and 3x-4y=7 are diameters of a circle having area 154 sq.units. Then find the equation of Circle.

2 Considerations

The input parameters are the lengths r, c and angle θ.

Symbol	Value	Description
O	$\begin{pmatrix}0\\0\end{pmatrix}$	Origin
r		Radius of the Circle
C	$\begin{pmatrix}x\\y\end{pmatrix}$	Center of the Circle

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3 Plotting the Circle with center and radius

Plot of the Circle with center $\mathbf{C} = \begin{pmatrix}1\\-1\end{pmatrix}$ and radius r=7 is shown in figure 1, where the point C is the crossing point of the given diameter lines, 2x-3y=5 and 3x-4y=7.

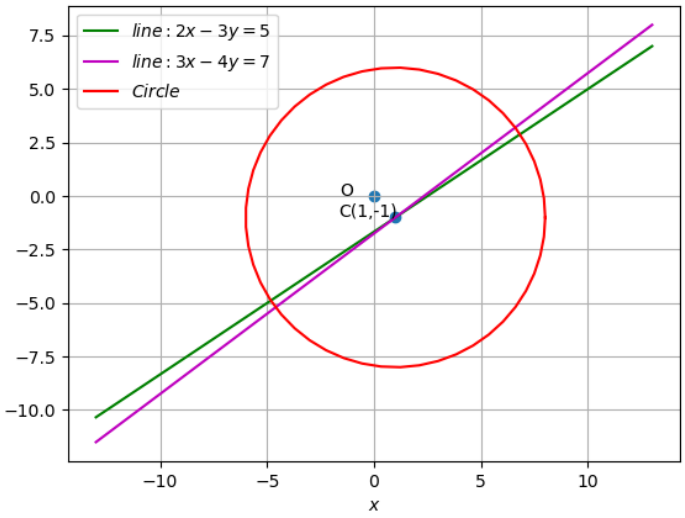

Figure 1: Circle with radius r=7 and center C(1, -1)

4 Solution

4.1 Finding the center C of the Circle

Let O be the origin and its coordinates are

$$\mathbf{O} = \begin{pmatrix}0\\0\end{pmatrix}$$

Given line equations are

$$2x - 3y = 5 \tag{4.1.1}$$

$$3x - 4y = 7 \tag{4.1.2}$$

The above equations can be written in matrix form as,

$$\begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

The augmented matrix can be expressed as,

$$\begin{pmatrix} \textcircled{2} & -3 & 5 \\ 3 & -4 & 7 \end{pmatrix}$$

Through pivoting, the augmented matrix will become as,

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

On solving above equation the crossing point of the given equations will be,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Therefore the Center of the Circle is,

$$\mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{4.1.3}$$

4.2 Calculation of radius of the Circle

As per the given data, the area of the Circle is 154 sq.units
Let r be the radius of circle,

$$\pi r^2 = 154 \implies r = 7 \tag{4.2.1}$$

4.3 Deriving equation for Circle in matrix form

The equation of circle in matrix form is,

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{4.3.1}$$

Where

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \mathbf{f} = -47$$

$$\implies \mathbf{x}^T\mathbf{I}\,\mathbf{x} + 2\begin{pmatrix} -1 \\ 1 \end{pmatrix}^T \mathbf{x} - 47 = 0$$

Therefore, the circle equation can be written as

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} -1 \\ 1 \end{pmatrix}^T \mathbf{x} - 47 = 0 \tag{4.3.2}$$

4.4 Deriving equation for Circle in quadratic form

In quadratic form, the expression for circle can be written as,

$$(x - x1)^2 + (y - y1)^2 = r^2$$

$$(x - 1)^2 + (y + 1)^2 = 7^2$$

$$x^2 + y^2 - 2x + 2y - 47 = 0 \tag{4.4.1}$$

5 Conclusion

1. At first, Center of the Circle has been found which is crossing point of the two diameter lines 2x-3y=5 and 3x-4y=7.

2. Radius of the center has been calculated from its area 154sq.units.

3. Matrix equation for $\mathbf{V}, \mathbf{U}, \mathbf{U^T}$ and $\mathbf{f}$ has been derived.

4. Finally, the Circle equation has been derived as,

$$\mathbf{x}.\mathbf{x}^T + 2\begin{pmatrix} -1 \\ 1 \end{pmatrix}^T \mathbf{x} - 47 = 0$$