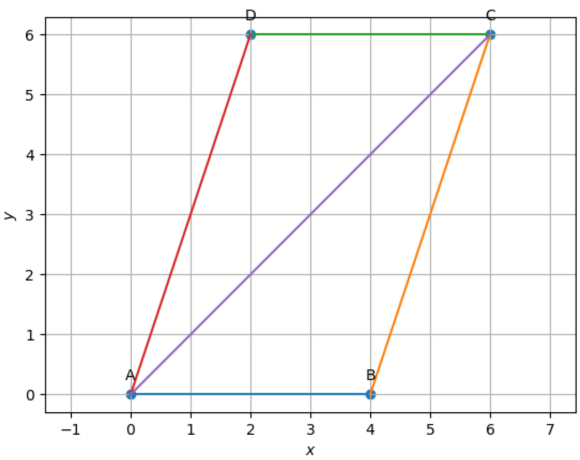
Line Assignment

Ginna Shreyani- FWC22006

September 2022

1 Problem:

Diagonal AC of a parallelogram ABCD bisects $\angle A$. Show that
(i) it bisects $\angle C$



2 Construction:

Variable	Point/Length	Description
A	$\begin{pmatrix}0\\0\end{pmatrix}$	Vertex A
B	$\begin{pmatrix}4\\0\end{pmatrix}$	Vertex B
C	$\begin{pmatrix}6\\6\end{pmatrix}$	Vertex C
D	$\begin{pmatrix}2\\6\end{pmatrix}$	Vertex D

3 Solution:

3.1 Theory:

Given a diagonal of a parallelogram bisects an angle,we need to prove that the same diagonal bisects the opposite angle of the parallelogram by using vector algebra.

3.2 Mathematical Calculation:

Here the diagonal joining vertices A and C can be represented as

$$\mathbf{C}-\mathbf{A} = (\mathbf{B}-\mathbf{A}) + (\mathbf{C}-\mathbf{B}) \tag{1}$$

The other diagonal joining vertices B and D can be represented as

$$\mathbf{D}-\mathbf{B} = (\mathbf{B}-\mathbf{C}) - (\mathbf{B}-\mathbf{A}) \tag{2}$$

(i) Let the $\angle CAB$ be θ_1 and the $\angle DAC$ be θ_2 and the $\angle DCA$ be θ_3 and $\angle ACB$ be θ_4
Since the diagonal $\mathbf{C}-\mathbf{A}$ bisects $\angle A$, $\angle CAB = \angle DAC$,therefore we get $\theta_1 = \theta_2$

$$cos\theta_1 = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{C}-\mathbf{A})}{||\mathbf{B}-\mathbf{A}||.||\mathbf{C}-\mathbf{A}||}$$
$$cos\theta_3 = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})}{||\mathbf{A}-\mathbf{B}||.||\mathbf{A}-\mathbf{C}||}$$
$$cos\theta_3 = \frac{-(\mathbf{B}-\mathbf{A})^T-(\mathbf{C}-\mathbf{A})}{||-(\mathbf{B}-\mathbf{A})||.||-(\mathbf{C}-\mathbf{A})||}$$
$$cos\theta_3 = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{C}-\mathbf{A})}{||\mathbf{B}-\mathbf{A}||.||\mathbf{C}-\mathbf{A}||}$$
$$cos\theta_1 = cos\theta_3$$
$$\theta_1 = \theta_3$$

Therefore,$\angle CAB = \angle DAC = \angle DCA$
Similarly, applying the same process to $\angle DAC$ and $\angle ACB$, we get $\theta_2 = \theta_4$ and as result $\angle DAC = \angle ACB$.
Therefore, $\angle DCA = \angle ACB$
Since both the angles $\angle DCA$ and $\angle ACB$ are equal, we can conclude that the diagonal $\mathbf{C}-\mathbf{A}$ bisects the $\angle C$.