PARALLELOGRAM

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Contents

1 Problem

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD . Show that

(i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ

2 Solution

The input parameters for this construction are

Symbol	Value
b	6
r	5
θ	$\frac{\pi}{3}$

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix}$$

$$\mathbf{C} = \mathbf{B} + \mathbf{C}$$

To Prove: AP = CQ

The line equation for diagonal BD is $x = \mathbf{B} + \lambda\mathbf{m}$

where $\mathbf{m} = \mathbf{B} - \mathbf{D}$

then,

$$\mathbf{P} = \mathbf{B} - \frac{\mathbf{m}^T\mathbf{B}}{\|\mathbf{m}\|^2}\mathbf{m}$$

$$\mathbf{Q} = \mathbf{B} - \frac{\mathbf{m}^T\mathbf{B-C}}{\|\mathbf{m}\|^2}\mathbf{m}$$

distance between A and P is $\|\mathbf{A} - \mathbf{P}\|$

distance between C and Q is $\|\mathbf{C} - \mathbf{Q}\|$

if $\|\mathbf{A} - \mathbf{P}\| = \|\mathbf{C} - \mathbf{Q}\|$

then AP = CQ..........(1)

To Prove: $\Delta APB \cong \Delta$ CQD

to prove $\angle APD = \angle CQD = 90°$

$\mathbf{m1} = \mathbf{A} - \mathbf{P}$

$\mathbf{m2} = \mathbf{P} - \mathbf{B}$

$\theta = \angle APD$

$\cos\theta = \frac{\mathbf{m1}^T\mathbf{m2}}{\|\mathbf{m1}\|\|\mathbf{m2}\|}$

$\theta = 90°, cos\theta = 0$

$\therefore m1^Tm2 = 0$

$\mathbf{n1} = \mathbf{C} - \mathbf{Q}$

$\mathbf{n2} = \mathbf{Q} - \mathbf{D}$

$\theta = \angle CQD$

$cos\theta = \frac{\mathbf{n1}^T\mathbf{n2}}{\|\mathbf{n1}\|\|\mathbf{n2}\|}$

f $\theta = 90°, cos\theta = 0$

$\therefore n1^Tn2 = 0$

if $m1^Tm2 = n1^Tn2 = 0$

then, $\angle APD = \angle CQD = 90°$..........(2)

to prove $\angle ABP = \angle CDQ$

$\mathbf{m2} = \mathbf{P} - \mathbf{B}$

$\mathbf{m3} = \mathbf{A} - \mathbf{B}$

$\theta 1 = \angle ABP$

$\theta 1 = \cos^{-}1 \frac{\mathbf{m2} \cdot \mathbf{m3}}{\|\mathbf{m2}\| \|\mathbf{m3}\|}$

$\mathbf{n2} = \mathbf{C} - \mathbf{D}$

$\mathbf{n3} = \mathbf{Q} - \mathbf{D}$

$\theta 2 = \angle CDQ$

$\theta 2 = \cos^{-}1 \frac{\mathbf{n2} \cdot \mathbf{n3}}{\|\mathbf{n2}\| \|\mathbf{n3}\|}$

if $\theta 1 = \theta 2$

then $\angle ABP = \angle CQD$..........(3)

$\therefore$ from (1),(2) and (3) $\Delta APB \cong \Delta CQD$

The below python code realizes the above construction:

https://github.com/sravani21vunnava/sravani21vunnava/blob/main/Matrices_line/codes/matrix_line.py

3 Construction