

circle Assignment

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I. QUESTION

Q(6), C , Section-A, Chapter-8: If a circle passes through the point (a,b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its center is.

II. SOLUTION

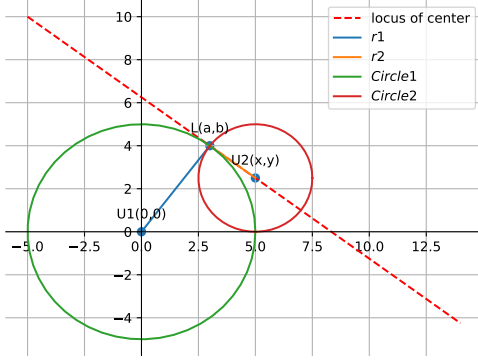


Figure 1: a circle passes through the point L and cuts the circle $x^2 + y^2 = k^2$ orthogonally

With the given circle equation $x^2 + y^2 = k^2$, we can find out centre U_1 and radius r_1 of Circle-1

STEP-1

Centre of Circle-1,

$$U_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1)$$

Radius of Circle-1,

$$r_1 = k \quad (2)$$

STEP-2

let, the center of the circle which passes through the point L and cuts the circle $x^2 + y^2 = k^2$ orthogonally is:

$$U_2 = \begin{pmatrix} x \\ y \end{pmatrix} \quad (3)$$

$$L = \begin{pmatrix} a \\ b \end{pmatrix} \quad (4)$$

Radius of Circle be r_2

As both the circles are orthogonal, we get:

$$||U_2 - U_1||^2 = r_1^2 + r_2^2 \quad (5)$$

where

$$\Rightarrow ||U_2 - U_1||^2 = ||U_2||^2 + ||U_1||^2 - 2U_1^T U_2 \quad (6)$$

$$\Rightarrow r_1^2 = k^2 \quad (7)$$

$$\begin{aligned} \Rightarrow r_2^2 &= ||U_2 - L||^2 \\ &= ||U_2||^2 + ||L||^2 - 2L^T U_2 \end{aligned} \quad (8)$$

substitute equation (6),(7),(8) in equation (5)

$$\begin{aligned} \Rightarrow ||U_2 - U_1||^2 &= r_1^2 + r_2^2 \\ \Rightarrow ||U_2||^2 + ||U_1||^2 - 2U_1^T U_2 &= k^2 + ||U_2||^2 + ||L||^2 - 2L^T U_2 \end{aligned}$$

by solving the above equation we get,

$$\begin{aligned} \Rightarrow 2L^T U_2 &= k^2 + ||L||^2 \\ \Rightarrow 2L^T U_2 &= k^2 + L^T L \end{aligned} \quad (9)$$

equation (9) is the required equation, which is a line equation $n^T X = c$

CONSTRUCTION

Symbol	Value	Description
U_1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	center of given circle
r_1	k	radius of given circle
U_2	$\begin{pmatrix} x \\ y \end{pmatrix}$	center of circle 2
L	$\begin{pmatrix} a \\ b \end{pmatrix}$	a point on circle 2
r_2	$ U_2 - L ^2$	radius of circle 2

Get the python code of the figures from

https://github.com/kkousar/KOUSAR_FWC/blob/main/circle_assignment/code/circle.py