

MATRICES

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IITH - Future Wireless Communication(FWC22012)

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1 Problem

Q.The circle passing through the point $(-1, 0)$ and touching the y -axis at $(0, 2)$ also passes through the point

2 Solution

Given, the point $(-1, 0)$ is passing through the circle

and the circle is touching the y -axis at $(0, 2)$ so the equation of circle using matrices can be written as,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{x}^T \mathbf{U} + f = 0 \quad (1)$$

$$\text{Let } \mathbf{A} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \text{ and } \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{A} \mathbf{A}^T + 2\mathbf{u}^T \mathbf{A} + f = 0 \quad (2)$$

$$\|\mathbf{A}\|^2 + 2\mathbf{A}^T \mathbf{u} + f = 0 \quad (3)$$

$$(2\mathbf{A}^T \ 1) \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\|\mathbf{A}\|^2 \quad (4)$$

$$\mathbf{B} \mathbf{B}^T + 2\mathbf{u}^T \mathbf{B} + f = 0 \quad (5)$$

$$(2\mathbf{B}^T \ 1) \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\|\mathbf{B}\|^2 \quad (6)$$

The equation of the tangent is

$$\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u}) = 0 \quad (7)$$

$$\mathbf{m}^T \mathbf{A} + \mathbf{m}^T \mathbf{u} = 0 \quad (8)$$

$$\mathbf{m}^T \mathbf{u} = -\mathbf{m}^T \mathbf{A} \quad (9)$$

from equations (4), (6) and (9), we can write as

$$\begin{pmatrix} \mathbf{m}^T & 0 \\ 2\mathbf{A}^T & 1 \\ 2\mathbf{B}^T & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -\mathbf{m}^T \mathbf{A} \\ -\|\mathbf{A}\|^2 \\ -\|\mathbf{B}\|^2 \end{pmatrix} \quad (10)$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 4 & 1 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ -1 \end{pmatrix}$$

solving the equations

$$\begin{pmatrix} 0 & 1 & 0 & -2 \\ 0 & 4 & 1 & -4 \\ -2 & 0 & 1 & -1 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_3}$$

$$\begin{pmatrix} -2 & 0 & 1 & -1 \\ 0 & 4 & 1 & -4 \\ 0 & 1 & 0 & -2 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 / -2}$$

$$\begin{pmatrix} 1 & 0 & -1/2 & 1/2 \\ 0 & 4 & 1 & -4 \\ 0 & 1 & 0 & -2 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 / 4}$$

$$\begin{pmatrix} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 1/4 & -1 \\ 0 & 1 & 0 & -2 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 - R_2}$$

$$\begin{pmatrix} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 1/4 & -1 \\ 0 & 0 & -1/4 & -1 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 * -4}$$

$$\begin{pmatrix} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1/4 & -1 \\ 0 & 0 & 1 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 1/4 R_3}$$

$$\begin{pmatrix} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 + 1/2 R_3}$$

$$\begin{pmatrix} 1 & 0 & 0 & 5/2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

By solving the above equations

$$\mathbf{u} = \begin{pmatrix} 5/2 \\ -2 \end{pmatrix}$$

$$\mathbf{c} = -\mathbf{u}$$

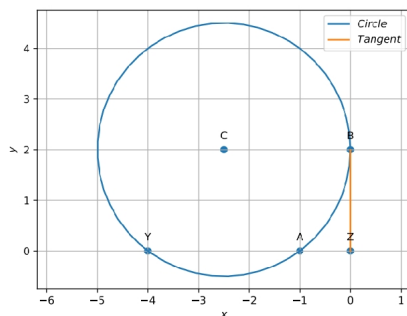
$\mathbf{c} = (-5/2 \ 2)$ is the center of circle
 $f=4$

as y-axis touches the circle the radius of
the circle becomes

$$r=5/2$$

by substituting given options in the above
equation $(-4,0)$ satisfies the equation
the third point which is passing through
given circle is $(-4,0)$

3 Plot



4 Software

We can plot the circle with the help of the
following code :

<https://github.com/Gowt-hami/fwc-1-module1/blob/main/par.py>