Matrix-Lines

R.Radhika

Contents

I. Problem Statement

A line passes through (x_1, y_1) and (h, k).If slope of the line is m show that $(k - y_1) = m(h - x_1)$.

II. Construction

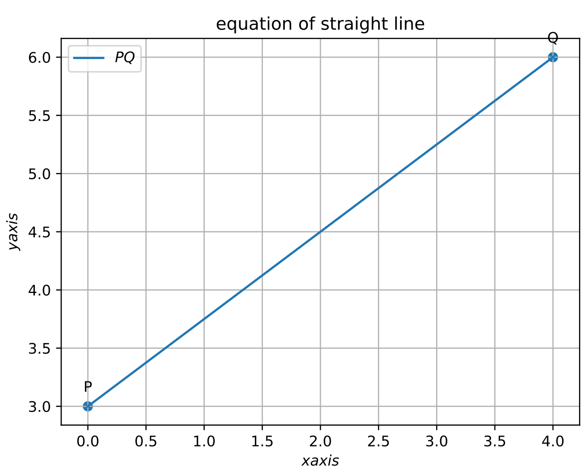

Fig. 1. Equation of the slope

Symbol	Value	Description
$\mathbf{A}$	$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$	Point on X-axis
$\mathbf{B}$	$\begin{pmatrix} h \\ k \end{pmatrix}$	Point on Y-axis
A	k-y1=m(h-x1)	Given Condition

TABLE I
PARAMETERS

III. Solution

Given that resultant line passes through point(x1,y1) and (h,k) (let prove the equation in vector form by line eqation)

given $\mathbf{A}$=$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\mathbf{B}$=$\begin{pmatrix} h \\ k \end{pmatrix}$

Equation of line is $\mathbf{n}^\top \mathbf{x} = c$.

First Method:
condition is

$$\mathbf{n}^\top \mathbf{m} = 0 \tag{1}$$

m is the direction vector
m=B-A

$$\mathbf{n}^\top (\mathbf{B} - \mathbf{A}) = 0 \tag{2}$$

$$\mathbf{n}^\top \begin{pmatrix} h - x_1 \\ k - y_1 \end{pmatrix} = 0 \tag{3}$$

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} h - x_1 \\ k - y_1 \end{pmatrix} = 0 \tag{4}$$

by solving eq-4

$$\left(-m(h - x_1)\right) + \left(k - y_1\right) = 0 \tag{5}$$

Then the equation becomes

$\left(k - y_1\right) = m\left(h - x_1\right)$
Therefore the Resultant Equation of line is

$$\left(k - y_1\right) = m\left(h - x_1\right) \tag{6}$$

$$\boxed{(k-y_1)=m(h-x_1)}$$

Second Method:
given $\mathbf{A}$=$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $\mathbf{B}$=$\begin{pmatrix} h \\ k \end{pmatrix}$
C=$\begin{pmatrix} \frac{x_1+h}{2} \\ \frac{y_1+k}{2} \end{pmatrix}$
m is the direction vector

m=C-A

m=B-C

m=$\begin{pmatrix} \frac{x_1+h}{2}-x_1 \\ \frac{y_1+k}{2}-y_1 \end{pmatrix}$
m=2$\begin{pmatrix} \frac{h-x_1}{2} \\ \frac{k-y_1}{2} \end{pmatrix}$
m=$\begin{pmatrix} h-x_1 \\ k-y_1 \end{pmatrix}$
condition is

$$\mathbf{n}^\top \mathbf{m} = 0 \tag{7}$$

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} h-x_1 \\ k-y_1 \end{pmatrix} = 0 \tag{8}$$

by solving eq-4

$$\big(-m(h-x_1)) + (k-y_1\big) = 0 \tag{9}$$

Then the equation becomes

$\big(k-y_1) = m(h-x_1\big)$
Therefore the Resultant Equation of line is

$$\big(k-y_1) = m(h-x_1\big) \tag{10}$$

Third Method:

$$\mathbf{n}^\top \mathbf{m} = 0 \tag{11}$$

$$\mathbf{n}^\top (\mathbf{x}-\mathbf{A}) = 0 \tag{12}$$

$$\mathbf{n}^\top (\mathbf{x}-\mathbf{B}) = 0 \tag{13}$$

The Equation of line through $\mathbf{A}$ from 1 is

$$\mathbf{n}^\top \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \right) = 0 \tag{14}$$

Equation of line passing through $\mathbf{B}$ from 2 is

$$\mathbf{n}^\top \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} h \\ k \end{pmatrix} \right) = 0 \tag{15}$$

Now by solving eq3,

$$\mathbf{n}^\top \begin{pmatrix} x-x_1 \\ y-y_1 \end{pmatrix} = 0 \tag{16}$$

Now by solving eq4,

$$\mathbf{n}^\top \begin{pmatrix} x-h \\ y-k \end{pmatrix} = 0 \tag{17}$$

From eq5 and eq6 we can prove the equation $\mathbf{n}$,

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x-x_1 & y-y_1 \\ x-h & y-k \end{pmatrix} = 0 \tag{18}$$

by solving 7 th equation

$$\big(k-y_1) = m(h-x_1\big) \tag{19}$$

Therefore the Resultant Equation of line is $\mathbf{n}^\top \mathbf{X} = c$

$$\big(k-y_1) = m(h-x_1\big) \tag{20}$$

IV. Software

Download the following code using,

https://github.com/Radhikarkv/fwcproject.git

and execute the code by using command

Python3 lineassign.py

V. Conclusion

prove the equation of a line passes trough a points(x_1, y_1),(h,k) if slope of the line is m i.e $(k-y_1) = m(h-x_1)$.