PARALLELOGRAM

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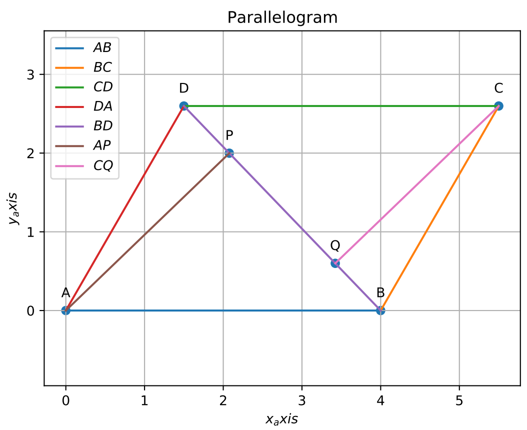
1 Construction



2 Problem

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD . Show that
(i) $\Delta APB \cong \Delta CQD$
(ii) AP = CQ

3 Solution

3.1 Considerations

The input parameters for this construction are

Symbol	Value	Description
b	6	length of AB
r	5	length of AC
θ	$\frac{\pi}{3}$	angle of parallelogram

$\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are the coordinates of the parallelogram

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad \mathbf{C} = \mathbf{B} + \mathbf{C}$$

3.2 Part 1

To Prove: AP = CQ

The line equation for diagonal BD is

$$x = \mathbf{B} + \lambda\mathbf{m} \tag{1}$$

$$\mathbf{m} = \mathbf{B} - \mathbf{D} \tag{2}$$

$\mathbf{P}, \mathbf{Q}$ are foot of perpendiculars drawn from A and C on to the diagonal BD

$$\mathbf{P} = \mathbf{B} - \frac{\mathbf{m}^T\mathbf{B}}{\|\mathbf{m}\|^2}\mathbf{m} \tag{3}$$

$$\mathbf{Q} = \mathbf{B} - \frac{\mathbf{m}^T\mathbf{B} - \mathbf{C}}{\|\mathbf{m}\|^2}\mathbf{m} \tag{4}$$

Distance between A and P is $\|\mathbf{A} - \mathbf{P}\|$
Distance between C and Q is $\|\mathbf{C} - \mathbf{Q}\|$

$$\|\mathbf{A} - \mathbf{P}\| = \|\mathbf{C} - \mathbf{Q}\| \tag{5}$$

$$AP = CQ \tag{6}$$

3.3 Part 2

To Prove: $\Delta APB \cong \Delta CQD$
To prove $\angle APB$ is equal to $\angle CQD$

$$\mathbf{m1} = \mathbf{A} - \mathbf{P}$$
$$\mathbf{m2} = \mathbf{P} - \mathbf{B}$$
$$\theta = \angle APB$$

$$\cos\theta = \frac{\mathbf{m1}^T\mathbf{m2}}{\|\mathbf{m1}\|\|\mathbf{m2}\|} \tag{7}$$

$$\theta = 90°, cos\theta = 0$$
$$\therefore m1^T m2 = 0$$
$$\angle APB = 90°$$

$$\mathbf{n1} = \mathbf{C} - \mathbf{Q}$$
$$\mathbf{n2} = \mathbf{Q} - \mathbf{D}$$
$$\theta = \angle CQD$$

$$cos\theta = \frac{\mathbf{n1}^T\mathbf{n2}}{\|\mathbf{n1}\|\|\mathbf{n2}\|} \tag{8}$$

$$\theta = 90^\circ, cos\theta = 0$$
$$\therefore n1^T n2 = 0$$
$$\angle CQD = 90^\circ$$

$$\angle APD = \angle CQD = 90^\circ \tag{9}$$

To prove$\angle ABP$ is equal to $\angle CDQ$

$$\mathbf{m2} = \mathbf{P} - \mathbf{B}$$
$$\mathbf{m3} = \mathbf{A} - \mathbf{B}$$
$$\theta 1 = \angle ABP$$

$$\theta 1 = \cos^- 1 \frac{\mathbf{m2} \cdot \mathbf{m3}}{\|\mathbf{m2}\| \|\mathbf{m3}\|} \tag{10}$$

$$\mathbf{n2} = \mathbf{C} - \mathbf{D}$$
$$\mathbf{n3} = \mathbf{Q} - \mathbf{D}$$
$$\theta 2 = \angle CDQ$$

$$\theta 2 = \cos^- 1 \frac{\mathbf{n2} \cdot \mathbf{n3}}{\|\mathbf{n2}\| \|\mathbf{n3}\|} \tag{11}$$
$$\theta 1 = \theta 2$$

$$\angle ABP = \angle CQD \tag{12}$$

$\therefore$ from (6),(9) and (12) $\Delta APB \cong \Delta CQD$

The below python code realizes the above construction:

https://github.com/sravani21vunnava/sravani21vunnava/blob/main/Matrices_line/codes/matrix_line.py