Comparision of Angles and Sides of Trapezium Using Matrices and lines
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1 Problem statement

ABCD is trapezium in which AB ∥ CD and AD=BC. Show that,

1. ∠ A = ∠ B
2. ∠ C = ∠ D
3. Diagonal AC = Diagonal BD
4. △ ABC = △ BAD

2 Considerations

The input parameters are the lengths r, c and angle θ.

Symbol	Value	Description
$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Origin
r	2.82	Distance of BC, AD
c	4	OC
$\mathbf{C}$	$\begin{pmatrix} c \\ 0 \end{pmatrix}$	Point C on X axes
θ	45 °	∠BOC

3 Plotting Trapezium

Plot of Trapezium is shown in figure 1, where point O is origin and points A, B, C and D are the vertices of Trapezium.

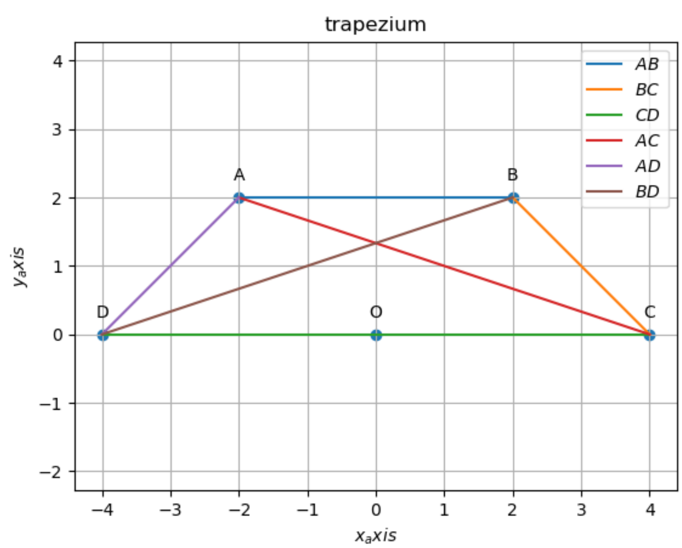

Figure 1: trapezium

4 Solution

4.1 Finding Co-ordinates O, A, B, C and D

Let O be the origin and its coordinates are

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$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Let C be the point on X-axes and it is expressed as

$$\mathbf{C} = c \tag{4.1.1}$$

$$\implies \mathbf{C} = \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{4.1.2}$$

Let D be the point on Negative X-axes and it will be the image of C,

$$\mathbf{D} = \text{-c}$$

Therefore, the coordinates of D are

$$\implies \mathbf{D} = \begin{pmatrix} -c \\ 0 \end{pmatrix} \tag{4.1.3}$$

Let r be the distance between point B and C, then

$$||\mathbf{B} - \mathbf{C}|| = \text{r and } ||\mathbf{A} - \mathbf{D}|| = \text{r}$$

Let θ be the angle at BOC, then

$$\angle\text{BOC} = \theta$$

According to the vector geometry formulaes, the vector B can be expresssed as

$$\mathbf{B} = r \begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix} \tag{4.1.4}$$

From ABCD trapezium,

$$\mathbf{A} = \mathbf{B} - \mathbf{C}$$

$$= \begin{pmatrix} rcos\theta \\ rsin\theta \end{pmatrix} \text{ - } \begin{pmatrix} c \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -c + rcos\theta \\ rsin\theta \end{pmatrix}$$

From triangle ODA, $\boldsymbol{OD} + \boldsymbol{DA} = \boldsymbol{OA}$,

$\implies$ -c+rcosθ = -r cos θ

$$= \begin{pmatrix} -rcos\theta \\ rsin\theta \end{pmatrix}$$

$$\implies \mathbf{A} = \text{r} \begin{pmatrix} -cos\theta \\ sin\theta \end{pmatrix}$$

Let c=4, r=2.82 and θ=45 $^\circ$

Then all the four coordinates will be,

$$\mathbf{O}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{A}=\begin{pmatrix} -2 \\ 2 \end{pmatrix}, \mathbf{B}=\begin{pmatrix} 2 \\ 2 \end{pmatrix}, \mathbf{C}=\begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{D}=\begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

4.2 Calculation of Angles A and B

To find angle $\angle$A:

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \text{ - } \begin{pmatrix} -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

and

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \text{ - } \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

$$\angle BAD = arccos\frac{(\mathbf{A} - \mathbf{D}).(\mathbf{A} - \mathbf{B})}{||\mathbf{A} - \mathbf{D}||.||\mathbf{A} - \mathbf{B}||} \tag{4.2.1}$$

$$\angle BAD = arccos\frac{(2 \quad 2)^T.(-4 \quad 0)}{\sqrt{2^2 + 2^2}.\sqrt{4^2 + 0}} \tag{4.2.2}$$

$$= arccos\frac{-8}{\sqrt{8}.\sqrt{16}} = arccos(-0.707) \tag{4.2.3}$$

$\implies \angle$ BAD = 135 $^\circ$

$\implies \angle$ A = 135 $^\circ$

To find angle $\angle$ B:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \text{-} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

and

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \text{ - } \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$\angle BAD = arccos\frac{(\mathbf{A} - \mathbf{D}).(\mathbf{A} - \mathbf{B})}{||\mathbf{A} - \mathbf{D}||.||\mathbf{A} - \mathbf{B}||} \tag{4.2.4}$$

$$\angle BAD = arccos\frac{(4, 0)^T.(-2, 2)}{\sqrt{4^2}.\sqrt{2^2 + 2^2}} \tag{4.2.5}$$

$$= arccos\frac{-8}{\sqrt{16}.\sqrt{8}} = arccos(-0.707) \tag{4.2.6}$$

$\implies \angle$ ABC = 135 $^\circ$

$\implies \angle$ B = 135 $^\circ$

Therefore $\angle A = \angle B$

4.3 Calculation of Angles C and D

To find angle $\angle$C:

$$\mathbf{C} - \mathbf{O} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \text{ - } \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

and

$$\mathbf{C}-\mathbf{B} = \begin{pmatrix}4\\0\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix} = \begin{pmatrix}2\\-2\end{pmatrix}$$

$$\angle OCB = arccos\frac{(\mathbf{C}-\mathbf{O}).(\mathbf{C}-\mathbf{B})}{||\mathbf{C}-\mathbf{O}||.||\mathbf{C}-\mathbf{B}||} \tag{4.3.1}$$

$$\angle BAD = arccos\frac{(4\ \ 0)^T.(2\ \ -2)}{\sqrt{4^2}.\sqrt{2^2+2^2}} \tag{4.3.2}$$

$$= arccos\frac{8}{\sqrt{16}.\sqrt{8}} = arccos(0.707) \tag{4.3.3}$$

$\implies \angle$ OCB = 45 $^\circ$

$\implies \angle$ C = 45 $^\circ$

To find angle $\angle$ D:

$$\mathbf{D}-\mathbf{O} = \begin{pmatrix}-4\\0\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}-4\\0\end{pmatrix}$$

and

$$\mathbf{D}-\mathbf{A} = \begin{pmatrix}-4\\0\end{pmatrix} - \begin{pmatrix}-2\\2\end{pmatrix} = \begin{pmatrix}-2\\-2\end{pmatrix}$$

$$\angle ODB = arccos\frac{(\mathbf{D}-\mathbf{O}).(\mathbf{D}-\mathbf{B})}{||\mathbf{D}-\mathbf{O}||.||\mathbf{D}-\mathbf{B}||} \tag{4.3.4}$$

$$\angle ODB = arccos\frac{(-4\ \ 0)^T.(-2\ \ -2)}{\sqrt{4^2}.\sqrt{2^2+2^2}} \tag{4.3.5}$$

$$= arccos\frac{8}{\sqrt{8}.\sqrt{16}} = arccos(0.707) \tag{4.3.6}$$

$\implies \angle$ ODB = 45 $^\circ$

$\implies \angle$ D = 45 $^\circ$

$$\text{Therefore } \angle C = \angle D$$

4.4 Calculation of Diagonals AC and BD

Calculation of Diagonal AC:

$$\mathbf{A}-\mathbf{C} = \begin{pmatrix}-2\\2\end{pmatrix} - \begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}-6\\2\end{pmatrix}$$

Legth of Diagonal AC,

$$||\mathbf{A}-\mathbf{C}|| = ||\begin{pmatrix}-6\\2\end{pmatrix}||$$

$$= \sqrt{(-6)^2+2^2} = 6.32$$

$$\implies ||\mathbf{A}-\mathbf{C}||=6.32$$

Calculation of Diagonal BD:

$$\mathbf{B}-\mathbf{D} = \begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}-4\\0\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$$

Length of Diagonal BD,

$$||\mathbf{B}-\mathbf{D}|| = ||\begin{pmatrix}6\\2\end{pmatrix}||$$

$$= \sqrt{6^2+2^2} = 6.32$$

$$\implies ||\mathbf{B}-\mathbf{D}||=6.32$$

Therefore both diagonals are equal, AC = BD

4.5 Comparing △ ABC and △ BAD

For Triangle ABC:

$\angle ABC = \angle AOC = \pi - \theta = 135$ $^\circ$

and

Base =Diagonal AC = $||\mathbf{A}-\mathbf{C}||$ = 6.32

For Triangle BAD:

$\angle BAD = \angle BOD = \pi - \theta = 135$ $^\circ$

and

Base =Diagonal AC = $||\mathbf{B}-\mathbf{D}||$ = 6.32

As base and opposite angles are same, both triangles are symmetrical.

Therefore △ ABC and △ BAD

5 Software

Download the codes given in the link below and execute them.

https://github.com/meertabresali-FWC-IITH/project/blob/main/Asgn4.matrixline/line.py

6 Conclusion

In this program, the following points have been verified.

1. $\angle$ A = $\angle$ B
2. $\angle$ C = $\angle$ D
3. Diagonal AC = Diagonal BD
4. △ ABC = △ BAD