Matrix-Conic

SHREYASH CHANDRA PUTTA

October 27, 2022

Contents

1 Problem Statement **1**

2 Solution **1**
2.1 verification 1

3 Software **2**

4 Plotting **2**

5 Conclusion **2**

1 Problem Statement

To find the locus of mid point of **P Q** where **P** is (1,0) and **Q** is a point on the locus $y^2 = 8x$.

2 Solution

Let **X** be any point on the Locus formed by the midpoint joining the point **P** and any point on the given locus say, point **Q**

Where, $\mathbf{P}=\begin{pmatrix}1\\0\end{pmatrix}$,$\mathbf{Q}=\begin{pmatrix}x'\\y'\end{pmatrix}$ and $\mathbf{X}=\begin{pmatrix}x\\y\end{pmatrix}$

The given equation of parabola $y^2 = 8x$ can be written in the general quadratic form as

$$\mathbf{x}^\top\mathbf{V}\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}, \tag{2}$$

$$\mathbf{u} = \begin{pmatrix}-4\\0\end{pmatrix}, \tag{3}$$

$$f = 0 \tag{4}$$

Substitute **Q** and data in (1).

$$\mathbf{Q}^\top\mathbf{V}\mathbf{Q} + 2\mathbf{u}^\top\mathbf{Q} = 0 \tag{5}$$

By section formula mid point of line joining **P** and **Q** as **X** is:

$$\mathbf{X} = \frac{\mathbf{Q}+\mathbf{P}}{2} \tag{6}$$

$$\mathbf{Q} = 2\mathbf{X} - \mathbf{P} \tag{7}$$

From (5) and (7) We get

$$(2\mathbf{X} - \mathbf{P})^\top\mathbf{V}(2\mathbf{X} - \mathbf{P}) + 2\mathbf{u}^\top(2\mathbf{X} - \mathbf{P}) = 0 \tag{8}$$

$$(2\mathbf{X}^\top\mathbf{V} - \mathbf{P}^\top\mathbf{V})(2\mathbf{X} - \mathbf{P}) + 2\mathbf{u}^\top 2\mathbf{X} - 2\mathbf{u}^\top\mathbf{P} = 0 \tag{9}$$

$$(2\mathbf{X}^\top\mathbf{V}2\mathbf{X} - 2\mathbf{X}^\top\mathbf{V}\mathbf{P}) + 2\mathbf{u}^\top 2\mathbf{X} - 2\mathbf{u}^\top\mathbf{P} = 0 \tag{10}$$

$$4\mathbf{X}^\top\mathbf{V}\mathbf{X} + 4\mathbf{u}^\top\mathbf{X} - 2\mathbf{X}^\top\mathbf{V}\mathbf{P} - 2\mathbf{u}^\top\mathbf{P} = 0 \tag{11}$$

$$4\mathbf{X}^\top\mathbf{V}\mathbf{X} + 4\mathbf{u}^\top\mathbf{X} + 8 = 0 \tag{12}$$

$$\mathbf{X}^\top\mathbf{V}\mathbf{X} + \mathbf{u}^\top\mathbf{X} + 2 = 0 \tag{13}$$

Therefore, required Locus equation of the mid point of given point **P** and **Q** is obtained as:

$$\mathbf{X}^\top\mathbf{V}\mathbf{X} + 2\mathbf{u'}^\top\mathbf{X} + f' = 0 \tag{14}$$

where

$$\mathbf{V} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}, \tag{15}$$

$$\mathbf{u'} = \begin{pmatrix}-2\\0\end{pmatrix}, \tag{16}$$

$$f' = 2 \tag{17}$$

2.1 verification

Comparing generate point on the obtained locus as **X** then,
The intersection of given (1) with a line along **P** and **X**.
And find If (6) is satisfied or not

the point of intersection of the line with the conic section is considered as **Q**

From the above considerations below vectors are taken

$$\mathbf{q} = \mathbf{X},\ \mathbf{m} = (\mathbf{X} - \mathbf{P}) \tag{18}$$

The points of intersection of the line,

$$L: \quad \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \quad \mu \in \mathbb{R} \tag{19}$$

with the conic section,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{20}$$

are given by

$$\mathbf{x}_i = \mathbf{q} + \mu_i \mathbf{m} \tag{21}$$

where

$$\mu_i = \frac{1}{\mathbf{m}^T \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T \mathbf{V}\mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f\right)\left(\mathbf{m}^T \mathbf{V}\mathbf{m}\right)}\right) \tag{22}$$

On substituting $\mathbf{V}, \mathbf{u}, \mathbf{q}, \mathbf{m}$ in the above equation, we get the values of μ. By substituting the values of μ in (21),
we get the points of intersection of line with the given curve.

$i.e., \mathbf{x1}, \mathbf{x2}$

we take only one of the suitable point in consideration to verify (6) in this way obtained locus is verified

3 Software

Download the following code using,

```
svn co https://github.com/chanduputta/
FWC-Module1Assignments/blob/
main/conic/code/conic.py
```

and execute the code by using command

cmd1: Python3 conic.py
cmd2: Input y-coordinate to generate point on locus
Observe: the obtained locus is verified or not

4 Plotting

Symbol	Value	Description
$\mathbf{P}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	given point
$\mathbf{Q}$	$\begin{pmatrix} x' \\ y' \end{pmatrix}$	point on given locus
$\mathbf{X}$	$\begin{pmatrix} x \\ y \end{pmatrix}$	mid point of **PQ**

Table 1: Parameters

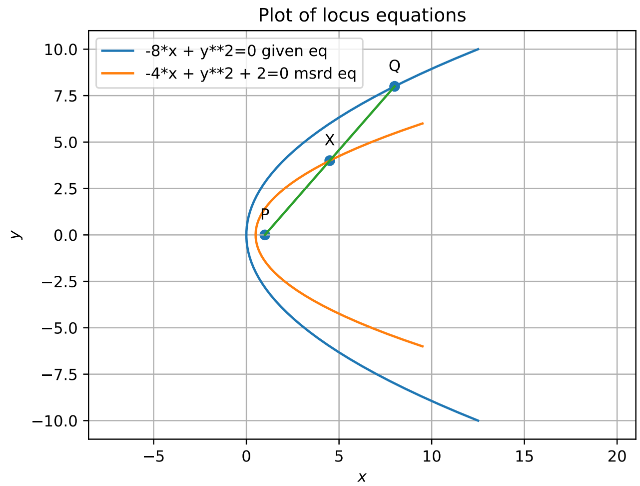

Figure 1: Found the locus equation

5 Conclusion

We found the locus of mid point of **P Q** where **P** is (1,0) and **Q** is a point on the locus $\mathbf{y^2} = \mathbf{8x}$ as $\mathbf{y^2} = \mathbf{4x} - \mathbf{2}$ and Verified.