

# Line Assignment

Ravi Sumanth Muppana- FWC22003

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## 1 Problem:

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus

## 2 Solution:

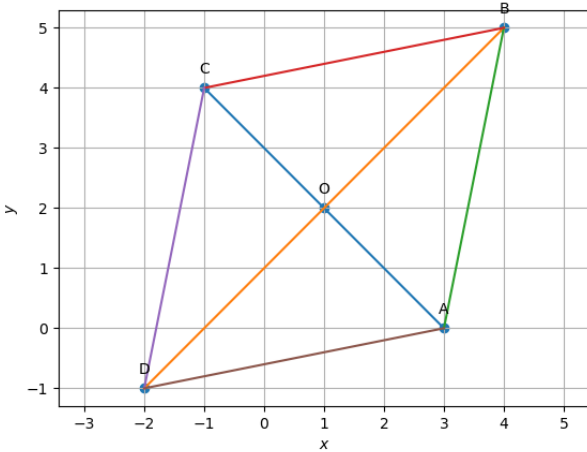


Figure 1: Rhombus

### 2.1 Theory:

Let us assume two vectors  $\mathbf{a}$  and  $\mathbf{b}$  for sides  $BA$  and  $CB$ . The diagonals  $AC, BD$  are the addition and subtraction of the two vectors:

$$(\mathbf{B} - \mathbf{A}) = \mathbf{a} \quad (1)$$

$$(\mathbf{C} - \mathbf{B}) = \mathbf{b} \quad (2)$$

$$(\mathbf{C} - \mathbf{A}) = (\mathbf{C} - \mathbf{B}) + (\mathbf{B} - \mathbf{A}) \quad (3)$$

$$(\mathbf{C} - \mathbf{A}) = \mathbf{b} + \mathbf{a} \quad (4)$$

$$(\mathbf{D} - \mathbf{B}) = \mathbf{b} - \mathbf{a} \quad (5)$$

### 2.2 Mathematical Calculation:

Let the two diagonals be  $\mathbf{a} + \mathbf{b}$ ,  $\mathbf{b} - \mathbf{a}$ . Since the diagonals are at right angle to each other,

$$0 = (\mathbf{a} + \mathbf{b})^T(\mathbf{b} - \mathbf{a}) \quad (7)$$

$$\|\mathbf{b}\|^2 - \|\mathbf{a}\|^2 = 0 \quad (8)$$

$$\|\mathbf{b}\| = \|\mathbf{a}\| \quad (9)$$

$$(10)$$

Hence, the two sides of the quadrilateral are equal. We need to prove the third side is also equal. Now, In triangle BOA and AOD;

$$\mathbf{B} - \mathbf{O} = \mathbf{p} \quad (11)$$

$$\mathbf{D} - \mathbf{O} = -\mathbf{p} \quad (12)$$

$$\mathbf{A} - \mathbf{O} = \mathbf{r} \quad (13)$$

$$(14)$$

$$\mathbf{a} = (\mathbf{B} - \mathbf{O}) - (\mathbf{A} - \mathbf{O}) \quad (15)$$

$$\mathbf{d} = (\mathbf{D} - \mathbf{O}) - (\mathbf{A} - \mathbf{O}) \quad (16)$$

$$\|\mathbf{a}\|^2 = \|\mathbf{p}\|^2 + \|\mathbf{r}\|^2 - 2\mathbf{p}^T\mathbf{r} \quad (17)$$

$$\|\mathbf{d}\|^2 = \|-\mathbf{p}\|^2 + \|\mathbf{r}\|^2 + 2\mathbf{p}^T\mathbf{r} \quad (18)$$

$$(19)$$

The terms  $\mathbf{p}^T\mathbf{r}$  is equal to zero as they are perpendicular. Therefore,

$$\|\mathbf{a}\|^2 = \|\mathbf{p}\|^2 + \|\mathbf{r}\|^2 \quad (20)$$

$$\|\mathbf{d}\|^2 = \|\mathbf{p}\|^2 + \|\mathbf{r}\|^2 \quad (21)$$

$$\text{Clearly, } \|\mathbf{a}\| = \|\mathbf{d}\| \quad (22)$$

$$(23)$$

Hence, all three sides are equal, it's a parallelogram. A parallelogram with its diagonals as perpendicular bisectors is a rhombus.

## 3 Construction:

Consider any three vertices of the rhombus. Using the vertices, find the midpoint of the diagonals, then find the fourth point using the midpoint and remaining vertex.

variable	length/point	Description
A	[3,0]	Vertex A
B	[4,5]	Vertex B
C	[-1,4]	Vertex C
D	[D-x,D-y]	Vertex D
M	$(A+B)/2$	midpoint
(D-x,D-y)	$(2*M[0]-B[0], 2*M[1]-B[1])$	vertex of D