

PROBLEM:

ABCD, DCFE and ABFE are parallelograms. Show that $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$

Theory: Parallelograms on the same base and in between the same parallels are equal in area.

Given: ABCD, DCFE and ABFE are parallelograms.

Solution Statement:

We can see that the sides of a triangle ADE and BCF are also the opposite sides of a given parallelogram. Now we can show both the triangles are congruent using congruency property. We know that congruent triangles are equal areas.

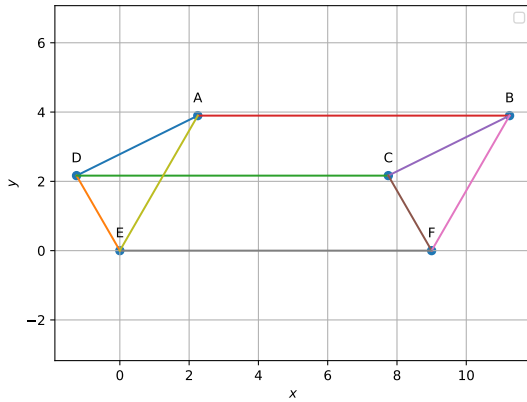
0.1 Construction

Figure of Construction

0.2 Table :

The input parameters for this construction are

Symbol	Value	Description
a	3	EA
b	4.5	EF
c	2	ED
θ_1	$1\pi/3$	$\angle AEF$
θ_2	$2\pi/3$	$\angle DEF$
E	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Point E

1. Considering point 'E' as origin.
2. From E, with some angle of 60 degrees, mark the point 'A'.
3. From E, with some angle of 120 degrees, mark the point 'D'.

4. With the distance of 'b' locate the point 'F'.
5. To locate a point 'B'

$$\mathbf{B} = \mathbf{A} + \mathbf{F} - \mathbf{E}$$

6. To locate a point 'C'

$$\mathbf{C} = \mathbf{D} + \mathbf{F} - \mathbf{E}$$

7. Joining all the lines from the figure.

0.3 Solution

In ABCD,

$$\mathbf{A} - \mathbf{B} = \mathbf{D} - \mathbf{C} \quad (1)$$

$$\mathbf{A} - \mathbf{D} = \mathbf{B} - \mathbf{C} \quad (2)$$

In DEFC,

$$\mathbf{D} - \mathbf{C} = \mathbf{E} - \mathbf{F} \quad (3)$$

$$\mathbf{D} - \mathbf{E} = \mathbf{C} - \mathbf{F} \quad (4)$$

In ABEF,

$$\mathbf{A} - \mathbf{B} = \mathbf{E} - \mathbf{F} \quad (5)$$

$$\mathbf{A} - \mathbf{E} = \mathbf{B} - \mathbf{F} \quad (6)$$

To Prove:

$$\text{Ar}(\triangle ADE) = \text{Ar}(\triangle BCF) \quad (7)$$

Area of the triangle $\triangle ADE$ is given by

$$\text{Ar}(\triangle ADE)$$

$$= \frac{1}{2} \|\mathbf{A} - \mathbf{D} \times \mathbf{D} - \mathbf{E}\| \quad (8)$$

Area of the triangle $\triangle BCF$ is given by

$$\text{Ar}(\triangle BCF)$$

$$= \frac{1}{2} \|\mathbf{B} - \mathbf{C} \times \mathbf{C} - \mathbf{F}\| \quad (9)$$

substituting (2) and (4) in (9),

$$= \frac{1}{2} \|\mathbf{A} - \mathbf{D} \times \mathbf{D} - \mathbf{E}\| \quad (10)$$

from (8) and (10),

$$\text{Ar}(\triangle ADE) = \text{Ar}(\triangle BCF) \quad (11)$$

The below python code realizes the above construction:

https:

[//github.com/9705701645/FWC/blob/main/lines4.py](https://github.com/9705701645/FWC/blob/main/lines4.py)