Matrix-Conics

Kukunuri Sampath Govardhan

Contents

I. Problem Statement

In an Ellipse, the distance between its foci is 6 and minor axis is 8. Then find its Eccentricity.

II. Construction

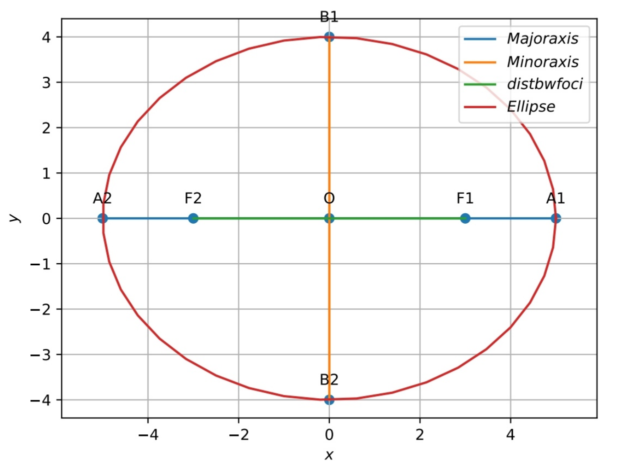

Fig. 1. Ellipse

Symbol	Value	Description
$\|\|\mathbf{F_1}-\mathbf{F_2}\|\|$	6	Distance between foci
$\|\|\mathbf{B_1}-\mathbf{B_2}\|\|$	8	Length of Minor axis
$\mathbf{e_1}$	$\begin{pmatrix}1\\0\end{pmatrix}$	Standard Basis Vector
e		Eccentricity

TABLE I
PARAMETERS

III. Solution

The length of minor axis in ellipse is

$$\|\mathbf{B_1}-\mathbf{B_2}\| = 2\sqrt{\left|\frac{f_0}{\lambda_2}\right|} \tag{1}$$

Therefore,

$$8 = 2\sqrt{\left|\frac{f_0}{\lambda_2}\right|}$$

Yielding,

$$\frac{f_0}{\lambda_2} = 16 \tag{2}$$

The focal points

$$\mathbf{F_1} = \frac{\frac{1}{e\sqrt{1-e^2}}e^2\sqrt{\frac{\lambda_2}{f_0}}\mathbf{e_1}}{\frac{\lambda_2}{f_0}} \tag{3}$$

$$\mathbf{F_2} = -\frac{\frac{1}{e\sqrt{1-e^2}}e^2\sqrt{\frac{\lambda_2}{f_0}}\mathbf{e_1}}{\frac{\lambda_2}{f_0}} \tag{4}$$

$$\|\mathbf{F_1}-\mathbf{F_2}\| = 2\frac{\frac{1}{e\sqrt{1-e^2}}e^2\sqrt{\frac{\lambda_2}{f_0}}\|\mathbf{e_1}\|}{\frac{\lambda_2}{f_0}}$$

$$6 = \frac{\frac{1}{\sqrt{1-e^2}}e\sqrt{\frac{\lambda_2}{f_0}}}{\frac{\lambda_2}{f_0}}$$

Substuting $\frac{\lambda_2}{f_0}$ from eq-2,

$$6 = 2\frac{e}{\sqrt{1-e^2}}\frac{1}{4}\frac{16}{1} \tag{5}$$

Yielding,

$$e = \frac{3}{5}$$

Therefore,

$$\boldsymbol{Eccentricity, e = \frac{3}{5}}$$