Matrix Assignment

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***Problem Statement* -ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.**

Solution

Using triangle law of vector addition:

$$\overline{PQ} = \overline{PB} + \overline{BQ} \tag{1}$$

$$\overline{PQ} = \frac{1}{2}\overline{AB} + \frac{1}{2}\overline{BC} \tag{2}$$

$$\overline{PQ} = \frac{1}{2}(\overline{AB + BC}) \tag{3}$$

$$\overline{PQ} = \frac{1}{2}\overline{AC} \tag{4}$$

Similarly,using triangle law of vector addition:

$$\overline{PS} = \overline{PA} + \overline{AS} \tag{5}$$

$$\overline{PS} = \frac{1}{2}\overline{BA} + \frac{1}{2}\overline{AD} \tag{6}$$

$$\overline{PS} = \frac{1}{2}(\overline{BA + AD}) \tag{7}$$

$$\overline{PS} = \frac{1}{2}\overline{BD} \tag{8}$$

ABCD is a rectangle and has the following properties:

1. Opposite sides are equal
2. All angles are equal
3. lengths of diagonals are equal
4. Diagonals are perpendicular to each other.

$$AC = BD \tag{9}$$

$$AC \perp BD \tag{10}$$

$$\boldsymbol{PQ} = \boldsymbol{PS} \tag{11}$$

similarly, the vector equations of QR,SR can be derived as:

$$\overline{QR} = \overline{QC} + \overline{CR} \tag{12}$$

$$\overline{QR} = \frac{1}{2}\overline{BC} + \frac{1}{2}\overline{CD} \tag{13}$$

$$\overline{PQ} = \frac{1}{2}(\overline{BC + CD}) \tag{14}$$

$$\overline{PQ} = \frac{1}{2}\overline{BD} \tag{15}$$

$$\overline{SR} = \overline{SD} + \overline{DR} \tag{16}$$

$$\overline{SR} = \frac{1}{2}\overline{AD} + \frac{1}{2}\overline{DC} \tag{17}$$

$$\overline{SR} = \frac{1}{2}(\overline{AD} + \overline{DC}) \tag{18}$$

$$\overline{SR} = \frac{1}{2}\overline{AC} \tag{19}$$

from the equations (),():

$$SR = QS \tag{20}$$

Thus, all sides of the quadrilateral PQRS are equal
The diagonals of the Quadrilateral PQRS are given by:

$$PR = PS + PQ \because \tag{21}$$

parallelogram law of addition

$$QS = PS - PQ \because \tag{22}$$

triangle law of addition

$$PR = \frac{1}{2}\overline{AC} + \frac{1}{2}\overline{BD} \tag{23}$$

$$QS = \frac{1}{2}\overline{AC} - \frac{1}{2}\overline{BD} \tag{24}$$

For a quadrilateral to be rhombus the diagonals should perpendicularly bisect each other

$$PR.QS = \frac{1}{4}((\overline{AC} + \overline{BD})(\overline{AC} - \overline{BD})) \tag{25}$$

$$PR.QS = 0 \tag{26}$$

Since the dot product of the diagonals is 0, the diagonals are perpendicular

Construction

Since , all sides of the qudarilateral PQRS are equal and the digonals are perpendicular to each other, the Quadrilateral is a Rhombus

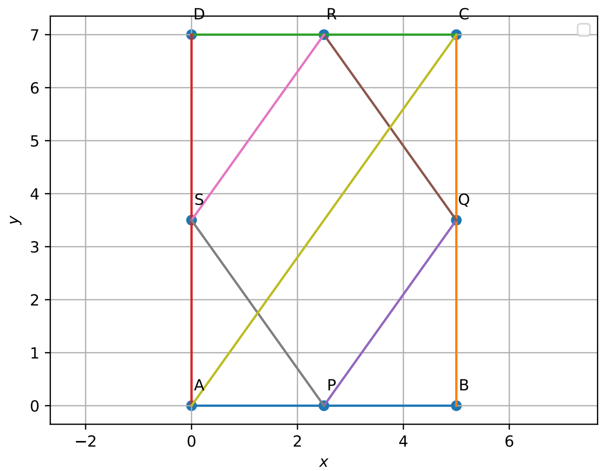

Figure 1: Rhombus PQRS formed by midpoints of Rectangle ABCD

Construction

The dimensions of the rectangle are taken as below

vertex	**co-ordinates**
A	(0,0)
B	(5,0)
C	(5,7)
D	(0,7)