Line Assignment

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Problem: In Fig.1, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

1. **ar (PQRS) = ar(ABRS)**
2. **ar(AXS) = 1/2 ar(PQRS)**

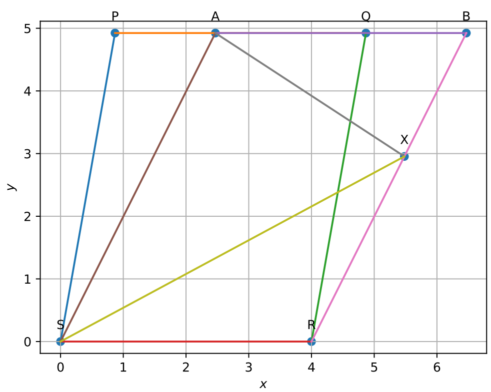

Fig 1. Parallelogram

Solution 1:

Two parallelograms PQRS and ABRS, on the same base SR and between the same parallels PB and SR are given (see Fig.1).
We need to prove that ar(PQRS) = ar(ABRS).
Let, In PQRS

$$\boldsymbol{S} - \boldsymbol{R} = \boldsymbol{p1} \tag{1}$$

$$\boldsymbol{S} - \boldsymbol{P} = \boldsymbol{p2} \tag{2}$$

$$\boldsymbol{P} - \boldsymbol{Q} = \boldsymbol{q1} \tag{3}$$

$$\boldsymbol{R} - \boldsymbol{Q} = \boldsymbol{q2} \tag{4}$$

According to parrallelogram condition

$$\|\mathbf{p1}\| = \|\mathbf{q1}\| \&\& \|\mathbf{p2}\| = \|\mathbf{q2}\| \tag{5}$$

Let,In ABRS

$$\boldsymbol{S} - \boldsymbol{R} = \boldsymbol{s1} \tag{6}$$

$$\boldsymbol{S} - \boldsymbol{A} = \boldsymbol{s2} \tag{7}$$

$$\boldsymbol{A} - \boldsymbol{B} = \boldsymbol{r1} \tag{8}$$

$$\boldsymbol{R} - \boldsymbol{B} = \boldsymbol{r2} \tag{9}$$

According to parallelogram condition

$$\|\mathbf{s1}\| = \|\mathbf{r1}\| \&\& \|\mathbf{s2}\| = \|\mathbf{r2}\| \tag{10}$$

Area of parallelogram PQRS

$$\boldsymbol{p1} X \boldsymbol{p2} \tag{11}$$

Now, area of parallelogram ABRS

$$= \boldsymbol{SR} X \boldsymbol{SA} \tag{12}$$

$$= \boldsymbol{s1} X \boldsymbol{s2} \tag{13}$$

$$= \boldsymbol{SR} X (\boldsymbol{SP} + \boldsymbol{PA}) \tag{14}$$

$[\boldsymbol{PA} \parallel \boldsymbol{PQ} \therefore \boldsymbol{PA}$ =k $\boldsymbol{p1}]$

$$= \boldsymbol{p1} X (\boldsymbol{p2} + k\boldsymbol{p1}) \tag{15}$$

[from [5] and [10] $\|\boldsymbol{p1}\|$ =$\|\boldsymbol{s1}\|$]

$$= \boldsymbol{p1} X \boldsymbol{p2} + \boldsymbol{p1} X k\boldsymbol{p1} \tag{16}$$

$$= \boldsymbol{p1} X \mathbf{p2} + k(\boldsymbol{p1} X \boldsymbol{p1}) \tag{17}$$

$$= \boldsymbol{p1} X \boldsymbol{p2} \tag{18}$$

$[\therefore \boldsymbol{p1}$ X $\boldsymbol{p1}$ =0]
=Area of parallelogram ABRS

Hence proved

So, from [11] and [18] PQRS and ABRS parallalograms are equal in area.

solution 2:

Let ΔAXS and parallelogram ABRS be on the same base AS and between the same parallels $\boldsymbol{AS}$ and $\boldsymbol{BR}$ (see Fig. 1).
You wish to prove that

$$\text{ar (AXS)} = \frac{1}{2} \text{ ar (PQRS)}$$

Draw BY ∥ AS to obtain another parallelogram AXYS as in Fig 2. Now parallelograms ABRS and AXYS are on the same base AS and between the same parallels AS and BY.

$$\therefore ar(ABRS) = ar(AXYS)\text{(By Solution 1)} \tag{19}$$

But $\Delta AXS \cong \Delta XYS$ (Diagonal SX divides parallelogram AXYS into two congruent triangles.)

$$\boldsymbol{A} - \boldsymbol{X} = \boldsymbol{a1} \tag{20}$$
$$\boldsymbol{S} - \boldsymbol{X} = \boldsymbol{d1} \tag{21}$$
$$\boldsymbol{X} - \boldsymbol{Y} = \boldsymbol{x1} \tag{22}$$
$$\boldsymbol{Y} - \boldsymbol{S} = \boldsymbol{x2} \tag{23}$$
$$\boldsymbol{S} - \boldsymbol{A} = \boldsymbol{x3} \tag{24}$$

from parallelogram condition

$$\boldsymbol{a1} = \boldsymbol{x2} \&\& \boldsymbol{x3} = \boldsymbol{x1} \tag{25}$$

$$ar(AXS) = ar(SXY) \tag{26}$$

Therefore,

ar (AXYS) =ar (AXS) +ar (XYS)

ar (AXS)=$\frac{1}{2}$ * ($\boldsymbol{x3}$ X $\boldsymbol{a1}$)

ar (AXYS)=($\boldsymbol{x3}$ X $\boldsymbol{a1}$) $\implies$ $\boldsymbol{SA}$ X$\boldsymbol{AX}$

ar (AXS)=1/2 ar (AXYS)
=$\frac{1}{2}$($\boldsymbol{SA}$ X$\boldsymbol{AX}$)
=$\frac{1}{2}$ ($\boldsymbol{SA}$ X ($\boldsymbol{AB} + \boldsymbol{BX}$))
=$\frac{1}{2}$ (($\boldsymbol{s2}$ + k $\boldsymbol{s1}$) X ($\boldsymbol{s1}$+ k1($\boldsymbol{s2}$ + k$\boldsymbol{s1}$)))

[=$\frac{1}{2}$ ($\boldsymbol{s1}$ X $\boldsymbol{s2}$) [$\boldsymbol{a}$ X $\boldsymbol{a}$ =0]
=$\frac{1}{2}$ ($\boldsymbol{SR}$ X$\boldsymbol{SA}$)

This gives ar (AXS) = $\frac{1}{2}$ ar (ABRS) [From (1) and (3)]
ar (AXS) = $\frac{1}{2}$ ar (PQRS) [From solution 1]

Construction

The following python code is used for constructing PQRS and ABRS parallalograms.

```
https://github.com/AnushaJella/
assigment_line/blob/main/code/asgn1.
py
```

See Fig 1 for the input parameters in Table 1.

Symbol	Value	Description
S	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	S Point
a	4	SR
θ	80°	∠RSP
b	5	SP
k	1.5	Point A

Table 1

For construction, let $\boldsymbol{S}=0$, $\boldsymbol{P}$,$\boldsymbol{R}$ are input vectors.
the fourth point be

$$\boldsymbol{Q}\text{=}\boldsymbol{P}\text{+}\boldsymbol{R}\text{-}\boldsymbol{S}$$

choose k value and define

$$\boldsymbol{A} = \frac{k\boldsymbol{P}}{k+1}$$

B is fourth vertex of ABRS parallelogram

$$\boldsymbol{B}\text{=}\boldsymbol{A}\text{+}\boldsymbol{R}\text{-}\boldsymbol{S}$$

X be any point on BR k value and define

$$\boldsymbol{X} = \frac{k\boldsymbol{B}}{k+1}$$

draw a line parallel from S such that AX ∥ SY.
So,AXYS is a parallalogram

$$\boldsymbol{Y}\text{=}\boldsymbol{S}\text{+}\boldsymbol{X}\text{-}\boldsymbol{A}$$

Proof: Two parallelograms PQRS and ABRS, on the same base SR and between the same parallels PB and SR are given (see Fig.1).
We need to prove that ar (PQRS) = ar (ABRS).
In Δ PSA and Δ QRB,

$$\angle SPA = \angle RQB \tag{27}$$

(Corresponding angles from PS ∥ RQ and transversal PB)

$$\angle PAS = \angle QBR \tag{28}$$

(Corresponding angles from AS ∥ BR and transversal PB)

$$Therefore, \angle PSA = \angle QRB \tag{29}$$

(Angle sum property of a triangle)

$$Also, PS = QR \tag{30}$$

(Opposite sides of the parallelogram PQRS)
So, Δ PSA $\cong$ Δ QRB
(By ASA rule, using (27), (29), and (30))

$$Therefore, ar(PSA) = ar(QRB) \tag{31}$$

(Congruent figures have equal areas)

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Now,

ar (PQRS) = ar (PSA) + ar (AQRS)

= ar (QRB) + ar (AQRS) [From(33)]

= ar (ABRS)

So, parallelograms PQRS and ABRS are equal in area.