Circle Assignment

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***Problem Statement* - let $2x^2 + y^2 - 3xy = 0$ equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA..**

Figure

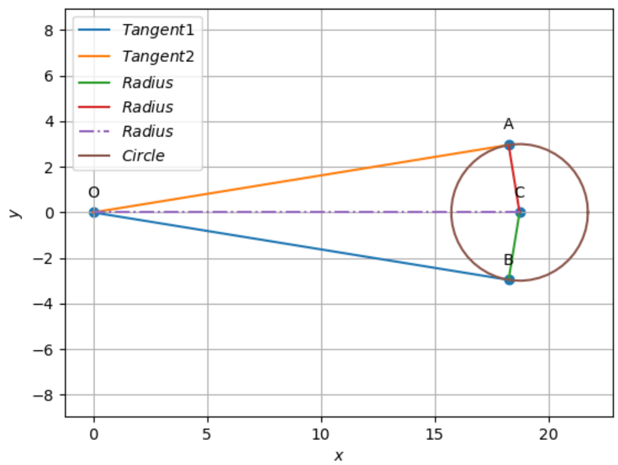

Figure 1: length of OA = 18.728

Solution

The equation of the pair of tangents is $2x^2 + y^2 - 3xy = 0$ and r=3.
The equation in quadratic form is

$$\mathbf{x^T v x + 2u^T x} + f = 0$$

from above v,u,f are

$$\mathbf{v} = \begin{pmatrix} 2 & \frac{-3}{2} \\ \frac{-3}{2} & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = 0$$

$$\mathbf{x^T v x} = 0$$

$$\mathbf{v = P^T D P}$$

on solving we get $\lambda_\mathbf{1}, \lambda_\mathbf{2}, \mathbf{P}$

$$\lambda_1 = 3.081,\ \lambda_2 = -0.081,\ P = \begin{pmatrix} 0.811 & 0.584 \\ -0.584 & 0.811 \end{pmatrix}$$

the pair of straight line can be expressd as

$$(\sqrt{|\lambda_\mathbf{1}|} \ \pm \sqrt{|\lambda_\mathbf{2}})\mathbf{P^T(X - C) = 0}$$

where,

$$\mathbf{C = v^{-1} u}$$

$$\mathbf{C = 0}$$

the normal vectors of the lines are

$$\mathbf{n_1 = P} \begin{pmatrix} \sqrt{\lambda_1} \\ \sqrt{\lambda_2} \end{pmatrix}$$

$$\mathbf{n_2 = P} \begin{pmatrix} \sqrt{\lambda_1} \\ -\sqrt{\lambda_2} \end{pmatrix}$$

on solving normal vectors are

$$\mathbf{n_1} = \begin{pmatrix} 1.59 \\ -0.79 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 1.25 \\ -1.25 \end{pmatrix}$$

the angle between the two vectors $n_1 and n_2$ is given by

$$cos\theta = \frac{\mathbf{n_1^T n_2}}{\|\mathbf{n_1}\|\|\mathbf{n_2}\|}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{n_1^T n_2}}{\|\mathbf{n_1}\|\|\mathbf{n_2}\|}\right)$$

$$\theta = 18.434$$

to find the length of the OA

$$OA = r * cosec\frac{\theta}{2}$$

on solving the length of OA

$$OA = 18.728$$