CONIC ASSIGNMENT

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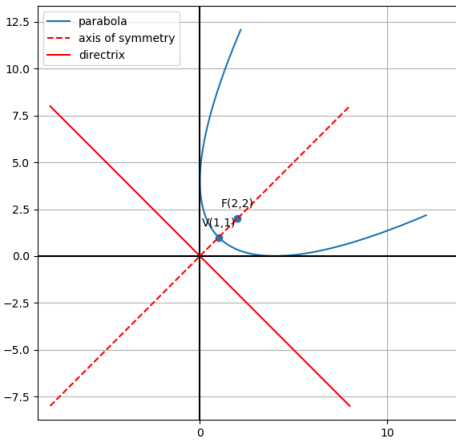
FWC22097 - IITH Future Wireless Communication (FWC)

1 Question

Q(6), C , Section-A, Chapter-8: **The axis of a parabola is along the line y=x and the distance of its vertex and focus from the origin are $\sqrt{2}$ and $2\sqrt{2}$ respectively. If vertex and focus both lie in the first quadrant, then the equation of the parabola is**

2 Solution



Given the parabola has the axis of symmetry along the line y=x. And focus and vertex lies in first quadrant.

The distance from the origin to the vertex is given as $\sqrt{2}$.

Let O be the origin,v be the vertex and F be the focus of the parabola.

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$$

We can write

$$||\mathbf{v} - \mathbf{O}|| = \sqrt{2}$$

$$(\mathbf{v} - \mathbf{O})^\top (\mathbf{v} - \mathbf{O}) = 2$$

which can be written as

$$\implies (\mathbf{v} - \mathbf{O})^\top (\mathbf{v} - \mathbf{O}) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\implies \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

From the above matrix, we can find x and y values as

$$\mathbf{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$||\mathbf{v} - \mathbf{O}|| = 2\sqrt{2}$$

$$(\mathbf{F} - \mathbf{O})^\top (\mathbf{v} - \mathbf{O}) = 8$$

$$\implies (\mathbf{F} - \mathbf{O})^\top (\mathbf{F} - \mathbf{O}) = \begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\implies \begin{bmatrix} x1 & y1 \end{bmatrix} \begin{bmatrix} x1 \\ y1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

From the above matrix, we can find x1 and y1 values as

$$\mathbf{F} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Given the axis of symmetry of parabola as x-axis.Therefore,directrix of the parabola will be the line perpendicular to the axis of symmetry i.e,

$$x + y = 0$$

Comparing with the line equation,

$$\mathbf{n}^\top x = c$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

and c=0.

By using $\mathbf{v}$,$\mathbf{F}$ and $\mathbf{n}$ values,we can find $\mathbf{V}$,$\mathbf{u}$,f values of parabola.

The equation of a conic with directrix $\mathbf{n}^\top x = c$, eccentricity e and focus F is given by

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

$$\mathbf{V} = ||\mathbf{n}||^2 I - e^2 \mathbf{n}\mathbf{n}^\top \tag{2}$$

Where I=Identity matrix,e=1.

By substituting I,e,$\mathbf{n}$ values in equation (2),we get

$$\mathbf{V} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{u} = ce^2\mathbf{n} - ||\mathbf{n}||^2\mathbf{F} \tag{3}$$

By substituting c,e,$\mathbf{n}$.$\mathbf{F}$ values in qquation (3),we get

$$\mathbf{u} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$$

$$f = ||\mathbf{n}||^2||\mathbf{F}||^2 - c^2e^2 \tag{4}$$

By substituting c,e,$\mathbf{n}$.$\mathbf{F}$ values in equation (4),we get

$$f = 16$$

By substituting $\mathbf{V}$,$\mathbf{u}$ and f values in equation (1),we get the equation of parabola.

3 Software

The below python code realizes the above construction:

https://github.com/PanjugalaShashikala/FWC_2022097/tree/main/Lines