# Circle Assignment

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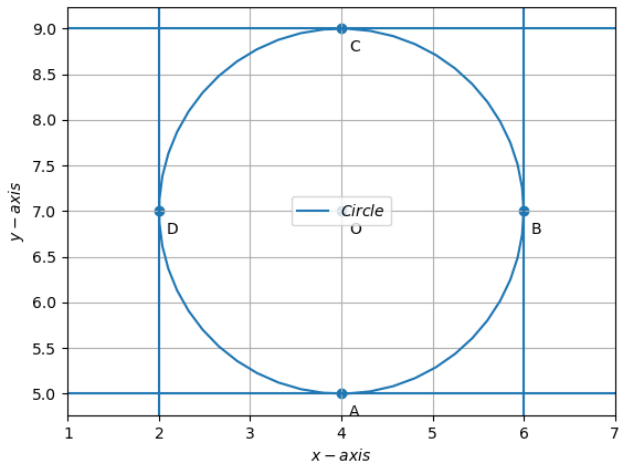
***Problem Statement*** **- The centre of circle inscribed in square formed by the lines** $x^2 - 8x + 12 = 0$ **and** $y^2 - 14y + 45 = 0$ **is ?**

## Solution




The Standard Equations of the Circle are :

$$\mathbf{x}^\top \mathbf{V_1} \mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f_1 = 0 \tag{1}$$

$$\mathbf{V_1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2}$$

$$\mathbf{u_1} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{3}$$

$$f_1 = 12 \tag{4}$$

$$\mathbf{x}^\top \mathbf{V_2} \mathbf{x} + 2\mathbf{u_2}^\top \mathbf{x} + f_2 = 0 \tag{5}$$

$$\mathbf{V_2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

$$\mathbf{u_2} = \begin{pmatrix} 0 \\ -7 \end{pmatrix} \tag{7}$$

$$f_2 = 45 \tag{8}$$

The lines in given pairs of straight lines are :

$$\mathbf{m_1}^\top (\mathbf{V_2 q_1} + \mathbf{u_2}) = 0 \tag{9}$$

$$e_2^T x = k_1 \tag{10}$$

Yielding,

$$\mathbf{q_1} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \tag{11}$$

$$\mathbf{m_1}^\top (\mathbf{V_2 q_2} + \mathbf{u_2}) = 0 \tag{12}$$

$$e_2^T x = k_2 \tag{13}$$

Yielding,

$$\mathbf{q_2} = \begin{pmatrix} 6 \\ 7 \end{pmatrix} \tag{14}$$

$$\mathbf{m_2}^\top (\mathbf{V_1 q_3} + \mathbf{u_1}) = 0 \tag{15}$$

$$e_2^T x = k_3 \tag{16}$$

Yielding,

$$\mathbf{q_3} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \tag{17}$$

$$\mathbf{m_2}^\top (\mathbf{V_1 q_4} + \mathbf{u_1}) = 0 \tag{18}$$

$$e_2^T x = k_4 \tag{19}$$

Yielding,

$$\mathbf{q_4} = \begin{pmatrix} 4 \\ 9 \end{pmatrix} \tag{20}$$

The centre of circle is given by:

$$\mathbf{O} = \frac{\mathbf{q_1} + \mathbf{q_2}}{2} \tag{21}$$

$$\mathbf{O} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$

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## Construction

| Symbol | Value | Description |
|---|---|---|
| $\mathbf{e_1}$ | $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ | basis vector |
| $\mathbf{e_2}$ | $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ | basis vector |
| $\mathbf{m_1}$ | $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ | directional vector of e1 |
| $\mathbf{m_2}$ | $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ | directional vector of e2 |