Conic section Assignment

Somisetty Kedareswari

October 2022

Problem Statement **- The area between $x = y^2$ and x = 4 is divided into two equal parts by the line x = a, find the value of a**

Solution

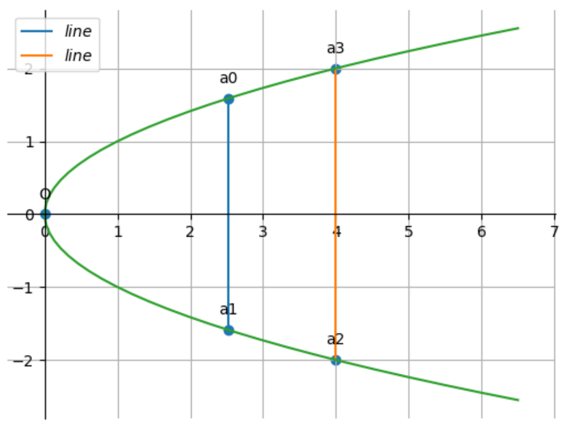

Figure 1: The parabola formed by the curve $y^2 = x$ and the line x=4

The given equation of parabola $y^2 = x$ can be written in the general quadratic form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} -0.5 \\ 0 \end{pmatrix}, \tag{3}$$

$$f = 0 \tag{4}$$

The point of intersection of the lines x=a and x=4 to the parabola is given by

The points of intersection of the line

$$L: \quad \mathbf{x} = \mathbf{q} + \mu \mathbf{m} \quad \mu \in \mathbf{R} \tag{5}$$

with the conic section are given by

$$\mathbf{x}_i = \mathbf{q} + \mu_i \mathbf{m} \tag{6}$$

where

$$\mu_i = \frac{1}{\mathbf{m}^T \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T \mathbf{V}\mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f\right)\left(\mathbf{m}^T \mathbf{V}\mathbf{m}\right)} \right) \tag{7}$$

From the line x-a=0 the vectors q,m are taken,

$$\mathbf{q_2} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{8}$$

$$\mathbf{omat} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{9}$$

$$n^T x = c \tag{10}$$

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{11}$$

$$m_2 = omat \cdot n_1 \tag{12}$$

$$\mathbf{m_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{13}$$

by substituting eq(2),(3),(4),(8),(9) in eq(7)

$$\mu_i = a, -a \tag{14}$$

substituting eq(8),(9),(10) in eq(6) the intersection points on the parabola are

$$\mathbf{a_0} = \begin{pmatrix} a \\ a \end{pmatrix}, \tag{15}$$

$$\mathbf{a_1} = \begin{pmatrix} a \\ -a \end{pmatrix} \tag{16}$$

From the line x-4=0 the vectors q,m are taken,

$$\mathbf{q_1} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{17}$$

$$\mathbf{omat} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{18}$$

$$\mathbf{n_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{19}$$

$$m_1 = omat \cdot n_2 \tag{20}$$

$$\mathbf{m_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{21}$$

by substituting eq(2),(3),(4),(13),(14) in eq(7)

$$\mu_i = 2, -2 \tag{22}$$

substituting eq(13),(14),(15) in eq(6) the intersection points on the parabola are

$$\mathbf{a_3} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \tag{23}$$

$$\mathbf{a_2} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \tag{24}$$

Area between parabola and the line x=4 is divided equally by the line x=a

$$\implies A_1 = \int_0^a \sqrt{x}\,dx \tag{25}$$

$$\implies A_2 = \int_a^4 \sqrt{x}\,dx \tag{26}$$

$$\implies A_1 = A_2 \tag{27}$$

$$\int_0^a \sqrt{x}\,dx = \int_a^4 \sqrt{x}\,dx \tag{28}$$

$$\implies a = 4^{\frac{2}{3}} \tag{29}$$

Construction

Points	**intersection points**
a0	$\begin{pmatrix} a \\ a \end{pmatrix}$
a1	$\begin{pmatrix} a \\ -a \end{pmatrix}$
a3	$\begin{pmatrix} 4 \\ 2 \end{pmatrix}$
a2	$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$