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ELECTRICAL ENGINEERING
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Line Assignment

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Problem Statement:

The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1)(1, 1)and (1,0) is:

(a)2+$\sqrt{2}$ (b)2-$\sqrt{2}$
(c)1+$\sqrt{2}$ (d)1-$\sqrt{2}$

Construction

vertex	coordinates
P	$\begin{pmatrix}1\\0\end{pmatrix}$
Q	$\begin{pmatrix}0\\1\end{pmatrix}$
R	$\begin{pmatrix}1\\1\end{pmatrix}$
k	1
A	$\begin{pmatrix}0\\0\end{pmatrix}$
B	$\begin{pmatrix}2\\0\end{pmatrix}$
C	$\begin{pmatrix}0\\2\end{pmatrix}$

Solution: If a point P divides the line segment AB in the ratio 1: 1 is given by

$$P = \frac{B+A}{2}$$

$$A + B = 2P$$

$$\begin{pmatrix}ABC\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix} = 2\text{P}$$

$$R = \frac{B+C}{2}$$

$$B + C = 2R$$

$$\begin{pmatrix}ABC\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix} = 2\text{R}$$

$$Q = \frac{A+C}{2}$$

$$A + C = 2Q$$

$$\begin{pmatrix}ABC\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = 2\text{Q}$$

$$\begin{pmatrix}ABC\end{pmatrix}\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix} = 2\begin{pmatrix}PQR\end{pmatrix}$$

$$\begin{pmatrix}ABC\end{pmatrix}\text{V}=2\begin{pmatrix}PQR\end{pmatrix}$$

$$\begin{pmatrix}ABC\end{pmatrix} = 2\begin{pmatrix}PQR\end{pmatrix}\mathbf{V}^{-1}$$

$$\mathbf{V}^{-1}=\tfrac{-1}{2}\begin{pmatrix}-1&-1&1\\-1&1&-1\\1&-1&-1\end{pmatrix}$$

$$\begin{pmatrix}ABC\end{pmatrix} = 2(\tfrac{-1}{2})\begin{pmatrix}PQR\end{pmatrix}\begin{pmatrix}-1&-1&1\\-1&1&-1\\1&-1&-1\end{pmatrix}$$

$$\begin{pmatrix}ABC\end{pmatrix} = \begin{pmatrix}-P-Q+R & -P+Q-R & P-Q-R\end{pmatrix}$$

Given P $\begin{pmatrix}1\\0\end{pmatrix}$ Q $\begin{pmatrix}0\\1\end{pmatrix}$ R $\begin{pmatrix}1\\1\end{pmatrix}$

By substituting P,Q and R in above equqtion, we will get A,B and C as

A $\begin{pmatrix}0\\0\end{pmatrix}$ B $\begin{pmatrix}2\\0\end{pmatrix}$ C $\begin{pmatrix}0\\2\end{pmatrix}$

Vector representation of A,B,C are as follows

$$\mathbf{A} = \mathbf{0} \tag{1}$$

$$\mathbf{B} = \mathbf{2i} \tag{2}$$

$$\mathbf{C} = \mathbf{2j} \tag{3}$$

The vectors of AB,BC and CA linesegments are

$$\mathbf{V1} = \mathbf{B} - \mathbf{A} = \mathbf{2i} \tag{4}$$

$$\mathbf{V2} = \mathbf{C} - \mathbf{B} = \mathbf{2j} - \mathbf{2i} \tag{5}$$

$$\mathbf{V3} = \mathbf{A} - \mathbf{C} = -\mathbf{2j} \tag{6}$$

Norms of the vectors V1, V2 and V3 are

$$\|\mathbf{V1}\| = 2 \tag{7}$$

$$\|\mathbf{V2}\| = 2\sqrt{2} \tag{8}$$

$$\|\mathbf{V3}\| = 2 \tag{9}$$

The incenter is the intersection of three angle bisectors,

$$I = \frac{\|\mathbf{V1}\|\,\mathbf{C} + \|\mathbf{V2}\|\,\mathbf{A} + \|\mathbf{V3}\|\,\mathbf{B}}{\|\mathbf{V1}\| + \|\mathbf{V2}\| + \|\mathbf{V3}\|} \tag{10}$$

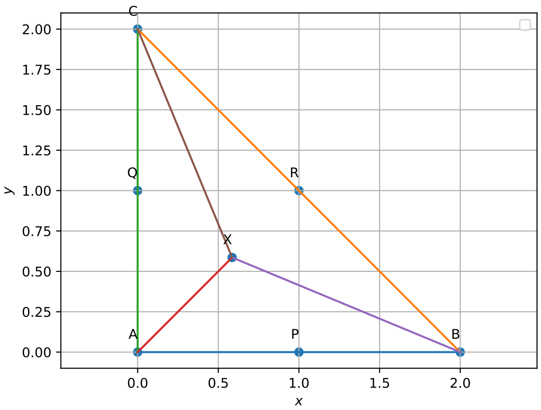
$$I = \frac{2(\mathbf{2i}) + 2(\mathbf{2j}) + 2\sqrt{2}(\mathbf{0})}{2 + 2 + 2\sqrt{2}}$$

$$I = \frac{\mathbf{4i} + \mathbf{4j}}{4 + 2\sqrt{2}}$$

The x-coordinate of the incentre of the triangle is

$$x = \frac{\mathbf{4i}}{4 + 2\sqrt{2}}$$

$$x = 2 - \sqrt{2} \tag{11}$$



Download the code from Github link: Assignment-4.