

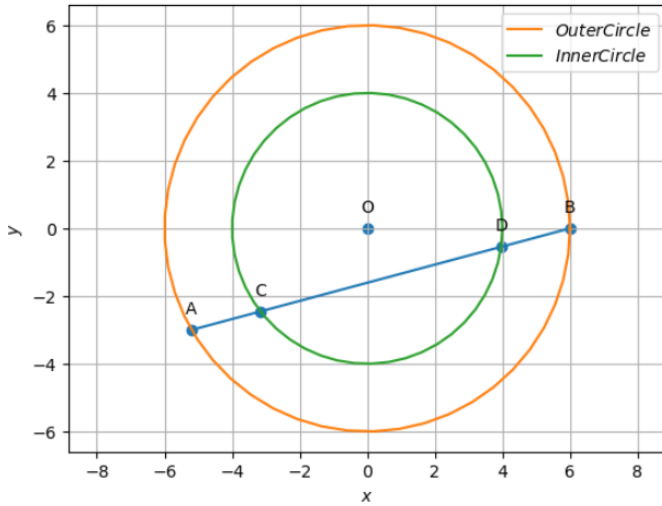
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Assignment-4

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Problem Statement:

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AC = DB$.



SOLUTION:

Let A and B be the end vertices of the chord intersecting two concentric circles, therefore C and D are points of the chord intersecting inner circle. We solve for the required points C and D as follows.

STEP-1

Calculating co-ordinates A and B :

Outer Circle has a radius, $r_o = 6$

Inner Circle has a radius, $r_i = 4$

We get coordinates of vertices A and B as follows :

$$\mathbf{A} = r_o \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix}, \mathbf{B} = r_o \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} \quad (1)$$

By taking $\theta_1 = 159^\circ$ and $\theta_2 = 0^\circ$,

We get,

$$\mathbf{A} = \begin{pmatrix} -5.2 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (2)$$

STEP-2

Finding the equation of line AB :

Using the parametric equation of the line

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \quad (3)$$

We know the equation of circle is,

$$x^2 + y^2 = r^2$$

The above equation can be expressed in vector form as

$$\mathbf{x}^T \mathbf{x} = r^2 \quad (4)$$

Substituting (2) in above Eq.,

$$(\mathbf{A} + \lambda \mathbf{m})^T (\mathbf{A} + \lambda \mathbf{m}) = r^2 \quad (5)$$

$$\Rightarrow \lambda^2 \|\mathbf{m}\|^2 + 2\lambda \mathbf{m}^T \mathbf{A} + \|\mathbf{A}\|^2 - r^2 = 0 \quad (6)$$

yielding

$$\lambda = \frac{-\mathbf{m}^T \mathbf{A} \pm \sqrt{(\mathbf{m}^T \mathbf{A})^2 - \|\mathbf{m}\|^2 (\|\mathbf{A}\|^2 - r^2)}}{\|\mathbf{m}\|^2} \quad (7)$$

For this problem, the numerical values are

$$\mathbf{A} = \begin{pmatrix} -5.2 \\ -3 \end{pmatrix}, r = 4, \quad (8)$$

$$\mathbf{m} = \mathbf{B} - \mathbf{A}, \mathbf{m} = \begin{pmatrix} 11.2 \\ 3 \end{pmatrix} \quad (9)$$

Substituting above in (6),

$$\lambda_1 = 0.18, \lambda_2 = 0.82 \quad (10)$$

Thus substituting λ_1 and λ_2 in (2) would give the desired points of intersection C and D.

For point C,

$$\mathbf{x} = \begin{pmatrix} -5.2 \\ -3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 11.2 \\ 3 \end{pmatrix} \quad (11)$$

$$= \begin{pmatrix} -3.16 \\ -2.45 \end{pmatrix} \quad (12)$$

For point D,

$$\mathbf{x} = \begin{pmatrix} -5.2 \\ -3 \end{pmatrix} + \lambda_2 \begin{pmatrix} 11.2 \\ 3 \end{pmatrix} \quad (13)$$

$$= \begin{pmatrix} 3.96 \\ -0.55 \end{pmatrix} \quad (14)$$

Thus, points C and D

$$\mathbf{C} = \begin{pmatrix} -3.16 \\ -2.45 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 3.96 \\ -0.55 \end{pmatrix} \quad (15)$$

Table

The input parameters are the lengths points A and C.

| Symbol | Value | Description |
|--------------|---|------------------------------|
| r_o | 6 | Outer Radius |
| r_i | 4 | Inner Radius |
| \mathbf{m} | $\begin{pmatrix} 11.2 \\ 3 \end{pmatrix}$ | Direction Vector of the Line |

Construction

| Vertex | Co-ordinates |
|--------|---------------|
| O | (0,0) |
| A | (-5.2,-3) |
| B | (6,0) |
| C | (-3.16,-2.45) |
| D | (3.96,-0.55) |