Matrix-Circle

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1 Problem Statement

To find angle QPR of the triangle PQR which is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3,4) and (-4,3) respectively .

2 Construction

Symbol	Value	Description
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Center point
r	5	Radius
Q	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$	given point
R	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$	given point
P	$\begin{pmatrix} rcos(\alpha) \\ rsin(\alpha) \end{pmatrix}$	user defined point

Table 1: Parameters

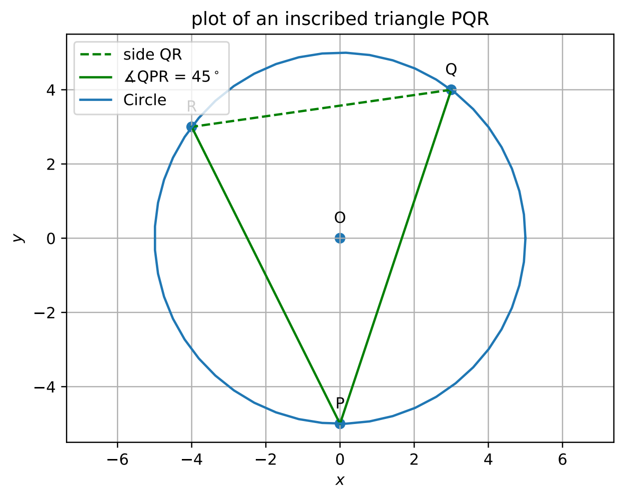

Figure 1: triangle inscribed in Circle and its angle QPR

3 solution

Given that Points given are on the circle forms an inscribed triangle the points **Q**, **R** and any point**P** on the given circle

The circle given,

$$\mathbf{x}^\top \mathbf{x} = 25 \tag{1}$$

From General Equation:

$$\mathbf{x}^\top \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

where

$$\mathbf{u} = -\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{3}$$

$$f = -25 \tag{4}$$

Centre and Radius of given circle are :

$$\mathbf{O} = -\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

$$r = \sqrt{\mathbf{u}^\top \mathbf{u} - f} = 5 \tag{6}$$

Point on the circle should satisfy the circle equation so, we take

$$\mathbf{P} = \begin{pmatrix} rcos(\alpha) \\ rsin(\alpha) \end{pmatrix} \tag{7}$$

$$\mathbf{Q} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{8}$$

$$\mathbf{R} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{9}$$

Considering **P**,**Q**and **R** as the Coordinates of the Inscribed Triangle

We know, The direction vector of the line joining two points **A**, **B** is given by

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \tag{10}$$

we Know The angle between two vectors is given by

$$\theta = cos^{-1} \frac{\mathbf{A}^\top \mathbf{B}}{\|\mathbf{A}\|\|\mathbf{B}\|} \tag{11}$$

By the equations (10) and (11) for $\angle\mathbf{QPR}$ we consider,

$$\measuredangle\mathbf{QPR} = cos^{-1} \frac{(\mathbf{P}-\mathbf{Q})^\top(\mathbf{P}-\mathbf{R})}{\|\mathbf{P}-\mathbf{Q}\|\|\mathbf{P}-\mathbf{R}\|} \tag{12}$$

3.1 Case 1: Major Arc

If **P** lies on major Arc of the circle for given **Q** and **R** points

$$\measuredangle\mathbf{QPR} = 45° \tag{13}$$

3.2 Case 2: Minor Arc

If **P** lies on minor Arc of the circle for given **Q** and **R** points.

$$\measuredangle\mathbf{QPR} = 135° \tag{14}$$

where,**P** is a random point on the given circle for its $\boldsymbol{\alpha}$

4 Software

Download the following code using,

https://github.com/chanduputta/FWC-Module1Assignments/blob/main/circle/circle.py

and execute the code by using command
cmd1:Python3 circle.py
cmd2:Input your $\boldsymbol{\alpha}$ value (0 to 360°)

5 Conclusion

We found the $\measuredangle\mathbf{QPR}$ of the $\triangle\mathbf{PQR}$ which is inscribed in the circle $\mathbf{x^2} + \mathbf{y^2} = \mathbf{25}$.
Where **P** is point on the circle as
i) **45°**, If **P** lies on Major Arc of Given Circle
ii) **135°**, If **P** lies on Minor Arc of Given Circle