Conic Assignment

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Problem Statement

- On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line 8x = 9y are .

Solution

Construction

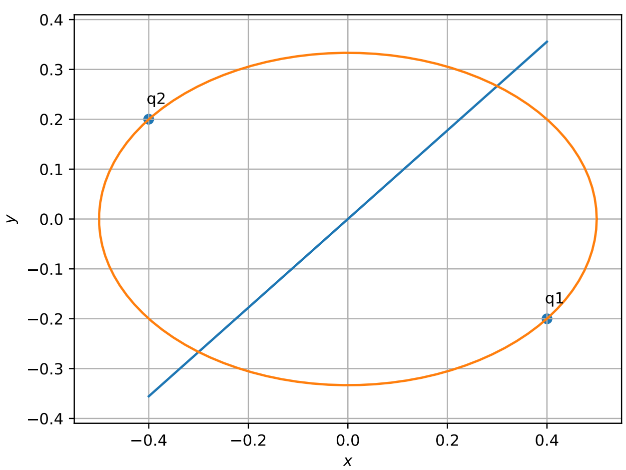

Figure 1: Figure

The dimensions of the figure is taken as below

symbol	value
u	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
a	1/4
b	1/9
V	$\begin{pmatrix} 1/4 & 0 \\ 0 & 1/9 \end{pmatrix}$
n	$\begin{pmatrix} 8 \\ -9 \end{pmatrix}$

***Given* :** Ellipse Equation

$$4x^2 + 9y^2 = 1 \tag{1}$$

Line Equation : 8x = 9y

The standard equation of the conic is given by

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

The given circle can be expressed as conics with Prameters

$$\lambda_1 = \frac{1}{4}, \lambda_2 = \frac{1}{9} \tag{3}$$

$$\mathbf{V} = \begin{pmatrix} \lambda_2 & 0 \\ 0 & \lambda_1 \end{pmatrix} = \begin{pmatrix} 1/9 & 0 \\ 0 & 1/4 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -\lambda_1\lambda_2 = \frac{1}{36} \tag{4}$$

To find the Points on ellipse which forms a tangent parallel to the line

$$8x - 9y = 0 \tag{5}$$

The points are given by the following equation:

$$\mathbf{q} = \mathbf{V}^{-1}(k_i\mathbf{n} - \mathbf{u}) \tag{6}$$

And the intermediate parameters are given by

$$k_i = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\mathbf{n}\mathbf{V}^{-1}\mathbf{n}}} \tag{7}$$

n is the normal vector of tangent from point **q1** and **q2**

$$\mathbf{n} = \begin{pmatrix} 8 \\ -9 \end{pmatrix} \tag{8}$$

Now to obtain the values of k_1 and k_2 , substitute **n** in equation 7

$$V^{-1} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \tag{9}$$

$$k_1 = \sqrt{\frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}{\begin{pmatrix} 8 \\ -9 \end{pmatrix}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 8 \\ -9 \end{pmatrix}}} \tag{10}$$

$$k_1 = 0.0055 \tag{11}$$

$$k_2 = -\sqrt{\frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}{\begin{pmatrix} 8 \\ -9 \end{pmatrix}^T \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 8 \\ -9 \end{pmatrix}}} \tag{12}$$

$$k_2 = -0.0055 \tag{13}$$

$$\mathbf{q_1} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} (k_1 * \begin{pmatrix} 8 \\ -9 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \tag{14}$$

$$\mathbf{q_1} = \begin{pmatrix} 0.4 \\ -0.2 \end{pmatrix} \tag{15}$$

$$\mathbf{q_2} = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} (k_2 * \begin{pmatrix} 8 \\ -9 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \tag{16}$$

$$\mathbf{q_2} = \begin{pmatrix} -0.4 \\ 0.2 \end{pmatrix} \tag{17}$$