Matrix Assignment

A L U R U A J A Y

September 2022

***Problem Statement* - Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$**

1. **parallel to the line 2x-y+9=0.**
2. **perpendicular to the line 5y-15x=13.**

1 Solution

According to the question equation of a parabola is

$y = x^2 - 2x + 7$

The standard equation of the parabola is given as :

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -1 \\ -0.5 \end{pmatrix}, \quad f = 7$$

1.1 parallel to the line 2x-y+9=0

Normal vector of a given line $\mathbf{n}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\mathbf{V}$ is not invertible,Then given the normal vector $\mathbf{n}$ the point of contact to is given by the matrix equation

$$\begin{pmatrix} (\mathbf{u} + k_1\mathbf{n}_1)^T \\ \mathbf{V} \end{pmatrix} \mathbf{q}_1 = \begin{pmatrix} -f \\ k_1\mathbf{n}_1 - \mathbf{u} \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} (\begin{pmatrix} -1 \\ -0.5 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2 \\ -1 \end{pmatrix})^T \\ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \mathbf{q}_1 = \begin{pmatrix} -7 \\ \frac{1}{2}\begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ -0.5 \end{pmatrix} \end{pmatrix}$$

By solving the above equation,we can get the point of contact as

$$\mathbf{q}_1 = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

Tangent Equation is

$$\mathbf{n}_1^T(\mathbf{x} - \mathbf{q}_1) = 0 \tag{3}$$

By substituting the values of $\mathbf{n}_1^T$ & $\mathbf{q}_1$ in Eq(3),Then

$$2x - y + 3 = 0 \tag{4}$$

1.2 perpendicular to the line 5y-15x=13

Normal vector of a given line $\mathbf{n}_2 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\mathbf{V}$ is not invertible,Then given the normal vector $\mathbf{n}$ the point of contact to is given by the matrix equation

$$\begin{pmatrix} (\mathbf{u} + k_2\mathbf{n}_2)^T \\ \mathbf{V} \end{pmatrix} \mathbf{q}_2 = \begin{pmatrix} -f \\ k_2\mathbf{n} - \mathbf{u} \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} (\begin{pmatrix} -1 \\ -0.5 \end{pmatrix} + \frac{3}{2}\begin{pmatrix} 2 \\ -1 \end{pmatrix})^T \\ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \mathbf{q}_2 = \begin{pmatrix} -7 \\ \frac{3}{2}\begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ -0.5 \end{pmatrix} \end{pmatrix}$$

By solving the above equation,we can get the point of contact as

$$\mathbf{q}_2 = \begin{pmatrix} \frac{-5}{6} \\ \frac{217}{36} \end{pmatrix}$$

Tangent Equation is

$$\mathbf{n}_2^T(\mathbf{x} - \mathbf{q}_2) = 0 \tag{6}$$

By substituting the values of $\mathbf{n}_2^T$ & $\mathbf{q}_2$ in Eq(6),Then

$$12x + 36y - 227 = 0 \tag{7}$$

2 Diagram

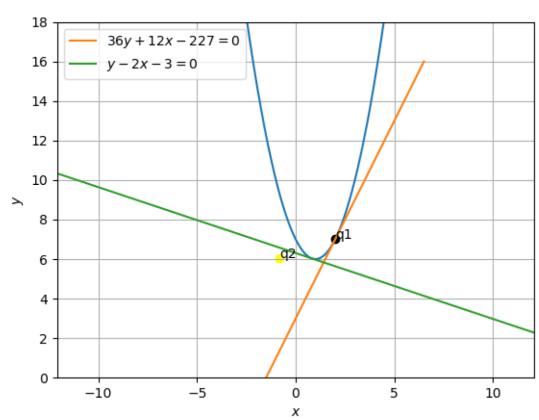

Figure 1: Parabola