Circle Assignment

A L U R U A J A Y

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***Problem Statement* -A square is inscribed in the circle x^2+y^2-2x+4y+3=0.Whose sides are parallel to the coordinate axes,Find out the vertexes of the square.**

1 CONSIDERATIONS

The input parameters are the r, a and c.

Symbol	Value	Description
r	r	radius of a circle
a	a	side of a square
c	c	centre of a circle

2 DIAGRAM

Plot of square in a circle is shown in figure 1, where point O is Center and points A, B, C and D are the vertices of Square.

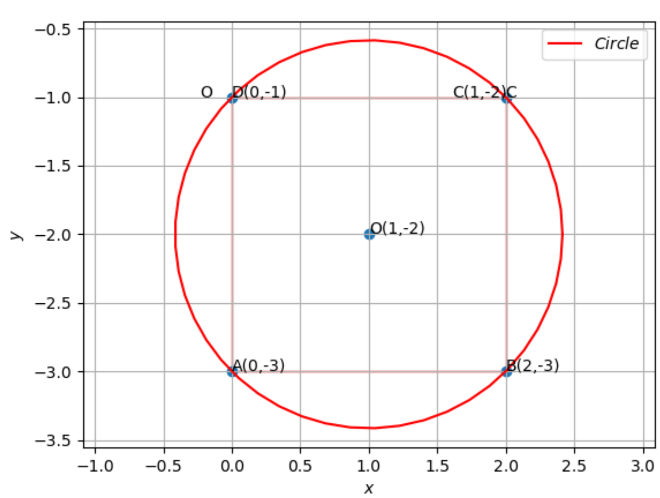

Figure 1: Circle

3 Solution

3.1 Calculation of Centre and Radius of a Circle

Equation of the circle is

$$x^2 + y^2 - 2x + 4y + 3 = 0 \tag{1}$$

The equation of circle in matrix form is,

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2}$$

Where

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, f = 3$$

We know that

radius (r)= $\sqrt{\mathbf{u}^T\mathbf{u} - f}$

centre(O)=$-\mathbf{u}$

3.2 Finding Co-Ordinate C

let The parametric equation of a line is given by

$$\boldsymbol{x} = \mathbf{e} + \lambda_1(\mathbf{m}_1) \tag{3}$$

Point on the $||\mathbf{B} - \mathbf{C}|| = \mathbf{e} = 1 \times \mathbf{e}_1 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

Direction Vector of a line $||\mathbf{B} - \mathbf{C}|| = \mathbf{m}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

from eq(3) The parametric equation of a line can be written as

$$\boldsymbol{x} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

let The parametric equation of a line is given by

$$\boldsymbol{x} = \mathbf{f} + \lambda_2(\mathbf{m}_2) \tag{5}$$

Point on the $||\mathbf{C} - \mathbf{D}|| = \mathbf{f} = 1 \times \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Direction Vector of a line $||\mathbf{C} - \mathbf{D}|| = \mathbf{m}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

from eq(5) The parametric equation of a line can be written as

$$\boldsymbol{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{6}$$

By solving eq(4) & eq(6),we get

$$\begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix} = \begin{pmatrix}1\\ 1\end{pmatrix}$$

$\lambda_1 = 1$ & $\lambda_2 = 1$

Sub the value of $\lambda_1$in eq(4),then we get the Co-ordinate of C

$$\text{C} = \begin{pmatrix}2\\ -2\end{pmatrix} + 1\begin{pmatrix}0\\ 1\end{pmatrix}$$

$$\text{C} = \begin{pmatrix}2\\ -1\end{pmatrix}$$

3.3 Finding Co-Ordinate A

let The parametric equation of a line is given by

$$\boldsymbol{x} = \mathbf{g} + \lambda_3(\mathbf{m}_1) \tag{7}$$

Point on the $||\mathbf{A} - \mathbf{D}|| = \mathbf{g} = 1 \times \mathbf{e}_1 = \begin{pmatrix}0\\ -2\end{pmatrix}$

Direction Vector of a line $||\mathbf{A} - \mathbf{D}|| = \mathbf{m}_1 = \begin{pmatrix}0\\ 1\end{pmatrix}$

from eq(7) The parametric equation of a line can be written as

$$\boldsymbol{x} = \begin{pmatrix}0\\ -2\end{pmatrix} + \lambda_3\begin{pmatrix}0\\ 1\end{pmatrix} \tag{8}$$

let The parametric equation of a line is given by

$$\boldsymbol{x} = \mathbf{h} + \lambda_4(\mathbf{m}_2) \tag{9}$$

Point on the $||\mathbf{C} - \mathbf{D}|| = \mathbf{h} = 1 \times \mathbf{e}_2 = \begin{pmatrix}1\\ -3\end{pmatrix}$

Direction Vector of a line $||\mathbf{C} - \mathbf{D}|| = \mathbf{m}_2 = \begin{pmatrix}1\\ 0\end{pmatrix}$

from eq(9) The parametric equation of a line can be written as

$$\boldsymbol{x} = \begin{pmatrix}1\\ -3\end{pmatrix} + \lambda_4\begin{pmatrix}1\\ 0\end{pmatrix} \tag{10}$$

By solving eq(8) & eq(10),we get

$$\begin{pmatrix}\lambda_3\\ \lambda_4\end{pmatrix} = \begin{pmatrix}-1\\ -1\end{pmatrix}$$

$\lambda_3 = -1$ & $\lambda_4 = -1$

Sub the value of $\lambda_1$in eq(8),then we get the Co-ordinate of A

$$\text{A} = \begin{pmatrix}0\\ -2\end{pmatrix} + \text{-1}\begin{pmatrix}0\\ 1\end{pmatrix}$$

$$\text{A} = \begin{pmatrix}0\\ -3\end{pmatrix}$$

3.4 Finding Co-Ordinate B

Let The parametric equation of a line is given by

$$\boldsymbol{x} = \mathbf{h} + \lambda_4(\mathbf{m}_2) \tag{11}$$

Point on the $||\mathbf{C} - \mathbf{D}|| = \mathbf{h} = 1 \times \mathbf{e}_2 = \begin{pmatrix}1\\ -3\end{pmatrix}$

Direction Vector of a line $||\mathbf{C} - \mathbf{D}|| = \mathbf{m}_2 = \begin{pmatrix}1\\ 0\end{pmatrix}$

from eq(11) The parametric equation of a line can be written as

$$\boldsymbol{x} = \begin{pmatrix}1\\ -3\end{pmatrix} + \lambda_4\begin{pmatrix}1\\ 0\end{pmatrix} \tag{12}$$

t The parametric equation of a line is given by

$$\boldsymbol{x} = \mathbf{e} + \lambda_1(\mathbf{m}_1) \tag{13}$$

Point on the $||\mathbf{B} - \mathbf{C}|| = \mathbf{e} = 1 \times \mathbf{e}_1 = \begin{pmatrix}2\\ -2\end{pmatrix}$

Direction Vector of a line $||\mathbf{B} - \mathbf{C}|| = \mathbf{m}_1 = \begin{pmatrix}0\\ 1\end{pmatrix}$

from eq(13) The parametric equation of a line can be written as

$$\boldsymbol{x} = \begin{pmatrix}-1\\ 2\end{pmatrix} + \lambda_1\begin{pmatrix}0\\ 1\end{pmatrix} \tag{14}$$

By solving eq(12) & eq(14),we get

$$\begin{pmatrix}\lambda_4\\ \lambda_1\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

$\lambda_4 = 1$ & $\lambda_1 = -1$

Sub the value of $\lambda_1$in eq(12),then we get the Co-ordinate of B

$$\text{B} = \begin{pmatrix}1\\ -3\end{pmatrix} + 1\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$\text{B} = \begin{pmatrix}2\\ -3\end{pmatrix}$$

3.5 Finding Co-Ordinate D

let The parametric equation of a line is given by

$$\boldsymbol{x} = \mathbf{g} + \lambda_3(\mathbf{m}_1) \tag{15}$$

Point on the $||\mathbf{A} - \mathbf{D}|| = \mathbf{g} = 1 \times \mathbf{e}_1 = \begin{pmatrix}0\\ -2\end{pmatrix}$

Direction Vector of a line $||\mathbf{A} - \mathbf{D}|| = \mathbf{m}_1 = \begin{pmatrix}0\\ 1\end{pmatrix}$

from eq(15) The parametric equation of a line can be written as

$$\boldsymbol{x} = \begin{pmatrix}0\\ -2\end{pmatrix} + \lambda_3\begin{pmatrix}0\\ 1\end{pmatrix} \tag{16}$$

let The parametric equation of a line is given by

$$\boldsymbol{x} = \mathbf{f} + \lambda_2(\mathbf{m}_2) \tag{17}$$

Point on the $||\mathbf{C} - \mathbf{D}|| = \mathbf{f} = 1 \times \mathbf{e}_2 = \begin{pmatrix}0\\ 1\end{pmatrix}$

Direction Vector of a line $||\mathbf{C} - \mathbf{D}|| = \mathbf{m}_2 = \begin{pmatrix}0\\ 1\end{pmatrix}$

from eq(17) The parametric equation of a line can be written as

$$\boldsymbol{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{18}$$

By solving eq(4) & eq(6),we get

$\begin{pmatrix} \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\lambda_2 = -1$ & $\lambda_3 = 1$

Sub the value of $\lambda_1$in eq(4),then we get the Co-ordinate of C

D $= \begin{pmatrix} 0 \\ -2 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

D $= \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

3.6 Conclusion

The Vertexes of square are

A $= \begin{pmatrix} 0 \\ -3 \end{pmatrix}$ B $= \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ C $= \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ D $= \begin{pmatrix} 0 \\ -1 \end{pmatrix}$