circle Assignment

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Problem Statement **-If the tangent at the point P on the circle $x^2+y^2+6x+6y=2$ meets a straight line $5x-2y+6=0$ at a point Q on the y-axis then the length of PQ is**

Solution

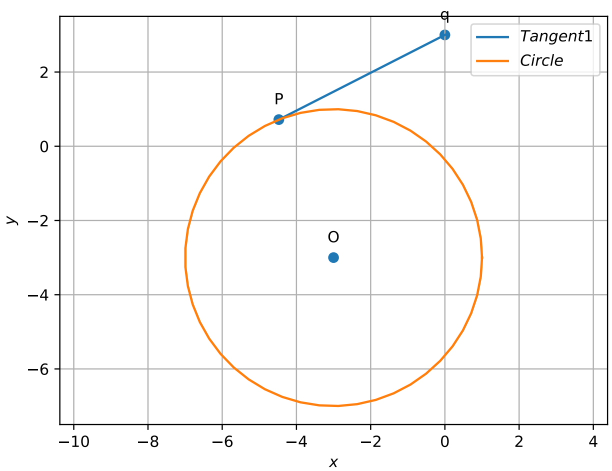

Figure 1:

1 construction

Point	Value	Description
q	(0,3)	given point
o	(-3,-3)	centre of circle

The line equation is

$$5x-2y+6=0 \tag{1}$$

for point Q

$$\begin{pmatrix}5 & -2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=-6 \tag{2}$$

$$\mathbf{n^T}\mathbf{x}=c \tag{3}$$

$$\begin{pmatrix}5 & -2\end{pmatrix}\begin{pmatrix}0\\k\end{pmatrix}=-6 \tag{4}$$

-2k=-6
k=3

$$\mathbf{Q}=\begin{pmatrix}0\\3\end{pmatrix} \tag{5}$$

Given circle equation is

$$x^2+y^2+6x+6y-2=0 \tag{6}$$

$$\mathbf{x}^T\mathbf{V}\mathbf{x}+2\mathbf{U}^T\mathbf{x}+f=0 \tag{7}$$

$$V=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\ u=\begin{pmatrix}3\\3\end{pmatrix}\ f=-2 \tag{8}$$

If $\mathbf{V}^{-1}$ exists, given normal vector $\mathbf{n}$, the tangent points of contact to equation(2) are given by,

$$\mathbf{q_i}=\mathbf{V}^{-1}(k_i\mathbf{n}-\mathbf{u})^T \tag{9}$$

$$where, k_i=\pm\sqrt{\frac{f_0}{\mathbf{n}^T\mathbf{V}^{-1}\mathbf{n}}} \tag{10}$$

$$f_0=\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f \tag{11}$$

$$\tag{12}$$

The normal vectors of tangents from a point $\mathbf{h}$ to the conic(7) are given by'

$$\mathbf{n_1}=\mathbf{P}\begin{pmatrix}\sqrt{\lambda_1}\\\sqrt{\lambda_2}\end{pmatrix},\mathbf{n_2}=\mathbf{P}\begin{pmatrix}\sqrt{\lambda_1}\\-\sqrt{\lambda_2}\end{pmatrix} \tag{13}$$

where $\lambda_i,\mathbf{P}$ are eigen parameters of

$$\sum=(\mathbf{Vh}+\mathbf{u})(\mathbf{Vh}+\mathbf{u})^T-\mathbf{V}(\mathbf{h}^T\mathbf{Vh}+2\mathbf{u}^Th+f) \tag{14}$$

So, by solving above equation,

$$\mathbf{n_1}=\begin{pmatrix}-6.4721\\-1.7639\end{pmatrix},\mathbf{n_2}=\begin{pmatrix}-2.472\\6.236\end{pmatrix} \tag{15}$$

By solving equation(10), using (8),(13) and (11), we get,

$$k=\pm 0.596 \tag{16}$$

Solving equation(9), using (8),(13) and (16), we get,

$$\mathbf{p} = \begin{pmatrix} -4.474 \\ 0.718 \end{pmatrix} \tag{17}$$

Distance between two points:

$$\|\mathbf{P} - \mathbf{Q}\| \tag{18}$$

Length of **PQ**=5