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Conic Assignment

Roll No. : FWC22048

Problem Statement:

Find the area of the smaller part of the circle $x^2+y^2=a^2$ cut off by the line x=$\frac{a}{\sqrt{2}}$

SOLUTION:

Given:

Equation of Circle is

$$x^2+y^2=a^2 \tag{1}$$

Equation of line is

$$x=\frac{a}{\sqrt{2}} \tag{2}$$

To Find

To find the intersection points and area of shaded region shown in figure

STEP-1

The given circle can be expressed as conics with parameters,

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

$$\mathbf{u}=0 \tag{4}$$

$$f=-a^2 \tag{5}$$

STEP-2

the given line equation can be written as

$$\mathbf{x}=\begin{pmatrix} \frac{a}{\sqrt{2}} \\ 0 \end{pmatrix}+k\begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{6}$$

STEP-3

The points of intersection of the line,

$$L: \quad \mathbf{x}=\mathbf{q}+\kappa\mathbf{m} \quad \kappa\in\mathbb{R} \tag{7}$$

with the conic section,

$$\mathbf{x}^\top\mathbf{V}\mathbf{x}+2\mathbf{u}^\top\mathbf{x}+f=0 \tag{8}$$

are given by

$$\mathbf{x}_i=\mathbf{q}+\kappa_i\mathbf{m} \tag{9}$$

where,

$$\kappa_i=\frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\left(-\mathbf{m}^T(\mathbf{V}\mathbf{q}+\mathbf{u}) \pm\sqrt{\left[\mathbf{m}^T(\mathbf{V}\mathbf{q}+\mathbf{u})\right]^2-\left(\mathbf{q}^T\mathbf{V}\mathbf{q}+2\mathbf{u}^T\mathbf{q}+f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\right) \tag{10}$$

On substituting

$$\mathbf{q}=\begin{pmatrix} \frac{a}{\sqrt{2}} \\ 0 \end{pmatrix} \tag{11}$$

$$\mathbf{m}=\begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{12}$$

With the given circle as in eq(3),(4),(5),
The value of κ ,

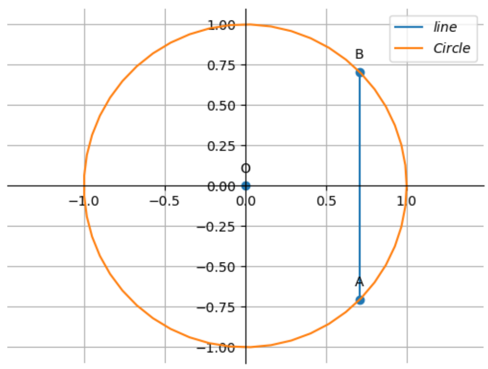
$$\kappa=\frac{a}{\sqrt{2}},\frac{-a}{\sqrt{2}} \tag{13}$$

by substituting eq(13) in eq(6)we get the points of intersection of line with Circle

$$\mathbf{A}=\begin{pmatrix} \frac{a}{\sqrt{2}} \\ \frac{-a}{\sqrt{2}} \end{pmatrix} \tag{14}$$

$$\mathbf{B}=\begin{pmatrix} \frac{a}{\sqrt{2}} \\ \frac{a}{\sqrt{2}} \end{pmatrix} \tag{15}$$

Result



From the figure,
Total area of portion is given by,
area **APQ**=2* area of **APR**

Area of APR

Since **APR** is in first Quadrant

$$y = \sqrt{a^2 - x^2} \tag{16}$$

Area of Circle

$$\implies APQ = 2 * \int_0^{\frac{a}{\sqrt{2}}} \sqrt{a^2 - x^2}\, dx \tag{17}$$

Get the python code of the figures from

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https://github.com/manasa/MANASA_FWC/blob/main/conics/code/conic.py
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by solving the above equation we get area of smaller part of the circle

$\implies APQ = \frac{a^2}{2}[1 + \frac{\pi}{2}]$ **Construction**

Points	**coordinates**
B	$\begin{pmatrix} \frac{a}{\sqrt{2}} \\ \frac{a}{\sqrt{2}} \end{pmatrix}$
A	$\mathbf{A} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ \frac{-a}{\sqrt{2}} \end{pmatrix}$