

Matrix Problems

Conics

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I. PROBLEM STATEMENT

Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

- 1) $2(\pi - 2)$
- 2) $\pi - 2$
- 3) $2\pi - 1$
- 4) $2(\pi + 2)$

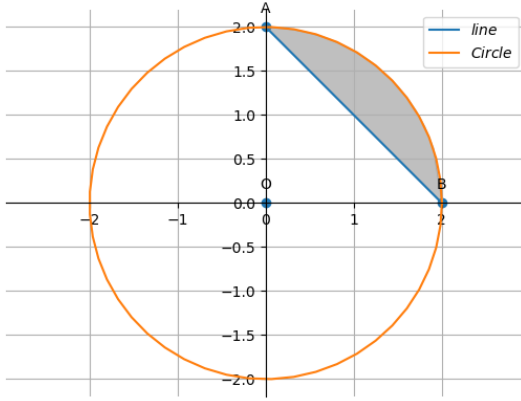


Fig. 1: Smaller region between Circle and Line

Given:

Equation of circle is

$$x^2 + y^2 = 4 \quad (1)$$

Equation of line is

$$x + y = 2 \quad (2)$$

To Find:

To find the intersection points and area of shaded region shown in figure

II. CONSTRUCTION

Points	coordinates
A	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
B	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

III. SOLUTION

The given circle can be expressed as conics with parameters,

$$\mathbf{V} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \quad (3)$$

$$\mathbf{u} = 0 \quad (4)$$

$$f = -16 \quad (5)$$

The given line equation can be written as

$$\mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + k \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \quad (6)$$

The points of intersection of the line,

$$L : \quad \mathbf{x} = \mathbf{q} + \kappa \mathbf{m} \quad \kappa \in \mathbb{R} \quad (7)$$

with the conic section,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (8)$$

are given by

$$\mathbf{x}_i = \mathbf{q} + \kappa_i \mathbf{m} \quad (9)$$

where,

$$\kappa_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u})]^2 - (\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f) (\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) \quad (10)$$

On substituting

$$\mathbf{q} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (11)$$

$$\mathbf{m} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \quad (12)$$

With the given as in eq(3),(4),(5),
The value of κ ,

$$\kappa = 0, -4 \quad (13)$$

By substituting eq(13) in eq(6) we get the points
of intersection of line with circle

$$\mathbf{A} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (14)$$

$$\mathbf{B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (15)$$

From the figure

Total area of portion is given by,

Total Area=(area of circle in first quadrant)-(area
of a triangle **AOB**)

Area of triangle

$$\Rightarrow A_1 = \int_0^2 (2 - x) dx \quad (16)$$

By solving the above equation we get area of
triangle as 2 units

Area of circle

$$\Rightarrow A_2 = \int_0^2 \sqrt{4 - x^2} dx \quad (17)$$

By solving the above equation we get area of
circle π

The total area is $\Rightarrow A = \pi - 2$

Get Python Code for image from

<https://github.com/ManojChavva/FWC/blob/main/Matrix/conics/code/conic.py>

Get LaTeX code from

<https://github.com/ManojChavva/FWC/blob/main/Matrix/conics/conic.tex>