Name: Cheenepalli Chandana

Conic Assignment

Roll No. : FWC22062

Problem Statement:

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

SOLUTION:

Given:

Equation of parabola is

$$4y = 3x^2 \tag{1}$$

Equation of line is

$$2y = 3x + 12 \tag{2}$$

To Find

To find the intersection points and area enclosed by the parabola and line shown in figure

STEP-1

The given parabola can be expressed as conics with parameters,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{3}$$

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \tag{4}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{5}$$

$$f = 0 \tag{6}$$

STEP-2

the given line equation can be written as

$$\mathbf{n}^\top \mathbf{X} = c \tag{7}$$

Where

$$\mathbf{n} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{8}$$

STEP-3

The points of intersection of the line,

$$L: \quad \mathbf{x} = \mathbf{q} + \kappa \mathbf{m} \quad \kappa \in \mathbb{R} \tag{9}$$

with the conic section,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{10}$$

are given by

$$\mathbf{x}_i = \mathbf{q} + \kappa_i \mathbf{m} \tag{11}$$

where,

$$\kappa_i = \frac{1}{\mathbf{m}^T \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T \mathbf{V}\mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f\right)\left(\mathbf{m}^T \mathbf{V}\mathbf{m}\right)}\right) \tag{12}$$

On substituting

$$\mathbf{q} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{13}$$

$$\mathbf{m} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{14}$$

With the given parabola as in eq(3),(4),(5),
The value of κ ,

$$\kappa = -2.5, 2.7 \tag{15}$$

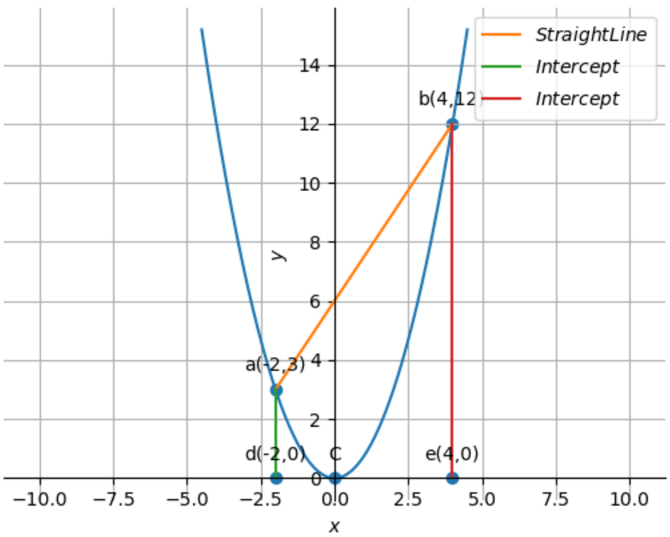
by substituting eq(13) in eq(6)we get the points of intersection of line with parabola

$$\mathbf{A} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{16}$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} \tag{17}$$

Result

by solving the above equation we get area of triangle $45m^2$



From the figure,

Total area of portion is given by,

Total Area=(area enclosed by the line)-(area of parabola under the line)

Area Under the line

$$\implies A1 = \int_{-2}^{4} \frac{3x+12}{2}\, dx \tag{18}$$

Area of enclosed ny the parabola under line

$$\implies A2 = \int_{-2}^{4} \frac{3x^2}{4}\, dx \tag{19}$$

by solving the above equation we get area of parabola under the line $18m^2$

the total area is

$$\implies A = 63m^2$$

The area enclosed by the parabola and line is ,

$$A = 27m^2 \tag{20}$$

Construction

Points	coordinates
B	$\begin{pmatrix} 4 \\ 12 \end{pmatrix}$
A	$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Get the python code of the figures from

```
https://github.com/chandana531/cchandana_fwc/blob/main/conic_assignment/code/conic.py
```