Conics Assignment

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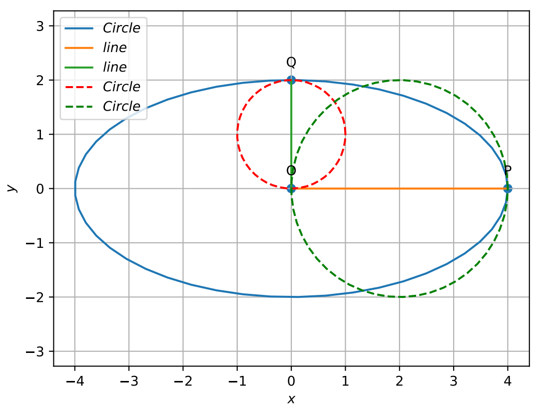
FWC22012 IITH Future Wireless Communication (FWC) ASSIGN-6

1 Problem

An ellipse is drawn by taking a diameter of the circle $(x-1)^2+y^2=1$ as its semi-minor axis and a diameter of the circle $x^2+(y-2)^2=4$ is semi-major axis.If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is?

2 Construction



3 Solution

Conics equation is

$$x^T\mathbf{V}x+2\mathbf{u}^Tx+f=0 \tag{1}$$

To find the lengths of semi-major axis and semi-minor axis, From given circle equation

$x^2+(y-2)^2=4$

$x^2+y^2-4y+4=4$

$x^2+y^2-4y=0.....(i)$

By comparing (i) with (1) we will get

$\mathbf{U1}=\begin{pmatrix}0\\-2\end{pmatrix}$ and f1=0

$$Radius\ (R)=\sqrt{\mathbf{u^T}.\mathbf{u}-f} \tag{2}$$

$$\sqrt{\begin{pmatrix}0 & -2\end{pmatrix}\begin{pmatrix}0\\-2\end{pmatrix}}=2 \tag{3}$$

semi-major axis=2*R=a=4

From given circle equation

$(x-1)^2+(y)^2=1$

$x^2+y^2-2x+1=1$

$x^2+y^2-2x=0.....(ii)$

By comparing (ii) with (1) we will get

$\mathbf{U2}=\begin{pmatrix}-1\\0\end{pmatrix}$ and f2=0

$$Radius\ (R)=\sqrt{\mathbf{u^T}.\mathbf{u}-f} \tag{4}$$

$$\sqrt{\begin{pmatrix}-1 & 0\end{pmatrix}\begin{pmatrix}-1\\0\end{pmatrix}}=1 \tag{5}$$

semi-minor axis=2*R=b=2

For ellipse given that

$$\mathbf{U}=\begin{pmatrix}0\\0\end{pmatrix} \tag{6}$$

From major axes equation of ellipse

$$a=\sqrt{(\mathbf{u^T}.V^-1.\mathbf{u}-f)/\lambda 1} \tag{7}$$

$$a=\sqrt{(-f)/\lambda 1} \tag{8}$$

$$a^2=-f/\lambda 1 \tag{9}$$

$$\therefore \lambda 1=-f/a^2 \tag{10}$$

From minor axes equation of ellipse

$$b=\sqrt{(\mathbf{u^T}.V^-1.\mathbf{u}-f)/\lambda 2} \tag{11}$$

$$b=\sqrt{(-f)/\lambda 2} \tag{12}$$

$$b^2=-f/\lambda 2 \tag{13}$$

$$\therefore \lambda 2 = -f/b^2 \tag{14}$$

$$\therefore \lambda 1 = -f/a^2 \ \ and \ \ \lambda 2 = -f/b^2 \tag{15}$$

$$\mathbf{V} = \begin{pmatrix} \lambda 1 & 0 \\ 0 & \lambda 2 \end{pmatrix} \tag{16}$$

$$\mathbf{V} = \begin{pmatrix} -f/a^2 & 0 \\ 0 & -f/b^2 \end{pmatrix} \tag{17}$$

By substituting (17) in (1) we will get

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} -f/a^2 & 0 \\ 0 & -f/b^2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + f = 0 \tag{18}$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} -f/a^2 & 0 \\ 0 & -f/b^2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -f \tag{19}$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \tag{20}$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1/16 & 0 \\ 0 & 1/4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \tag{21}$$

$\therefore$The equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{4} = 1 \tag{22}$$

4 Execution

Verify the above problem in the following code.

https://github.com/gowripriya-2002/FWC/blob/main/Matrix/conic_assignment/code/conics.py