Circle Assignment

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Symbol	Value	Description
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Center $\mathbf{O}$
θ_1	60°	$\angle Q_1 P Q_2$
r	5	radius of circle

Table 1

Problem: Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

In ΔOqh

$$\boldsymbol{h} = r \csc \frac{\theta}{2} e1 \tag{1}$$

Fig 1. Circle

Solution

The equation of a conic with directrix $\mathbf{n}^\top \mathbf{x} = c$, eccentricity e and focus $\mathbf{F}$ is given by

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

for circle eccentricity e=0 then,

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -r^2. \tag{3}$$

Point q on conic is given by

$$q = \mathbf{V}^{-1}(k_i \mathbf{n} - \mathbf{u}^T)^T$$
$$where,$$
$$k_i = \pm \sqrt{\frac{f_0}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}}$$
$$f_0 = f + \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u}$$
$$n = P \begin{pmatrix} \sqrt{\lambda_1} \\ \pm\sqrt{\lambda_2} \end{pmatrix}$$

$\mathbf{P}, \lambda_{1,2}$ are eigen parameters of

$$\sum = (\mathbf{V}\mathbf{h} + \mathbf{u})(\mathbf{V}\mathbf{h} + \mathbf{u})^T - \mathbf{V}(\mathbf{h}^T \mathbf{V}\mathbf{h} + 2\mathbf{u}^T h + f)$$

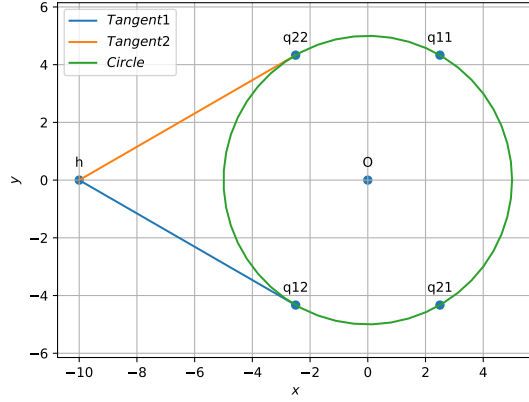## Construction

SOLUTION: The following python code is used for constructing circle with pair of tangents.

```
https://github.com/AnushaJella/
assignment_circle/blob/main/circle1.
py
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See Fig 1 for the input parameters in Table 1.

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