Conic section Assignment

T.Varsha Reddy

September 2022

1 Problem Statement

Find the equation of the tangent to the curve

2 Solution

The given equation of parabola $y^2 = 3x - 2$ can be written in the general quadratic form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} -3/2 \\ 0 \end{pmatrix}, \tag{3}$$

$$f = 2 \tag{4}$$

which is the equation of a parabola. Thus can be expressed as by choosing

$$Ki = \frac{\mathbf{Pi}^T\mathbf{u}}{\mathbf{Pi}^T\mathbf{n}} \tag{5}$$

where

$$\mathbf{pi} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{6}$$

$$\mathbf{n} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \tag{7}$$

If V is non invertible,given the normal vector η,the point of contact is given by the matrix equation.

$$\begin{pmatrix} (\mathbf{u} + \mathbf{kin})^T \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ (\mathbf{Kin} - \mathbf{u}) \end{pmatrix} \qquad |V| = 0 \tag{8}$$

Substituting appropriate values from (2), (3), (4), and into (8), the below matrix equation is obtained

$$\begin{pmatrix} -3 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} -41/16 \\ 0 \\ 3/4 \end{pmatrix} \tag{9}$$

$$\tag{10}$$

The augmented matrix for (9) can be expressed as

$$\xleftrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{cc|c} -3 & 0 & -41/16 \\ 0 & 1 & 0 \\ 0 & 0 & 3/4 \end{array}\right) \tag{11}$$

$$\xleftrightarrow{-\frac{R_1}{-3} \leftarrow R_2} \left(\begin{array}{cc|c} 1 & 0 & 41/48 \\ 0 & 1 & 0 \\ 0 & 0 & 3/4 \end{array}\right) \tag{12}$$

$$\implies \mathbf{q} = \begin{pmatrix} 41/48 \\ 3/4 \end{pmatrix} \tag{13}$$

By the equation of tangent is

$$(\mathbf{V}\mathbf{q} + \mathbf{u})^\top \mathbf{X} + \mathbf{u}^\top \mathbf{q} + \mathbf{f} = 0 \tag{14}$$

3 Construction