# Circle Assignment

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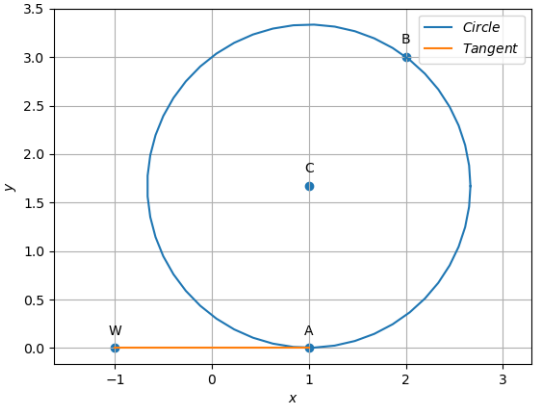
FWC22012 IITH Future Wireless Communication (FWC) ASSIGN-5

## 1 Problem

The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3) is?

## 2 Construction




## 3 Solution

The equation of the circle is

$$x^T\mathbf{V}x + 2\mathbf{u}^T x + f = 0 \tag{1}$$

Circle passes through $\begin{pmatrix}2\\3\end{pmatrix}$ and touches the x-axis at $\begin{pmatrix}1\\0\end{pmatrix}$

Let $\mathbf{A}=\begin{pmatrix}1\\0\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}2\\3\end{pmatrix}$ and $\mathbf{m}=\begin{pmatrix}1\\0\end{pmatrix}$

$$\mathbf{A}\mathbf{A}^T + 2\mathbf{u}^T\mathbf{A} + f = 0 \tag{2}$$

$$\|\mathbf{A}\|^2 + 2\mathbf{A}^T\mathbf{u} + f = 0 \tag{3}$$

$$\begin{pmatrix}2\mathbf{A}^T & 1\end{pmatrix}\begin{pmatrix}\mathbf{u}\\f\end{pmatrix} = -\|A\|^2 \tag{4}$$

$$\mathbf{B}\mathbf{B}^T + 2\mathbf{u}^T\mathbf{B} + f = 0 \tag{5}$$

$$\|\mathbf{B}\|^2 + 2\mathbf{B}^T\mathbf{u} + f = 0 \tag{6}$$

$$\begin{pmatrix}2\mathbf{B}^T & 1\end{pmatrix}\begin{pmatrix}\mathbf{u}\\f\end{pmatrix} = -\|\mathbf{B}\|^2 \tag{7}$$

The equation of the tangent is

$$\mathbf{m}^T(\mathbf{V}q + \mathbf{u}) = 0 \tag{8}$$

$$\mathbf{m}^T\mathbf{A} + \mathbf{m}^T\mathbf{u} = 0 \tag{9}$$

$$\mathbf{m}^T\mathbf{u} = -\mathbf{m}^T\mathbf{A} \tag{10}$$

from equations (4),(7) and (10),we can write as

$$\begin{pmatrix}\mathbf{m}^T & 0\\2\mathbf{A}^T & 1\\2\mathbf{B}^T & 1\end{pmatrix}\begin{pmatrix}\mathbf{u}\\f\end{pmatrix} = \begin{pmatrix}-\mathbf{m}^T\mathbf{A}\\-\|\mathbf{A}\|^2\\-\|\mathbf{B}\|^2\end{pmatrix} \tag{11}$$

$$\begin{pmatrix}1&0&0&-1\\2&0&1&-1\\4&6&1&-13\end{pmatrix} \xrightarrow{R_2\leftarrow R3}$$

$$\begin{pmatrix}1&0&0&-1\\4&6&1&-13\\2&0&1&-1\end{pmatrix} \xrightarrow{R_2\leftarrow R2/6}$$

$$\begin{pmatrix}1&0&0&-1\\4/6&1&1/6&-13/6\\2&0&1&-1\end{pmatrix} \xrightarrow{R_3\leftarrow R_3-2R_1}$$

$$\begin{pmatrix}1&0&0&-1\\4/6&1&1/6&-13/6\\0&0&1&1\end{pmatrix} \xrightarrow{R_2\leftarrow R_2-4/6R_1}$$

$$\begin{pmatrix}1&0&0&-1\\0&1&1/6&-9/6\\0&0&1&1\end{pmatrix} \xrightarrow{R_2\leftarrow R_2-1/6R_3}$$

$$\begin{pmatrix}1&0&0&-1\\0&1&0&-10/6\\0&0&1&1\end{pmatrix}$$

By solving the above equations

$$\mathbf{u} = \begin{pmatrix}-1\\-10/6\end{pmatrix} = \begin{pmatrix}-1\\-5/3\end{pmatrix}$$

The center is $\mathbf{C}$=-$\mathbf{u}$

$$\therefore \mathbf{C} = \begin{pmatrix}1\\5/3\end{pmatrix} \text{ and } f = 1$$

$$Radius\ (R) = \sqrt{\mathbf{u^T}.\mathbf{u} - f} \tag{12}$$

$$\sqrt{\begin{pmatrix}-1 & -5/3\end{pmatrix}\begin{pmatrix}-1\\-5/3\end{pmatrix} - 1} = 5/3 \tag{13}$$

$\therefore$ R=5/3=1.67

D=2*R

$\therefore$ Diameter (D)=3.34

## 4 Execution

Verify the above problem in the following code.

https://github.com/gowripriya-2002/FWC/blob/main/Matrix