MATRICES USING PYTHON

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1 Problem

A circle touches the line y=x at a point P such that OP=4$\sqrt{2}$, where O is the origin. The circle contains the point (-10,2) in its interior and the length of its chord on the line x+y=0 is 6$\sqrt{2}$

2 Figure

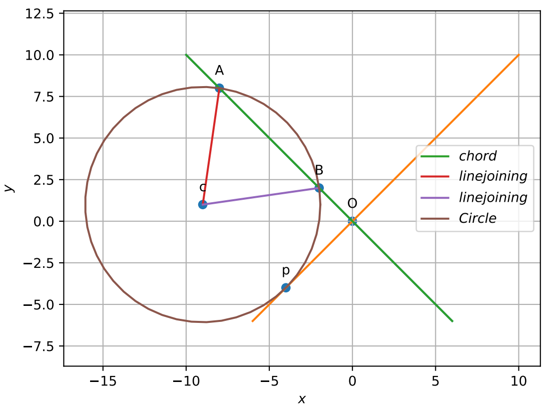

Figure of construction

3 Solution

Let c(α,β) be the center of the circle touching OP at P and making intercept AB = 6$\sqrt{2}$ on the line x+y=0.
If r is the radius of the circle then

In ΔACL

$$r^2 = \|\mathbf{A}-\mathbf{C}\|^2 = \|\mathbf{C}-\mathbf{L}\|^2 + \|\mathbf{L}-\mathbf{A}\|^2 ....(1)$$

$$\|\mathbf{C}-\mathbf{L}\|^2 = \frac{\mathbf{n^T P} - \mathbf{C}}{\|\mathbf{n}\|}$$

$$\|\mathbf{L}-\mathbf{A}\| = 3\sqrt{2}$$

n $=\begin{pmatrix}1\\1\end{pmatrix}$, $n^T = \begin{pmatrix}1 & 1\end{pmatrix}$, P $=\begin{pmatrix}\alpha\\\beta\end{pmatrix}$, $\|\mathbf{n}\| = \sqrt{2}$,c $= 0$
Given
$\|\mathbf{A}-\mathbf{L}\|^2 = \|\mathbf{3\sqrt{2}}\|$
substitute all above equation in (1)

In ΔOCP
r=5$\sqrt{2}$

$$\|\mathbf{O}-\mathbf{C}\|^2 = \|\mathbf{C}-\mathbf{P}\|^2 + \|\mathbf{P}-\mathbf{O}\|^2 ....(2)$$

Given
$\|\mathbf{O}-\mathbf{C}\|^2 = \alpha^2 + \beta^2$,
$\|\mathbf{C}-\mathbf{p}\|^2 = r^2$,
$\|\mathbf{P}-\mathbf{O}\| = 4\sqrt{2}$
Solve equation (1) ,(2)

$$\alpha - \beta = +/- 10....(3)$$

Distance from center to x-y=0 line

$$\|\mathbf{C}-\mathbf{P}\|^2 = \frac{\mathbf{n^T P} - \mathbf{C}}{\|\mathbf{n}\|}$$

$$r^2 = \frac{(\alpha-\beta)^2}{2} ....(4)$$

$n^T = \begin{pmatrix}1 & -1\end{pmatrix}$, P $=\begin{pmatrix}\alpha\\\beta\end{pmatrix}$, $\|\mathbf{n}\| = \sqrt{2}$,c $= 0$
Solve equation (3),(4)

then r=5$\sqrt{2}$....(5)
Substitute equation (5) in (1)
we get
α = -9 , β = 1
α = 9 , β = -1
α = 1 , β = -9
α = -1 , β = 9
The standard equation of the conics is given as : we choose α and β values from which the line passes through the point (-10,2)
so, α= -9 and β=1

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (1)$$

Where $\mathbf{Vx} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{u}$=$\begin{pmatrix} -9 \\ 1 \end{pmatrix}$, f = 32.

4 Construction

We considered midpoint as L on the chord AB, from that we found the distance between the center to L.And also given OP length,Again from centre to OP finding the distance.

$$\|\mathbf{C}-\mathbf{P}\|^{\mathbf{2}} = \frac{\mathbf{n^T P - C}}{\|\mathbf{n}\|}$$

$$\|\mathbf{C}-\mathbf{L}\|^{\mathbf{2}} = \frac{\mathbf{n^T P - C}}{\|\mathbf{n}\|}$$

from the above 2 equations we get the centre and radius of the circle. $\alpha =$ -9 , $\beta = 1$
r=$5\sqrt{2}$