CIRCLE ASSIGNMENT

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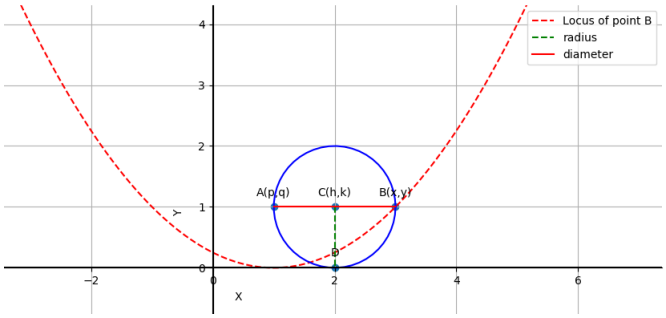
FWC22097 - IITH Future Wireless Communication (FWC)

1 Question

Q(6), C , Section-A, Chapter-8: **A variable circle passes through the fixed point A(p,q) and touches the x-axis. The locus of the other end of the diameter through A is**

2 Solution



Given the circle passes through the point

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}$$

Let the other end of the diameter through A be

$$\mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix}$$

and the centre be $\mathbf{C}$.

By using section formula, we can find the centre C as

$$\mathbf{C} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) \tag{1}$$

$$\mathbf{C} = \frac{1}{2}\begin{pmatrix} p+x \\ q+y \end{pmatrix}$$

We know that AB is the diameter of the circle.And from figure we can find the radius as $r = \frac{q+y}{2}$.

We can write

$$||\mathbf{B} - \mathbf{A}|| = 2r \tag{2}$$

Squaring on both sides, we get

$$||\mathbf{B} - \mathbf{A}||^2 = 4r^2 \tag{3}$$

$$||\mathbf{B} - \mathbf{A}||^2 = ||\mathbf{A}||^2 + ||\mathbf{B}||^2 - 2\mathbf{B}^\top\mathbf{A} \tag{4}$$

substituting equation (4), in equation (3)

$$||\mathbf{A}||^2 + ||\mathbf{B}||^2 - 2\mathbf{B}^\top\mathbf{A} = 4r^2 \tag{5}$$

By solving the above equation we get,

$$\mathbf{A}^\top\mathbf{A} + \mathbf{B}^\top\mathbf{B} - 2\mathbf{B}^\top\mathbf{A} = 4r^2 \tag{6}$$

$$\mathbf{A}^\top\mathbf{A} + \mathbf{B}^\top\mathbf{B} - 2\mathbf{B}^\top\mathbf{A} - 4r^2 = 0 \tag{7}$$

Equation (7) is the required equation,which is the equation of parabola

$$\mathbf{x}^\top\mathbf{V}\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0$$

where,

$$\mathbf{V} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{u} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$f = p^2$$

Get the python code of the figures from

https://github.com/PanjugalaShashikala/FWC_2022097/tree/main/Circle/code/circle.py