

Line Assignment

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Problem Statement - ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig). AC is a diagonal. Show that :

1. $SR \parallel AC$ and $SR = 1/2 (AC)$
2. $PQ = SR$
3. PQRS is a parallelogram.

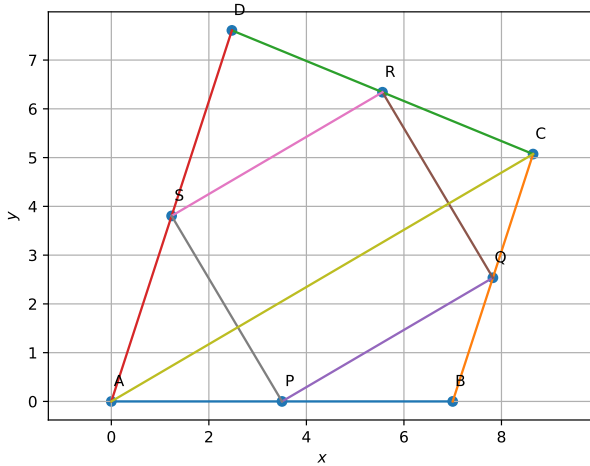


Figure 1: Figure

Solution

Given : ABCD is a Quadrilateral P,Q,R and S are the midpoints of line AB,BC,CD,DA. We can obtain the points P,Q,R and S from A,B,C and D and are given by

$$\begin{aligned} P &= \frac{(A+B)}{2} \\ Q &= \frac{(C+B)}{2} \\ R &= \frac{(C+D)}{2} \\ S &= \frac{(D+A)}{2} \end{aligned}$$

(3) To prove that PQRS is a parallelogram we need to prove $PQ \parallel SR$ To prove $SR \parallel PQ$

Direction vector of line SR $(R - S) = \frac{(C-A)}{2}$

Direction vector of line PQ $(Q - P) = \frac{(C-A)}{2}$

$$(R - S) = (Q - P) = \frac{(C - A)}{2} \quad (1)$$

Since the direction vectors of line SR and PQ are in same direction

$SR \parallel PQ$

Therefore, **PQRS** is a parallelogram

(1) Directional vector of line SR $= (R - S) = \frac{(C-A)}{2}$
Directional vector of line AC $= (C - A)$

It is observed that the constant k is $\frac{1}{2}$
Therefore

$$SR \parallel AC \quad (2)$$

and from equation 1

$$SR = \frac{1}{2} AC \quad (3)$$

(2) To prove $PQ = SR$
From equation 1

$$(Q - P) = (R - S) = \frac{(C - A)}{2} \quad (4)$$

1 Execution

The below python code realizes the construction:

https://github.com/bhavani360/FWC_assignments

Construction

The dimensions of the Quadrilateral ABCD are taken as below

symbol	value
r	8
θ	pi/2.5
d	7
A	(0,0)
B	(d,0)
D	(rcos θ ,rsin θ)
C	(D/1.5)+B