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Assignment-4

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Problem Statement:

The slope of a line is double of the slope of another line. If tangent of the angle between them is 1/3, find the slopes of the lines.

SOLUTION:

Given:

Slope of one line is double of the slope of the other line.

The direction vector of a line is expressed as

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{1}$$

where m is defined to be the slope of the line.

Also given that the tangent of the angle between them is $\frac{1}{3}$.

$$\tan\theta = \frac{1}{3} \tag{2}$$

(3)

$$\implies \cos\theta = \frac{3}{\sqrt{10}} \tag{4}$$

Input Parameters:

Symbol	Value	Description
$\mathbf{m_1}$	$\begin{pmatrix} 1 \\ m \end{pmatrix}$	Direction vector
$\mathbf{m_2}$	$\begin{pmatrix} 1 \\ 2m \end{pmatrix}$	Direction vector
$\tan\theta$	1/3	Angle

The angle between two vectors is expressed as

$$\cos\theta = \frac{\mathbf{A}^\top \mathbf{B}}{\|\mathbf{A}\| \, \|\mathbf{B}\|} \tag{5}$$

$$\frac{3}{\sqrt{10}} = \frac{\mathbf{m_1}^\top \mathbf{m_2}}{\|\mathbf{m_1}\| \, \|\mathbf{m_2}\|} \tag{6}$$

(7)

$$\frac{3}{\sqrt{10}} = \frac{\begin{pmatrix} 1 & m \end{pmatrix} \begin{pmatrix} 1 \\ 2m \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ m \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ 2m \end{pmatrix} \right\|} \tag{8}$$

(9)

$$\frac{3}{\sqrt{10}} = \frac{2m^2+1}{\sqrt{m^2+1}\sqrt{4m^2+1}} \tag{10}$$

Squaring on both sides,

$$\frac{9}{10} = \frac{4m^4+4m^2+1}{4m^4+5m^2+1} \tag{11}$$

(12)

$$4m^4 - 5m^2 + 1 = 0 \tag{13}$$

Let m^2 = x and substituting it in above equation we get a quadratic equation.

$$4x^2 - 5x + 1 = 0 \tag{14}$$

From the formula of fining roots of a quadratic equation

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} \tag{15}$$

(16)

$$\frac{5 \pm \sqrt{(-5)^2-16}}{8} \tag{17}$$

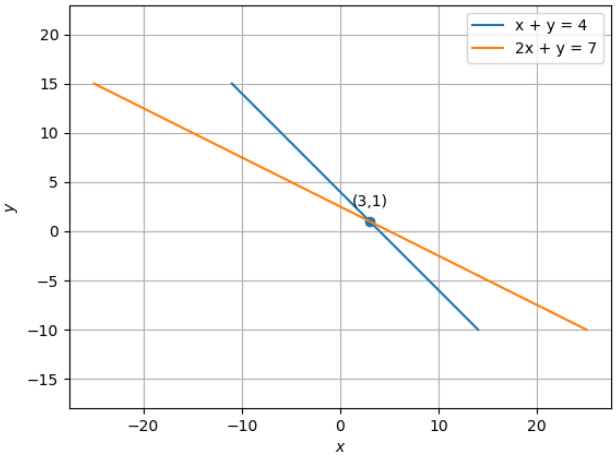
$$x = 1 (or) x = 1/4 \tag{18}$$

The slope of the first line is

$$\therefore m = \pm\tfrac{1}{2}$$
$$(\text{or})$$
$$m = \pm 1$$

$\therefore$ Slope of second line is

$$2m = \pm 1$$
(or)
$$2m = \pm 2$$



Download the code
Github link:Assignment-4.