

# Line Assignment

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October 18, 2022

## Problem Statement:

The straight line  $5x + 4y = 0$  passes through the point of intersection of the straight lines  $x + 2y - 10 = 0$  and  $2x + y + 5 = 0$ . Find the point of intersection of 2 and 3 lines. and then passes 1st equation in 3rd equation.

## Solution:

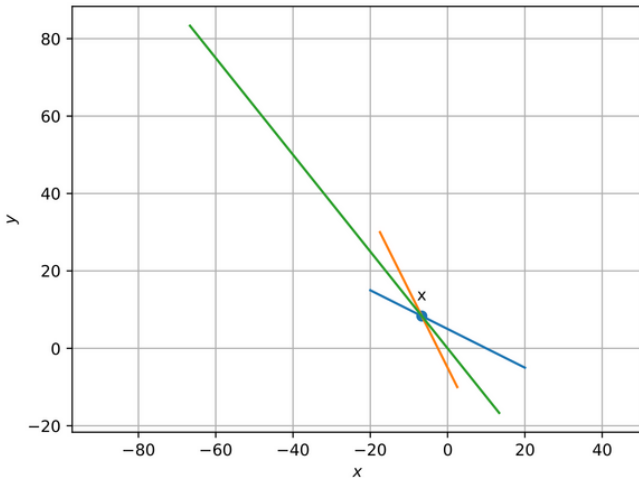


Figure 1: Diagram generated using python

### 0.1 Theory:

They given three lines one line i.e)  $5x + 4y = 0$  passes through the point of intersection of the straight lines  $x + 2y - 10 = 0$  and  $2x + y + 5 = 0$ .

### 0.2 Mathematical Calculation:

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Given line equations are

$$x + 2y = 10$$

$$2x + y = -5 \quad (2)$$

The parametric equation of a line is given by

$$\mathbf{n}_1^\top \mathbf{x} = \mathbf{c}_1 \quad (3)$$

$$\mathbf{n}_2^\top \mathbf{x} = \mathbf{c}_2 \quad (4)$$

$$\mathbf{n}_3^\top \mathbf{x} = \mathbf{c}_3 \quad (5)$$

$$\begin{pmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n}_3 \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{pmatrix} \quad (6)$$

The vector equation of the line  $x + 2y - 10 = 0$  is

$$\mathbf{n}_1^\top \mathbf{x} = \mathbf{c}_1 \quad (7)$$

$$(1 \ 2) (\mathbf{X}) = 10 \quad (8)$$

The vector equation of the line  $2x + y + 5 = 0$  is

$$\mathbf{n}_2^\top \mathbf{x} = \mathbf{c}_2 \quad (9)$$

$$(2 \ 1) (\mathbf{X}) = -5 \quad (10)$$

The vector equation of the line  $5x + 4y = 0$  is

$$\mathbf{n}_3^\top \mathbf{x} = \mathbf{c}_3 \quad (11)$$

$$(5 \ 4) (\mathbf{X}) = 0 \quad (12)$$

The above equations can be written in matrix form

(1) as,

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 5 & 4 \end{pmatrix} (\mathbf{X}) = \begin{pmatrix} 10 \\ -5 \\ 0 \end{pmatrix}$$

The augmented matrix can be expressed as,

$$\begin{pmatrix} 1 & 2 & 10 \\ 2 & 1 & -5 \\ 5 & 4 & 0 \end{pmatrix}$$

Through pivoting, the augmented matrix will become as,

$$\left( \begin{array}{cc|c} 1 & 2 & 10 \\ 2 & 1 & -5 \\ 5 & 4 & 0 \end{array} \right)$$

$$\boxed{R_2 \leftarrow R_2 - 2R_1} \left( \begin{array}{cc|c} 1 & 2 & 15 \\ 0 & -3 & 20 \\ 5 & 4 & 0 \end{array} \right) \quad (13)$$

$$\boxed{R_1 \leftarrow 2R_1 - R_3} \left( \begin{array}{cc|c} 1 & 2 & 10 \\ 0 & -3 & -25 \\ 5 & 4 & 0 \end{array} \right) \quad (14)$$

$$\boxed{R_2 \leftarrow R_2 - 2R_1} \left( \begin{array}{cc|c} 1 & 2 & 15 \\ 0 & -3 & 20 \\ 5 & 4 & 0 \end{array} \right) \quad (15)$$

$$\boxed{R_3 \leftarrow 3R_3 + 5R_1} \left( \begin{array}{cc|c} -3 & 0 & 20 \\ 0 & -3 & -25 \\ 0 & 12 & 0 \end{array} \right) \quad (16)$$

$$\boxed{R_3 \leftarrow 4R_2 + R_3} \left( \begin{array}{cc|c} -3 & 0 & 15 \\ 0 & -3 & 20 \\ 0 & 0 & 0 \end{array} \right) \quad (17)$$

On solving above equation the crossing point of the given equations will be,

$$\mathbf{X} = \begin{pmatrix} -20/3 \\ 25/3 \\ 0 \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} -6.8 \\ 8.3 \\ 0 \end{pmatrix}$$

hence the straight are passing through the concentric

## 1 Construction:

The construction of given lines can be done only the x and y of given equations

variable	point	Description
X	(x/y)	point of intersection
line1	$n_1 = (1, 2)c_1 = (-10)$	-6,8
line2	$n_2(2, 1)c_2 = (5)$	-6,8
line3	$n_3 = (5, 4)c_3 = (0)$	-6,8