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Assignment-5

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Problem Statement:

Construct the family of circles with fixed radius 5 units and center on the line $y=2$.

SOLUTION:

Given:

Radius of a circle is

$$r = 5$$

Center of circle lies on $y=2$

So,

$$\beta = 2$$

To Find

Constructing the family of circles with different values of α

STEP-1

Let r be the radius of circles which is given $r=5$

Let \mathbf{O} be the center of circle and the coordinates are,

$$\mathbf{O} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad (3)$$

From given, we know that $\beta = 2$

So,

$$\mathbf{O} = \begin{pmatrix} \alpha \\ 2 \end{pmatrix} \quad (4)$$

Let \mathbf{X} be the any point on the circle

$$\mathbf{X} = r \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \quad (5) \text{ If } \alpha = 0,$$

STEP-2

The equation of circle is given by,

$$\|\mathbf{X} - \mathbf{O}\| = r \quad (6)$$

$$\sqrt{(\mathbf{X} - \mathbf{O})^\top (\mathbf{X} - \mathbf{O})} = r$$

Squaring on both the sides

$$(\sqrt{(\mathbf{X} - \mathbf{O})^\top (\mathbf{X} - \mathbf{O})})^2 = r^2$$

$$(\mathbf{X} - \mathbf{O})^\top (\mathbf{X} - \mathbf{O}) = r^2$$

Expanding the above equation,

$$\|\mathbf{X}\|^2 - 2\mathbf{X}^\top \mathbf{O} + \|\mathbf{O}\|^2 = r^2 \quad (7)$$

(1) STEP-3

Let θ be any angle from 0 to 2π

$$\theta \in [0, 2\pi) \quad (8)$$

(2) Let us assume,

$$\theta = \frac{2\pi}{3} \quad (9)$$

Using equation (5) any point on circle $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ is,

Here ,

$$x = r \cos\theta \quad (10)$$

$$y = r \sin\theta \quad (11)$$

$$\mathbf{X} = \begin{pmatrix} -2.5 \\ 4.3301 \end{pmatrix} \quad (12)$$

(4) Let α be any values ranging from 0 to 10 with the incrementation of +2

So,

$$\alpha = 0, 2, 4, 6, 8 \quad (13)$$

(5) If $\alpha = 0$,

$$\mathbf{O} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (14)$$

when $\alpha = 2$ and so on till $\alpha = 8$,

$$\mathbf{O} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} \quad (15)$$

Construction

vertex	coordinates
O	$\begin{pmatrix} \alpha \\ 2 \end{pmatrix}$
X	$\begin{pmatrix} x \\ y \end{pmatrix}$

Download the code
Github link: Assignment-5.

