

Matrix Problems

Circles

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I. PROBLEM STATEMENT

The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ ($c > 0$) touch each other if:

- 1) $2|a| = c$
- 2) $|a| = c$
- 3) $a = 2c$
- 4) $|a| = 2c$

II. CONSTRUCTION

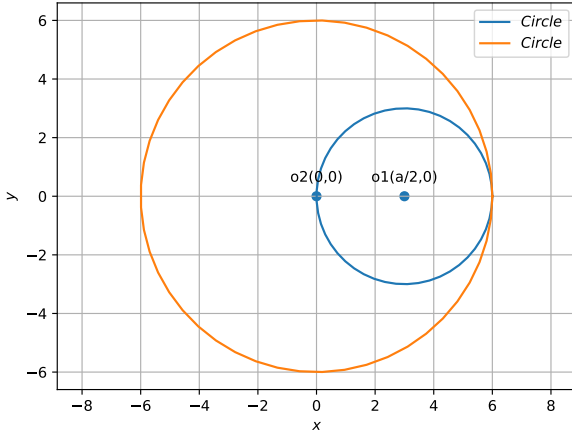


Fig. 1: Figure of construction

III. SOLUTION

Circle equation 1 : $x^2 + y^2 = ax$

Circle equation 2 : $x^2 + y^2 = c^2$

The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (1)$$

The given circle 1 can be expressed as conics with parameters

$$\mathbf{V}_1 = \mathbf{I}, \mathbf{u}_1 = \left(\frac{a}{2} \right), f_1 = 0 \quad (2)$$

Radius and Centre are

$$r_1 = \sqrt{\mathbf{u}^\top \mathbf{u} - f}, \mathbf{a} = -u \quad (3)$$

$$r_1 = \sqrt{\frac{a}{2} * \frac{a}{2}}, \quad (4)$$

$$r_1 = \pm \frac{a}{2} \quad (5)$$

The given circle 2 can be expressed as conics with parameters

$$\mathbf{V}_2 = \mathbf{I}, \mathbf{u}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f_2 = -c^2 \quad (6)$$

Radius and Centre are

$$r_2 = \sqrt{\mathbf{u}^\top \mathbf{u} - f} \quad (7)$$

$$r_2 = \sqrt{0 - (-c^2)} \quad (8)$$

$$r_2 = \pm c \quad (9)$$

Symbol	Value	Description
a	6	Input Circle Radius
\mathbf{u}_1	$\begin{pmatrix} \frac{a}{2} \\ 0 \end{pmatrix}$	Centre of circle 1
\mathbf{u}_2	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Centre of Circle2

Distance between centres u_1 and u_2 is given by

$$\|\mathbf{u}_1 - \mathbf{u}_2\| = \pm \frac{a}{2} \quad (10)$$

The two circles will touch each other iff.

$$r_1 \pm r_2 = \|\mathbf{u}_1 - \mathbf{u}_2\| \quad (11)$$

$$r_1 \pm r_2 = \pm \frac{a}{2} \quad (12)$$

$$\pm \frac{a}{2} + c = \pm \frac{a}{2} \quad (13)$$

$$c = |a| \quad (14)$$

Hence, option 2 is correct.

Get Python Code for image from

<https://github.com/ManojChavva/FWC/blob/main/Matrix/circle/codes/circle.py>

Get LaTeX code from

<https://github.com/ManojChavva/FWC/blob/main/Matrix/circle/circle.tex>