PARABOLA

SRAVANI VUNNAVA
sravani21vunnava@gmail.com
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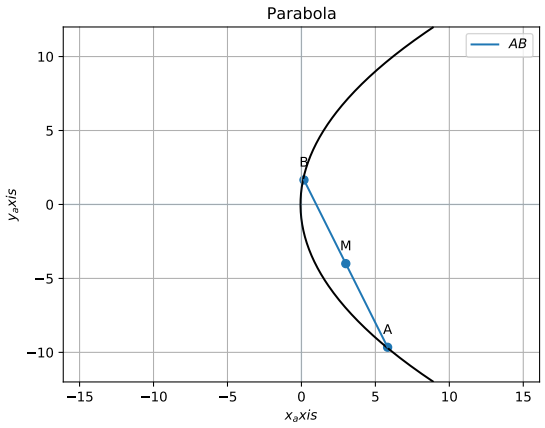
1 Problem

If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint $\begin{pmatrix} h \\ k \end{pmatrix}$, then which of the following is (are) possible value(s) of p,h and k

2 Construction



3 Solution

The given equation of line is

$$x = \begin{pmatrix} 0 \\ p \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{1}$$

The equation of parabola is:

$$\mathbf{X}^T \mathbf{V} \mathbf{X} + 2\mathbf{u}^T \mathbf{X} + f = u \tag{2}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

$$\mathbf{u} = \begin{pmatrix} -8 \\ 0 \end{pmatrix} \tag{4}$$

If line (1) is chord to the parabola (2) then

$$\mu_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{V}\mathbf{q} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^T (\mathbf{V}\mathbf{q} + \mathbf{u})\right]^2 - (\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f)(\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) \tag{5}$$

Let $\mathbf{q} = \begin{pmatrix} 0 \\ p \end{pmatrix}$ $m = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$$\mu_i = \frac{p+4}{2} \pm \sqrt{2p+4} \tag{6}$$

for $p = -2$
(1) gives tangent, so $p > -2$
consider $p = 2$ and $\mathbf{A}$, $\mathbf{B}$ are the points of intersection of (1) and (2)

$$\mathbf{A} = \mathbf{q} + \mu_1 \mathbf{m} \tag{7}$$

$$\mathbf{B} = \mathbf{q} + \mu_2 \mathbf{m} \tag{8}$$

The midpoint $\mathbf{M} = \begin{pmatrix} h \\ k \end{pmatrix}$

$$M = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{9}$$

for $p = 2$ $h = 3$ and $k = -4$