MATRIX ANALYSIS USING PYTHON

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FWC22034 IITH Future Wireless Communication (FWC) Assignment

October 28, 2022

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1 Problem

Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are

(i) parallel to x-axis
(ii) parallel to y-axis

2 Construction

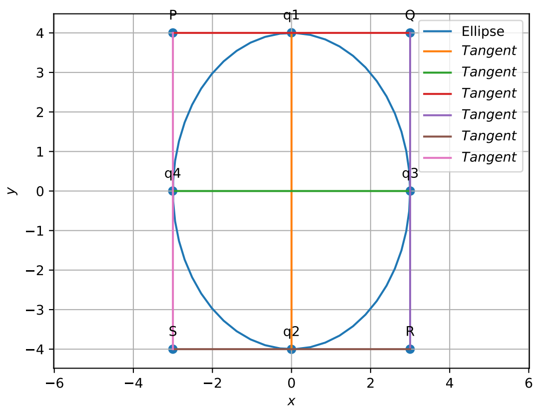

Figure of construction

The dimensions of the figure is taken as below

Symbol	Value
a	3
b	4

3 Solution

Ellipse equation :

$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \tag{1}$$

The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

The given circle can be expressed as conics with parameters

$$\lambda_1 = 16, \lambda_2 = 9 \tag{3}$$

$$\mathbf{V} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -144 \tag{4}$$

(i) Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to x-axis

The points are given by the following equation

$$\mathbf{q} = \mathbf{v}^{-1}(k_i \mathbf{n_1} - \mathbf{u}) \tag{5}$$

And the intermediate parameters are given by

$$k_i = \pm\sqrt{\frac{\mathbf{u}^T \mathbf{v}^{-1}\mathbf{u} - f}{\mathbf{n_1}^T \mathbf{v}^{-1}\mathbf{n_1}}} \tag{6}$$

Here $\mathbf{n_1}$ is normal vector which is parallel to x-axis.

$$\mathbf{n_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Now to obtain the k1 and k2 values substitute n1 value in equation (6)

$$\mathbf{v}^{-1} = \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \tag{7}$$

$$k_1 = \sqrt{\frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}} \tag{8}$$

$$k_1 = 31.17 \tag{9}$$

k1 value substitute in equation (5) we get q1

$$\mathbf{q_1} = \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} (k_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \tag{10}$$

$$\mathbf{q_1} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{11}$$

$$k_2 = -\sqrt{\frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}} \tag{12}$$

$$k_2 = -31.17 \tag{13}$$

k2 value substitute in equation (5) we get q1

$$\mathbf{q_2} = \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} (k_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \tag{14}$$

$$\mathbf{q_2} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{15}$$

(ii) Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to y-axis

The points are given by the following equation

$$\mathbf{q} = \mathbf{v}^{-1}(k_i \mathbf{n_2} - \mathbf{u}) \tag{16}$$

And the intermediate parameters are given by

$$k_i = \pm\sqrt{\frac{\mathbf{u}^T \mathbf{v}^{-1} \mathbf{u} - f}{\mathbf{n_2}^T \mathbf{v}^{-1} \mathbf{n_2}}} \tag{17}$$

Here $\mathbf{n_2}$ is normal vector which is parallel to y-axis.

$$\mathbf{n_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Now to obtain the k3 and k4 values substitute n2 value in equation (17)

$$\mathbf{v}^{-1} = \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \tag{18}$$

$$k_3 = \sqrt{\frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}} \tag{19}$$

$$k_3 = 55.42 \tag{20}$$

k3 value substitute in equation (16) we get q3

$$\mathbf{q_3} = \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} (k_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \tag{21}$$

$$\mathbf{q_3} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{22}$$

$$k_4 = -\sqrt{\frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}} \tag{23}$$

$$k_4 = -31.17 \tag{24}$$

k4 value substitute in equation (16) we get q1

$$\mathbf{q_4} = \begin{pmatrix} 1/16 & 0 \\ 0 & 1/9 \end{pmatrix} (k_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \tag{25}$$

$$\mathbf{q_4} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{26}$$

The points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to x-axis and parallel to y-axis

$$\mathbf{q_1} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{q_2} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{27}$$

$$\mathbf{q_3} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{q_4} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{28}$$

Below python code realizes the above construction :

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https://github.com/soundaryanaru/
FWC-assignments/blob/main/Matrix/Conic_
assignment/code/ellipse.py
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