Matrix Assignment

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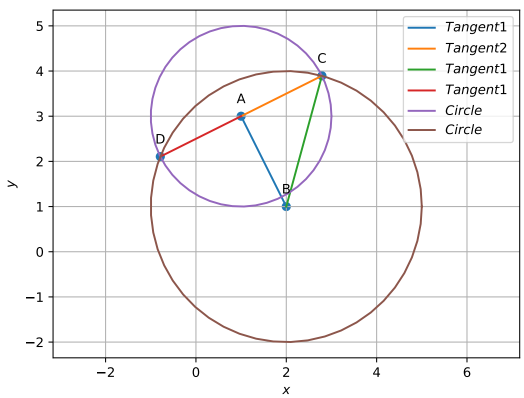
***Problem Statement* - If one the diameters of a circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre(2,1),then radius the circle is.**

Construction



The input parameters for this construction are

Symbol	Value	Description
$\mathbf{B}$	$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$	centre of circle 2

Solution

Statement: The equation of a conic is given by

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.0.1}$$

From the given information,

$$x^2 + y^2 - 2x - 6y + 6 = 0 \tag{0.0.2}$$

The circle can be expressed as conics,

$$\mathbf{V} = I \tag{0.0.3}$$

$$\mathbf{u} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{0.0.4}$$

$$f = 6 \tag{0.0.5}$$

$$\mathbf{A} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{0.0.6}$$

equation of diameter

$$\mathbf{n}^\top (\mathbf{X} - \mathbf{A}) = 0 \tag{0.0.7}$$

$$\mathbf{n} = \mathbf{A} - \mathbf{B} \tag{0.0.8}$$

Thus, the desired solution is the point of intersection of the line with the circle in the first quadrant as shown in Fig.

Using the parameteric equation of the line

$$\left(\mathbf{A_1} + \lambda\mathbf{m}\right)^\top \left(\mathbf{A_1} + \lambda\mathbf{m}\right) = r^2$$
$$\implies \lambda^2 \left\|\mathbf{m}\right\|^2 + 2\lambda\mathbf{m}^\top \mathbf{A_1} + \left\|\mathbf{A_1}\right\|^2 - r^2 = 0 \tag{0.0.9}$$

$$\lambda = \frac{-\mathbf{m}^\top \mathbf{A_1} \pm \sqrt{\left(\mathbf{m}^\top \mathbf{A_1}\right)^2 - \left\|\mathbf{m}\right\|^2 \left(\left\|\mathbf{A_1}\right\|^2 - r^2\right)}}{\left\|\mathbf{m}\right\|^2} \tag{0.0.10}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, c = -5, \tag{0.0.11}$$

$$\mathbf{m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}, \tag{0.0.12}$$

$$\mathbf{A_1} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}, r^2 = 4 \tag{0.0.13}$$

By substituting the values in above ,we get

$$\lambda_1 = -2.1 \tag{0.0.14}$$

$$\lambda_2 = -3.9 \tag{0.0.15}$$

$$\mathbf{C} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} - 3.8 \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{0.0.16}$$

$$= \begin{pmatrix} 2.7 \\ 3.8 \end{pmatrix} \tag{0.0.17}$$

$$\mathbf{D} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} - 2.1 \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{0.0.18}$$

$$= \begin{pmatrix} -0.7 \\ 2.1 \end{pmatrix} \tag{0.0.19}$$

Thus, the points of intersection are C and D.
The distance between the vectors

$$\mathbf{C} = \begin{pmatrix} 2.7 \\ 3.8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{0.0.20}$$

Using the definition of the norm,

$$\|\mathbf{C} - \mathbf{B}\| = 3 \tag{0.0.21}$$

By substituting the values of C and B in the above equation . The distance between the point C and B will be the radius of circle2 i.e 3.