

Line Assignment

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Problem

The area of triangle is 5. Two of its vertices are A(2,1) and B(3,-2). The third vertex lies on $y=x+3$. Find C.

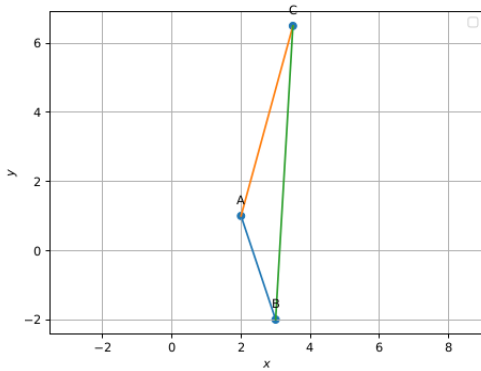


Figure of Construction

The following python code is used for finding third vertex of triangle

Githublink : <https://github.com/RupaSaiSreshta/FWC>

$$\Rightarrow \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 17 \\ -3 \end{pmatrix} \quad (2)$$

The above matrix is in the form of $\mathbf{Ax}=\mathbf{B}$

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c} 3 & 1 & 17 \\ 1 & -1 & -3 \end{array} \right) \quad (3)$$

$$\xleftrightarrow{R_2 \leftarrow 3R_2 - R_1} \left(\begin{array}{cc|c} 3 & 1 & 17 \\ 0 & -4 & -20 \end{array} \right) \quad (4)$$

$$\xleftrightarrow{R_1 \leftarrow 4R_1 + R_2} \left(\begin{array}{cc|c} 12 & 0 & 42 \\ 0 & -4 & -20 \end{array} \right) \quad (5)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 * 1/12} \left(\begin{array}{cc|c} 1 & 0 & 3.5 \\ 0 & -4 & -20 \end{array} \right) \quad (6)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 * 1/4} \left(\begin{array}{cc|c} 1 & 0 & 3.5 \\ 0 & 1 & 6.5 \end{array} \right) \Rightarrow \mathbf{x} = \begin{pmatrix} 3.5 \\ 6.5 \end{pmatrix} \quad (7)$$

Now the vertex of C is

$$\mathbf{C} \begin{pmatrix} 3.5 \\ 6.5 \end{pmatrix}$$

Solution

Construction:

Input parameters for this construction

Symbol	Value	Description
A	$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$	Given vertex of triangle
B	$\begin{pmatrix} 3 \\ -2 \end{pmatrix}$	Given vertex of triangle
Ar	5	Area of triangle

Solution:

Let us assume $\mathbf{C} \begin{pmatrix} x \\ y \end{pmatrix}$

Area of triangle is

$$\mathbf{Ar} = \frac{1}{2} \|((\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C}))\| \quad (1)$$

By solving we will get the below matrix

Proof

Proof: Here The area of triangle is given as 5. Now we have to calculate the area of triangle using three vertices A, B and C. We need to prove that the area of triangle is 5. Area of triangle is

$$\mathbf{Ar} = \frac{1}{2} \|((\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C}))\| \quad (8)$$

By substituting the values of A, B and C

$$\mathbf{Ar} = \frac{1}{2} [2(-8.5) + 3(5.5) + 3.5(3)] \quad (9)$$

$$\mathbf{Ar} = \frac{1}{2} [-17 + 16.5 + 10.5] \quad (10)$$

$$\mathbf{Ar} = \frac{1}{2} [10.5 - 0.5] \quad (11)$$

$$\boxed{\mathbf{Ar} = 5}$$

Hence verified.