

# Line Assignment

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**Problem Statement** -Two sides of a rhombus are along the lines  $x-y+1=0$  and  $7x-y-5=0$ . If diagonals intersect at  $(-1,-2)$  then which is the vertex of the rhombus .

**Figure**

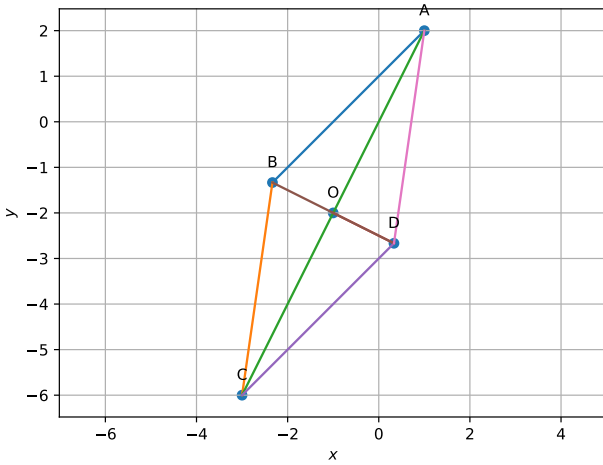


Figure 1: Diagonals intersect at point  $O(-1,-2)$

**Solution**

The vector equation of the line  $x-y+1=0$  AB is

$$(1 \ -1) \begin{pmatrix} x \\ y-1 \end{pmatrix} = 0 \quad (1)$$

The vector equation of the line  $7x-y-5=0$  AD is

$$(7 \ -1) \begin{pmatrix} x \\ y+5 \end{pmatrix} = 0 \quad (2)$$

The point of intersection of eq(1) and eq(2) is

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (3)$$

The midpoint gives the vertex c

$$\mathbf{O} = \frac{\mathbf{A} + \mathbf{C}}{2} \quad (4)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -6 \end{pmatrix} \quad (5)$$

The slope of AC is the direction vector of AC

$$\mathbf{m1} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} \quad (6)$$

Since  $AC \perp BD$  the slope of diagonal BD becomes direction vector

$$\mathbf{m2} = \begin{pmatrix} -8 \\ 4 \end{pmatrix} \quad (7)$$

Since,  $AC \perp BD$  the equation of line in vector along the diagonal BD

$$(-8 \ 4) \begin{pmatrix} x+1 \\ y+2 \end{pmatrix} = 0 \quad (8)$$

The point of intersection of eq(8) and eq(1) gives vertex B

$$\mathbf{B} = \begin{pmatrix} -7/3 \\ -4/3 \end{pmatrix} \quad (9)$$

The point of intersection of eq(8) and eq(2) gives vertex D

$$\mathbf{D} = \begin{pmatrix} 1/3 \\ -8/3 \end{pmatrix} \quad (10)$$

Hence the vertices of the rhombus are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -7/3 \\ -4/3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1/3 \\ -8/3 \end{pmatrix} \quad (11)$$

**Construction**

The Vertices of rhombus are

Symbol	Value	Description
A	(1,2)	vertex at A
B	(1/3,-8/3)	vertex at B
C	(-3,-6)	vertex at C
D	(-7/3,-4/3)	vertex at D
O	(-1,-2)	diagonals intersection point