Conic section Assignment

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1 Problem Statement

Find the equation of the circle passing through the points (2,3)and (-1,1) and whose centre is on the line x-3y-11=0

2 Solution

To Find : The equation of circle. Given , points passing through circle (2,3) and (-1,1), And equation of line passing through the center of circle x-3y-11=0

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} h \\ k \end{pmatrix}, \tag{3}$$

$$f = f \tag{4}$$

which is the equation of a circle. And h,k,C are unknown values we should find

$$\begin{pmatrix} 4 & 6 & 1 \\ -2 & 2 & 1 \\ -1 & 3 & 0 \end{pmatrix} \begin{pmatrix} h \\ k \\ f \end{pmatrix} = \begin{pmatrix} -13 \\ -2 \\ 11 \end{pmatrix} \tag{5}$$

(6)

The augmented matrix for (5) can be expressed as

$$\xleftrightarrow{1/4R_1 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 3/2 & .1/4 & -13/4 \\ -2 & 2 & 1 & -2 \\ -1 & 3 & 0 & 11 \end{array}\right) \tag{7}$$

$$\xleftrightarrow{-1/2R_2 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 3/2 & .1/4 & -13/4 \\ 1 & -1 & -1/2 & 1 \\ -1 & 3 & 0 & 11 \end{array}\right) \tag{8}$$

$$\xleftrightarrow{-1R_3 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/4 & -13/4 \\ 1 & -1 & -1/2 & 1 \\ 1 & -3 & 0 & -11 \end{array}\right) \tag{9}$$

$$\xleftrightarrow{R_2 - 1.R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/4 & -13/4 \\ 0 & -5/2 & -3/4 & 17 \\ 1 & -3 & 0 & -11 \end{array}\right) \tag{10}$$

$$\xleftrightarrow{R_3 - 1.R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/4 & -13/4 \\ 0 & -5/2 & -3/4 & 17/4 \\ 0 & 9/2 & -1/4 & -31/4 \end{array}\right) \tag{11}$$

$$\xleftrightarrow{-2/5R_2 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/4 & -13/4 \\ 0 & 1 & 3/10 & 17/10 \\ 0 & 9/2 & -1/4 & -31/4 \end{array}\right) \tag{12}$$

$$\xleftrightarrow{-2/9R_3 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/4 & -13/4 \\ 1 & -1 & -1/2 & 1 \\ 1 & -3 & 0 & -11 \end{array}\right) \tag{13}$$

$$\xleftrightarrow{R_3 - 1.R_2 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/4 & -13/4 \\ 0 & 1 & 3/10 & -17/10 \\ 0 & 0 & -11/45 & 154/45 \end{array}\right) \tag{14}$$

$$\xleftrightarrow{-45/11R_3 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/4 & -13/4 \\ 0 & 1 & 3/10 & -17/10 \\ 0 & 0 & 1 & -14 \end{array}\right) \tag{15}$$

$$\xleftrightarrow{R_2 - 3/10R_3 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 3/2 & 1/4 & -13/4 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 1 & -14 \end{array}\right) \tag{16}$$

$$\xleftrightarrow{R_1 - 1/4R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 3/2 & 0 & 1/4 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 1 & -14 \end{array}\right) \tag{17}$$

$$\xleftrightarrow{R_1 - 3/2R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -7/2 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 1 & -14 \end{array}\right) \tag{18}$$

(19)

$$\mathbf{u} = \begin{pmatrix} -7/2 \\ 5/2 \end{pmatrix}$$

$$f = -14 \tag{20}$$

from u and f we can find radius

$$r = \sqrt{\|(\mathbf{u})\|^2 - f} \tag{21}$$

$$r = \sqrt{65/4} \tag{22}$$

And, from them we can find the equation of circle.

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{23}$$

$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{u}$=$\begin{pmatrix} 7/2 \\ -5/2 \end{pmatrix}$ f = 14 steps for constructing above figure are:

Symbol	Value	Description
r	$\sqrt{65/4}$	Radius of the circle
$\mathbf{C}$	$\begin{pmatrix} 7/2 \\ -5/2 \end{pmatrix}$	center of circle

3 Construction