**Name:** GANGA GOPINATH

# Assignment

**Roll No. :** FWC22050

**Problem Statement:**

Consider a family of circles which are passing through the point (-1, 1) and are tangent to x-axis. If (h, k) are the coordinates of the center of the circles, then the set of values of k is given by interval.

**SOLUTION**:

**Given**:

**O** be the center of circle and the coordinates are,

$$\mathbf{O} = \begin{pmatrix} h \\ k \end{pmatrix} \tag{1}$$

Let **X** be the point on the circle

$$\mathbf{X} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{2}$$

**To Find**

Constructing the family of circles with different values of k

**STEP-1**

The perpendicular distance of center from tangent of the circle is equal to its radius. Let r be the radius of circles

So that r=k
Let **R** be the any point on the circle

$$\mathbf{R} = r\begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix} \tag{3}$$

For the input parameters in Table 1.

| Symbol | Value | Description |
|---|---|---|
| O | $\begin{pmatrix} \gamma \\ \alpha \end{pmatrix}$ | Center |
| X | $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ | Passing point |
| R | r $\begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix}$ | point on circle |
| r | $\alpha$ | radius |

Table 1

**STEP-2**

The distance between the point **X** and **O** is given by,

$$\|\mathbf{X} - \mathbf{O}\| = r \tag{4}$$

which can be expressed as

$$\sqrt{(\mathbf{X} - \mathbf{O})^\top (\mathbf{X} - \mathbf{O})} = r$$

Squaring on both the sides

$$(\sqrt{(\mathbf{X} - \mathbf{O})^\top (\mathbf{X} - \mathbf{O})})^2 = r^2$$

$$(\mathbf{X} - \mathbf{O})^\top (\mathbf{X} - \mathbf{O}) = r^2$$

Expanding the above equation,

$$\|\mathbf{X}\|^2 - 2\mathbf{X}^\top \mathbf{O} + \|\mathbf{O}\|^2 = r^2 \tag{5}$$

Upon substituting numerical values,

$$(-1)^2 + 1^2 - 2\begin{pmatrix} -1 \\ 1 \end{pmatrix}^\top \begin{pmatrix} \gamma \\ \alpha \end{pmatrix} + \gamma^2 + \alpha^2 = \alpha^2 \tag{6}$$

$$(-1)^2 + 1^2 - 2\begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} \gamma \\ \alpha \end{pmatrix} + \gamma^2 + \alpha^2 = \alpha^2 \tag{7}$$

$$2 + 2\gamma - 2\alpha + \gamma^2 = 0 \tag{8}$$

Solving the above equation ,

$$\alpha \geq \frac{1}{2} \tag{9}$$

Let $\alpha$ be any value from $\frac{1}{2}$ to $\infty$

$$\alpha \in [\frac{1}{2}, \infty) \tag{10}$$

which can be expressed as,
Let $\theta$ be any angle from 0 to $2\pi$

$$\theta \in [0, 2\pi) \tag{11}$$

Let us assume,

$$\theta = \frac{\pi}{3} \tag{12}$$

Using equation (3) any point on circle $\mathbf{R} = \begin{pmatrix} x \\ y \end{pmatrix}$ is,

Here ,

$$x = rcos\theta \tag{13}$$
$$y = rsin\theta \tag{14}$$

$$\mathbf{R} = \begin{pmatrix} 0.5 \\ .86 \end{pmatrix} \tag{15}$$

Let $\alpha$ be any values ranging from $\frac{1}{2}$ to $\infty$ with the incrementation of $+2$
So,

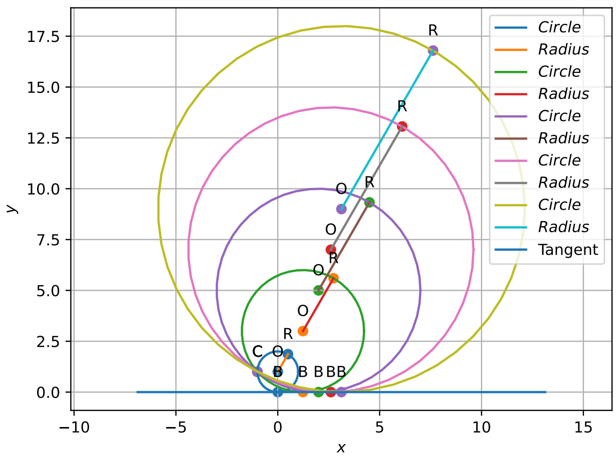
$$\alpha = 1, 3, 5, 7, 9, 11 \tag{16}$$

If $\alpha = 1$ ,

$$\mathbf{O} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{17}$$

when $\alpha = 3$ and so on till $\alpha = 11$,

$$\mathbf{O} = \begin{pmatrix} 1.23 \\ 3 \end{pmatrix} \tag{18}$$




**Construction**

| vertex | coordinates |
|---|---|
| $\mathbf{O}$ | $\begin{pmatrix} \gamma \\ \alpha \end{pmatrix}$ |
| $\mathbf{X}$ | $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ |

Download the code
https://github.com/Gangagopinath/ASSIGNMENT/tree/main/assignment5