Matrix-circle

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I. Problem Statement

If circles $x^2+y^2+2x+2ky+6 = 0$, $x^2+y^2+2ky+k = 0$ intersect orthogonally then find k.

II. Construction

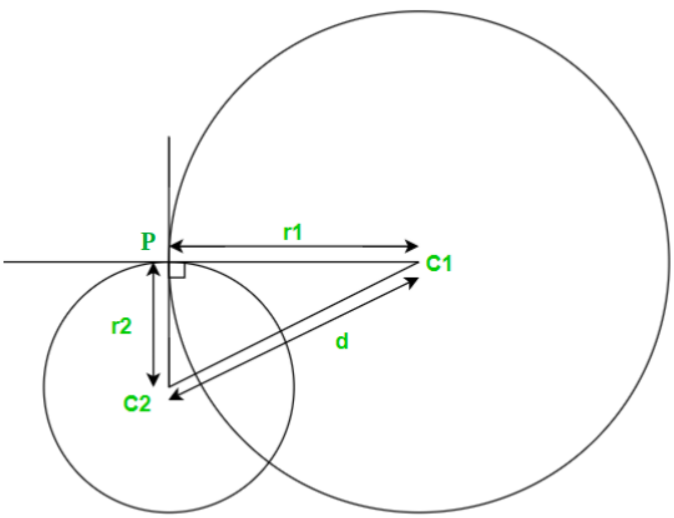

Fig. 1. Orthogonal Circles

Symbol	Value	Description
C1	$\begin{pmatrix}-1\\-k\end{pmatrix}$	Center of circle C1
C2	$\begin{pmatrix}0\\-k\end{pmatrix}$	Center of circle C2
P	**X**	Radius of circle C1
θ	90°	Given that C1 and C2 are Orthogonal
d	1	Distance between centers of the circles

TABLE I
PARAMETERS

Standard form of a circle in matrix form is

$$\mathbf{x}\mathbf{V}\mathbf{x}^{\mathbf{T}} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + \mathrm{f} = 0$$

where,

$$\mathbf{V} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, Center\mathbf{C} = -\mathbf{V}^{-1}\mathbf{u}^{\mathbf{T}},$$

$$\text{Radius of a circle is r} = \sqrt{\mathbf{u}^{\mathbf{T}}\mathbf{u} - f}$$

Equation of given circles can be represented in matrix form as

$$\mathbf{x}\mathbf{x}^{\mathbf{T}} + 2\begin{pmatrix}1 & k\end{pmatrix}\mathbf{x} + 6 = 0 \tag{1}$$

$$\mathbf{x}\mathbf{x}^{\mathbf{T}} + 2\begin{pmatrix}0 & k\end{pmatrix}\mathbf{x} + k = 0 \tag{2}$$

where,

$$\mathbf{u_1} = \begin{pmatrix}1\\k\end{pmatrix}, \mathbf{u_2} = \begin{pmatrix}0\\k\end{pmatrix}, f_1 = 6, f_2 = k$$

$$\mathbf{C_1} = \begin{pmatrix}-1 & -k\end{pmatrix}, \mathbf{C_2} = \begin{pmatrix}0 & -k\end{pmatrix}$$

The radius of first circle is $r_1 = \sqrt{k^2 - 5}$

The radius of Second Circle is $r_2 = \sqrt{k^2 - k}$

Given that, the circles are orthogonal so the angle between the radii r1 and r2 is 90°.

From figure $\triangle$PC1C2 is a Right angled triangle with hypotenuse d = $||C1 - C2||$.

So, by using **Pythagoraus theorem**

$$||\mathbf{C_1} - \mathbf{C_2}||^2 = ||\mathbf{C_1} - \mathbf{P}||^2 + ||\mathbf{C_2} - \mathbf{P}||^2 \tag{3}$$

Therefore,

$$||\mathbf{C_1} - \mathbf{C_2}||^2 = \mathbf{r_1^2} + \mathbf{r_2^2}$$

$$1 = k^2 - 5 + k^2 - k \tag{4}$$

Yielding, k = 2 or -$\frac{3}{2}$
Therefore, the value of k is

k = 2 or -$\mathbf{\frac{3}{2}}$