MATRICES USING PYTHON

R.Ramesh
rameshrandhiglra@gmail.com
FWC22076 IITH Future Wireless Communication (FWC) September

Problem statement:

The focus of an ellipse is at origin .The directrix is the line x=4 and eccentricity is 1/2. Then the length of the semi major axis is.

Construction

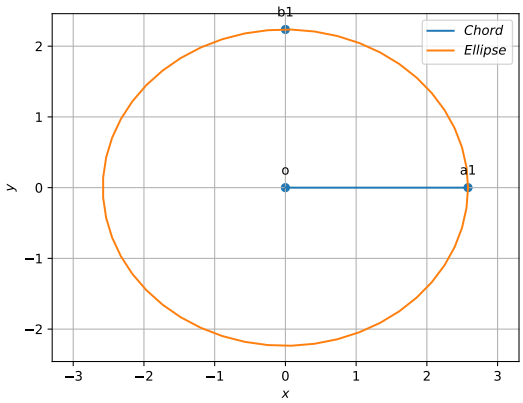

Figure of construction

The input parameters for this construction are

Symbol	Value	Description
$\mathbf{F}$	$\begin{pmatrix}0\\0\end{pmatrix}$	focus point
d	x=4	line of directrix
e	1/2	value of eccntricity

Solution

Statement: The equation of a conic with directrix $\mathbf{n}^\top\mathbf{x} = c$, eccentricity e and focus $\mathbf{F}$ is given by

$$\mathbf{x}^\top\mathbf{V}\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \|\mathbf{n}\|^2\mathbf{I} - e^2\mathbf{n}\mathbf{n}^\top, \tag{2}$$
$$\mathbf{u} = ce^2\mathbf{n} - \|\mathbf{n}\|^2\mathbf{F}, \tag{3}$$
$$f = \|\mathbf{n}\|^2\|\mathbf{F}\|^2 - c^2e^2 \tag{4}$$

$$\mathbf{n} = \begin{pmatrix}1\\0\end{pmatrix}, c = 4, \tag{5}$$

$$\mathbf{F} = \begin{pmatrix}0\\0\end{pmatrix} \tag{6}$$

By substituting the values in above ,we get

$$\mathbf{V} = \begin{pmatrix}\frac{3}{4} & 0\\0 & 1\end{pmatrix} \tag{7}$$

$$\mathbf{u} = \begin{pmatrix}1\\0\end{pmatrix} \tag{8}$$

$$f = -4 \tag{9}$$

since diagnoal elements of **V** are eigen values

$$\lambda_2 = \|\mathbf{n}\|^2 = 1 \tag{10}$$

$$\lambda_1 = (1-e^2)\lambda_2 = \frac{3}{4} \tag{11}$$

Now,

$$f_0 = \mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u} - f \neq 0 \tag{12}$$

$$a = \sqrt{\frac{|f_0|}{|\lambda_1|}} \tag{13}$$

$$b = \sqrt{\frac{|f_0|}{|\lambda_2|}} \tag{14}$$

By substituting the values in above ,we get

$$f_0 = 5 \tag{15}$$
$$a = 2.58 \tag{16}$$
$$b = 2.23 \tag{17}$$

so, the length of semi major axis is 2.58