

Circle Assignment

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Problem Statement:

Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Solution

Construction

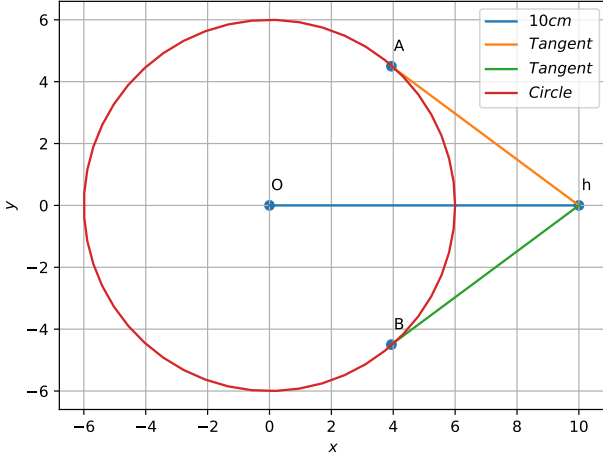


Figure 1: Figure

The dimensions of the figure is taken as below

Symbol	Value	Description
r	6	Radius
h	10	distance
O	$d \begin{pmatrix} \cos(0) \\ \sin(0) \end{pmatrix}$	centre
A	$a \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	point of contact
B	$b \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	point of contact

The equation of a conic with directrix $\mathbf{n}^T \mathbf{x} = c$, eccentricity e and focus \mathbf{f} is given by

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

for circle eccentricity $e = 0$ then,

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -r^2 \quad (2)$$

Point q on conic is given by

$$\mathbf{q} = \mathbf{V}^{-1}(\mathbf{n} - \mathbf{u}) \quad (3)$$

where, \mathbf{n} is the normal vectors of the tangents from a point h to the conic are given by

$$\mathbf{n} = \frac{\mathbf{e}_1}{\mathbf{e}_1^T \mathbf{h}} + \mu_i (\mathbf{R} \mathbf{h}) \quad (4)$$

where μ_i 's are given by the following equation

$$\mu_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} (-\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u})) \quad (5)$$

$$\pm \sqrt{\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u})^2 - (\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f)(\mathbf{m}^T \mathbf{V} \mathbf{m})}$$

μ_i 's are obtained by substituting the following in equation 6

$$\mathbf{m} = (\mathbf{R} \mathbf{h}); \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}; \mathbf{q} = \frac{\mathbf{e}_1}{\mathbf{e}_1^T \mathbf{h}} \quad (6)$$

$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ The obtained μ_i 's are substituted in equation 5 and equation 5 is substituted in equation 6 the required points on conic A and B are obtained.

Now the point A and B are formed and tangents are drawn

To find the length of point h and point A

The distance between h and A is $\|\mathbf{h} - \mathbf{A}\|$

$$(\mathbf{h} - \mathbf{A})(\mathbf{h} - \mathbf{A})^T = d^2 \quad (7)$$

By solving equation(7) we get
distance $d=8\text{cm}$

$$(\mathbf{h} - \mathbf{A})(\mathbf{h} - \mathbf{A})^T = d^2 \quad (8)$$

By solving equation(7) we get
distance $d=8\text{cm}$

$$\begin{aligned} \|\mathbf{h} - \mathbf{A}\| &= 8\text{cm} \\ &= 8\text{cm} \end{aligned} \quad (9)$$

To find the length of point h and point B

The distance between h and B is $\|\mathbf{h} - \mathbf{B}\|$

$$(\mathbf{h} - \mathbf{B})(\mathbf{h} - \mathbf{B})^T = d^2 \quad (10)$$

By solving equation(10) we get
distance $d=8cm$

$$\|\mathbf{h} - \mathbf{B}\| = 8cm \quad (11)$$

from equation (9) and (11)

$$\|\mathbf{h} - \mathbf{A}\| = \|\mathbf{h} - \mathbf{B}\| \quad (12)$$

Hence,the above equation (12) we can prove that the lenght of the tangents to a circle of radius 6cm,from a point 10cm away from the centre of the circle,is 8cm.

The below python code realizes the above construction:

https://github.com/soundaryanaru/FWC-assignments/blob/main/Matrix/circle_assignment/code/circle.py