Matrix Assignment - Conic

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Get Python code for the figure from

https://github.com/SurabhiSeetha/Fwciith2022/tree/main/Assignment%201/codes/src

Get LaTex code from

https://github.com/SurabhiSeetha/Fwciith2022/tree/main/avr%20gcc

1 Question-Class 12, Exercise 6.3, Q(11)

Find the equation of all lines having slope 2 which are tangents to the curve $y = \frac{1}{x-3}$, $x \neq 3$

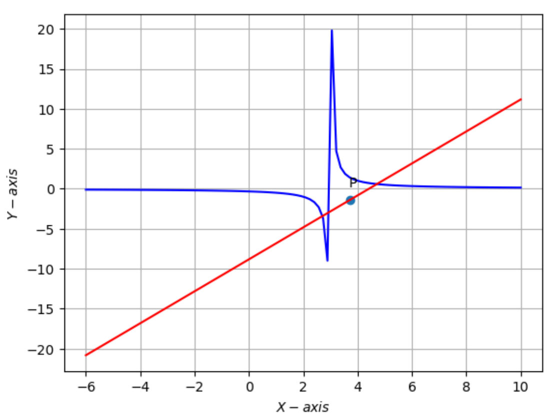

Fig 1. Curve

Construction

See Fig 1 for the input parameters in Table 1.

Symbol	Value	Description
C	$y = \frac{1}{x-3}$	Given Conic C
P	$x_i = q + \mu_i m$	Point of Contact P

Table 1

2 Solution

The equation of a conic with directrix $\mathbf{n}^\top \mathbf{x} = c$, eccentricity e and focus $\mathbf{F}$ is given by

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2.0.1}$$

Given,

$$\mathbf{V} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix} f = -1, m = 2 \tag{2.0.2}$$

$$n = \begin{pmatrix} -m \\ 1 \end{pmatrix} \tag{2.0.3}$$

$$q = \mathbf{V}^{-1}(k_i \mathbf{n} - \mathbf{u}) \tag{2.0.4}$$

$$k_i = \pm \sqrt{\frac{f_0}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \tag{2.0.5}$$

$$f_0 = f + \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} \tag{2.0.6}$$

$$n = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \tag{2.0.7}$$

By substituting Eq. (2.0.2) in Eq. (2.0.6) we get,

$$f_o = -1 + \begin{pmatrix} 0 & \frac{-3}{2} \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{-3}{2} \end{pmatrix}$$
$$f_o = -1 + \begin{pmatrix} 0 & \frac{-3}{2} \end{pmatrix} \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$
$$f_o = -1 + 0$$

$$f_o = -1 \tag{2.0.8}$$

substituting Eq. (2.0.8) in Eq. (2.0.5) as,

$$k_i = \pm \sqrt{\frac{-1}{\begin{pmatrix} -2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix}}}$$
$$= \pm \sqrt{\frac{-1}{\begin{pmatrix} 2 & -4 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix}}}$$

$$= \pm \sqrt{\frac{1}{8}} \tag{2.0.9}$$

Substituting Eq. (2.0.9) in (2.0.4),

$$q = (\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \pm \sqrt{\frac{1}{8}} \begin{pmatrix} -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{-3}{2} \end{pmatrix})$$

$$q = \begin{pmatrix} 3.707 \\ -1.414 \end{pmatrix} \tag{2.0.10}$$

We know that,

$$\mu_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}(-\mathbf{m}^T(\mathbf{V}\mathbf{q} + \mathbf{u}) \\ \pm \sqrt{(\mathbf{m}^T(\mathbf{V}\mathbf{q} + \mathbf{u}))^2 - (\mathbf{q}^T + 2\mathbf{u}^T\mathbf{q} + f)(\mathbf{m}^T\mathbf{V}\mathbf{m})} \tag{2.0.11}$$

By substituting Eq. (2.0.2) and (2.0.10) in (2.0.11) we get,

$$\mu_i = 0 \tag{2.0.12}$$

Now the point of contact is given by,

$$\mathbf{x_i} = \mathbf{q} + \mu_\mathbf{i}\mathbf{m} \tag{2.0.13}$$

By substituting Eq. (2.0.10) and (2.0.12) in Eq. (2.0.13) we get,

$$P = \begin{pmatrix} 3.707 \\ -1.414 \end{pmatrix}$$

But when we plot the tangent through this point P with the given slope 2, we observe that it touches the curve at 2 points.

Hence tangents are not possible for the given curve with slope 2.