

Line Assignment

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Problem Statement - Two sides of a rhombus ABCD are parallel to the lines $y=x+2$ and $y=7x+3$. If the diagonals of the rhombus intersect at the point (1,2) and the vertex A is on the y-axis, find possible coordinate of A.

Figure

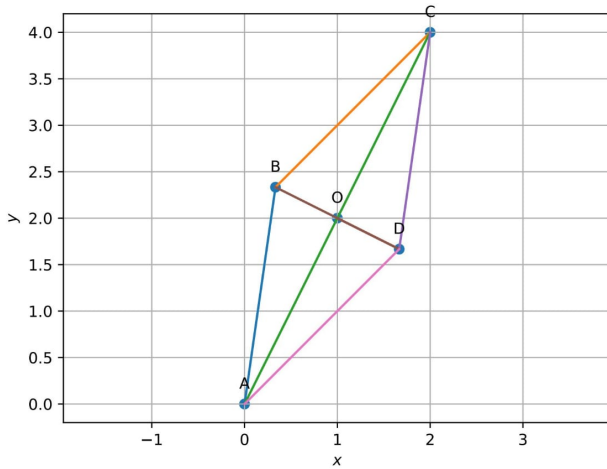


Figure 1: Diagonals intersect at point O(1,2)

Solution

The equation of the line1 $y=x+2$ and line2 $y=7x+3$. we know that vector equation of the line is

$$\mathbf{n}^T \mathbf{x} = c \quad (1)$$

The vector equation of the line1 and line2 is

$$(-1 \ 1)\mathbf{x} = 2 \quad (2)$$

$$(-7 \ 1)\mathbf{x} = 3 \quad (3)$$

from above equations the direction vectors of AB and AD are

$$\mathbf{m}_1 = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \quad (4)$$

$$\mathbf{m}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (5)$$

To find the coordinate of A we use the parametric equation of the line

$$\mathbf{X} = \mathbf{P} + \lambda \left(\frac{\mathbf{m}_1}{\|\mathbf{m}_1\|} + \frac{\mathbf{m}_2}{\|\mathbf{m}_2\|} \right) \quad (6)$$

where

$$\mathbf{x} = \mathbf{A} = \alpha \mathbf{e}_2, \mathbf{P} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (7)$$

$$\alpha \mathbf{e}_2 = \mathbf{P} + \lambda \left(\frac{\mathbf{m}_1}{\|\mathbf{m}_1\|} + \frac{\mathbf{m}_2}{\|\mathbf{m}_2\|} \right) \quad (8)$$

multiplying the above equation with \mathbf{e}_1 transpose

$$\lambda = -1.178 \quad (9)$$

after finding the lambda substitute in the eq(12) solve for the equation for vertex A

$$\alpha \mathbf{e}_2 = \mathbf{P} + \lambda \left(\frac{\mathbf{m}_1}{\|\mathbf{m}_1\|} + \frac{\mathbf{m}_2}{\|\mathbf{m}_2\|} \right) \quad (10)$$

the vertex A is

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (11)$$

The midpoint gives the vertex c

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{C}}{2} \quad (12)$$

$$\mathbf{C} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (13)$$

the normal vectors are

$$\mathbf{n}_1 = \text{omat} * \mathbf{m}_1, \mathbf{n}_2 = \text{omat} * \mathbf{m}_2 \quad (14)$$

on sloving, we get

$$\mathbf{n}_1 = \begin{pmatrix} 7 \\ -1 \end{pmatrix}, \mathbf{n}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (15)$$

the vertex B is the intersection of the lines

$$\mathbf{n}_1^T (\mathbf{x} - \mathbf{A}) = 0, \quad \mathbf{n}_2^T (\mathbf{x} - \mathbf{C}) = 0 \quad (16)$$

on solving, we get

$$\mathbf{B} = \begin{pmatrix} 1/3 \\ 7/3 \end{pmatrix} \quad (17)$$

the vertex D is the intersection of the lines

$$\mathbf{n}_1^T (\mathbf{x} - \mathbf{C}) = 0, \quad \mathbf{n}_2^T (\mathbf{x} - \mathbf{A}) = 0 \quad (18)$$

on solving, we get

$$\mathbf{D} = \begin{pmatrix} 5/3 \\ 5/3 \end{pmatrix} \quad (19)$$

Hence the vertices of the rhombus are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (20)$$

$$\mathbf{B} = \begin{pmatrix} 1/3 \\ 7/3 \end{pmatrix} \quad (21)$$

$$\mathbf{C} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (22)$$

$$\mathbf{D} = \begin{pmatrix} 5/3 \\ 5/3 \end{pmatrix} \quad (23)$$