భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Name: Ganga Gopinath

Assignment-6

Roll No. : FWC22050

Problem Statement:

Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

SOLUTION:

Given:

Equation of Parabola is

$$x^2 = 4y \tag{1}$$

Equation of line is

$$y = |x| \tag{2}$$

From (1) we can say that Parabola is symmetric about the positive y axis.

To Find

To find the intersection points and area of shaded region shown in figure

STEP-1

The given parabola and line can be expressed as conics with parameters,
For Parabola,

$$\mathbf{V}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{3}$$

$$\mathbf{u_1} = -\begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} \tag{4}$$

$$f_1 = 0 \tag{5}$$

For line,

$$\mathbf{V}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{6}$$

$$\mathbf{u_2} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{7}$$

$$f_2 = 0 \tag{8}$$

STEP-2

The points of intersection of the line is given by,

$$L: \quad \mathbf{x} = \mathbf{q} + \kappa \mathbf{m} \quad \kappa \in \mathbb{R} \tag{9}$$

with the conic section,

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{10}$$

are given by

$$\mathbf{x}_i = \mathbf{q} + \kappa_i \mathbf{m} \tag{11}$$

where,

$$\kappa_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + \mathbf{u} \right) \right]^2 - \left(\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f \right) \left(\mathbf{m}^T \mathbf{V} \mathbf{m} \right)} \right) \tag{12}$$

On substituting

$$\mathbf{q} = \begin{pmatrix} 0 \\ 0.25 \end{pmatrix} \tag{13}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{14}$$

With the given Parabola,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{15}$$

$$\mathbf{u} = -\begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} \tag{16}$$

$$f = 0 \tag{17}$$

The value of κ ,

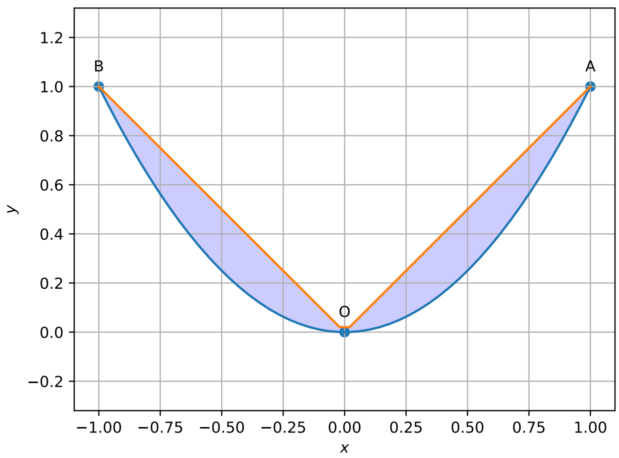
$$\kappa = 1, -1 \tag{18}$$

The points of intersection with Parabola along circle are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{19}$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{20}$$

Result



From the figure,
Total area of portion is given by,

$$A = \int_0^1 g(x) - f(x)\, dx \tag{21}$$

Where g(x) is area under line and f(x) is the area of parabola

$$A = \int_0^1 y dx - \int_0^1 y_1 dx \tag{22}$$

$$A = \int_0^1 x dx - \int_0^1 x^2 dx \tag{23}$$

$$\tag{24}$$

Area A is,

$$A = .333\, m^2 \tag{25}$$

Construction

Points	**coordinates**
A	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
B	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Download the code
https://github.com/Gangagopinath/ASSIGNMENT/tree/main/assignment6