PYTHON PROGRAMMING ON MATRICES

DUDEKULA USENI
r171099@rguktrkv.ac.in

FWC220   IITH Future Wireless Communication (FWC)   Matrix:Circle

Contents

1 Problem

From the origin, chords are drawn to the circle $(x-1)^2 + y^2 = 1$., find the equation of the locus of the mid-points of these chords
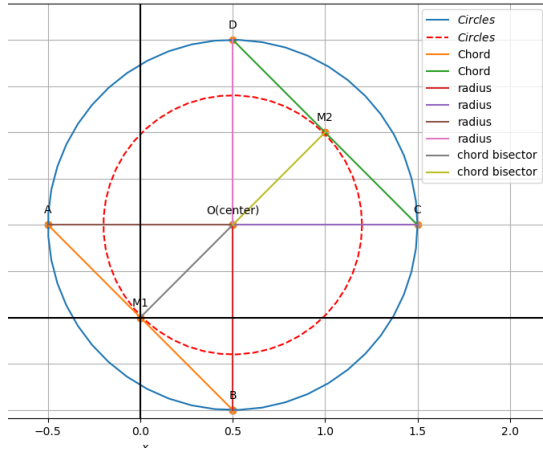
2 Construction

Figure of construction

3 Solution

Circle equation : $(x-1)^2 + y^2 = 1$
The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

The given circle can be expressed as conics with parameters

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = -\begin{pmatrix} -1 \\ 0 \end{pmatrix}, f = 0 \tag{2}$$

Radius and Centre are

$$r = \sqrt{\mathbf{u}^\top \mathbf{u} - f}, \mathbf{C} = -u \tag{3}$$

$$r = 1 \tag{4}$$

From the figure

$$\mathbf{A}^\top \mathbf{B} = 0 \tag{5}$$

Let $\mathbf{R}$ is the rotation matrix of given circle

$$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{6}$$

Let $\mathbf{B}$ be the another end point of chord

$$\mathbf{B} = \mathbf{R}\mathbf{A} \tag{7}$$

Let $\mathbf{P}$ be th mid point of chord of the circle

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{8}$$

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{R}\mathbf{A}}{2} \tag{9}$$

$$\mathbf{P} = \frac{\mathbf{A}(\mathbf{I} + \mathbf{R})}{2} \tag{10}$$

$$\mathbf{A} = 2\mathbf{P}[\mathbf{I} + \mathbf{R}]^{-1} \tag{11}$$

STEPS TO FIND THE LOCUS OF THE MIDPOINT OF CHORD OF THE CIRCLE:
By substituting $\mathbf{A}$ value in quadratic form of the circle we get

$$[2\mathbf{P}(\mathbf{I}+\mathbf{R})^{-1}]^\top[2\mathbf{P}(\mathbf{I}+\mathbf{R})^{-1}] + 2[2\mathbf{P}(\mathbf{I}+\mathbf{R})^{-1}]\left(\mathbf{u}\right)^\top + 0 = 0 \tag{12}$$

$$\mathbf{P}^\top[2(\mathbf{I}+\mathbf{R})^{-1}]^\top\mathbf{P}[2(\mathbf{I}+\mathbf{R})^{-1}]+4\mathbf{P}[(\mathbf{I}+\mathbf{R})^{-1}]\left(\mathbf{u}\right)^\top+0 = 0$$

$$\mathbf{P}^\top \mathbf{V} \mathbf{P} + 2\mathbf{u}^\top \mathbf{P} + 0 = 0 \tag{13}$$

$$\mathbf{V} = \mathbf{I}, \mathbf{u}^\top = (-0.5 - 0.5) \tag{14}$$

Finally Equation of the locus of the midpoint of chord of the given circle:

$$\mathbf{P}^\top \mathbf{V} \mathbf{P} + 2\mathbf{u}^\top \mathbf{P} = 0 \tag{15}$$

$$\mathbf{X}^\top \mathbf{V} \mathbf{X} + 2\mathbf{u}^\top \mathbf{X} = 0 \tag{16}$$

4 Software

Below python code realizes the above construction :
https://github.com/dudekulauseni123/FWC0982022