

Matrix Assignment

Mannava Venkatasai

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Problem Statement - Show that the diagonals of a parallelogram divide it into four triangles of equal area.

1. $AB=DC$ and $AD=BC$
2. O is the midpoint of AC and BD

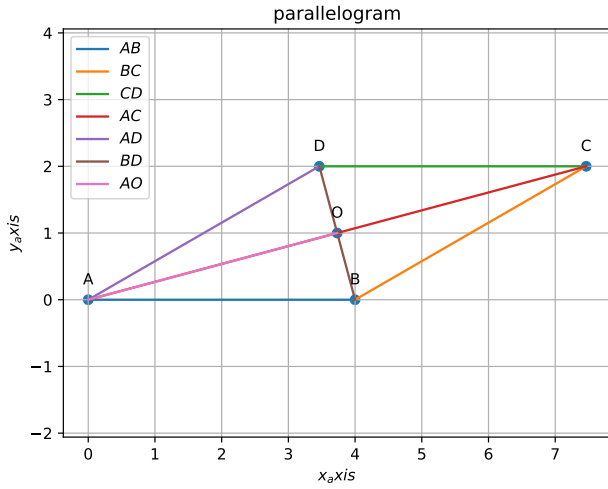


Figure 1: Parallelogram ABCD With centre O

Solution

Part 1

Construction

The input parameters are the lengths of AB and AD and angle between AB and AD s

Symbol	Value	Description
d	4	AB
r	3	AD
θ	$\pi/6$	$\angle A$
D	$r \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	Point D
C	B+D	Point C

Part 2

The diagonals of a parallelogram bisect each other.

$$\mathbf{O} - \mathbf{A} = \mathbf{C} - \mathbf{O}$$

$$\mathbf{O} - \mathbf{D} = \mathbf{B} - \mathbf{O} \quad (2)$$

$$(\mathbf{C} - \mathbf{A}) = (\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{B}) \quad (3)$$

$$(\mathbf{B} - \mathbf{D}) = (\mathbf{B} - \mathbf{A}) - (\mathbf{C} - \mathbf{B}) \quad (4)$$

$$(\mathbf{O} - \mathbf{A}) = (\mathbf{C} - \mathbf{A})/2 \quad (5)$$

$$(\mathbf{O} - \mathbf{D}) = (\mathbf{B} - \mathbf{D})/2 \quad (6)$$

In parallelogram ABCD

1. consider $\triangle AOB$

Area of triangle is given as

$$\frac{1}{2} |(\mathbf{O} - \mathbf{A} \times \mathbf{B} - \mathbf{O})| \quad (7)$$

$$\frac{1}{2} |(\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{B})/2 \times (\mathbf{B} - \mathbf{A}) - (\mathbf{C} - \mathbf{B})/2| \quad (8)$$

$$\text{Area of } \triangle AOB = \frac{1}{4} |(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \quad (9)$$

2. consider $\triangle BOC$

Area of triangle is given as

$$\frac{1}{2} |(\mathbf{B} - \mathbf{O}) \times (\mathbf{C} - \mathbf{O})| \quad (10)$$

$$\frac{1}{2} |((\mathbf{B} - \mathbf{A}) - (\mathbf{C} - \mathbf{B}))/2 \times ((\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{B}))/2| \quad (11)$$

$$\frac{1}{4} |(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \quad (12)$$

$$\text{Area of } \triangle BOC = \frac{1}{4} |(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \quad (13)$$

$$(1) \quad 3. \text{ consider } \triangle COD$$

Area of triangle is given as

$$\frac{1}{2}|(\mathbf{C} - \mathbf{O}) \times (\mathbf{D} - \mathbf{O})| \quad (14)$$

$$\frac{1}{4}|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \quad (15)$$

$$\text{Area of } \triangle \text{COD} = \frac{1}{4}|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \quad (16)$$

4. consider $\triangle \text{DOA}$

Area of triangle is given as

$$\frac{1}{2}|(\mathbf{A} - \mathbf{O}) \times (\mathbf{D} - \mathbf{O})| \quad (17)$$

$$\frac{1}{2}|(-(\mathbf{B} - \mathbf{A}) - (\mathbf{C} - \mathbf{B}))/2 \times (-(\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{B}))/2| \quad (18)$$

$$\frac{1}{4}|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \quad (19)$$

$$\text{Area of } \triangle \text{AOD} = \frac{1}{4}|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \quad (20)$$

Hence from the above results we can conclude that

$$\begin{aligned} \text{ar}(\triangle \text{AOB}) &= \text{ar}(\triangle \text{BOC}) = \text{ar}(\triangle \text{COD}) = \text{ar}(\triangle \text{DOA}) \\ &= 1/4 * |\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{B}| \end{aligned}$$

Hence We proved that the diagonals of a parallelogram divide it into four triangles of equal area.