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Conic Assignment

Roll No. : FWC22063

Problem Statement:

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

SOLUTION:

Given:

Equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (1)$$

Equation of line is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad (2)$$

To Find

To find the intersection points and area of shaded region shown in figure

STEP-1

The given ellipse can be expressed as conics with parameters,

$$\mathbf{V} = \begin{pmatrix} b^2 & 0 \\ 0 & a^2 \end{pmatrix} \quad (3)$$

$$\mathbf{u} = 0 \quad (4)$$

$$f = -(a^2b^2) \quad (5)$$

STEP-2

the given line equation can be written as

$$\mathbf{x} = \begin{pmatrix} a \\ 0 \end{pmatrix} + k \begin{pmatrix} \frac{1}{b} \\ -\frac{1}{a} \end{pmatrix} \quad (6)$$

STEP-3

The points of intersection of the line,

$$L : \mathbf{x} = \mathbf{q} + \kappa \mathbf{m} \quad \kappa \in \mathbb{R} \quad (7)$$

with the conic section,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (8)$$

are given by

$$\mathbf{x}_i = \mathbf{q} + \kappa_i \mathbf{m} \quad (9)$$

where,

$$\kappa_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{q} + \mathbf{u})]^2 - (\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f) (\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) \quad (10)$$

On substituting

$$\mathbf{q} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (11)$$

$$\mathbf{m} = \begin{pmatrix} \frac{1}{b} \\ -\frac{1}{a} \end{pmatrix} \quad (12)$$

With the given ellipse as in eq(3),(4),(5),
The value of κ ,

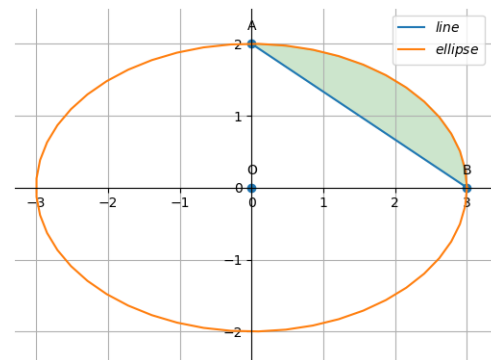
$$\kappa = 0, -6 \quad (13)$$

by substituting eq(13) in eq(6) we get the points of intersection of line with ellipse

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (14)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad (15)$$

Result



From the figure,

Total area of portion is given by,

Total Area=(area of ellipse in first quadrant)-(area of a triangle **AOB**)

Area of triangle

$$\Rightarrow A1 = \int_0^a \frac{b}{a}(a-x) dx \quad (16)$$

by solving the above equation we get area of triangle $\frac{ab}{2}$

Area of ellipse

$$\Rightarrow A2 = \int_0^a \frac{b}{a} \sqrt{a^2 - x^2} dx \quad (17)$$

Get the python code of the figures from

https://github.com/kkousar/KOUSAR_FWC/blob/main/conics/code/conic.py

by solving the above equation we get area of ellipse $\frac{\pi ab}{4}$
the total area is

$$\Rightarrow A = \frac{\pi ab}{4} - \frac{ab}{2}$$

The area of the smaller region is ,

$$A = \frac{ab}{2} \left(\frac{\pi}{2} - 1 \right) \quad (18)$$

Construction

Points	coordinates
B	$\begin{pmatrix} a \\ 0 \end{pmatrix}$
A	$\begin{pmatrix} 0 \\ b \end{pmatrix}$