ASSIGNMENT-MATRICES

1 PROBLEM

The circle

$$x^2 + y^2 - 4x - 4y + 4 = 0$$

is inscribed in a triangle which has two of its sides along the co-ordinate axes.The locus of the circumcenter of the triangle is

$$x + y - xy + k(x^2 + y^2)^{1/2} = 0$$

Find k.

2 SOLUTION

1. It is given that the two sides of the triangle are the coordinate axes and the third side is $\frac{x}{a} + \frac{y}{b} = 1$ can be represented as

$$\mathbf{n}^\top \mathbf{x} = C \tag{1}$$

$$\begin{pmatrix} \frac{1}{a} & \frac{1}{b} \end{pmatrix} \mathbf{x} = 1 \tag{2}$$

2.The circle equation is given as

$$x^2 + y^2 - 4x - 4y + 4 = 0$$

so the center of the circle will be

$$\mathbf{c} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{3}$$

3. The distance of the point c to the line is '2'.

$$\mathbf{d} = \frac{|\mathbf{n}^\top - c|}{\|\mathbf{n}\|} \tag{4}$$

from equation 2

$$\|\mathbf{n}\| = \sqrt{\mathbf{n}^\top \mathbf{n}}$$

$$\|\mathbf{n}\| = \sqrt{\begin{pmatrix} \frac{1}{a} & \frac{1}{b} \end{pmatrix} \begin{pmatrix} \frac{1}{a} \\ \frac{1}{b} \end{pmatrix}}$$

$$\|\mathbf{n}\| = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} \tag{5}$$

Distance of point $\mathbf{p}$ to the line(2) is

$$2 = \frac{\begin{pmatrix} \frac{1}{a} & \frac{1}{b} \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} - 1}{\|\mathbf{n}\|}$$

$$2 = \frac{\begin{pmatrix} \frac{1}{a} & \frac{1}{b} \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$$2\sqrt{\frac{a^2 + b^2}{ab^2}} = |\frac{2}{a} + \frac{2}{b} - 1|$$

$$\frac{(a^2 + b^2)^{1/2}}{ab} = -[\frac{2a + 2b - ab}{ab}]$$

$$2(a^2 + b^2)^{1/2} + 2a + 2b - ab = 0$$

$$2(a^2 + b^2)^{1/2} + 2a + 2b - ab = 0 \tag{6}$$

4. Since it is the right angled triangle consider the circumcenter to be $(\frac{a}{2}, \frac{b}{2})$ = (h,k).
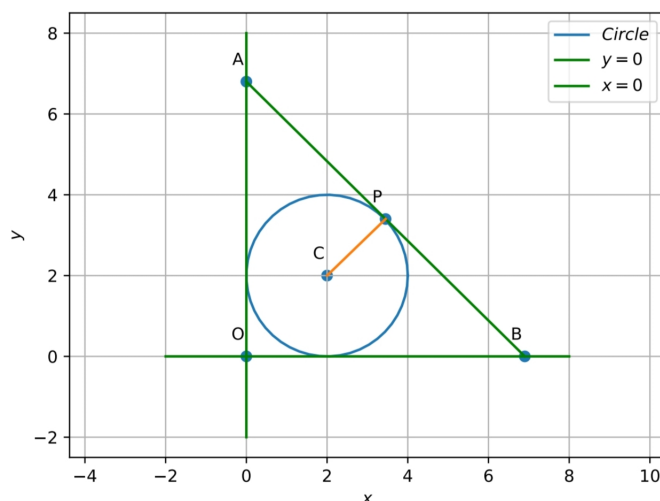a = 2h, b = 2k
substituting the a and b values in the eq(6)

$$2(4h^2 + 4k^2)^{1/2} + 4h + 4h - 4hk = 0 \tag{7}$$

$$h + k - hk + (h^2 + k^2)^{1/2} = 0 \tag{8}$$

5. Compare the given locus equation and the

equation 8 then the value of the k is 1.

Figure

3 Construction

The circle and triangle are constructed with,

Symbol	Co-ordinates	Description
r	2	radius
C	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$	center of the circle
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	origin
A	$\begin{pmatrix} 0 \\ b \end{pmatrix}$	point of intersection with the Y-axis
B	$\begin{pmatrix} 0 \\ a \end{pmatrix}$	point of intersection with the X-axis
P	$\frac{A+B}{2}$	circumcenter

The figure above is generated using python code provided in the below source code link.

https://github.com/sivagayathri
/FWC/blob/main/matrices/circles/cir.py