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Assignment-5

Roll No. : FWC22048

0.1 Problem Statement:

If the lines 2x+3y =5 and 3x-4y = 7 lie along diameter of a circle having area of 49π sq.units then the equation of the circle is.

0.2 SOLUTION:

Given:

Two line equations are

$$\mathbf{n_1}^\top \mathbf{x} = c_1 \tag{1}$$
$$\mathbf{n_2}^\top \mathbf{x} = c_2 \tag{2}$$

Above two equations are diameters of the circle.

We know that the diameters intersect at the **centre** of the circle.

So solving those two equations, we get the centre of the circle.

Let $\mathbf{x}$ be the centre of the circle.

$$\mathbf{x} = \begin{pmatrix} \mathbf{n_1} & \mathbf{n_2} \end{pmatrix}^{-\top} \mathbf{c} \tag{3}$$

where,

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} \tag{4}$$
$$\mathbf{x} = \begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix}^{-\top} \begin{pmatrix} 5 \\ 7 \end{pmatrix} \tag{5}$$

To Find

We can find the centre of the circle by solving the above equation through finding the inverse

From the above equation we get the centre of the circle i.e,

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{6}$$

STEP-1

Given that the area of the circle is 49πsq.units

$$\pi r^2 = 49\pi \tag{7}$$
$$\tag{8}$$
$$r = 7 \tag{9}$$

STEP-2

The general equation of the circle is given by,

$$\mathbf{X}^\top \mathbf{V} \mathbf{X} + 2\mathbf{u}^\top \mathbf{X} + f = 0 \tag{10}$$

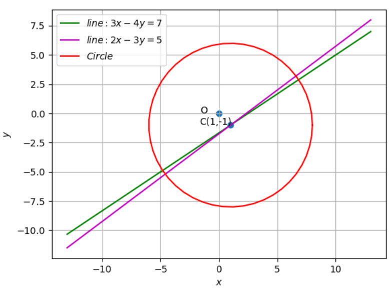
where,

$$f = \|\mathbf{u}\|^2 - r^2 = -47 \tag{11}$$
$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{12}$$
$$\mathbf{u} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{13}$$

Substituting all the values in the above equation, we get The final equatin of circle,

$$\mathbf{X}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{X} - 47 = 0 \tag{14}$$

0.3 Construction



Download the code

Github link:https://github.com/manasareddy/FWC.