circle Assignment

Dulla Srinivas - FWC22041

October 20, 2022

Problem Statement:

Two circles of radii 5cm and 3cm intersect at two points and the distance between their center is 4cm c1=(0,0) c2=(4,0) . Find the length of the common chord.
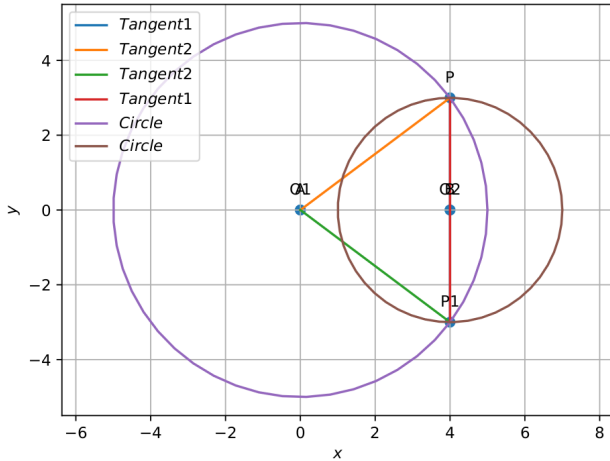
Solution:

Figure 1: Diagram generated using python

0.1 Theory:

They given two circles radius first circle radii is 5cm(Q1) and second circle radii is 3cm(Q2) distance between circle 1 and circle 2 is 4cm. we have find the length of the chord.

0.2 Mathematical Calculation:

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

0.3 Deriving equation for Circle in matrix form

The equation of circle in matrix form is,

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \qquad (1)$$

Where

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{f} = -25$$

$$\implies \mathbf{x}^T\mathbf{I}\,\mathbf{x} + 2\begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \mathbf{x} - 25 = 0$$

Therefore, the circle equation can be written as

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \mathbf{x} - 25 = 0 \qquad (2)$$

The equation of circle in matrix form is,

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \qquad (3)$$

Where

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \mathbf{f} = 7$$

$$\implies \mathbf{x}^T\mathbf{I}\,\mathbf{x} + 2\begin{pmatrix} -4 \\ 0 \end{pmatrix}^T \mathbf{x} + 7 = 0$$

Therefore, the circle equation can be written as

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} -4 \\ 0 \end{pmatrix}^T \mathbf{x} + 7 = 0 \qquad (4)$$

Here we have to Find the Intersection of Two conics

$$\mathbf{x}^T\mathbf{V_i}\mathbf{x} + 2\mathbf{u_i}^T\mathbf{x} + f_i = 0 \qquad (5)$$

The locus of their Intersection is a pair of straight lines

$$\begin{pmatrix} v_1 + \mu v_2 & u_1 + \mu u_2 \\ u_1 + \mu u_2 & (f_1 + f_2) \end{pmatrix}$$

Construction

Symbol	Value	Description
r_1	5	Radius
r_2	3	Radius
O	$(0,0)$	Center
O_1	$(4,0)$	Center
P	$(4,3)$	Point Of intersection
P_1	$(4,-3)$	Point Of intersection
P-P_1	6	Length of the common chord

$$\mathbf{x}^T(\mathbf{v_1} + \mu\mathbf{v_2})(f1 + f2) =$$
$$(u_1 + \mu u_2)(u1 + \mu u2)(6)\begin{pmatrix} 1 + \mu & 0 \\ 0 & 1 + \mu \end{pmatrix}$$
$$7\mu\begin{pmatrix} 4,0 \end{pmatrix} = \begin{pmatrix} -4\mu, 0 \end{pmatrix}\begin{pmatrix} -4\mu, 0 \end{pmatrix}$$

$$\begin{pmatrix} 7\mu(1+\mu) & 0 \\ 0 & 7\mu(1+\mu) \end{pmatrix} = \begin{pmatrix} 16\mu^2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mu = -1$$

0.4 According to the equation 6

$$2\left(-1\begin{pmatrix} -4 \\ 0 \end{pmatrix}x\right)\text{-25-7=0}$$

$$2\begin{pmatrix} 4,0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 32$$

$$8\text{x} = 32$$

$$\text{x=4}$$

So, p =(4,3) $p_1 = (4,-3)$

0.5 So The length of the common chord is 6cm

$$= ||p - p_1||$$

$$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$=6$$