MATRICES USING PYTHON(CONIC)

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Contents

1 Problem

The area bounded by the curve y=$x|x|$ x-axis and the ordinates x=-1 and x=1,is given by [Hint: y=x^2 if $x > 0$ and y=$-x^2$ if $x < 0$]

2 Construction

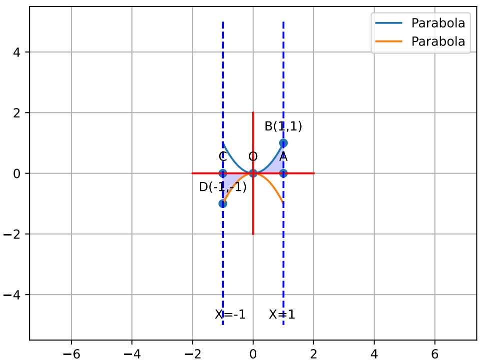

Figure of construction

3 Solution

Draw the ordinates by using x=1 and x=-1. Then we need to draw two parabolas using given hint [Hint: y=x^2 if $x > 0$ and y=$-x^2$ if $x < 0$]for that we need to find out the area bounded by the curve y=$x|x|$.

Then the limits from -1 to 1 and the points(-1,-1),(1,1)

The standard conic equation

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f_1 = 0 \tag{2}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u_1} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, f_1 = 0 \tag{3}$$

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u_2}^\top \mathbf{x} + f_2 = 0 \tag{4}$$

$$\mathbf{V} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u_2} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, f_2 = 0 \tag{5}$$

The intersection of two conics

$$\left| \begin{pmatrix} \mathbf{V_1} + \mu \mathbf{V_2} & \mathbf{u_1} + \mu \mathbf{u_2} \\ \mathbf{u_1} + \mu \mathbf{u_2} & 0 \end{pmatrix} \right| \tag{6}$$

substitute eq 3 and 4 in eq 6

$$\begin{pmatrix} 1-\mu & 0 & 0 \\ 0 & 0 & -\frac{1}{2} - \frac{\mu}{2} \\ 0 & -\frac{1}{2} - \frac{\mu}{2} & 0 \end{pmatrix} \tag{7}$$

by solving eq-7
yielding,

$$\mu^3 + \mu^2 - \mu - 1 = 0 \tag{8}$$

After solving eq-8
we get
$\mu = -1, 1, 1$

$$|\mathbf{V_1} + \mu \mathbf{V_2}| < 0 \tag{9}$$

substitute $\mathbf{V_1}$ and $\mathbf{V_2}$ in eq-9
we get 0

$$\mathbf{x} = \mathbf{q} + \mu \mathbf{m} \tag{10}$$

$$q = \mathbf{V^{-1}}(k\mathbf{n} - \mathbf{u}) \tag{11}$$

$$k = \pm\sqrt{\frac{\|\mathbf{u_2}\|^2 \mathbf{V} - f}{\mathbf{n}^\top \mathbf{V^{-1}} \mathbf{n}}} \tag{12}$$

$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
by solving eq 10 and 11 we get

$$\mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{13}$$

Given equation : y=$x|x|$
We know that

$$|x| = \begin{cases} x, & x \geq 0 \\ -x & x < 0 \end{cases} \tag{14}$$

Therefore,

$$y = x|x| = \begin{cases} xx, & x \geq 0 \\ x(-x) & x < 0 \end{cases} \tag{15}$$

$$y = \begin{cases} x^2, & x \geq 0 \\ -x^2 & x < 0 \end{cases} \tag{16}$$

Area Required=Area ABO+Area DCO
Area of DCO

Area :

$$\int_{-1}^{1} y\, dx$$

Here, y=$x|x|$

Therefore Area DCO:

$$\int_{-1}^{0} -x^2\, dx$$

yielding ,
-1/3
$|(-1/3)|$=1/3
Area of DCO= 1/3

Area of ABO:

$$\int_{0}^{1} x^2\, dx$$

yielding 1/3
Area of ABO= 1/3

Required Area=ABO+DCO: 1/3+1/3=2/3
Below python code realizes the above construction

https://github.com/Radhikarkv/fwcproject.git