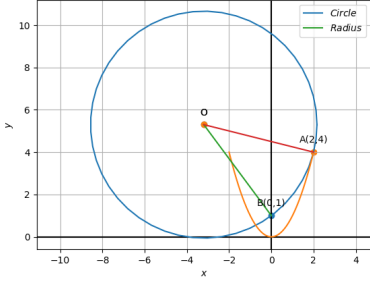


Assignment on circles

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I. QUESTION

Find the centre of the circle passing through the point (0,1) and touching the curve $y = x^2$ at the point (2,4).



$$\mathbf{m}^T \mathbf{u} = -\mathbf{m}^T \mathbf{A} \quad (10)$$

Where \mathbf{m} is the directional vector of tangent found from the equation of tangent $\mathbf{n}^T(\mathbf{x} - \mathbf{q}) = 0$ i.e $4x - y = 4$

\mathbf{n} is the normal vector of the curve $y = x^2$ given as $\mathbf{n} = \mathbf{V}\mathbf{q} + \mathbf{u}$

The directional vector is given as $\mathbf{m}^T = \text{omat} * \mathbf{n}$

$$\mathbf{m} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

from equations (4),(7) and (10),we can write as

$$\begin{pmatrix} \mathbf{m}^T & 0 \\ 2\mathbf{A}^T & 1 \\ 2\mathbf{B}^T & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -\mathbf{m}^T \mathbf{A} \\ -\|\mathbf{A}\|^2 \\ -\|\mathbf{B}\|^2 \end{pmatrix} \quad (11)$$

II. SOLUTION

The equation of the circle is

$$x^T \mathbf{V}x + 2\mathbf{u}^T x + f = 0 \quad (1)$$

Circle passes through $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and touches the curve $y = x^2$ at $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ Let $\mathbf{A} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\mathbf{A}\mathbf{A}^T + 2\mathbf{u}^T \mathbf{A} + f = 0 \quad (2)$$

$$\|\mathbf{A}\|^2 + 2\mathbf{A}^T \mathbf{u} + f = 0 \quad (3)$$

$$(2\mathbf{A}^T \quad 1) \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\|\mathbf{A}\|^2 \quad (4)$$

$$\mathbf{B}\mathbf{B}^T + 2\mathbf{u}^T \mathbf{B} + f = 0 \quad (5)$$

$$\|\mathbf{B}\|^2 + 2\mathbf{B}^T \mathbf{u} + f = 0 \quad (6)$$

$$(2\mathbf{B}^T \quad 1) \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\|\mathbf{B}\|^2 \quad (7)$$

The equation of the tangent is

$$\mathbf{m}^T (\mathbf{V}\mathbf{q} + \mathbf{u}) = 0 \quad (8)$$

$$\mathbf{m}^T \mathbf{A} + \mathbf{m}^T \mathbf{u} = 0 \quad (9)$$

$$\begin{pmatrix} 1 & 4 & 0 & -18 \\ 4 & 8 & 1 & -20 \\ 0 & 2 & 1 & -1 \end{pmatrix} \xrightarrow{R_1 \leftarrow -R_1} \begin{pmatrix} 1 & 4 & 0 & -18 \\ 1 & 2 & 1/4 & -5 \\ 0 & 2 & 1 & -1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/4} \begin{pmatrix} 1 & 4 & 0 & -18 \\ 0 & -2 & 1/4 & -13 \\ 0 & 2 & 1 & -1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 4 & 0 & -18 \\ 0 & 1 & -1/8 & -13/2 \\ 0 & 2 & 1 & -1 \end{pmatrix} \xrightarrow{R_2 \leftarrow -1/2 R_2} \begin{pmatrix} 1 & 4 & 0 & -18 \\ 0 & 1 & -1/8 & -13/2 \\ 0 & 1 & 1/2 & -1/2 \end{pmatrix} \xrightarrow{R_3 \leftarrow 1/2 R_3} \begin{pmatrix} 1 & 4 & 0 & -18 \\ 0 & 1 & -1/8 & -13/2 \\ 0 & 0 & 5/8 & 6 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 4 & 0 & -18 \\ 0 & 1 & -1/8 & -13/2 \\ 0 & 0 & 1 & 48/5 \end{pmatrix} \xrightarrow{R_3 \leftarrow 8/5 R_3} \begin{pmatrix} 1 & 4 & 0 & -18 \\ 0 & 1 & 0 & -53/10 \\ 0 & 0 & 1 & 48/5 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + 1/8 R_3} \begin{pmatrix} 1 & 4 & 0 & -18 \\ 0 & 1 & 0 & -53/10 \\ 0 & 0 & 1 & 48/5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 16/5 \\ 0 & 1 & 0 & -53/10 \\ 0 & 0 & 1 & 48/5 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - 4R_2}$$

By solving the above equations $\mathbf{u} = \begin{pmatrix} 16/5 \\ -53/10 \end{pmatrix}$

The center is $\mathbf{C} = -\mathbf{u}$ therefore $\mathbf{C} = \begin{pmatrix} -16/5 \\ 53/10 \end{pmatrix}$ and
 $f = 48/5$

Get the python code of the figures from

<https://github.com/Sireesha1602/sireesha/blob/main/circleassignment>