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Indian Institute of Technology Hyderabad

FWC22025

MATRICES-LINES

Contents

1 Problem

If E,F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that

$$\text{ar(EFGH)} = \tfrac{1}{2}\ \text{ar(ABCD)}$$

2 Solution

1. Construct a parallelogram with vertices A,B,C and D.

2. Point mid-points E,F,G and H on sides AB,BC,CD and DA.

$$\mathbf{E} = \frac{\mathbf{A+B}}{\mathbf{2}}$$
$$\mathbf{F} = \frac{\mathbf{B+C}}{\mathbf{2}}$$
$$\mathbf{G} = \frac{\mathbf{C+D}}{\mathbf{2}}$$
$$\mathbf{H} = \frac{\mathbf{D+A}}{\mathbf{2}}$$

3. By joining the midpoints of adjacent sides of parallelogram ABCD, another parallelogram EFGH is formed.

4. The area of parallelogram ABCD is given as,

$$\mathbf{ar(ABCD)} = \big(\mathbf{(A-B)x(A-D)}\big) \quad (1)$$
$$\mathbf{(A-B)} = \begin{pmatrix}\mathbf{10}\\ \mathbf{0}\end{pmatrix} \quad (2)$$
$$\mathbf{(A-D)} = \begin{pmatrix}\mathbf{1.25}\\ \mathbf{4.8}\end{pmatrix} \quad (3)$$
$$\big(\mathbf{(A-B)x(A-D)}\big) = \begin{vmatrix}\mathbf{10} & \mathbf{0}\\ \mathbf{1.25} & \mathbf{4.8}\end{vmatrix} \quad (4)$$

From (4),

$$\text{ar(ABCD)} = 48$$

5. The area of parallelogram EFGH is given as,

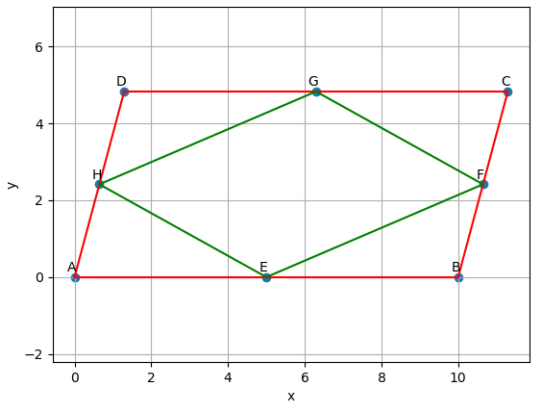
$$\mathbf{ar(EFGH)} = \big(\mathbf{(E-F)x(E-H)}\big) \quad (5)$$
$$\mathbf{(E-F)} = \begin{pmatrix}\mathbf{5.625}\\ \mathbf{2.4}\end{pmatrix} \quad (6)$$
$$\mathbf{(E-H)} = \begin{pmatrix}\mathbf{-4.375}\\ \mathbf{2.4}\end{pmatrix} \quad (7)$$
$$\big(\mathbf{(E-F)x(E-H)}\big) = \begin{vmatrix}\mathbf{5.625} & \mathbf{2.4}\\ \mathbf{-4.375} & \mathbf{2.4}\end{vmatrix} \quad (8)$$

From (8),

$$\text{ar(EFGH)} = 24$$

Hence,

$$\mathbf{ar(EFGH)} = \tfrac{1}{2}\ \mathbf{ar(ABCD)}$$



Figure

3 Construction

The parallelogram is constructed with l=10 and j=5,

Symbol	Co-ordinates	Description
l	10	AB
j	5	AD
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	point vector A
B	$\begin{pmatrix} l \\ 0 \end{pmatrix}$	point vector B
D	$\begin{pmatrix} j.cos(\theta) \\ j.sin(\theta) \end{pmatrix}$	point vector D
C	$\left(\mathbf{B}+\mathbf{D}\right)$	point vector C

The figure above is generated using python code provided in the below source code link.

https://github.com/madind5668/FWC/blob/main/matrices/lines/codes/main.py