Conic Assignment

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1 Problem Statement

Show that the locus of a point that divides a chord joining the points P and Q(0,0) of the parabola $y^2 = 4x$ internally in the ratio 1:3 is a parabola. Find the vertex of a parabola.

2 Solution

Let $\mathbf{X}$ be Locus point that divides the chord joining $\mathbf{P}$ and $\mathbf{Q}$ internally in the ratio 1:3.

The given equation of parabola $y^2 = 4x$ can be written in the general quadratic form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \tag{3}$$

$$f = 0 \tag{4}$$

By section formula the point that divides the line joining $\mathbf{P}$ and $\mathbf{Q}$ as $\mathbf{X}$ is:

$$\mathbf{X} = \frac{\mathbf{Q} + 3\mathbf{P}}{4} \tag{5}$$

$$\mathbf{P} = \frac{4\mathbf{X} - \mathbf{Q}}{3} \tag{6}$$

Substituting P in (1), we get

$$\mathbf{P^T V P} + 2\mathbf{u^T P} + f = 0 \tag{7}$$

Substituting (6) in (7), We get

$$\left(\frac{4\mathbf{X} - \mathbf{Q}}{3}\right)^\top \mathbf{V}\left(\frac{4\mathbf{X} - \mathbf{Q}}{3}\right) \ + \ 2\mathbf{u}^\top\left(\frac{4\mathbf{X} - \mathbf{Q}}{3}\right) \ = \ 0 \tag{8}$$

$$\frac{1}{9}(4\mathbf{X}^\top \mathbf{V}4\mathbf{X} - 4\mathbf{X}^\top \mathbf{V}\mathbf{Q} - \mathbf{Q}^\top \mathbf{V}4\mathbf{X} + \mathbf{Q}^\top \mathbf{V}\mathbf{Q}) + \frac{2}{3}\mathbf{u}^\top 4\mathbf{X} - \frac{2}{3}\mathbf{u}^\top \mathbf{Q} \tag{9}$$

$$\frac{16}{9}\mathbf{X}^\top \mathbf{V}\mathbf{X} - \frac{4}{9}\mathbf{X}^\top \mathbf{V}\mathbf{Q} - \frac{4}{9}\mathbf{Q}^\top \mathbf{V}\mathbf{X} + \frac{1}{9}\mathbf{Q}^\top \mathbf{V}\mathbf{Q} + \frac{8}{3}\mathbf{u}^\top \mathbf{X} - \frac{2}{3}\mathbf{u}^\top \mathbf{Q} = 0 \tag{10}$$

$$\frac{16}{9}\mathbf{X}^\top \mathbf{V}\mathbf{X} + \frac{8}{3}\mathbf{u}^\top \mathbf{X} = 0$$

$$\mathbf{X}^\top \mathbf{V}\mathbf{X} + 2\mathbf{u'}^\top \mathbf{X} = 0 \tag{11}$$

where

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{12}$$

$$\mathbf{u'} = \begin{pmatrix} -3/2 \\ 0 \end{pmatrix} \tag{13}$$

$$\tag{14}$$

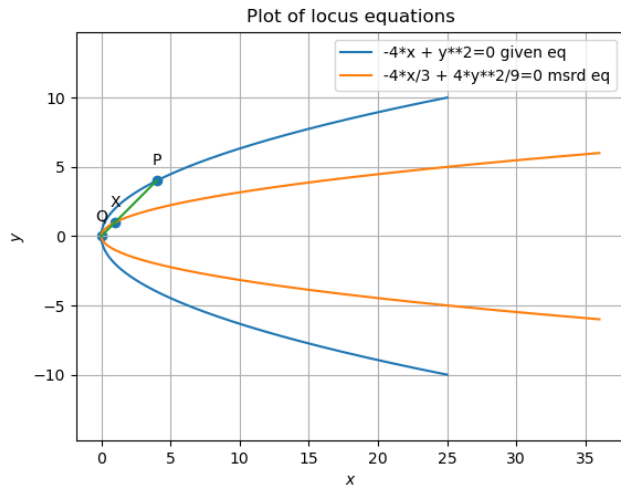| Symbol	Value	Description
$\mathbf{Q}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	given point
$\mathbf{P}$	$\begin{pmatrix} x \\ y \end{pmatrix}$	point on given locus
$\mathbf{X}$	$\begin{pmatrix} x \\ y \end{pmatrix}$	divides in the ratio 1:3 $\mathbf{PQ}$

Table 1: Parameters

Figure 1: Found the locus equation