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## Circle Assignment

For circle 2

 $\mathbf{r_2}^2 = \|\mathbf{U_2}\|^2 - \mathbf{f_2} \tag{9}$ 

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By solving the equations (6) and (7)

By substituting (a,b) in equation (13)

$$\mathbf{f_2} = \mathbf{p}^2 \tag{10}$$

## **Problem Statement:**

If a circle passes through the point (a,b) and cuts the circle  $x^2 + y^2 = p^2$  orthogonally, then the equation of the locus of its centre is

## Construction

Symbol	Value	Description
$\mathrm{U}_1$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	center of given circle
$\mathbf{r_1}$	2	radius of given circle
$\begin{pmatrix} a & b \end{pmatrix}$	$\begin{pmatrix} 1 & 2 \end{pmatrix}$	point on circle

**Solution:** With the given circle equation  $x^2 + y^2 = p^2$ , we can find out centre  $U_1$  and radius  $r_1$  of Circle-1

$$\mathbf{U_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

Radius of Circle-1,

Centre of Circle-1,

 $\mathbf{r_1} = \mathbf{p}$  (2) The locus is

The general form of conic is

$$\mathbf{X}^{\mathsf{T}}\mathbf{V}\mathbf{X} + 2\mathbf{U}^{\mathsf{T}}\mathbf{X} + \mathbf{f} = 0 \tag{3}$$

For circle 1

$$\mathbf{X}^{\mathsf{T}}\mathbf{V}_{\mathbf{1}}\mathbf{X} + 2\mathbf{U}_{\mathbf{1}}^{\mathsf{T}}\mathbf{X} + \mathbf{f}_{\mathbf{1}} = 0 \tag{4}$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + f1 = 0 \qquad (5)$$

as circles are orthogonal

$$\mathbf{r_1}^2 + \mathbf{r_2}^2 = \|\mathbf{U_1} - \mathbf{U_2}\|^2$$
 (7)

$$||U_1||^2 + ||U_2||^2 - 2\mathbf{U_1}^{\mathsf{T}}\mathbf{U_2} = \mathbf{p}^2 + \mathbf{r_2}^2$$
 (8)

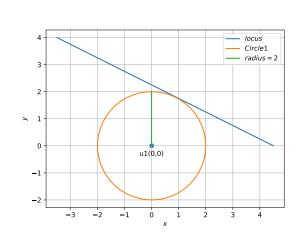
$$\mathbf{X}^{\top}\mathbf{V_2}\mathbf{X} + 2\mathbf{U_2}^{\top}\mathbf{X} + \mathbf{f_2} = 0 \tag{11}$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} -g & -t \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + p^2 = 0 \quad (12)$$

$$\mathbf{x}^2 + \mathbf{y}^2 - 2\mathbf{g}\mathbf{x} - 2\mathbf{t}\mathbf{y} + \mathbf{p}^2 = 0 \tag{13}$$

$$\mathbf{a}^2 + \mathbf{b}^2 - 2\mathbf{g}\mathbf{a} - 2\mathbf{t}\mathbf{b} + \mathbf{p}^2 = 0 \tag{14}$$

$$2ga + 2tb - (a^{2} + b^{2} + p^{2}) = 0$$
 (15)



(6)