Conic Assignment

Anusha Jella

October 10, 2022

Problem: Find the equation of all lines having slope – 1 that are tangents to the curve $\mathbf{y} = \frac{1}{\mathbf{x-1}}$, $\mathbf{x} \neq \mathbf{1}$

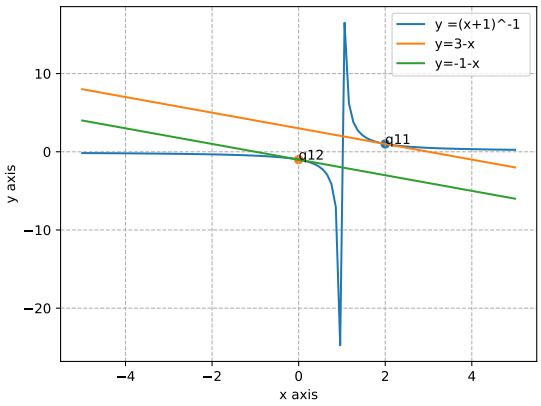

Fig 1. Curve

Construction

SOLUTION: The following python code is used for constructing conic with tangents.

https://github.com/AnushaJella/assignment_conic/blob/main/conic1.py

See Fig 1 for the input parameters in Table 1.

Symbol	Value	Description
x	[-5,5,100]	to find **y**

Table 1

Solution

The equation of a conic with directrix $\mathbf{n}^\top\mathbf{x} = c$, eccentricity e and focus $\mathbf{F}$ is given by

$$\mathbf{x}^\top\mathbf{V}\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \|\mathbf{n}\|^2\mathbf{I} - e^2\mathbf{n}\mathbf{n}^\top, \tag{2}$$
$$\mathbf{u} = ce^2\mathbf{n} - \|\mathbf{n}\|^2\mathbf{F}, \tag{3}$$
$$f = \|\mathbf{n}\|^2\|\mathbf{F}\|^2 - c^2e^2 \tag{4}$$

Given,

$$\mathbf{V} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix} f = -1, m = -1 \tag{5}$$

$$n = \begin{pmatrix} -m \\ 1 \end{pmatrix}$$
$$q = \mathbf{V}^{-1}(k_i\mathbf{n} - \mathbf{u}^T)^T$$
$$where,$$
$$k_i = \pm\sqrt{\frac{f_0}{\mathbf{n}^T\mathbf{V}^{-1}\mathbf{n}}}$$
$$f_0 = f + \mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}$$

by substituting (5) in above eqtions we get q=$(0,-1)$ and $(2,1)$.
Then equation of tangent is:

$$(\mathbf{V}\mathbf{q} + \mathbf{u})^T\mathbf{x} + \mathbf{u}^T\mathbf{q} + f = 0 \tag{6}$$

here 2 lines are possible with slope -1 that are tangent to given line equation.
They are

$$\mathbf{x} + \mathbf{y} + 1 = 0$$
$$\mathbf{x} + \mathbf{y} - 3 = 0$$

2