CIRCLE ASSIGNMENT

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Contents

1 Problem

Let C be the circle with centre $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and radius 3 units. Find the equation of the locus of the mid-points of the chords which subtend an angle of $\frac{2\pi}{3}$ at its center.

2 Construction

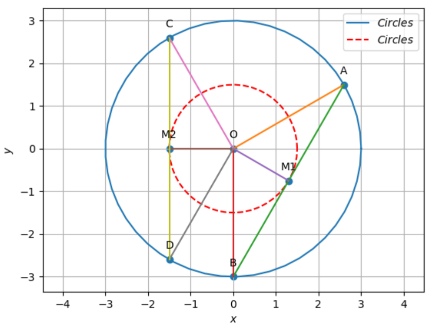

Figure of construction

3 Solution

Circle equation : $x^2 + y^2 = 9$
The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

The given circle can be expressed as conics with parameters

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = -\begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -9 \tag{2}$$

Radius and Centre are

$$r = \sqrt{\mathbf{u}^\top \mathbf{u} - f}, \mathbf{O} = -u \tag{3}$$

$$r = 3 \tag{4}$$

Angle between A and B is cos$\theta = \frac{(\mathbf{A})^\top \mathbf{B}}{||A||||B||}$
cos$120° = \frac{(\mathbf{A})^\top \mathbf{B}}{9}$

$$(\mathbf{A})^\top \mathbf{B} = \frac{-9}{2} \tag{5}$$

Let $\mathbf{R}$ is the rotation matrix of given circle

$$\mathbf{R} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{6}$$

Let $\mathbf{B}$ be the another end point of chord

$$\mathbf{B} = \mathbf{R}\mathbf{A} \tag{7}$$

Let $\mathbf{M}$ be th mid point of chord of the circle

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{8}$$

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{R}\mathbf{A}}{2} \tag{9}$$

$$\mathbf{M} = \frac{\mathbf{A}(\mathbf{I} + \mathbf{R})}{2} \tag{10}$$

$$\mathbf{A} = \mathbf{2}[\mathbf{I} + \mathbf{R}]^{-\mathbf{1}}\mathbf{M} \tag{11}$$

STEPS TO FIND THE LOCUS OF THE MIDPOINT OF CHORD OF THE CIRCLE:
By substituting A value in quadratic form of the circle we get

$$(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}\mathbf{M})^\top(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}\mathbf{M}) + 2(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}\mathbf{M})\begin{pmatrix} 0 & 0 \end{pmatrix} + f = 0 \tag{12}$$

$$(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}\mathbf{M})^\top(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}\mathbf{M}) + f = 0 \tag{13}$$

$$\left\|(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-\mathbf{1}}\mathbf{M})\right\|^2 + f = 0 \tag{14}$$

$$(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}\mathbf{M})^\top(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}\mathbf{M}) + f = 0 \tag{15}$$

$$(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1})^\top(\mathbf{M})^\top\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}\mathbf{M} + f = 0 \tag{16}$$

$$(\mathbf{M})^\top(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1})^\top\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}\mathbf{M}+f=0 \tag{17}$$

Let

$$\mathbf{V}=(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1})^\top\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1} \tag{18}$$

Where

$$\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1}=\begin{pmatrix}1 & 1.72\\ -1.72 & 1\end{pmatrix} \tag{19}$$

$$(\mathbf{2}(\mathbf{I}+\mathbf{R})^{-1})^\top=\begin{pmatrix}1 & -1.72\\ 1.72 & 1\end{pmatrix} \tag{20}$$

By solving this we get

$$\mathbf{V}=I \tag{21}$$

FINALLY THE LOCUS OF MIDPOINT OF CHORD OF THE GIVEN CIRCLE IS:

$$\mathbf{M}^\top\mathbf{V}\mathbf{M}+f=0 \tag{22}$$

where

$$\mathbf{V}=\mathbf{I}, f=-9 \tag{23}$$

Radius

$$r=\sqrt{-f}=\sqrt{3} \tag{24}$$

termux commands :

bash sh2.sh............using shell command

Below python code realizes the above construction :

https://github.com/kedareswari200/fwc-module1/blob/Matrix1/cir.py