Line Assignment

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Contents

I Problem 1

II Construction 1

III Solution 1

I. Problem

In figure below, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (BPC) = ar (DPQ). [Hint : Join AC.]

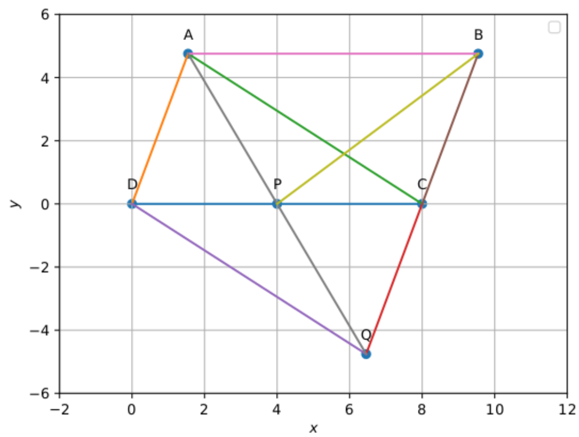

Figure of Construction

II. Construction

Symbol	Value	Description
r1	5	DA
r2	8	DC
θ_1	$2\pi/5$	$\angle$ADC

$$\mathbf{B} = \mathbf{A} + \mathbf{C} - \mathbf{D} \tag{1}$$

$$\mathbf{Q} = \mathbf{D} + \mathbf{C} - \mathbf{A} \tag{2}$$

$$\mathbf{P} = (\mathbf{D} + \mathbf{C})/2 \tag{3}$$

III. Solution

Construction: Join AC

To Prove: ar(Δ BPC) = ar(Δ DPQ)

We need to prove that ar(BPC)=ar(DPQ)

Given BC is extended to Q, so

$$\mathbf{AD} \,||\, \mathbf{CQ} \tag{4}$$

and given

$$\mathbf{AD} = \mathbf{CQ} \tag{5}$$

from (4) and (5), ACQD is a parallelogram

In ABCD,

$$\mathbf{A} - \mathbf{B} = \mathbf{D} - \mathbf{C} \tag{6}$$

$$\mathbf{A} - \mathbf{D} = \mathbf{B} - \mathbf{C} \tag{7}$$

In ACQD,

$$\mathbf{A} - \mathbf{C} = \mathbf{D} - \mathbf{Q} \tag{8}$$

$$\mathbf{A} - \mathbf{D} = \mathbf{C} - \mathbf{Q} \tag{9}$$

Area of triangle BPC

$$= \mathbf{1/2x} \left\| (\mathbf{P} - \mathbf{C}) \times (\mathbf{B} - \mathbf{C}) \right\| \tag{10}$$

Substituiting (3) in (10),

$$= \mathbf{1/2x} \left\| (\frac{\mathbf{D} - \mathbf{C}}{2}) \times (\mathbf{B} - \mathbf{C}) \right\| \tag{11}$$

$$= \mathbf{1/4x} \left\| (\mathbf{D} - \mathbf{C}) \times (\mathbf{B} - \mathbf{C}) \right\| \tag{12}$$

Area of triangle DPQ

$$= \mathbf{1/2x} \left\| (\mathbf{D} - \mathbf{P}) \times (\mathbf{Q} - \mathbf{D}) \right\| \tag{13}$$

Substituiting (3) in (13),

$$= \mathbf{1/2x} \left\| (\frac{\mathbf{D} - \mathbf{C}}{2}) \times (\mathbf{Q} - \mathbf{D}) \right\| \tag{14}$$

from (6),

$$\mathbf{C} - \mathbf{A} = \mathbf{Q} - \mathbf{D} \tag{15}$$

And from Δ ABC,

$$\mathbf{C} - \mathbf{A} = \mathbf{A} - \mathbf{B} + \mathbf{B} - \mathbf{C} \tag{16}$$

Substituiting (15) and (16) in (14),

$$= \mathbf{1/4x} \left\|(\mathbf{D} - \mathbf{C}) \times ((\mathbf{A} - \mathbf{B}) + (\mathbf{B} - \mathbf{C}))\right\| \tag{17}$$

$$= \mathbf{1/4x} \left\|((\mathbf{D} - \mathbf{C}) \times (\mathbf{A} - \mathbf{B})) + ((\mathbf{D} - \mathbf{C}) X (\mathbf{B} - \mathbf{C}))\right\| \tag{18}$$

Substituiting (6) in (18),

$$= \mathbf{1/4x} \left\|((\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{B})) + ((\mathbf{D} - \mathbf{C}) X (\mathbf{B} - \mathbf{C}))\right\| \tag{19}$$

$$= \mathbf{1/4x} \left\|(\mathbf{D} - \mathbf{C}) \times (\mathbf{B} - \mathbf{C})\right\| \tag{20}$$

from (12) and (20),

$$\mathbf{ar}(\boldsymbol{\Delta}\mathbf{BPC}) = \mathbf{ar}(\boldsymbol{\Delta}\mathbf{DPQ})$$

hence proved