MATRIX ANALYSIS USING PYTHON

DUDEKULA USENI
r171099@rguktrkv.ac.in
IITH Future Wireless Communication (FWC)

FWC22098 Assignment- Conics

October 21, 2022

Contents

1 **Problem** 1

2 **Construction** 1

3 **Solution** 1

4 **Software** 2

1 Problem

A hyperbola, having the transverse axis of length $\mathbf{2sin\theta}$, is confocal with the ellipse $3x^2 + 4y^2 = 12$, then its equation is

2 Construction

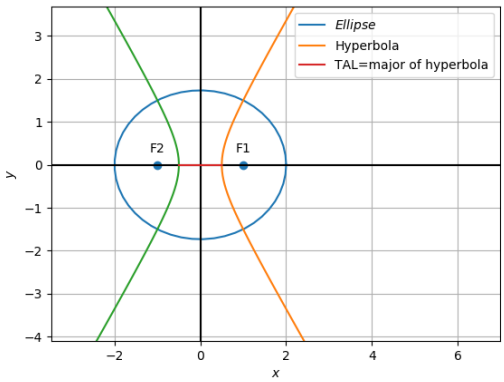

Figure of construction

3 Solution

Ellipse equation :

$$\mathbf{3x^2 + 4y^2 = 12} \tag{1}$$

The standard equation of the conics is given as :

$$\mathbf{x^\top V x + 2u^\top x + f = 0} \tag{2}$$

The given ellipse can be expressed as conics with parameters

$$\lambda_1 = 3, \lambda_2 = 4 \tag{3}$$

$$\mathbf{V} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -12 \tag{4}$$

Eccentricity:

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \tag{5}$$

$$\implies e = 1/2 \tag{6}$$

Foci:

$$f_0 = -f, \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{7}$$

$$\mathbf{F}_1 = e\sqrt{\frac{|f_0|}{\lambda_2 (1 - e^2)}}\mathbf{e}_1 \tag{8}$$

$$\mathbf{F}_2 = -e\sqrt{\frac{|f_0|}{\lambda_2 (1 - e^2)}}\mathbf{e}_1 \tag{9}$$

$$\implies \mathbf{F}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{F}_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{10}$$

Hyperbola equation :

The standard equation of the conics is given as :

$$\mathbf{x^\top V_1 x + 2u_1^\top x + f_1 = 0} \tag{11}$$

The given Hyperbola can be expressed as conics with parameters

$$\mathbf{V}_1 = \begin{pmatrix} \lambda_3 & 0 \\ 0 & \lambda_4 \end{pmatrix}, \mathbf{u}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{12}$$

Transverse axis length is : $\mathbf{2sin\theta}$
It is a distance between two vertices of hyperbola and also major axis of Hyperbola now we take semi major axis of hyperobla :

$$\mathbf{sin\theta} = \sqrt{\frac{\mathbf{u_1}^\top \mathbf{v_1}^{-1} \mathbf{u_1} - f_1}{\lambda_3}} \tag{13}$$

where b is a minor axis of Hyperbola

$$\mathbf{b} = \sqrt{\frac{f_1 - \mathbf{u_1}^\top \mathbf{V_1}^{-1} \mathbf{u_1}}{\lambda_4}} \tag{14}$$

$$\lambda_3 = \frac{1}{sin^2\theta}, \lambda_4 = -\frac{1}{b^2}, f_1 = -1 \tag{15}$$

$$\frac{x^2}{sin^2\theta} - \frac{y^2}{b^2} = 1 \tag{16}$$

Eccentricity:

$$e = \sqrt{1 - \frac{\lambda_3}{\lambda_4}} \tag{17}$$

$$e = \sqrt{1 + \frac{b^2}{sin^2\theta}} \tag{18}$$

Foci:

$$f_0 = -f_1, \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{19}$$

$$\mathbf{F}_4 = e\sqrt{\frac{|f_0|}{\lambda_4 (1 - e^2)}}\mathbf{e}_1 \tag{20}$$

$$\mathbf{F}_5 = -e\sqrt{\frac{|f_0|}{\lambda_4 (1 - e^2)}}\mathbf{e}_1 \tag{21}$$

Then equate the $\mathbf{F}_1 = \mathbf{F}_4$we get the b^2

$$b^2 = 1 - sin^2\theta \tag{22}$$

$$b^2 = cos^2\theta \tag{23}$$

There fore equation of hyperbola is :

$$\frac{x^2}{sin^2\theta} - \frac{y^2}{cos^2\theta} = 1 \tag{24}$$

4 Software

Below python code realizes the above construction :
https://github.com/dudekulauseni123/FWC0982022