

Matrix Problems

Straight Lines

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I. PROBLEM STATEMENT

The base of an equilateral triangle with side $2a$ lies along the y-axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

II. SOLUTION

Given ABC is an equilateral triangle i.e

$$AB = BC = CA \quad (1)$$

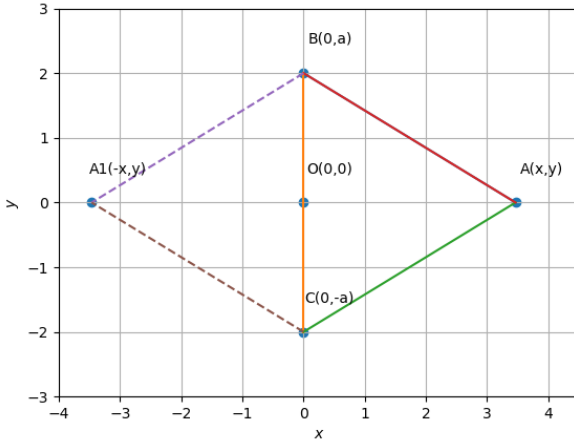


Fig. 1: Equilateral Triangle ABC

Since base with $2a$ is lies on the y-axis with the mid-point of the base is at origin. The vertices of the two points on y-axis will be

$$\mathbf{B} = \begin{pmatrix} 0 \\ a \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ -a \end{pmatrix} \quad (2)$$

The distance between the two points B and A is

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 0 - x \\ a - y \end{pmatrix} \quad (3)$$

Using the definition of the norm,

$$\|\mathbf{B} - \mathbf{A}\| = \left\| \begin{pmatrix} -x \\ a - y \end{pmatrix} \right\| \quad (4)$$

Since, the side of an equilateral triangle is $2a$

$$2a = \sqrt{\begin{pmatrix} -x & a - y \end{pmatrix} \begin{pmatrix} -x \\ a - y \end{pmatrix}} \quad (5)$$

$$2a = \sqrt{(x)^2 + (a - y)^2} \quad (6)$$

Squaring on both sides

$$4a^2 = (x)^2 + (a - y)^2 \quad (7)$$

$$4a^2 = x^2 + a^2 + y^2 - 2ay \quad (8)$$

$$3a^2 = x^2 + y^2 - 2ay \quad (9)$$

Similarly, The distance between the two points C and A is

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 - x \\ -a - y \end{pmatrix} \quad (10)$$

Using the definition of the norm,

$$\|\mathbf{C} - \mathbf{A}\| = \left\| \begin{pmatrix} -x \\ -a - y \end{pmatrix} \right\| \quad (11)$$

Since, the side of an equilateral triangle is $2a$

$$2a = \sqrt{\begin{pmatrix} -x & -a - y \end{pmatrix} \begin{pmatrix} -x \\ -a - y \end{pmatrix}} \quad (12)$$

$$2a = \sqrt{(x)^2 + (a + y)^2} \quad (13)$$

Squaring on both sides

$$4a^2 = (x)^2 + (a + y)^2 \quad (14)$$

$$4a^2 = x^2 + a^2 + y^2 + 2ay \quad (15)$$

$$3a^2 = x^2 + y^2 + 2ay \quad (16)$$

Using equation (9) and (16),

Hence, the coordinates of the vertices of triangle are

$$\mathbf{A} = \begin{pmatrix} \pm\sqrt{3}a \\ 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ a \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ -a \end{pmatrix} \quad (17)$$

III. CONSTRUCTION

B and C are the inputs.

Symbol	Value	Description
B	(0, 2)	Vertex B
C	(0, -2)	Vertex C
A	(x,y)	Vertex A
A1	(x1, y1)	Vertex A1

Get Python Code for image from

<https://github.com/ManojChavva/FWC/blob/main/Matrix/line/code-py/triangle.py>