MATRICES

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Contents

1 Problem

Q.

$$x^2 + 4y^2 = 4 \tag{1}$$

is the equation of ellipse which is inscribed in a rectangular aligned with the coordinate axes,which is turn is inscibed in another ellipse that passes through the point (4,0).Then the equation of ellipse

2 Solution

Given, the equation of ellipse is

$$x^2 + 4y^2 = 4 \tag{2}$$

and point passing through another ellipse

$$\mathbf{Q}=\begin{pmatrix}4\\0\end{pmatrix}$$

$$x^2/4 + y^2 = 1 \tag{3}$$

$$\lambda_1 = 1/4 \lambda_2 = 1 \tag{4}$$

hence a=2 and b=1
the standard equation of ellipse using matrices can be written as,

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$$\mathbf{x}^\top \mathbf{V} \mathbf{x} = 1 \tag{6}$$

where

$$\mathbf{V} = \begin{pmatrix}\lambda 1 & 0\\0 & \lambda 2\end{pmatrix} \tag{7}$$

$$\mathbf{V} = \begin{pmatrix}1/4 & 0\\0 & 1\end{pmatrix} \tag{8}$$

$$\mathbf{x}^\top \begin{pmatrix}1/4 & 0\\0 & 1\end{pmatrix} \mathbf{x} = 1 \tag{9}$$

the equation of tangents that passing through $\mathbf{l}=(a,0)$ and $\mathbf{m}=(0,b)$ is

$$(\mathbf{V}\mathbf{l} + \mathbf{u})^\top \mathbf{X} + \mathbf{u}^\top \mathbf{l} + f = 0 \tag{10}$$

$$(\mathbf{V}\mathbf{m} + \mathbf{u})^\top \mathbf{X} + \mathbf{u}^\top \mathbf{m} + f = 0 \tag{11}$$

as $\mathbf{u}=\begin{pmatrix}0\\0\end{pmatrix}$ and f=-1 we get

$$\begin{pmatrix}(\mathbf{V}\mathbf{l})^\top\\(\mathbf{V}\mathbf{m})^\top\end{pmatrix} \mathbf{X} = \begin{pmatrix}1\\1\end{pmatrix} \tag{12}$$

by solving them we get

$$\begin{pmatrix}1/2 & 0 & 1\\0 & 1 & 1\end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 * 2} \begin{pmatrix}1 & 0 & 2\\0 & 1 & 1\end{pmatrix}$$

point of intersection of two tangents is

$$\mathbf{P}=\begin{pmatrix}2\\1\end{pmatrix}$$

from the given information **p** touches the second ellipse and **Q** is also passing through it.so we can write the equaion od ellipse as

$$\mathbf{P}^\top \mathbf{V} \mathbf{P} = 1 \tag{13}$$

$$\mathbf{Q}^\top \mathbf{V} \mathbf{Q} = 1 \tag{14}$$

we can write them as

$$\mathbf{P}^\top \mathbf{vp} = 1 \tag{15}$$

$$\mathbf{Q}^\top \mathbf{vq} = 1 \tag{16}$$

as

$$\mathbf{v} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \ \mathbf{p} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \ \mathbf{q} = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \tag{17}$$

the matrix form of above equations is

$$\begin{pmatrix} \mathbf{P}^\top \mathbf{p} \\ \mathbf{Q}^\top \mathbf{q} \end{pmatrix} \mathbf{v} = 1 \tag{18}$$

substituting appropriate values we get

(19)

$$\begin{pmatrix} 4 & 1 \\ 16 & 0 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{20}$$

solving the above equation

$$\begin{pmatrix} 4 & 1 & 1 \\ 16 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1/4} \tag{21}$$

$$\begin{pmatrix} 1 & 1/4 & 1/4 \\ 16 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow -16R_1 + R_2} \tag{22}$$

$$\begin{pmatrix} 1 & 1/4 & 1/4 \\ 0 & -4 & -3 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/-4} \tag{23}$$

$$\begin{pmatrix} 1 & 1/4 & 1/4 \\ 0 & 1 & 3/4 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - R_2/4} \tag{24}$$

$$\begin{pmatrix} 1 & 0 & 1/16 \\ 0 & 1 & 3/4 \end{pmatrix} \tag{25}$$

By solving the above equations we get

$$\lambda_1 = 1/16 \ and \ \lambda_2 = 3/4 \tag{26}$$

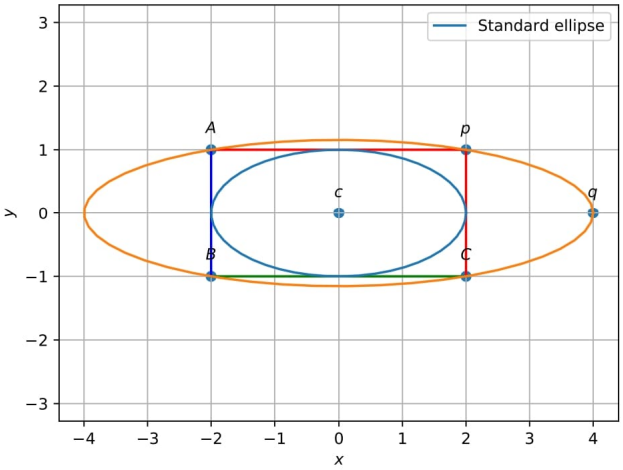
the required equation of ellipse is

$$\mathbf{x}^\top \begin{pmatrix} 1/16 & 0 \\ 0 & 3/4 \end{pmatrix} \mathbf{x} = 1 \tag{27}$$

we get

$$x^2/16 + 3y^2/4 = 1 \tag{28}$$

3 Plot



4 Software

We can plot the cicle with the help of the following code :

https://github.com/Gowt-hami/fwc-1-module1/blob/main/par.py