Matrix Assignment - Circle

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CONTENTS

I. PROBLEM

The Circles x^2 + y^2 - 10x +16 = 0 and x^2 + y^2 = r^2 intersect each other at two distinct points if .
A) $r < 2$
B) $r > 8$
C) $2 < r < 8$
D) $2 \leq r \leq 8$

II. SOLUTION

Given equation of circle 1 : $x^2 + y^2 - 10x + 16 = 0$
The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

The given circle 1 can be expressed as conics with parameters

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = -\begin{pmatrix} 5 \\ 0 \end{pmatrix}, f = 16 \tag{2}$$

Radius and Centre are

$$r = \sqrt{\mathbf{u}^\top \mathbf{u} - f}, \tag{3}$$

$$r = \sqrt{25 - 16} = 3 \tag{4}$$

$$Center = \mathbf{A} = -u = (5, 0) \tag{5}$$

Given equation of circle 2 : $x^2 + y^2 = r^2$
The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{6}$$

The given circle 1 can be expressed as conics with parameters

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = -\begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = 0 \tag{7}$$

Radius and Centre are

$$r = \sqrt{\mathbf{u}^\top \mathbf{u} - f}, \tag{8}$$

$$Radius = r \tag{9}$$

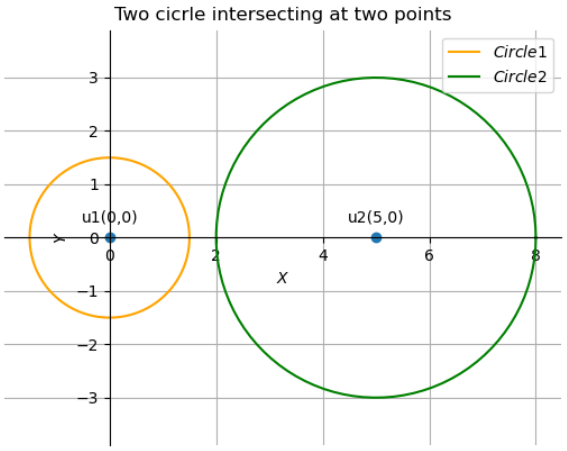
$$Center = \mathbf{A} = -u = (0, 0) \tag{10}$$

III. CONDITIONS

Here we will vary the radius of circle 1 and one by one we will verify all the given options

A. Conditon 1

When Radius r is less than 2 i.e $(r < 2)$ here we have taken radius r = 1.5

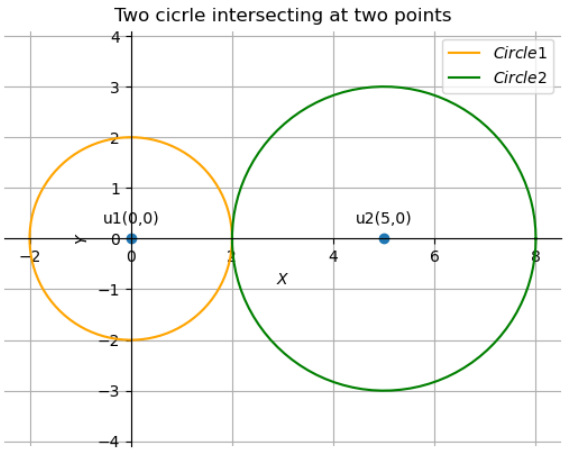


By watching the above image , we can conclude that ,the Circle 1 and Circle 2 will not intersect each other at any point when $r < 2$

Conclusion 1 : option A is discarded as the Circle 1 and Circle 2 will not intersect each other at any point when $r < 2$

B. Conditon 2

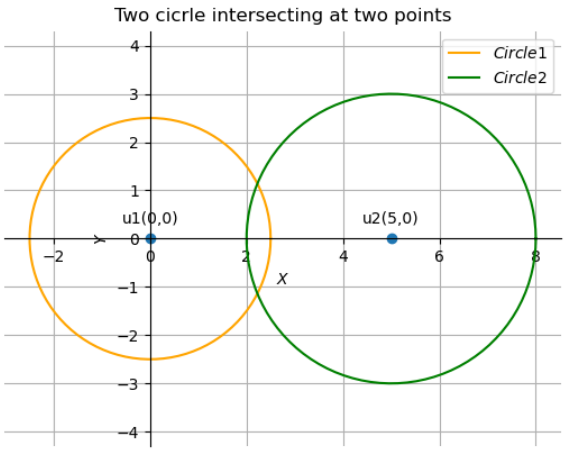
When Radius r is equal to 2 i.e $(r = 2)$
here we have taken radius r = 2



By watching the above image , we can conclude that ,the Circle 1 and Circle 2 will only touch each other but not intersect each other at any point when $r = 2$, So option **D** is also discarded

C. Conditon 3

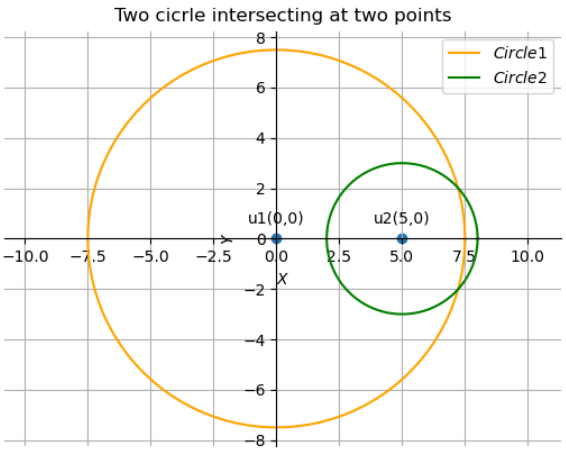
When Radius r is greater than 2 i.e $(r > 2)$
here we have taken radius r = 2.5



By watching the above image , we can conclude that ,the Circle 1 and Circle 2 will intersect each other at two point when $r > 2$

D. Conditon 4

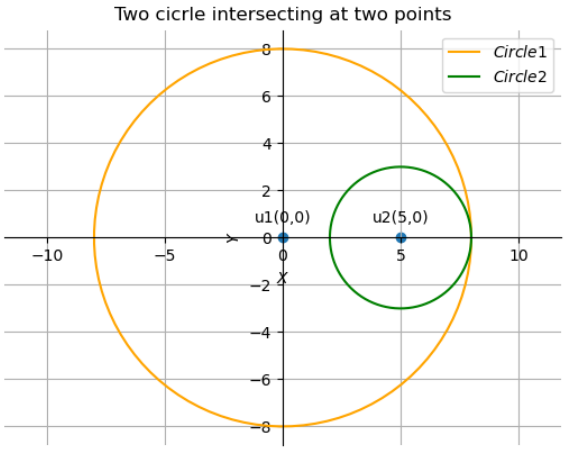
When Radius r is less than 8 i.e $(r < 8)$
here we have taken radius r = 7.5



By watching the above image , we can conclude that ,the Circle 1 and Circle 2 will intersect each other at two point when $r < 8$

E. Conditon 5

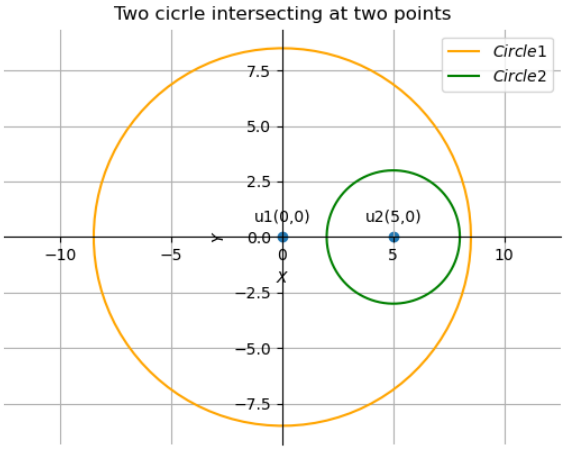
When Radius r is equal to 8 i.e $(r = 8)$
here we have taken radius r = 8



By watching the above image , we can conclude that ,the Circle 1 and Circle 2 will only touch each other but not intersect each other at any point when $r = 8$, So option **D** is also discarded

F. Conditon 6

When Radius r is greater than 8 i.e $(r > 8)$ here we have taken radius r = 8.5



By watching the above image , we can conclude that ,the Circle 1 and Circle 2 will not intersect each other at any point when $r > 8$, So option **B** is also discarded.

Finally we can conclude that the two circles will intersect at two points only if Radius r is between 2 and 8 i.e $2 < r < 8$

So we can conclude that option C is the only Correct answer to this problem

IV. Code Link

https://github.com/aadrshptel/Fwc_module1/tree/main/Assignments/Matrix%20assignments/Circles/codes

Execute the code by using the command
python3 circle.py