Assignment-4

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1 Problem

In Δ ABC and Δ DEF, AB = DE, AB ∥ DE, BC = EF and BC ∥ EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Figure).
Show that
(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD ∥ CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi)$\Delta ABC \cong \Delta$ DEF.

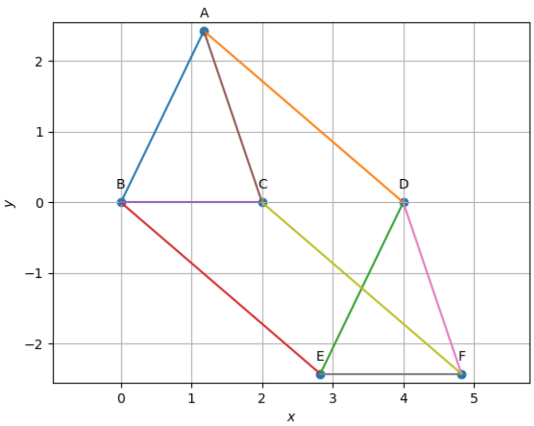

Figure 1: Given Figure

2 Solution

The input parameters for this construction are

Symbol	Value
r1	2
r2	3
θ	$\frac{3\pi}{10}$

$$\vec{A} = \begin{pmatrix} r1\cos\theta \\ r2\sin\theta \end{pmatrix}$$

$$\vec{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\vec{D} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\vec{C} = \vec{B} + \vec{D}/2$$

$$\vec{E} = \vec{B} + \vec{D} - \vec{A}$$

$$\vec{F} = \vec{E} + \vec{C} - \vec{B}$$

Direction vectors

The Direction vectors are

$$\vec{m_1} = \vec{A} - \vec{B}$$

$$\vec{m_2} = \vec{B} - \vec{C}$$

$$\vec{m_3} = \vec{C} - \vec{A}$$

$$\vec{n_1} = \vec{D} - \vec{E}$$

$$\vec{n_2} = \vec{E} - \vec{F}$$

$$\vec{n_3} = \vec{F} - \vec{D}$$

$$\vec{o_1} = \vec{A} - \vec{D}$$

$$\vec{o_2} = \vec{C} - \vec{F}$$

To proove

i.Quadrilateral ABED is a parallelogram

Distance between A and B is $\left\|A \vec{-} B\right\|$

Distance between D and E is $\left\|D \vec{-} E\right\|$

if $\left\|A \vec{-} B\right\| = \left\|D \vec{-} E\right\|$
then AB = DE..........(1)
if $\vec{m_1} \times \vec{n_1} = 0$
then AB ∥ DE...........(2)
Because,Two vectors ara parallel when cross product of that two vectors is zero.
From (1) and (2) we can say that ABED is a parallelogram.Because,If one pair of opposite sides of a quadrilateral are equal and parallel to each other,then it is a parallelogram.
∴ Quadrilateral ABED is a parallelogram.

ii.Quadrilateral BEFC is a parallelogram

Distance between B and C is $\left\|B \vec{-} C\right\|$

Distance between E and F is $\left\|E \vec{-} F\right\|$

if $\left\|B \vec{-} C\right\| = \left\|E \vec{-} F\right\|$
then BC = EF..........(3)
if $\vec{m_2} \times \vec{n_2} = 0$
then BC ∥ EF...........(4)
Because,Two vectors ara parallel when cross product of that two vectors is zero.
From (3) and (4) we can say that BEFC is a parallelogram.Because,If one pair of opposite sides of a quadrilateral are equal and parallel to each other,then it is a parallelogram.
∴ Quadrilateral BEFC is a parallelogram.

iii.AD∥CF and AD=CF

Distance between A and D is $\left\|A \vec{-} D\right\|$

Distance between C and F is $\left\|C \vec{-} F\right\|$

if $\left\|A \vec{-} D\right\| = \left\|C \vec{-} F\right\|$
then AD = CF..........(5)

if $\vec{O_1} \times \vec{O_2} = 0$
then AD $\parallel$ CF...........(6)
Because,Two vectors ara parallel when cross product of that two vectors is zero.
From (5) and (6) AD$\parallel$CF and AD=CF

iv.Quadrilateral ACFD is a parallelogram

From (iii) we can say that AD$\parallel$CF and AD=CF SO, we can say that ACFD is a parallelogram.Because,If one pair of opposite sides of a quadrilateral are equal and parallel to each other,then it is a parallelogram.
$\therefore$ Quadrilateral ACFD is a parallelogram.

v.AC=DF

Distance between A and C is $\left\|A \vec{-} C\right\|$
Distance between D and F is $\left\|D \vec{-} F\right\|$
if $\left\|A \vec{-} C\right\| = \left\|D \vec{-} F\right\|$
then AC = DF
$\therefore$ AC=DF

vi.$\Delta ABC \cong \Delta DEF$

If $\left\|A \vec{-} B\right\| = \left\|D \vec{-} E\right\|$ and $\left\|B \vec{-} C\right\| = \left\|E \vec{-} F\right\|$ and $\left\|A \vec{-} C\right\| = \left\|D \vec{-} F\right\|$
Then,$\Delta ABC \cong \Delta DEF$ .Because, If three sides of one triangle are equal to three sides of another triangle, the triangles are congruent.(By SSS Rule)
$\therefore$ $\Delta ABC \cong \Delta DEF$

3 Execution

*Verify the above proofs in the following code.

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https://github.com/gowripriya-2002/FWC/blob/main/line_assignment/line.py
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