Conic Assignment

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Problem

A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B. If AB subtends a right angle at the vertex of the parabola, find the slope of AB.

Solution

Symbol	Value	Description
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	vertex
A	$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$	point on parabola
B	$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$	point on parabola
F	$\begin{pmatrix} 2, 0 \end{pmatrix}$	Focus of the parabola

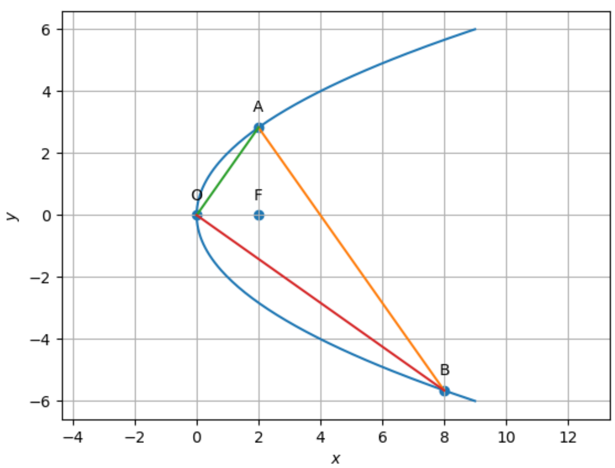

Figure 1: parabola with normal at A

Construction

Input taken for the construction of the parabola are it's focus and directrtx.

From the given equation of the parabola, we can assume the points $\mathbf{A}\&\mathbf{B}$ as

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2\sqrt{2} \end{pmatrix} \tag{1}$$

The given equation of the parabola is $y^2 = 4ax$, which can be expressed also as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} = 0 \tag{2}$$

where

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{4}$$

$$f = 0 \tag{5}$$

We know that the point $\mathbf{A}$ lies on the parabola and the normal at $\mathbf{A}$ can be calculated as

$$\mathbf{n} = \mathbf{V}\mathbf{A} + \mathbf{u} \tag{6}$$

Given that the normal passes through $\mathbf{A}\&\mathbf{B}$

$$L : \mathbf{x} = \mathbf{A} + \mu_i \mathbf{n} \tag{7}$$

intersection with conic in eqn (2)

$$\mu_1 = 0 \tag{8}$$

$$\mu_2 = -3 \tag{9}$$

the point $\mathbf{B}$

$$\mathbf{B} = \mathbf{A} + \mu_2 \mathbf{n} \tag{10}$$

The slope of $\mathbf{AB}$ can be calculated as

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 6 \\ -6\sqrt{2} \end{pmatrix} \tag{11}$$

$$\implies \mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{12}$$

$$\implies m = -\sqrt{2} \tag{13}$$

Hence, the slope of the normal $\mathbf{AB}$ is $-\sqrt{2}$.