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Assignment-5

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Problem Statement:

The lines 2x-3y=5 and 3x-4y=7 are diameters of a circle of area 154 square units.Then find the equation of the circle.

SOLUTION:

Given:

Two line equations are

$$\mathbf{n_1}^\top \mathbf{x} = c_1 \tag{1}$$

$$\mathbf{n_2}^\top \mathbf{x} = c_2 \tag{2}$$

Above two equations are diameters of the circle.

We know that the diameters intersect at the **centre** of the circle.

So solving those two equations, we get the centre of the circle.

Let $\mathbf{x}$ be the centre of the circle.

$$\mathbf{x} = \begin{pmatrix} \mathbf{n_1} & \mathbf{n_2} \end{pmatrix}^{-\top} \mathbf{c} \tag{3}$$

where,

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{4}$$

$$, \mathbf{n_2} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{5}$$

$$\mathbf{c} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} \tag{6}$$

$$\mathbf{x} = \begin{pmatrix} 2 & 3 \\ -3 & -4 \end{pmatrix}^{-\top} \begin{pmatrix} 5 \\ 7 \end{pmatrix} \tag{7}$$

To Find

We can find the centre of the circle by solving the above equation through finding the inverse

From the above equation we get the centre of the circle i.e,

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{8}$$

STEP-1

Given that the area of the circle is 154 square units.

$$\pi r^2 = 154 \tag{9}$$

$$\tag{10}$$

$$r = 7 units \tag{11}$$

STEP-2

The general equation of the circle is given by,

$$\mathbf{X}^\top \mathbf{V} \mathbf{X} + 2\mathbf{u}^\top \mathbf{X} + f = 0 \tag{12}$$

where,

$$f = \mathbf{u}^\top \mathbf{u} - r^2 \tag{13}$$

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{14}$$

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{15}$$
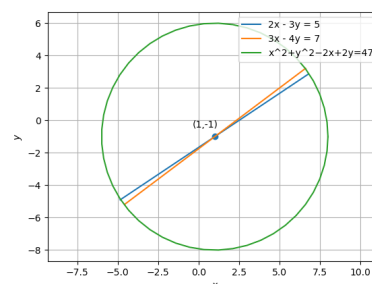
Substituting all the values in the above equation, we get

$$\mathbf{X}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{X} - 47 = 0 \tag{16}$$

The above equation gives the equation of the circle.

0.1 Construction

Input Parameters:

Symbol	Value	Description
$\mathbf{n_1}$	$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$	$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$
$\mathbf{n_2}$	$\begin{pmatrix} 3 \\ -4 \end{pmatrix}$	$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$
$\mathbf{c}$	$\begin{pmatrix} 5 \\ 7 \end{pmatrix}$	$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

Download the code

Github link:https://github.com/Siva Krishna/FWC.