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Matrix-Lines

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October 20, 2022

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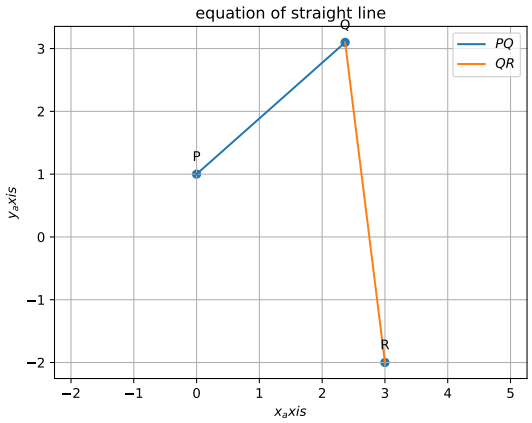

Figure 1: Equation of the Straight Line

1 Problem Statement

straight line L through point (3,-2) is inclined at an angle of 60 degres to the line $\sqrt{3}x + y = 1$.if also intersects the x axis, then equation of L is ?

2 Construction

3 Solution

Given that resultant line passes through point(3,-2) and intercepts on x axes and inclined at an angle of 60 degres (let x intercept is (x,0))

Symbol	Value	Description
$\mathbf{P}$	$\begin{pmatrix}0\\1\end{pmatrix}$	Point on Y-axis
$\mathbf{Q}$	$\begin{pmatrix}2.4\\3.1\end{pmatrix}$	Point of Intersection
$\mathbf{R}$	$\begin{pmatrix}3\\-2\end{pmatrix}$	Given Point
θ	60	Given Condition
$\mathbf{a}$	-	Point on X-axis
$\mathbf{e_1}$	$\begin{pmatrix}1\\0\end{pmatrix}$	basic vector

Table 1: Parameters

Let $\mathbf{P}=\begin{pmatrix}0\\1\end{pmatrix}$, $\mathbf{Q}=\begin{pmatrix}2.4\\3.1\end{pmatrix}$, $\mathbf{R}=\begin{pmatrix}3\\-2\end{pmatrix}$,$\mathbf{e_1}=\begin{pmatrix}1\\0\end{pmatrix}$

Equation of line is $\mathbf{n}^\top\mathbf{X}=\mathbf{c}$.

the above equation can be written as

$$\mathbf{X}^\top\mathbf{n}=\mathbf{c}$$

by given point $\mathbf{R}$ we can write equation of the line as

$$\begin{pmatrix}3 & -2\end{pmatrix}\mathbf{n_1}=1 \tag{1}$$

given statement that the line intersects on x axis let point of interscetion be $\mathbf{a}$

$$\mathbf{a}\mathbf{e_1}^\top\mathbf{n_1}=1 \tag{2}$$

by sloving

$$\begin{pmatrix}1 & 0\end{pmatrix}n_1=\frac{1}{a} \tag{3}$$

From eq1 and eq3,

$$\begin{pmatrix}3 & -2\\1 & 0\end{pmatrix}\mathbf{n_1}=\begin{pmatrix}1\\\frac{1}{a}\end{pmatrix} \tag{4}$$

$$\mathbf{n_1}=\begin{pmatrix}3 & -2\\1 & 0\end{pmatrix}^{-1}.\begin{pmatrix}1\\\frac{1}{a}\end{pmatrix} \tag{5}$$

by solving the matric we get

$$\mathbf{n_1}=\begin{pmatrix}\frac{1}{a}\\\frac{3-a}{2a}\end{pmatrix} \tag{6}$$

from given equation of line

$$\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\mathbf{X}=1 \tag{7}$$

$$\mathbf{n_2}=\begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{8}$$

Angle between two vectors is

$$cos\theta=\frac{\mathbf{n_1}^\top\mathbf{n_2}}{||\mathbf{n_1}||||\mathbf{n_2}||} \tag{9}$$

$$cos60=\frac{\begin{pmatrix}\frac{1}{a} & \frac{3-a}{2a}\end{pmatrix}\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{3-a}{2a}\right)^2 2}} \tag{10}$$

From angle between two vectors by solving we get the unknown a

$$\mathbf{a}=\frac{2}{\sqrt{3}}+3$$

Put $a = \frac{2}{\sqrt{3}} + 3$ in eq 6

$$\mathbf{n_1} = \begin{pmatrix} \frac{\sqrt{3}}{2+3\sqrt{3}} \\ \frac{-1}{2+3\sqrt{3}} \end{pmatrix} \tag{11}$$

by solving we get

$$\mathbf{n_1} = \frac{1}{2+3\sqrt{\mathbf{3}}} \begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix} \tag{12}$$

The Resultant Equation of line is $\mathbf{n_1^\top X} = 1$

Desired equation of the line is,

$$\frac{1}{2+\sqrt{3}} \begin{pmatrix} \sqrt{3} & -1 \end{pmatrix} \mathbf{X} = 1 \tag{13}$$

Or,

$$\begin{pmatrix} \sqrt{3} & -1 \end{pmatrix} \mathbf{X} = 2 + 3\sqrt{3} \tag{14}$$

4 Software

Download the following code using,

https://github.com/imran111888/fwc2/blob//matrix/line/20assignment/codes/1

and execute the code by using command

Python3 line.py

5 Conclusion

We found the equation of a line passing trough a point (3,-2) and intersects the x axis.